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NEGATIVE ENERGY WAVES IN MAGNETIC FLUX TUBES

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Negative Energy Waves in Magnetic Flux Tubes

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A B S T R A C T

The properties of oscillation of a magnetic flux tube in the presence of a shear flow of matter along its axis have been studied. The presence of flows is shown to cause a number of new effects: the appearance of negative energy waves, reversal of the sign of radiative damping, the development of explosive instability at the nonlinear stage, and the development of «coarse» (linear) hydrodynamic instability when the velocity exceeds a certain threshold. The corresponding processes have been classified. The calculations of the growth rates of dissipative instabilities associated with the radiation of sound waves and anomalous damping in the resonance layer have been performed. The conditions for the existence of explosive instability have been found.

The results obtained can be of interest in connection with the problem of energy accumulation and release in the Solar atmosphere as well as for better insight into the dynamics of various processes in the space and laboratory plasma with the filamentary structure of magnetic field.

1. INTRODUCTION

In various astrophysical objects and in laboratory plasma the situation is frequently met when the magnetic flux in a plasma is concentrated in a separate, relatively thin magnetic fluxtubes while magnetic field in a bulk plasma is small. In particular, according to the observational data all the Solar magnetic field is concentrated in a narrow fluxtubes usually far removed from each other. In the spots intense fluxtubes are assumed to be tightly settled (see for example [1] and References therein). For a better insight into various processes in the Solar atmosphere where the magnetic field plays a dominant part (e. g., the processes of energy transfer from the lower to the upper layers of atmosphere, the processes of energy storing and release and others), the properties of both the separate tubes and their ensembles should be analysed. Besides its significance for astrophysical objects, such a study is also of interest from the point of view of general physics due to a great variety of wave processes in such structures. Therefore, it is no wonder that this problem is attracting much attention.

One of the first theoretical works on this subject was that of Ref. [2], where the study of bending oscillations of a separate tubes has been made (in particular, their «radiative damping» associated with the radiation of secondary sound waves to the ambient medium) and the problem dealing with the propagation of long-wave sound oscillations in a plasma containing an ensemble randomly distributed magnetic tubes, has been treated. In addition, a specific, nondissipative mechanism of damping of such oscillations has been revealed that is connected with the transfer of their energy to the energy of bending oscillations of tubes. This effect is similar, to some extent, to the mechanism of Landau damping. In Ref. [3] attention has been drawn to the existence of specific quasi-longitudinal oscillations of a tube in which a longitudinal compression (expansion) of a plasma inside the tube is accompanied by an increase (decrease) of its cross-section so that the sum of the gas-kinetic and magnetic pressures is not perturbed. These oscillations are an analog of slow magnetosonic oscillations in a homogeneous plasma (and are sometimes called «slow» or «sausage» oscillations). They are of particular interest for having a very small radiative damping [4]. In Refs [4, 5] studies have been made of various damping mechanisms of oscillations of magnetic tubes, in particular, by the Alfvén resonance (see [6]) occurring in the region where the phase velocity of oscillations becomes equal to a local value of Alfvén velocity. In Ref. [7] the dispersion properties of a plasma containing an ensemble of tightly packed magnetic tubes are considered. The presence of random inhomogeneities (inhomogeneities aren't assumed to be small: the magnetic field, plasma density and pressure can change by the order of unity from tube to tube) gives rise to a more intense dissipation of energy of the long-wave oscillations in comparison with the homogeneous case.

The above studies deal with the systems in which the unperturbed plasma is at rest. On the other hand, it is often the case when the plasma outside a tube moves along the magnetic field with respect to the plasma inside the tube. In particular, according to observations, the plasma flows usually with different velocities inside and outside the magnetic structures are observed in all the regions of Solar atmosphere where magnetic field has a filamentary structure. In other words, there always exist shear flows along the fluxtubes in Solar atmosphere.

The presence of shear flows along the magnetic tubes leads to appearing of a quantitatively new effects. First of all, when the speed of the relative plasma motion exceeds a certain threshold value there arise negative energy waves*) in the system which can become unstable due to various dissipative processes (for instance,

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*) The possibility of existing the negative energy waves in nonequilibrium media was pointed out in Ref. [8] for the first time (in plasma); on the negative energy waves in hydrodynamics see, e. g., [9].

due to the radiation of sound wave to the environment). Besides, since there are also positive energy waves in the system a nonlinear, «explosive» instability can develop in it as well. At last, when the speed exceeding the second threshold (the first one is that for excitation of negative energy waves) there arises a «coarse» (linear) instability similar to the instability of tangential discontinuity.

We will consider long-wave oscillations of a tube, i. e. the oscillations whose wavelength $\lambda\!\!\!^{-}=1/k$ is large as compared with the radius R: $kR \ll 1$. Just these oscillations are most readily excited by large-scale plasma motions and have a relatively low damping rate.

The paper is arranged as follows. In Section 2 linear equations for bending and slow oscillations are investigated. The conditions for the existence of bending oscillations with negative energy as well as the condition of a «coarse» instability of a tube are found. In Section 3 the conditions of the dissipative instability of bending oscillations are formulated and its growth rate is estimated. In Section 4 the instability of bending and slow oscillations associated with the radiation of secondary sound waves are discussed. The nonlinear «explosive» instability of negative energy waves is considered in Section 5. Some properties of the «coarse» instability of bending oscillations which develops when the flow velocity exceeds the second threshold value, are analysed in Section 6. The results obtained are briefly discussed in Section 7. Some necessary calculations are given in Appendixes.

2. LINEAR THEORY OF BENDING AND SLOW OSCILLATIONS

Let's consider a model of an axisymmetric homogeneous magnetic tube in the presence of a flow along its axis. An analysis will be made in the coordinate system where the substance inside the tube is at rest, while the flow velocity outside it equals u and is directed towards the increasing z.

We will start with bending oscillations. The displacement of the tube relative to its unperturbed position will be described by a vector $\vec{\xi}_\perp(z, t)$ lying in a plane normal to the tube axis. One can show that the vector $\vec{\xi}_\perp(z, t)$ satisfies the equation (the procedure is similar to one in Ref. [2]):

$$\rho_i \frac{\partial^2 \vec{\xi}_\perp}{\partial t^2} = -\rho_e \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial z}\right)^2 \vec{\xi}_\perp + \frac{B^2}{4\pi}\frac{\partial^2 \vec{\xi}_\perp}{\partial z^2}, \qquad (1)$$

where ρ_i and ρ_e are the densities of plasma inside and outside the tube and B is the magnetic field strength inside the tube (just as in Ref. [2] we assume that there is no magnetic field outside the tube since this assumption makes the calculations simpler, having no influence on the essence of the problem).

Equation (1) has the energy integral which can be represented as follows:

$$I=\int dz\frac{\pi R^2}{2}\left\{(\rho_i+\rho_e)\left(\frac{\partial\xi_\perp}{\partial t}\right)^2+\left(\frac{B^2}{4\pi}-\rho_e u^2\right)\left(\frac{\partial\xi_\perp}{\partial z}\right)^2\right\}=\text{const}, \qquad (2)$$

where R is tube radius. The integrand has the meaning of oscillation energy per unit length of the tube.

For harmonic plane waves of the form $\exp(-i\omega t+ikz)$, the dispersion relation following from (1) has a form:

$$\omega^2+\frac{1}{\eta}(\omega-ku)^2-k^2a^2=0, \qquad (3)$$

where $\eta=\rho_i/\rho_e$, $a=(B^2/4\pi\rho_i)^{1/2}$ is the Alfvén velocity inside the tube. From (3) we have

$$\frac{\omega}{k}=\frac{1}{1+\eta}\{u\pm\sqrt{\eta[a^2(1+\eta)-u^2]}\}. \qquad (4)$$

Hence, it is seen that at

$$u>u_c^b=a\sqrt{1+\eta} \qquad (5)$$

(index b indicates that the point is concerned with the critical velocity of excitation of bending oscillations) the system becomes unstable. This instability may be referred to as a «coarse» one since its growth rate is comparable with the frequency at the above-threshold value of the order of unity (e. g., at $u-u_c^b\sim u_c^b$). We will dwell upon this instability in Section 6 in a more detail and now we are concerned with the phenomena occurring in the region $u<u_c^b$.

Using the dispersion relation (4), it is easy to obtain from (2) that for travelling waves the energy density W per unit length of the tube is equal to

$$W=\frac{\pi R^2\rho_e}{2}\xi_\perp^2 k^2\left[(1+\eta)\frac{\omega^2}{k^2}+a^2\eta-u^2\right],$$

or

$$W=\frac{1}{1+\eta}\pi R^2k^2\rho_e\xi_\perp^2(x^2\pm ux) \qquad (6)$$

with a new designation $x=\sqrt{\eta[a^2(1+\eta)-u^2]}$. The radicand is assumed to be positive, i. e. the «coarse» instability is absent. Since we also assume that $u>0$, only the wave corresponding to the sign minus in the dispersion relation (4) can have negative energy, i. e. the wave, which travels in the opposite direction of the z-axis if no flow exists. For this wave we get from (6):

$$W=\pi R^2k^2\rho_e\xi_\perp^2(\eta a^2-u^2)\frac{x}{x+u}.$$

It is seen that the wave energy becomes negative at

$$u>u_c^n=a\sqrt{\eta} \qquad (7)$$

(the index n indicates that the question is the threshold at which negative energy waves appear). Comparing (5) and (7) we see that $u_c^n<u_c^b$, i. e. negative energy waves (NEW) indeed appear in the plasma which is still stable with respect to the «coarse» hydrodynamic instability. At the lower boundary of the interval within which the NEW exist (at $u=u_c^n$) the phase velocity of the wave corresponding to «minus» in the dispersion relation (4) equals zero. At $u>u_c^n$ the NEW are propagated towards the mass flow.

Let us now consider slow oscillations [3, 4]. The peculiarity of these oscillations is that the sum of the magnetic (p_M) and gas-kinetic (p) pressures in them inside the tube almost is not perturbed, while each particular term is perturbed significantly:

$$\frac{\delta p}{p}\sim\frac{\delta\rho}{\rho};\quad \frac{\delta p_M}{p_M}\sim\frac{\delta\rho}{\rho};\quad \frac{\delta(p_M+p)}{p_M+p}\sim(kR)^2\frac{\delta\rho}{\rho}\ll\frac{\delta\rho}{\rho}.$$

Because of this property of slow oscillations the plasma parameters outside the tube have a little influence of their dispersion relation (giving the corrections of the order of $(kR)^2$). In particular, they are weakly influenced by the presence of external flow. Thus, we come to conclusion that in the presence of shear flow the dispersion relation for slow oscillations up to the small corrections remains the same as without a flow:

$$c_T=\left(\frac{\omega}{k}\right)_T=\frac{as_i}{\sqrt{a^2+s_i^2}}, \qquad (8)$$

where $s_i=(\gamma p_i/\rho_i)^{1/2}$ is the sound speed inside the tube (the index T is used to designate slow oscillations).

The complete equations of bending and slow oscillations are derived in Appendix 1.

3. DISSIPATIVE INSTABILITIES OF BENDING NEW

Within the interval

$$\sqrt{\eta}\, a<u<\sqrt{1+\eta}\, a \tag{9}$$

there can exist the instability of NEW caused by any dissipative processes in plasma. In the other words, taking into account dissipative effects (interaction with medium or with other waves) leads to losing the energy of NEW and, hence, to growing their amplitude. It is remarkable that for magnetic fluxtubes due to their specific features this («fine») instability can occur even in the absence of any dissipative processes (viscosity, thermal conductivity, Ohmic losses; of course, all of them lead to instability too). Namely, the instability takes place due to collisionless dissipation of bending oscillations. The mechanism of such dissipation has been studied in [5], where it is shown that for radially inhomogeneous magnetic tube there appears the effect of anomalous and strong absorption of oscillations at a resonance point, i. e. at the point where the phase velocity of oscillations becomes equal to a local value of Alfvén velocity. This effect also exists in the presence of shear flow along the tubes. Here we will follow the method used in Ref. [5]

In the long-wave approximation the fluid may be regarded with good accuracy as incompressible (that is, one may assume that $\operatorname{div}\bar{v}=0$). Then instead of velocity one can introduce the current function ψ:

$$v_r=-\frac{1}{r}\frac{\partial\psi}{\partial\varphi},\qquad v_\varphi=\frac{\partial\psi}{\partial r}.$$

In this case the general set of equations (I.6) reduces to a single equation for ψ:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\rho\Omega-\frac{k^2B^2}{4\pi\Omega}\right)r\frac{\partial\psi}{\partial r}-\left(\rho\Omega-\frac{k^2B^2}{4\pi\Omega}\right)\frac{\psi}{r^2}=0. \tag{10}$$

We assume that ω contains now a small imaginary part $\omega+i\nu$ introduced for a correct accounting of a singular point at $\rho\Omega=\frac{k^2B^2}{4\pi\Omega}$. From the physical point of view the appearance of imaginary part can be explained, for instance, by rare collisions between ions and plasma neutrals. To made our calculations more visual we take, just as in [5], a model of the tube uniform almost throughout the whole space expect a narrow transitional region (the smeared boundary of the tube) where plasma density and magnetic field (squared) depend linearly on the radius:

$$\rho\Omega^2=\rho_i\omega^2\frac{R-r+l}{l}+\rho_e\Omega^2\frac{r-R}{l},$$

$$B^2(r)=B^2\frac{R-r+l}{l}.$$

Note that $l\ll R$.

The solution of Eq. (10) at constant density, magnetic field and flow velocity is the Bessel functions in the internal region and the Hankel functions in the external one. In a first approximation over $kR\ll 1$, these solutions have correspondingly the form:

$$\psi=\begin{cases} Ar, & r<R\\ B/r, & r>R+l\end{cases}$$

To find the solution in the transition region $R<r<R+l$ let's introduce the variable $z=\frac{r-R}{l}$ $(0\leqslant z\leqslant 1)$. Taking the smallness of the parameter l/R into account Eq. (10) can be represented as follows:

$$\frac{d}{dz}(z-z_0-i\varepsilon)\frac{d\psi}{dz}-\frac{l^2}{R^2}(z-z_0-i\varepsilon)\psi=0, \tag{11}$$

where

$$z_0=\frac{k^2a^2-\omega^2}{k^2a^2+\frac{\rho_e}{\rho_i}\Omega^2-\omega^2}.$$

The imaginary addition $i\varepsilon$ has appeared here because of $i\nu$ (the concrete value of ε makes no difference since it is not enter in the final result).

Equation (11) has a single-valued solution in the complex plane z with a cut along the line $\operatorname{Im} z=i\varepsilon$, $-\infty<\operatorname{Re} z<z_0$. This solution

can be expressed in the Bessel functions. Expanding in the series over parameter l/R the solution can be represented approximately in the form:

$$\psi = C + D \ln (z - z_0 - i\varepsilon) .$$

Using now the continuity conditions for ψ and $\frac{d\psi}{dz}$ at the points $r=R$ and $r=R+l$ (i. e. performing the appropriate matching) and choosing the required branch of logarithm we get the following dispersion relation:

$$\ln \frac{z_0}{1-z_0} + \frac{R}{l}\left(\frac{1}{z_0} - \frac{1}{1-z_0}\right) + i\pi = 0 . \tag{12}$$

The real part of (12) gives

$$1 - z_0 = z_0 .$$

It is easy to check that this expression is just the same as the dispersion relation (4). For the imaginary part of the frequency we get from (12) the following expression:

$$\frac{\gamma}{\omega} = -\frac{\pi}{4}\frac{l}{R}\frac{\eta}{(1+\eta)^2}\frac{(\eta u \mp x)^2}{\pm x} .$$

It is seen that for positive energy waves (the upper sign) γ corresponds to the damping rate and in the case of negative energy waves (the lower sign) γ is the growth rate.

Thus, the growth rate of instability for NEW due to resonant absorption has a form:

$$\frac{\gamma_{res}}{\omega} = \frac{\pi}{4}\frac{l}{R}\frac{\eta}{(1+\eta)^2}\frac{(\eta u + \sqrt{\eta[a^2(1+\eta) - u^2]}\,)^2}{\sqrt{\eta[a^2(1+\eta) - u^2]}} . \tag{13}$$

One should bear in mind that the value of growth rate is valid in the region which is not too close to the threshold where the denominator in (13) vanishes.

It is worth noting that the effect of anomalous absorption which causes here the instability of NEW can occur also for a uniform magnetic fluxtube with the nonuniform shear flow.

4. INSTABILITY OF BENDING AND SLOW OSCILLATIONS CAUSED BY RADIATION OF SECONDARY SOUND WAVES

The dispersion relation (4) for bending oscillations has been derived neglecting the compressibility of media. Formally, allowance for compressibility corresponds to keeping the next-order terms with respect to the parameter $kR \ll 1$ in an exact dispersion relation (cf. [2]). The main effect arising when compressibility is taken into account is the radiation of secondary sound waves by the oscillating tube [2]. Under the conditions when there is no plasma flow this effect leads to «radiation» damping of bending oscillations, whereas in a case when outside a tube plasma has a finite velocity the radiation of secondary sound waves can lead to growing of the bending oscillations amplitude. This is possible in two cases: when a bending oscillation has a negative energy while a sound wave has a positive one, or when the former has a positive energy while the radiated sound wave has a negative one.

The dispersion relation for plane sound waves (the waves may be regarded as plane at large distances from the tube) is of the form:

$$\left(\frac{\omega}{k}\right)_s = u \pm s_e \sqrt{1 + \frac{k_\perp^2}{k^2}} , \tag{14}$$

where $k_\perp$ is a component of the wave vector normal to the z-axis and k is, as before, a component of wave vector in the direction of z-axis; the index s is used to designate sound waves. It easy to see that a sound wave which is propagated in the absence of the flow in the negative direction of the z-axis (which corresponds to the lower sign in the dispersion relation (14)), may have negative energy. Namely, it happens at

$$u \geqslant s_e \sqrt{1 + \frac{k_\perp^2}{k^2}} . \tag{15}$$

The transverse component of the wave vector of a sound wave is defined from the condition

$$(\omega/k)_b = (\omega/k)_s .$$

Let's first find the conditions under which bending oscillations with positive energy radiate negative energy sound waves, i. e.

under which the following conditions are satisfied:

$$\frac{1}{1+\eta}(u+x)=u-s_e\sqrt{1+\frac{k_\perp^2}{k^2}}>0. \qquad (16)$$

Simple calculations give that this is possible at the conditions

$$a>s_e/\sqrt{\eta}, \quad u>s_e+\sqrt{a^2-\frac{1}{\eta}s_e^2}. \qquad (17)$$

From the equilibrium condition of an unperturbed tube it follows that

$$s_e>a\sqrt{\frac{1}{2}\gamma\eta},$$

where γ is the specific heat ratio. Correspondingly, the regime (17) is realized only if $\gamma<2$.

Let's elucidate now whether a negative energy bending oscillations can radiate a positive energy wave, i. e. whether the condition

$$\frac{1}{1+\eta}(u-x)=u+s_e\sqrt{1+\frac{k_\perp^2}{k^2}}$$

can be satisfied. It is evident that this condition cannot be satisfied because it reduces to the equality

$$-x=\eta u+(1+\eta)\,s_e\sqrt{1+\frac{k_\perp^2}{k^2}}$$

whose l.h. side is negative while the r.h. side is positive. Note that this conclusion is a consequence of our assumption that there is no magnetic field outside the tube; in the general case such a mechanism of instability becomes possible.

The growth rate of instability in the situation when the bending positive energy wave becomes unstable with respect to radiation of secondary negative energy sound waves is calculated in Appendix 2 and is of the form

$$\frac{\gamma_{rad}}{\omega}=\frac{\pi}{2}\,\frac{(v_\varphi-u)^2[(v_\varphi-u)^2-s_e^2]}{s_e^2 v_\varphi[(1+\eta)\,v_\varphi-u]}\,k^2R^2.$$

One should bear in mind, of course, that the instability has a threshold with respect to the flow velocity (see (17)).

There exists a similar mechanism of instability for slow oscillations as well. As has been mentioned in Section 2, the outer flow has a weak influence on these oscillations; in particular, their energy remains positive in the presence of flow. So that, instability in this case can be caused by the radiation negative energy sound waves. The negative energy have those waves which are propagated against the flow in the coordinate system connected with the fluid; their energy becomes negative under the condition (13), i. e. when they are propagated along the flow in the laboratory system. Thus, we draw a conclusion from the phase synchronism condition that the instability condition has a form (cf. (16)):

$$c_T=u-s_e\sqrt{1+\frac{k_\perp^2}{k^2}}>0,$$

which is possible when fulfilling the requirement

$$u>c_T+s_e. \qquad (18)$$

So, a slow wave propagating along the flow can be unstable. The growth rate of this instability (see Appendix 2) is of the form

$$\frac{\gamma_{rad}}{\omega}=\frac{\pi}{4}\,\frac{\rho_e}{\rho_i}\,\frac{c_T^2(c_T-u)^2}{a_i^4}\,k^2R^2.$$

The threshold of this instability with respect to the flow speed is defined by the condition (18)

5. EXPLOSIVE INSTABILITY OF NEW

In the system containing the waves with different signs of energy there exists a specific nonlinear instability called explosive. This instability was first considered in Ref. [10] and illustrated by the waves with random phases. Later on, it was analysed in Ref. [11] for a triplet of coherent waves, where the term «explosive instability» was proposed. The main feature of this instability is that the amplitudes of interacting waves achieve infinitely large values for a finite period of time. This assertion, of course, is formal in some extence: higher-order nonlinear processes limit the growth of amplitudes at a finite level.

When studying nonlinear, in particular, three-wave processes it is convenient to assume that the sign of frequency corresponds to the sign of energy. With such an approach taken the explosive instability condition for a three-wave process may be written as follows:

$$\omega_1+\omega_2+\omega_3=0,$$
$$k_1+k_2+k_3=0, \qquad (19)$$
$$|m_1|\pm|m_2|\pm|m_3|=0,$$

where indices 1, 2 and 3 refer to three interacting waves. Since we analyse the oscillations with $m=0$, ± 1, it follows from the last relation in (19) that either all three waves must have $m=0$, or two of them must have $m=\pm 1$ while the third has $m=0$.

Let's show that the conditions (19) are satisfied and, therefore, the explosive instability can occur for the interaction of one slow wave ($m=0$) and two bending ones ($m=\pm 1$). As before, the indices assigned to slow and bending waves will be respectively T and b. Then, instead of (19) we have the following conditions:

$$\omega_T+\omega_{b_+}+\omega_{b_-}=0,$$
$$k_T+k_{b_+}+k_{b_-}=0. \qquad (20)$$

The signs «+» and «—» of index b correspond to the waves propagating along and against the flow, respectively.

As it was mentioned above, the environment properties have a weak influence on the slow oscillations and we can regard them as positive energy waves with unchanged dispersion relation (8). From the dispersion relation (4) for bending oscillations at the condition (15) b_- — waves propagating against the flow have negative energy while b_+ are positive energy waves. Thus, it is easy to verify that at $k_T>0$ conditions (20) are fulfilled if it is satisfied the inequality

$$u>\frac{c_T}{1+\eta}+\sqrt{a^2\eta-c_T^2\eta\frac{\eta^2+3\eta+3}{(1+\eta)^2}}. \qquad (21)$$

The condition (21) together with (9) provides the explosive instability.

The usual analysis of nonlinear three-wave interaction leads to following equations for the amplitudes of coupled waves (in our case these are radial components of velocity of tube's boundary in slow (v_T) and bending ($v_{b_\pm}$) oscillations):

$$\frac{\partial v_T}{\partial t}=J_1 v_{b_+} v_{b_-},$$
$$\frac{\partial v_{b_+}}{\partial t}=J_2 v_T v_{b_-}, \qquad (22)$$
$$\frac{\partial v_{b_-}}{\partial t}=J_3 v_T v_{b_+},$$

where J_1, J_2 and J_3 are matrix elements of interaction. It is easy to show from energy and momentum conservation that in our case matrix elements are approximately equal. Using the appropriate scale we can write instead of (22) the equations:

$$\frac{\partial G_T}{\partial t}=Q\,G_{b_+}G_{b_-},$$
$$\frac{\partial G_{b_+}}{\partial t}=Q\,G_T G_{b_-}, \qquad (23)$$
$$\frac{\partial G_{b_-}}{\partial t}=Q\,G_T G_{b_+},$$

where G_T and $G_{b_\pm}$ having the velocity dimension are of the order of v_T and $v_{b_\pm}$; Q is of the order of wave-vector of interacting modes.

If at the initial moment of time in the system is excited only one wave, for instance, T-wave with $k_T>0$ ($G_T|_{t=0}=G_0$) and the amplitudes two other waves are determined by the thermal noises and, therefore, are very small: $G_{b_+}|_{t=0}=G_1\ll G_0$, $G_{b_-}|_{t=0}=G_2\ll G_0$, then one can see from the first equation of system (23) that at the early stage G_T remains almost constant. This enables to solve easily two other equations of the system (23). The result is following:

$$G_{b_+}=\left(G_1+\frac{1}{2}G_2\right)e^{QG_0t}-\frac{1}{2}G_2e^{-QG_0t}$$
$$G_{b_-}=\left(G_2+\frac{1}{2}G_1\right)e^{QG_0t}-\frac{1}{2}G_1e^{-QG_0t},$$

and corresponds to the exponential growth of b_+ and b_- waves amplitudes at the initial stage of development of instability. The corresponding growth rate by the order of magnitude is equal to $k_T v_T$. In a time of several inverse growth rates when the amplitudes of all three waves become the quantities of the same order the instability passes into nonlinear stage and takes a character of explosion. Indeed, at the stage when $G_{b_+} \simeq G_{b_-} \simeq G_T = G$ the equation for G has a form

$$\frac{dG}{dt} = QG^2,$$

which yields

$$G \sim \frac{1}{t - t_0},$$

where t_0 is characteristic time of «explosion» which is of the order of $(k_T v_T)^{-1}$.

6. HYDRODYNAMIC INSTABILITY OF BENDING OSCILLATIONS

As has been shown in Section 2 when the velocity of shear flow exceeds the threshold $u_c^b = a\sqrt{1+\eta}$, defined by formula (5) there arises a new instability in the system. By the nature this instability is close to tangential discontinuity in magnetohydrodynamics. We have already mentioned that this instability is «coarse», implying that at the above-threshold value of the order of unity the growth rate becomes comparable with the frequency and the length of growth becomes comparable with the wave-length. Under the conditions when this instability develops, namely, at

$$u > a\sqrt{1+\eta}$$

the finer dissipative and nonlinear instabilities considered in Sections 3—5 become negligible.

It follows from dispersion relation (4) that the unstable perturbations propagate upwards along the flow:

$$\mathrm{Re}(\omega/k) = \frac{u}{1+\eta} > 0.$$

Therefore, if a certain part of a tube is «blown» by an upward flow of surrounding plasma the bending oscillations excited here are propagated further upwards. This instability must play an essential role in different astrophysical objects where there are high speed streams along the magnetic fields for being an important agent of excitation of oscillations. Particularly, in Solar atmosphere the excitation of magnetic fluxtube oscillations is thought as a rule to be

connected with the oscillatory motions of the point of intersection of the tube and photosphere bottom, which is due to the nonstationary convection in this region. Of course, there exist the oscillations of fluxtubes which are excited by the convective motions, but the frequency of these oscillations is of the order of inverse time of re-arrangement of the granulation picture, i. e. is of the order of $1/\tau \sim 10^{-2} \div 3 \cdot 10^{-3}\ \mathrm{s}^{-1}$. This frequency is very low and this circumstance presents a problem in an attempt to explain the energy transfer from photosphere to the upper layers of Solar atmosphere by means of these oscillations.

The instability described above leads to the existence of another mechanism of excitation of oscillations which is independent on the motions in the base of fluxtube and which can take place far from the convective zone. Now the oscillation frequency of fluxtube is, naturally, by no means related to the inverse time re-arrangement of granulation picture and can be considerably higher than $1/\tau$.

SUMMARY

We have shown that in the presence of a relative plasma motion inside and outside the magnetic fluxtube there arises in the system a rich spectrum of phenomena not occurring in a stable plasma and we have classified these phenomena.

First of all, in a system with shear flow there can appear bending negative energy waves and the dissipative processes can cause the instability of these waves. In particular, the dissipative instability can be caused by a collisionless absorption of bending oscillations in the layer of Alfvén resonance inside the fluxtube. The specific type of dissipative instability is connected with the radiation of secondary sound waves to the external plasma (in a system without flow this process leads to «radiative» damping of bending oscillations [2]); note that the instability is possible, in principle, in two cases: when the bending wave has a positive energy, while the energy of the radiated sound wave is negative and vice versa.

Dissipative processes are «weak» in a sense that their growth rate is usually small in comparison with the frequency. The «coarse» instability of bending oscillations (with growth rate roughly equal to the frequency) similar to the instability of tangential discontinuity appears as the velocity of shear flow further incre-

ases. In the reference frame where the plasma inside the tube is at rest the unstable waves are propagated along the external flow. This mechanism of excitation of bending oscillations can play an important role in energy transfer in the Solar atmosphere.

We have analysed also the three-wave interaction processes between bending NEW and positive energy slow oscillations. The conditions have been revealed of the existence of nonlinear explosive instability.

The effects described above are assumed to play an important role in the dynamics of various processes in Solar atmosphere, in particular, in the processes of energy transfer from the lower to the upper layers of atmosphere, in the processes of energy accumulation and release, in the evolution of magnetic structures, in the phenomena associated with solar wind and others.

Appendix I

Equations of Small Oscillations of Magnetic Fluxtubes

In the presence of plasma flow the linearized set of equations of single-fluid magnetohydrodynamics is as follows:

$$\rho\frac{\partial\vec{v}}{\partial t}+(\vec{u}\nabla)\vec{v}+(\vec{v}\nabla)\vec{u}=-\nabla\delta p+\frac{1}{4\pi}\{[\,\mathrm{rot}\,\vec{b},\vec{B}]+[\,\mathrm{rot}\,\vec{B},\vec{b}]\},$$

$$\frac{\partial\vec{b}}{\partial t}=\mathrm{rot}[\,\vec{v}\vec{B}]+\mathrm{rot}[\,\vec{u}\vec{b}]\,,$$

$$\frac{\partial\delta\rho}{\partial t}+\mathrm{div}\,\rho\vec{v}+\mathrm{div}\,\delta\rho\vec{u}=0\,, \qquad (\mathrm{I.1})$$

$$\frac{\partial\delta F}{\partial t}+(\vec{v}\nabla)F+(\vec{u}\nabla)\delta F=0\,.$$

Here $F=p\rho^{-\gamma}$, and v, b, $\delta\rho$ and δp are the perturbations of velocity, magnetic field, density and pressure. These equations should be complemented by the equilibrium condition of a magnetic fluxtube in the unperturbed state

$$p(r)+\frac{B^2(r)}{8\pi}=p_e\,. \qquad (\mathrm{I.2})$$

Here p_e is the gas-kinetic plasma pressure outside the tube.

We will consider a model of fluxtube which is axisymmetrical in the unperturbed state and homogeneous along the axis (coinciding with the z-axis in the cylindrical coordinates), i. e. we will assume that the unperturbed density $\rho(r)$, pressure $p(r)$ and magnetic field $B(r)$ depend only on the radius. The shear flow is also assumed to be directed along the z-axis: $\vec{u}=\vec{u}(0,0,u(r))$. All the perturbed quantities are assumed to be proportional to $\exp(-i\omega t+im\varphi+ikz)$. For such perturbations the first equation in the set (I.1) gives:

$$-i(\omega-ku)\rho v_r=-\frac{\partial}{\partial r}\left(\delta p+\frac{b_zB}{4\pi}\right)+\frac{ikB}{4\pi}b_r\,,$$

$$-i(\omega-ku)\rho v_\varphi=\frac{1}{r}\frac{\partial}{\partial\varphi}\left(\delta p+\frac{b_zB}{4\pi}\right)+\frac{ikB}{4\pi}b_\varphi\,, \qquad (\mathrm{I.3})$$

$$-i(\omega-ku)\rho v_z=-ik\delta p-\rho v_r\frac{\partial u}{\partial r}-\frac{kB}{4\pi(\omega-ku)}v_r\frac{\partial B}{\partial r}\,.$$

From the second equation of (I.1) we get

$$b_r=-\frac{kB}{\omega-ku}v_r\,,$$

$$b_\varphi=-\frac{kB}{\omega-ku}v_\varphi\,, \qquad (\mathrm{I.4})$$

$$-i(\omega-ku)b_z=-\frac{1}{r}\frac{\partial}{\partial r}rBv_r-\frac{imB}{r}v_\varphi+b_r\frac{\partial u}{\partial r}\,.$$

In deriving the last equation we have taken into account that $\mathrm{div}\,\vec{b}=0$. Correspondingly, the third and fourth equations of (I.1) take the forms:

$$-i(\omega-ku)\delta\rho+\frac{1}{r}\frac{\partial}{\partial r}r\rho v_r+\frac{im}{r}\rho v_\varphi+ik\rho v_z=0\,,$$

$$-i(\omega-ku)(\delta p-s^2\delta\rho)+v_r\left(\frac{\partial p}{\partial r}-s^2\frac{\partial\rho}{\partial r}\right)=0\,, \qquad (\mathrm{I.5})$$

where $s^2=\gamma p/\rho$ is the sound velocity.

To put the sets of equations (I.3), (I.4) and (I.5) into a more compact form, it is convenient to express all the perturbed quantities in terms of v_r, v_φ and the total pressure perturbation $\delta\mathscr{P}=\delta p+\frac{b_zB}{4\pi}$. After some algebra we obtain the following set of equations for small oscillations of fluxtube in the presence of shear flow:

$$i\delta\mathscr{P}=\rho\frac{\Omega^2(s^2+a^2)-k^2s^2a^2}{\Omega^2-k^2s^2}\left[\frac{1}{r}\frac{\partial}{\partial r}r\frac{v_r}{\Omega}+\frac{im}{r}\frac{v_\varphi}{\Omega}\right],$$

$$\frac{\partial\delta\mathscr{P}}{\partial r}=i\rho(\Omega^2-k^2a^2)\frac{v_r}{\Omega}, \qquad (I.6)$$

$$\frac{im}{r}\delta\mathscr{P}=i\rho(\Omega^2-k^2a^2)\frac{v_\varphi}{\Omega}.$$

Here $a=(B^2/4\pi)^{1/2}$ is Alfvén velocity and $\Omega=\omega-ku$.

The set (I.6) describes all the types of linear oscillations of fluxtube. In the present paper we only deal with the dipole mode $m=\pm1$, corresponding to bending oscillations, and axisymmetrical one $m=0$ with the phase velocity $c_T=\frac{as}{\sqrt{a^2+s^2}}$, corresponding to slow («sausage») oscillations.

For slow oscillations the set (I.6) reduces to a single equation for v_r. Indeed, excluding $\delta\mathscr{P}$ from (I.6) and taking into consideration that in this case $v_\varphi=0$, we get

$$\frac{1}{r}\frac{\partial}{\partial r}\rho\frac{(a^2+s^2)(\Omega^2-k^2c_T^2)}{\Omega^2-k^2s^2}\frac{1}{r}\frac{\partial}{\partial r}r\frac{v_r}{\Omega}+\rho(\Omega^2-k^2a^2)\frac{v_r}{\Omega}=0. \qquad (I.7)$$

For bending oscillations in the long-wave limit the set (I.6) reduces to Eq. (10) in the case of a non-uniform fluxtube and to Eq. (1) in the case of a homogeneous one, where we use the displacement vector instead of velocity: $\vec{v}=\frac{d\vec{\xi}}{dt}$. As has already been said, to the mode $m=0$ correspond also torsional vibrations of fluxtube and oscillations with the sine-phase change of gas-kinetic and magnetic pressures (analog of fast magnetosonic waves), but the first of them are weakly influenced by the longitudinal flows of matter, while the second have, even at $kR\ll1$, a very high frequency ($\sim a/R$) and, therefore, are rapidly damping due to the radiation of sound waves to the environment. As to higher azimuthal modes ($m=\pm2$, ±3, ...), they are not a matter of interest because of their weak influence on the «global» characteristics of fluxtubes.

Appendix II

Radiation of Sound Waves to Outer Space

The density perturbation in a sound wave outside the tube may be written as follows:

$$\delta\rho=\cos m\varphi\left[\frac{1}{2}f(r)\exp(-i\omega t+ikz)+\text{c.c}\right]. \qquad (II.1)$$

As usual $m=0$ corresponds to slow oscillations and $m=1$ to plane-polarized bending ones. The function $f(r)$ satisfies the equation

$$\frac{1}{r}\frac{d}{dr}r\frac{df}{dr}-\frac{m^2}{r^2}f-k^2f+\frac{(\omega-ku)^2}{s_e^2}f=0.$$

The solution of this equation which corresponds to divergent sound waves is

$$f(r)=A\,H_m^{(1)}(k_\perp r), \qquad (II.2)$$

where

$$k_\perp=\sqrt{\frac{(\omega-ku)^2}{s_e^2}-k^2}=\frac{k}{s_e}\sqrt{(v_\varphi-u)^2-s_e^2}. \qquad (II.3)$$

Of course, the radiation of sound waves is possible only if the radicand in (II.3) is not negative. The conditions under which this case is realized are formulated in Section 4.

At large distances from the tube ($k_\perp r\gg1$), the solution (II.2) has the asymptotics

$$f=A\sqrt{\frac{2}{\pi k_\perp r}}\exp\left(ik_\perp r-\frac{im\pi}{2}-\frac{i\pi}{4}\right). \qquad (II.4)$$

For $k_\perp r\gg1$ the solution (II.1) with f as (II.4) is close to plane waves. Now, let's calculate the energy density of a plane (in a local sense) sound wave of the form

$$\delta\rho=\frac{1}{2}\delta\rho_0\cos m\varphi\cdot\exp(-i\omega t+ikz+ik_\perp r)+\text{c.c.} \qquad (II.5)$$

We have

$$W_s=\frac{\rho_e s_e^2}{2}\left|\frac{\delta\rho_0}{\rho_e}\right|^2\frac{\sqrt{k^2+k_\perp^2}\,s_e-ku}{\sqrt{k^2+k_\perp^2}\,s_e}\cos^2 m\varphi. \qquad (II.6)$$

It is seen that W_s is negative under the condition (15). Using the relation (II.4) we have

$$W_s = \frac{s_e}{\pi k_\perp r} \frac{|A|^2}{\rho_e} \frac{\varkappa s_e - ku}{\varkappa} \cos^2 m\varphi . \qquad (II.7)$$

Here $\varkappa = \sqrt{k_\perp^2 + k^2}$. The energy flux from unit length of the tube

$$Q = 2\pi r \langle W_s \rangle_\varphi \frac{k_\perp s_e}{\varkappa}$$

equals

$$Q = \frac{2s_e^2 |A|^2}{\rho_e} \frac{\varkappa s_e - ku}{\varkappa^2} \begin{cases} 1, & m=0 \\ 1/2, & m=1 \end{cases} \qquad (II.8)$$

Now, we have to express the coefficient A in terms of the amplitude of oscillations of fluxtube. To do this, let's consider the solution (II.2) near the tube boundary, that is at $k_\perp r \ll 1$. For slow oscillations we have (see Ref. [4]):

$$f = \left[1 + \frac{2i}{\pi} \ln \frac{Ck_\perp r}{2}\right], \qquad (II.9)$$

where C is the Euler constant. It is seen from Eqs (I.1) that for outer region (which is free from magnetic field) the density perturbation is connected with the radial component of fluid displacement ξ_r by the following relation:

$$\xi_r = \frac{s_e^2}{\rho_e(\omega - ku)^2} \frac{\partial \delta\rho}{\partial r}. \qquad (II.10)$$

Writing now the displacement of tube boundary in slow oscillations as

$$\xi_r = \frac{1}{2} \xi_0 \exp(-i\omega t + ikz) + \text{c.c.} \qquad (II.11)$$

with help of Eqs (II.1), (II.9) and (II.10) one can find that

$$A = -\frac{i\pi}{2} \frac{\rho_e(\omega - ku)^2}{s_e^2} R\xi_0 . \qquad (II.12)$$

So that, for slow oscillations we have

$$Q_T = \frac{\pi^2 \rho_e}{2} v_\varphi (v_\varphi - u)^2 k^3 R^2 |\xi_0|^2.$$

Let's consider now the bending oscillations. Using the expansion of Hankel functions $H_1^{(1)}(k_\perp r)$ at $k_\perp r \ll 1$ from (II.2) we have at the small distances from fluxtube:

$$f \simeq -\frac{2iA}{\pi k_\perp r}. \qquad (II.13)$$

Writing the radial component of tube's displacement in bending oscillations as

$$\xi_r = \cos\varphi \left[\frac{1}{2} \xi_0 \exp(-i\omega t + ikz) + \text{c.c.}\right]$$

(where ξ_0 is the deviation amplitude of the tube axis from its unperturbed position) we obtain from (II.1), (II.10) and (II.13) that

$$A = -\frac{i\pi}{2} \frac{k_\perp R^2 \rho_e (\omega - ku)^2}{s_e^2} \xi_0 .$$

Correspondingly, the energy flux from unit length of the tube is equal to

$$Q_b = \frac{\pi^2 \rho_e}{4} v_\varphi (v_\varphi - u)^2 [(v_\varphi - u)^2 - s_e^2] \frac{k^5 R^4 |\xi_0|^2}{s_e^2}.$$

Let's find now the energy of slow and bending oscillations per unit length of fluxtube. For bending oscillations the result follows directly from the expression (2):

$$W_b = \frac{\pi R^2}{4} |\xi_0|^2 k^2 \left[(\rho_i + \rho_e) \frac{\omega^2}{k^2} + a^2 \rho_i - u^2 \rho_e\right].$$

For slow oscillations some calculations should be performed. In this case we have

$$W_T = \left\langle \frac{\rho \delta v_\parallel^2}{2} + \frac{\gamma p}{2} \left(\frac{\delta\rho}{\rho}\right)^2 + \frac{\delta B^2}{8\pi} \right\rangle \pi R^2,$$

where angular brackets indicate the averaging over the wavelength. When writing this expression we have taken into account that the transverse velocity of plasma inside the tube is small comparing with the longitudinal one. Using the equation of motion and continuity equation and bearing in mind that magnetic field is frozen we find that inside the tube

$$\delta v_{\parallel} = \frac{s_i}{\rho}\,\delta\rho\,, \qquad \delta B = B\,\frac{\xi}{2R}\,, \qquad \delta\rho = \frac{a^2}{2s_i^2}\,\frac{\xi}{R}\,,$$

where ξ is the radial displacement of the fluid boundary. So with ξ given as (II.10) we get

$$W_T = \frac{\pi\,|\xi_0|^2}{8}\;\frac{\rho_i\,a^2(a^2+s_i^2)}{s_i^2}\,.$$

Therefore, with the help of formula $\gamma = Q/2W$ we find the growth rate of instability respectively for bending oscillations

$$\frac{\gamma_b^{rad}}{\omega} = \pi k^2 R^2\,\frac{(v_\varphi - u)^2[\,(v_\varphi - u)^2 - s_e^2]}{2s_e^2 v_\varphi[\,(1+\eta)\,v_\varphi - u]}\,,$$

and for slow oscillations

$$\frac{\gamma_T^{rad}}{\omega} = \frac{\pi k^2 R^2}{4}\;\frac{\rho_e}{\rho_i}\;\frac{c_T^2(c_T-u)^2}{a^4}\,.$$

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