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CHIRAL SYMMETRY RESTORATION AT FINITE TEMPERATURE IN THE INSTANTON LIQUID

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НОВОСИБИРСК

Chiral Symmetry Restoration at Finite Temperature in the Instanton Liquid

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ABSTRACT

We extend the instanton model of the QCD vacuum with quark-induced interactions to nonzero temperature. Using the mean field approximation, we compare the free energies of the two phases, the «polymer» and «molecular» ones. We have found that the phase transition should happen in a narrow temperature interval around 200 MeV.

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1. INTRODUCTION

After the discovery of the topologically nontrivial fluctuations of non-abelian gauge fields [1], the instantons, there were a number of attempts to connect them with such an important phenomenon as spontaneous breakdown of the SU(N_f) chiral symmetry in QCD [2]. Unfortunately, the dilute gas approximation used in these works is not selfconsistent. Therefore the account for instanton interactions is inevitable.

The so called «instanton liquid model» suggested in Ref. [3] contains two free parameters: the density of pseudoparticles (instantons plus anti-instantons) n and their typical size ρ_0. The analysis of the data made in these works has lead to the following values

$$n \simeq (1\ \text{fm})^{-4}; \quad \rho_0 = 1/3\ \text{fm}. \tag{1}$$

These numbers imply the existence of a set of «small parameters» in the problem, in particular: (i) the smallness of the mean radius of the instanton compared to their typical separation, $\rho_0/R = 1/3$; (ii) the smallness of quantum corrections due to the rather large action of an instanton, $S(\rho_0) \simeq 10$; (iii) relatively small corrections due to the mutual interaction $|S_{int}| \ll S(\rho_0)$; (iv) which, however, are strong enough to enter the «statistical mechanics» of the problem $\exp |S_{int}| \gg 1$.

The first attempt to correct the dilute gas picture by means of «hard core» type interaction was made in Ref. [4]. The variational

theory of the interacting ensemble of instantons, suggesting a more realistic interaction law, was developed by Dyakonov and Petrov [5]. Recently one of us has performed detailed numericai studies of the interacting instanton system [6], correcting both the interaction law and the statistical mechanics of the problem. All the results support the qualitative features of the instanton liquid mentioned above.

The most important physical phenomena produced by instantons belong to the light quark sector. Due to existence of fermionic nearly degenerate (almost) zero modes, the quarks can propagate far enough, hopping from one pseudoparticle to another. As shown in [6], this phenomenon gives a good description for the quark condensate value, as well as for a variety of properties of the pseudoscalar mesons.

In this paper we study how this picture extends to finite temperature. In Refs [7, 6] it was numerically demonstrated that the instanton liquid with quark-induced interactions exists in two essentially different phases, depending on its density. In the dense phase it is a «polymer» in the sense that each pseudoparticle is connected with several neighbours by «quark bonds», while at low density it splits into a set of instanton—anti-instanton «molecules». The former phase is chirally asymmetric, posessing a nonzero quark condensate, while in the latter this symmetry is restored.

Here we propose a simplified but analytic description of both phases. For the polymer phase we assume that both gluonic and fermionic interactions can be studied by means of the mean-field approximation. For the molecular phase we assume that molecules are noninteracting. Also for simplicity of our formulae, we consider only massless quarks (although there is no problem to account for quark masses).

2. POLYMER PHASE

The partition function for the interacting instantons can be written as follows

$$Z=\sum_{N_+=N_-}\frac{1}{N_+!N_-!}\exp(-S_{int})\left[\det\begin{vmatrix}0 & T^f_{Al}\\ T^{+f}_{lA} & 0\end{vmatrix}\right]^{N_f}\prod_{i=1}^{N_++N_-}d\Omega_i\,\rho_i^{N_f}$$

$$d\Omega_i=C\,\rho_i^{b-5}d^4z_i\,d\rho_i\,d\Omega^i_{SU(3)}/\Omega_{SU(3)} \qquad (2)$$

where we have for simplicity used the one-loop instanton amplitude [8]. S_{int} is the gluonic interaction. The fermionic interaction is contained in the fermionic determinant, evaluated in the zero mode approximation [9, 6]:

$$T^f_{Al}=\int d^4x\,\psi_0^{+A}(x-Z_A)\,(i\hat{D})\,\psi_0^l(x-z_l)\simeq\frac{2\cos\theta R}{\rho_l\rho_A(2.58+R^2/\rho_l\rho_A)^2} \qquad (3)$$

where θ is the relative orientation angle and $R=|z_A-z_l|$ is the distance between instanton and anti-instanton. The constant C (including the gluonic zero-mode prefactors and the proper power of a certain lambda parameter) is treated as a free parameter, to be fixed from (1). The parameter $b=(11N_c/3)-(2N_f/3)$ is the usual one-loop beta function coefficient.

The gluonic interactions have a complicated space and orientational dependence [5, 6] which we take into account in a simplified manner. We average the binary pseudoparticle interaction over its orientational dependence and reproduce the residual repulsion by means of the constant k defined as follows*):

$$k\rho_1^2\rho_2^2=\int S_{int}\,d^4R\,d\Omega_{SU(3)}\,. \qquad (4)$$

The mean-field approach to fermionic interaction is based on the assumption that any given instanton randomly interacts with many neighbours. Writing the initial determinant as a $N_+\times N_+$ one, we assume that all non-diagonal elements of this matrix are random and average to zero, while the diagonal ones can be expressed as follows

$$A^l_{ll}=\sum_A T^l_{lA}\,T^l_{Al}\,. \qquad (5)$$

Only these are kept in the final statistical weight, and we can express tnem as a product of several factors. One is the angular average of $\cos^2\theta$, where θ is the relative orientation angle introduced in [6]

$$I_a=\int\cos^2\theta\,d\Omega_{SU(3)}/\Omega_{SU(3)}=\frac{1}{4}. \qquad (6)$$

Another one is the integral over distances R

*) D.I.Dyakonov and A.D.Mirlin have checked, that the gluonic interaction is practically temperature independent (privite communication).

$$I_R = 2\pi^2 \int \frac{z^5 dz}{(2.58+z^2)^4} = 0.494 . \qquad (7)$$

Finally, A contains the anti-instanton density

$$A_{II}^{I} = I_a I_R \rho_I \int n_A(\rho_A)\, d\rho_A . \qquad (8)$$

Proceeding in a symmetrical way for instantons and anti-instantons and redistributing the corresponding determinantal factors over the pseudoparticles, our mean field estimate for the zero mode determinant amounts to defining an effective one-instanton amplitude

$$d^{+}(\rho) = C\rho^{b-4} \prod_{f} (M_{det}^{+} \rho) \qquad (9)$$

in terms of a «determinantal» mass

$$M_{det}^{+} = | I_a I_R \rho \int n^{-}(\rho')\, d\rho' |^{1/2} . \qquad (10)$$

The temperature dependence of the amplitude was determined in Ref. [10], we take its simplified form

$$d(\rho) \to d(\rho) \exp\left[- \frac{2N_c + N_f}{3} (\pi\rho T)^2 \right] \qquad (11)$$

which has correct zero and large temperature limits (at small temperatures the expression is somewhat more complicated).

The free energy density is maximal for the following distribution

$$n^{\pm}(\rho) = C\rho^{\bar{b}-4} (Fe^{\rho/\bar{\rho}})^{N_f/2} \exp\left[- \frac{2N_c + N_f}{3} (\pi\rho T)^2 - k\rho^2 \bar{\rho}^2 n \right] \qquad (12)$$

where the constant k was defined in (3) and

$$F = I_a I_R n \bar{\rho}/2 ; \qquad \bar{b} = b + \frac{3}{2} N_f . \qquad (13)$$

For sake of analytic simplification, we neglect the small shift of the distribution due to the factor $\exp(\rho/\bar{\rho})$, and then get the mean square radius to be

$$\frac{\bar{b}-4}{2\bar{\rho}^2} \equiv \frac{2N_c + N_f}{3} \pi^2 T^2 + k\bar{\rho}^2 n . \qquad (14)$$

Finally, the selfconsistency equation determines the total density

$$n = C(Fe)^{N_f/2} \frac{\Gamma(\alpha)}{\alpha^{\alpha}} (\bar{\rho}^2)^{\alpha} ; \qquad \alpha \equiv \frac{\bar{b}-4}{2} . \qquad (15)$$

Note, that $F \sim n$, and at $N_f = 2$ the situation is quite specific: the density enters linearly into both sides. In this case, the initial parameter C is determined by the ρ_0 value only

$$C = 375/\rho_0^{9.66} \qquad (N_f = 2) \qquad (16)$$

while in general it depends on the total density, too.

If all parameters at $T=0$ are fixed, we may consider the temperature dependence of the density. From (14, 15) we get:

$$\left(\frac{n(T)}{n(0)}\right)^{4\frac{1-N_f/2}{b-4+2N_f}+1} + \frac{(2N_c+N_f)}{3(\bar{b}/2-2)} \pi^2 T^2 \rho_0^2 \left(\frac{n(T)}{n(0)}\right)^{\frac{1-N_f/2}{b-4+2N_f}2} = 1 . \qquad (17)$$

The non-zero solution of this equation exists only below some temperature T_0, so the polymer phase is a low temperature phenomenon. Note, that for $N_f > 2$ at this point the density drops to zero from a finite value. We give the corresponding temperatures for several values of N

$$T_0 = \begin{cases} 0.40/\rho_0 = 240 \text{ MeV} & N_f = 2 \\ 0.34/\rho_0 = 200 \text{ MeV} & N_f = 3 \\ 0.30/\rho_0 = 178 \text{ MeV} & N_f = 4 \end{cases} \qquad (18)$$

We have substituted here $\rho_0 = 1/3$ fm and got T_0 close to 200 MeV, which is a quite reasonable value of a critical temperature of the chiral transition. However, one should keep in mind, that even at arbitrary large temperature the instanton density is nonzero due to the existence of «molecules» mentioned above.

3. MOLECULAR PHASE

The amplitude for a single molecule can be written as follows

$$d_{mol} = C^2 (\rho_1 \rho_2)^{b-5} \exp\left[- \frac{2N_c + N_f}{3} \pi^2 T^2 (\rho_1^2 + \rho_2^2) \right] |T_{Al}|^{2N_f} d^4 R\, d\rho_1\, d\rho_2 \qquad (19)$$

where the «overlap integral» T is evaluated as [5, 6]

$$T_{Al} = \frac{2R\cos\theta}{\rho_1\rho_2(2.58+R^2/\rho_1^2\rho_2^2)^2}. \qquad (20)$$

The integral over relative distance R in (19) is taken from above the «core» size $R_0/\sqrt{\rho_1\rho_2} = \bar{R}_0 (\simeq 1)$

$$I_R = \int_{R_0}^{\infty} [R/(2.58+R^2)^2]^{2N_f} 2\pi^2 R^3 dR. \qquad (21)$$

The orientational integral over the SU(3) group is equal to

$$I_a(N_f) = \int \cos^{2N_f}\theta \, d\Omega_{SU(3)}/\Omega_{SU(3)} . \qquad (22)$$

Note, that these integrals contain some numerical smallness, which is the more pronounced the larger is N_f.

The final integral over ρ leads to the molecule density

$$n_{mol} = C^2 I_a I_R \Gamma^2((b-2)/2)/4\left[\left(\frac{2N_c+N_f}{3}\right)\pi^2 T^2\right]^{b-2} \qquad (23)$$

where the value of constants C, ρ were determined above (at zero temperature). Note that at large T the molecular contribution decreases as $T^{-(2b-4)}$, which is possible to get by mere dimensional arguments.

4. DISCUSSION OF THE RESULTS

In order to compare both phases we have to equate not the densities but free energies. For the non-interacting molecules the probability to have N molecules in the volume V is just Poissonian

$$P_N = \frac{(Vn_{mol})^N}{N!}\exp(-Vn_{mol}), \qquad (24)$$

and the free energy at extremal N is just the molecule density. Of course, if the density of molecules is too high, they are interacting, too. The simplest way to account for it is to include an «excluded volume» correction.

However, for the polymer phase the free energy is not just equal to the density, in our approximation it looks as follows

$$F(T) = -n(T)\left\{\frac{b}{4} - \frac{N_f}{8} - \frac{2N_c+N_f}{6}\pi^2 T^2 \rho_0^2 \left[\frac{n(T)}{n(0)}\right]^{\frac{2-N_f}{b-4+2N_f}}\right\}, \qquad (25)$$

and it decreases nc the end point $T = T_0$ stronger than the density.

In Figures 1—3 we show the free energy for both phases for $N_c = 3$ and $N_f = 2, 3, 4$ (for the last case there exist the most reliable lattice data). For $N_f = 3, 4$ the endpoint of the selfconsistency equation (18) is not real because already at smaller T_c the free energy changes its sign and the trivial solution dominates from this point. (This happens at temperatures 190 and 165 MeV, respectively.) For the molecular phase we have shown several dashed curves, corresponding to different excluded volumes per molecule.

Of course, at temperatures at which the free energies of both phases are comparable our approximations probably fail, so that we have neither a random polymer nor well separated molecules, but more complicated matter, may be with some finite clusters, strong correlations etc. We may hope that the true free energy is still in between of these two extremes. Only numerical simulations for this liquid, accounting for all competing interactions, can give an accurate description of the transition region. Let us remind in this respect, that simulations described in [7, 6] revealed a sharp structural transition with two coexisting phases, thus it is most probably a first order transition.

Considering the quark condensate and quark propagation in general we remind that it is most effectively described in terms of spectrum of eigenvalues of the Dirac operator. The polymer phase leads to a Gaussian shaped spectrum, the width of which (being of the order of the hopping amplitude or the determinantal mass) decreases with rising temperature. On the contrary, the molecular phase leads to a completely different double-humped shape, with zero density at zero eigenvalue. The transition between these phases cannot be expected to be smooth, and the true shape of the spectrum in the transition region is very complicated and sensitive to subtleties of the correlations between the pseudoparticles.

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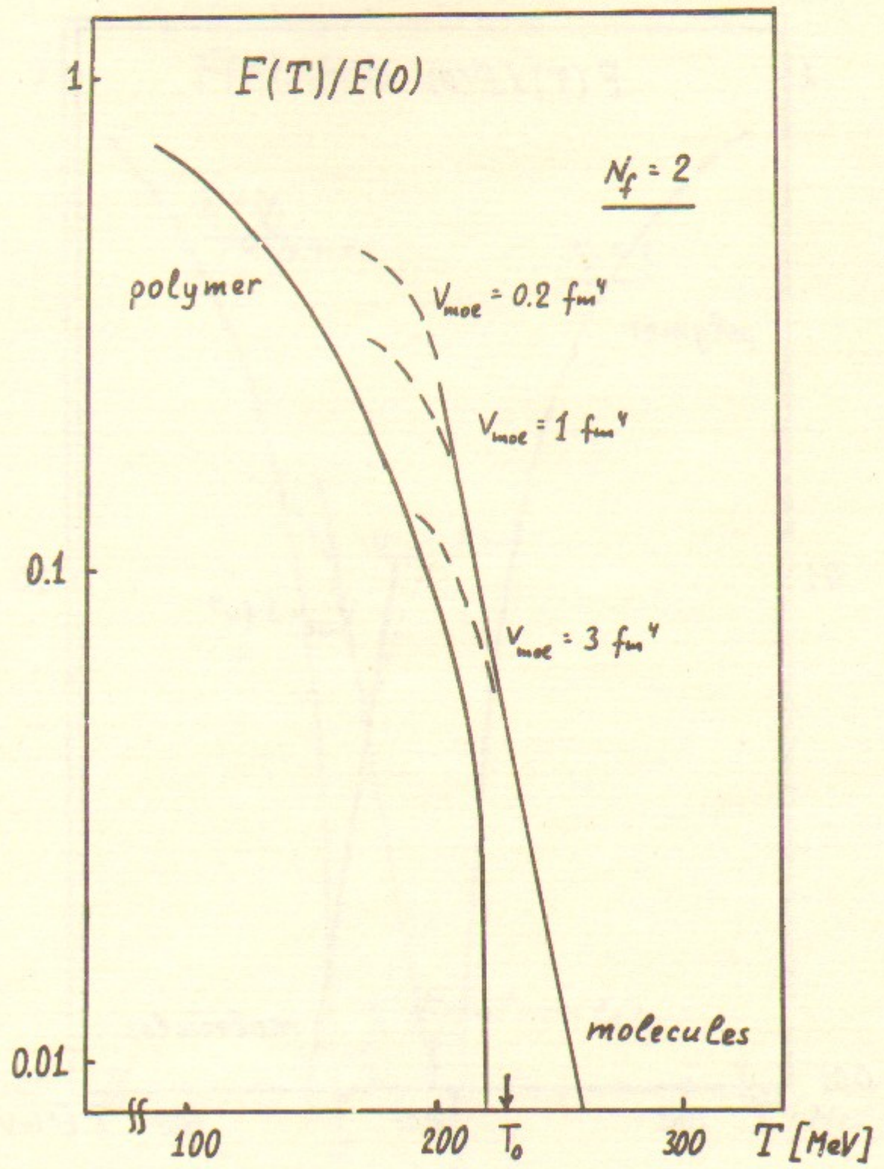

Fig. 1. Free energy, divided by its value at $T=0$, of the instanton liquid versus temperature T for 3 colours and 2 massless flavours. The two solid curves represent our results for the polymer and molecular phases, respectively. The dashed lines correspond to different excluded molecular volumes.

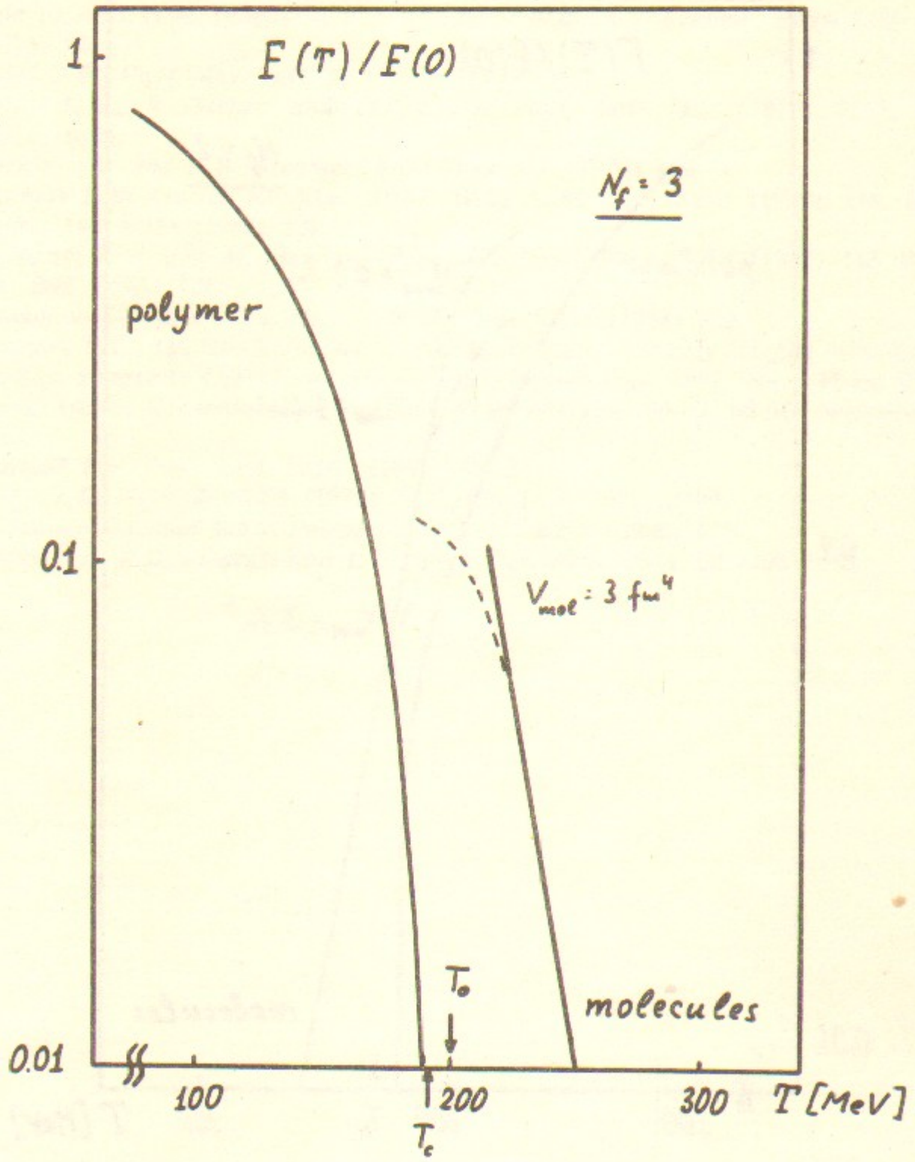

Fig. 2. The same as Fig. 1, but for 3 flavours.

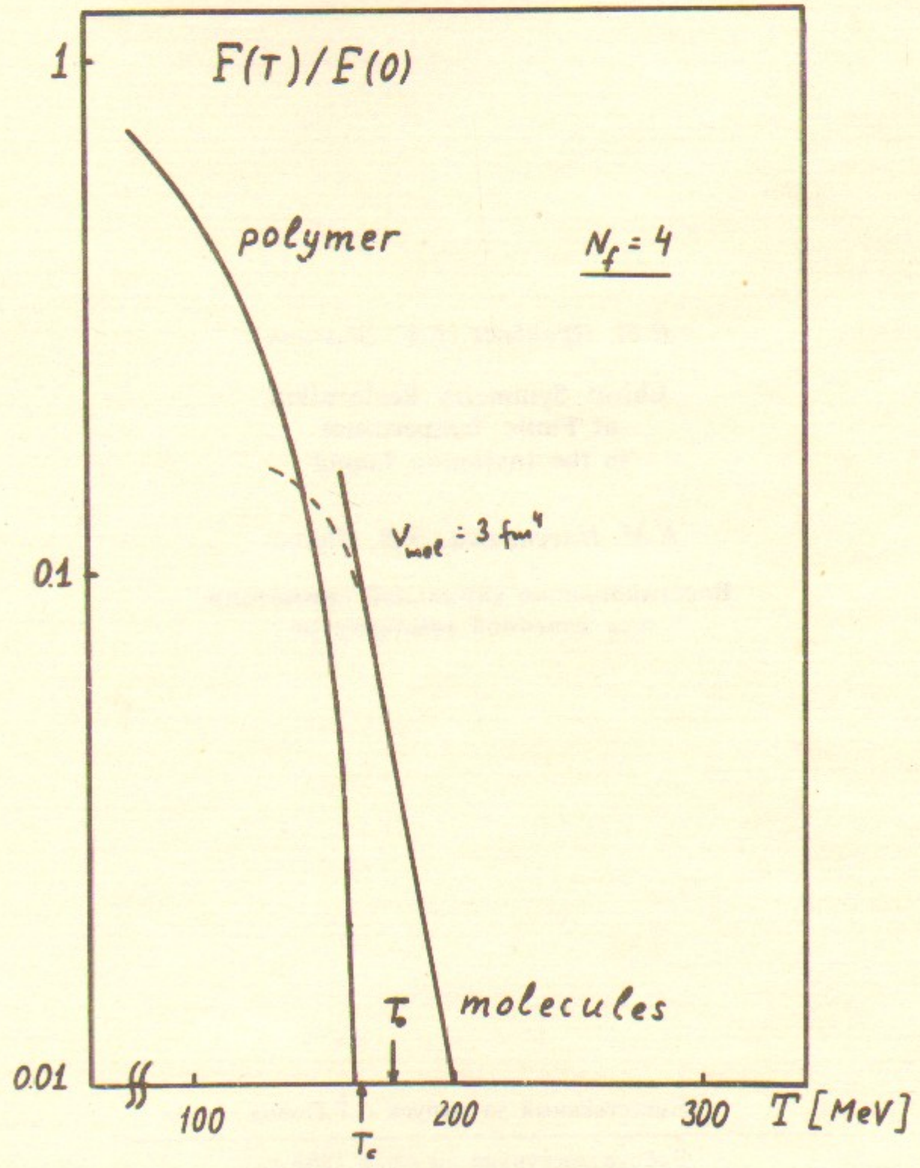

Fig. 3. The same as Fig. 1, but for 4 flavours.

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