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INSTANTONS IN QCD I. PROPERTIES OF THE «INSTANTON LIQUID»

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НОВОСИБИРСК

Instantons in QCD I.
Properties of the «Instanton Liquid»

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ABSTRACT

This is the first work of the series devoted to the instanton-induced effects in QCD. We report the results of numerical simulation of the ensemble of interacting pseudoparticles, or the «instanton liquid». All its qualitative features suspected earlier on the phenomenologal grounds are indeed reproduced. The chiral symmetry is broken, and the value of the quark condensate is measured.

1. INTRODUCTION

After discovery of the topologically nontrivial fluctuations of the non-Abelian gauge fields [1], the instantons, it was suspected that they are very essential ingredient of the vacuum of quantum chromodynamics, explaining such important phenomena as explicit breakdown of the U(1) chiral symmetry [2] and spontaneous breakdown of the SU(N_f) chiral symmetry [3]. Unfortunately, the very-small-size instantons, described by the semiclassical theory [2], produce too small effects to be noticeable, while for larger instantons, (with $\rho > 1\ \mathrm{GeV}^{-1} = 0.2$ fm), account for their interaction is inevitable [4].

The question about their role remained open for some time, till the analysis of the data [5] (by means of the QCD some rules [6]), leading to the so called *«instanton liquid model»*. (Below papers of this series are referred simply as AI—AIV.) The main consequence of this model was *existence of a set of «small parameters»* in the problem, in particular: (i) the smallness of the mean radius of the instanton compared to their typical separation, $\rho/R \sim 1/3$; (ii) the smallness of quantum corrections due to rather large action of an instanton, $S_0 \sim 10$; (iii) relatively small corrections due to the mutial interaction $|S_{int}| \ll S_0$; (iv) which, however, are strong enough to affect strongly the «statistical mechanics» of the problem $\exp|S_{int}| \gg 1$.

The «microscopic» theory of the interacting ensemble of instantons was started by the paper [7], in which the variational approach to this problem was developed. Although it was devoted to the simplest case possible, the SU(2) theory without quarks instead of real QCD, all features (i—iv) mentioned above were qualitatively reproduced.

However, this pioneer work [7] was based on a *number of assumptions*, in particular: (i) only the simplest trial function for the field configurations was considered; (ii) the so called «current-induced» corrections were neglected, although no their estimates were actually presented; (iii) the integral over collective variables, or the «statistical mechanics» of the problem was made in a very simplified way, by means of some mean-field approximation. Therefore, it was necessary to check whether the important conclusions of this paper do survive, if one makes more accurate calculations.

That was the aim of the previous series of our papers [8] (below referred as BI—BIV), were we have developed the more *quantitative theory of the instanton-induced phenomena* in gauge theories. Systematic application of powerful numerical methods made it possible to consider more complicated trial functions, to control the magnitude of the «current-induced» corrections and to make some estimates for the «non-binary» forces among the instantons and anti-instantons (below for brevity, pseudoparticles or *PPs*). As a result, much better understanding of the «gluonic» interaction was reached.

Important progress was also reached in the *fermionic* sector, producing even more complicated interactions among the *PPs*. As any diagrammatic methods proved to be too complicated here, and any type of approximations too suspicious, we had decided to perform the integration over all collective variables explicitly, by the rather time-consuming and straightforward Metropolis method, which is very reliable and avoids all types of approximations. It was not quite trivial that it can really work with so complicated weight function, but it did. And again, although quantitative corrections to [7] are sometimes large, the mentioned qualitative features (i—iv) of the «instanton liquid» model were reproduced once more.

By this paper we start the new series of papers (referred as CI, CII etc.) devoted to *applications of this theory to QCD* with three colors and three light quark flavors, or to the real strong interaction physics. Thus, unlike for the results of BI—BIV which could only be compared with the lattice data, now we are going to confront our theoretical predictions directly with the experiment.

2. THE MODEL

The motivations and details of our approach can be found in our previous works BI—BIV, and therefore we are not going to repeat them here. Instead, we just formulate our model for the interacting ensemble of *PPs*. First of all, their interactions can be split into three different categories, *the «classical»*, *the «quantum»* and *the «fermionic»* ones.

The «classical» type of the interaction was, in fact, the most difficult conceptual point of this theory. The problem is how to select few most important «collective» variables out of the infinite set of them, describing all configurations of the gauge field possessing *PPs*. It is desired to do it in such a way, that all integrals over the remaining variables be just Gaussian and can be calculated semiclassically, while all the nontrivial («non-Gaussian») part of the problem be included in the subsequent explicit integration over «collective» variables. In BI we have discussed the properties of the ideal set of configurations (called the «streamline») and in BII we have checked whether various «trial functions» are really close to it. As a result we have found such trial function which does the job, at least for interparticle separation $R > \rho$, where ρ is the PPs radius. (It will be shown that only few percent of the particles have more close neighbours in our ensemble.)

The resulting form of the «classical» binary interactions of the instanton—anti-instanton and instanton—instanton pairs is as follows

$$S^{IA}_{int}/S_0 = |u|^2\left[\frac{4d}{(4+R^2/\rho_1,\rho_2)^2} + \frac{16}{(3+R^2/\rho_1\rho_2)^3}\right],$$

$$S^{II}_{int}/S_0 = |u|^2\left(\frac{d+3}{4}\right)\frac{1.6}{(1+R^2/\rho_1\rho_2)^3}, \qquad (1)$$

where R is the distance between the centers, ρ_1, ρ_2 are the *PP* radii, and $|u|^2$, d are some parameters describing relative orientation of the *PPs* in the color space. (This point is generalized compared to BII in order to include the SU(N) gauge group.) Out of the «relative orientation matrix» $\hat{O} = U_1^+ U_2$ (where U_1, U_2 describe color orientations of the *PPs* considered) only the 2×2 upper left corner is relevant. One may write it down in the following «quaternion form»

$$\hat{O} = iu_4 + \vec{u}\vec{\tau} \qquad (2)$$

($\vec{\tau}$ are the Pauli matrices) and the parameters $|u|^2$, d entering (1) are defined as follows:

$$|u|^2 = u_\mu^* u_\mu ,$$
$$d = 1 - 4|R_\mu u_\mu|^2 / R^2 |u|^2. \tag{3}$$

In the SU(2) case the matrix $\hat{O}$ is unitary, therefore $|u|^2=1$. For the **SU(3)** case to be considered their variation range **is** as follows:

$$\frac{1}{2} < |u|^2 < 1 ,$$
$$-3 < d < 1 . \tag{4}$$

Now we come to *the «quantum» interaction*. Expressions (1) define relative action variation of a *PP*, S_{int}/S_0. The absolute action (in fact, in the Plank constant unites) is equal to $S_0 = 8\pi^2/g^2$, where, in the quantum theory, the coupling constant g is not actually a constant but is logarithmically scale-dependent. The question is then which particular value of g one should actually use.

This problem may be formulated in somewhat different form. Quantum fluctuations around the instanton in Gaussian approximation was taken into account by 't Hooft [2], and his expression for the size-ρ instanton probability

$$d\Omega_I(\rho) = C_{N_c} d\rho_I d^4 z_I / \rho_I^5 \, \beta_I^{2N_c - b'/2b} \left(\frac{b}{2}\right)^{b'/2b} \times$$
$$\times \exp[-\beta_I - (2N_c - b'/2b)\, b'/2b \ln \beta_I/\beta_I] \quad (1 + O(1/\beta_I)) ,$$
$$b = \frac{11}{3} N_c - \frac{2}{3} N_f \simeq 9 ; \qquad b' = \frac{34}{3} N_c^2 - \frac{13}{3} N_c N_f + \frac{N_f}{N_c} \simeq 64 ;$$
$$\beta_I \equiv b \log\left(\frac{1}{\rho_I \Lambda_{PV}}\right) ; \qquad C_{N_c} = \frac{4.66 \exp(-1.68 N_c)}{\pi^2 (N_c - 1)! (N_c - 2)!} \tag{5}$$

includes this scale-dependence of g (in fact, in (5) it was generalized [7], accounting for the two-loop effects as well).

Now, expression (5) holds for well-separated *PPs*, while our trial function prescribe also the shape of a somewhat «deformed» configurations appearing if two (or more) *PPs* are close. If one looks in BII for their explicit expressions he will find parameters also called ρ, the *PP* radii. However, they do not describe the field distribution in such fluctuations in exactly the same way as for the isolated instanton. Moreover, we have argued in BII that it is more

reasonable to use instead a somewhat corrected version, better reproducing the real field distribution. Such «renormalized» radius

$$\tilde{\rho}_I = \frac{\rho_I}{\left[1 + \sum_{I' \neq I} \frac{\exp(-0.2 R_{II'}^2)}{R_{II'}^2} + \sum_A \frac{\exp(-0.2 R_{IA}^2)}{R_{IA}^2}\right]^{1/2}} ,$$
$$R_{II'} = \frac{(z_I - z_{I'})^2}{\rho_I^2} ; \qquad R_{IA} = \frac{(z_I - z_A)^2}{\rho_I^2} \tag{6}$$

depends on the position of the neighbours, and if one uses $\tilde{\rho}$ in (5) instead of ρ he has what we call the «quantum» interaction. (Such terminology is, of course, a matter of convention.)

Coming finally to the «*fermionic interaction*», we remind the reader that in the theory with massless fermions each *PP* can be considered as some effective vertex, emitting a quark—antiquark pair of each light flavor [2]. Moreover, these lines for a single *PP* cannot be closed in loops, because chirality of emitted quarks and antiquarks are opposite. Therefore, in the massless limit the emitted quarks can only propagate to the *PP* of the opposite topology, and then be absorbed. Therefore, the probability for any configuration of *PPs* to occure in vacuum depends on the probability amplitudes of all such «jumps» of the light quarks. Unfortunately, diagrammatic expression for such probability is rather complicated, containing many contributions of different signs.

We have taken into account these effects as follows. We remind that formal integration over fermions leads to the standard Mattew—Salam «fermionic determinant» of the Dirac operator

$$\int D\bar{\psi}\, D\psi\, DA_\mu e^{-S} = \int DA_\mu \det(i\hat{D} + im)\, e^{-S_{glue}} , \tag{7}$$

which is evaluated in the following approximation. We assume that all nonzero fermionic modes (leading, in particular, to charge renormalization) are correctly included in the 't Hooft factor (5) for each *PP* individually. The zero-modes, describing such «jumps» of the light quarks, form a basis in which the Dirac operator and then its determinant is calculated explicitly. This is done by forming the matrix

$$i\hat{D} = \begin{pmatrix} 0 & T_{IA} \\ T_{AI}^+ & 0 \end{pmatrix} \tag{8}$$

out of the «overlap integrals» T_{IA} for all instanton—anti-instanton pairs. Its value in BIII was approximated by

$$T_{IA}=\frac{2u_\mu(z_I-z_A)_\mu}{\rho_I\rho_A}\,\frac{1}{[2.58+(z_I-z_A)^2/\rho_I\rho_A]^2}. \qquad (9)$$

The determinant of $i\hat{D}$ (8) is real because the matrix is Hermitean. Moreover as all its eigenvalues go in pairs due to the chiral symmetry, $\pm\lambda$, it is (up to the sign factor $(-1)^{N_{PP}}$) even positive, as the statistical weight function should be. If the quark masses are small but nonzero, the matrix (8) has on its diagonal also im. As we diagonalize $i\hat{D}$ numerically, there is no problem to account for the mass while a determinant is calculated.

Collecting all this, let us now summarize the statistical sum of our model. It is a product of 't Hooft factors (5), times exponent of the «classical» bynary interactions S_{int} (1), times the fermionic determinant (8) calculated in the zero-mode basis as explained above

$$Z=\int\left[\prod_{I,A=1}^{N} d\Omega_{I,A}(\rho_{I,A})\right]\exp(-S_{int})\times$$

$$\times\prod_{f=1}^{N_f}\left[1.34\det(i\hat{D}+im_f)\cdot(-1)^N\prod_{I,A=1}^{N}\rho_{I,A}\right], \qquad (10)$$

where N and N_f are the numbers of instantons and flavors, respectively.

3. THE INSTANTON—ANTI-INSTANTON MOLECULES

Before we dive into discussion of the «instanton liquid», let us present some results concerning its elementary «building block» As it was explained above, due to the fermionic zero modes the individual (or well separated) PP has vanishing probability to occure, and therefore the smallest element of our ensemble is the instanton — anti-instanton molecule.

The «*fermionic bonds*» binding it are scale invariant: the «overlap integrals» depend only on the ratio of the distance R to the PP radii. The gluonic interaction in principle brings in its own scale and therefore molecules of different size have slightly different properties. However, this effect was found very weak and actually all

results are quite close for all ρ. The physics is, as usual, some compromise between the «energy» (the weight function) and the «entropy» (or the multiplicity of configurations). As it is explained in BIII, the weight function is maximal at the distance $R=0.93\rho$ and corresponds to PP orientation such that $|u|^2=1$, $d=-3$. We wander how strong deviations from this point are produced by the quantum fluctuations.

There are 26 parameters describing a pair of pseudoparticles in the SU(3) case, but actually the weight function depends only on half of them (in particular, common displacement or common rotation is irrelevant). In order to simulate the distribution over all these parameters we have chosen very straightforward (and therefore very reliable) numerical method known as the Metropolis algorithm. It actually generates a random work in our configuration space, corresponding to the weight function described above, by means of subsequent «updating» one variable after another. Only the radii of both PPs were hold fixed, considered to be our input parameters (the results to be reported are, in fact, for $\rho=0.4\Lambda_{PV}^{-1}$).

In Fig. 1 we show the distribution over the overlap integral T_{IA}. Note that it is not peaked at its maximal possible value (marked by the arrow), which means that the «entropy» takes this system far from the weight function maximum. At the same time, too strong fluctuations ($T_{IA}\to 0$) are also strongly suppressed.

Fig. 2 shows the distribution over the «classical» gluonic interaction S_{int}. Compared to the action of a PP of such size, $S_0=8.2$, these modifications are rather modest. Note that they are shifted toward the negative values, which means that the attraction slightly wins over the repulsion. (This is not surprising: as noted in BIII, fermionic «bonds» prefer the relative orientation which leads to attraction.) It is important that the width of the distribution in Fig. 2 is not small, so the small mean value does not imply that one may just ignore this type of the interaction.

«*Polarization*» of the PPs in relative orientation is demonstrated by the distributions over the invariants $|u|^2$ and d (4) shown in Figs 3, 4. For comparison we have shown the corresponding distributions for all pairs of PPs in the «instanton liquid» (the dashed lines), more or less corresponding to random relative orientation. The distributions for molecules are essentially different, and the tendency toward the maximum values $|u|^2=1$, $d=-3$ is well seen.

Concluding this section we may say, that although quantum fluctuations are strong and 100% significant, the system is also very far from being completely random. The energy—entropy compromise is made somewhat in the middle, thus neither of the two extremes can be used as a reasonable approximation.

4. PROPERTIES OF THE «INSTANTON LIQUID»

We have studied ensemble of the interacting *PPs* in the same way as «molecules» reported above, namely by means of straightforward numerical integration over all collective variables using the *Metropolis algorithm*. For this we take some fixed number of pseudoparticles N_{PP} (in fact, 16 or 24) in a hypercube of (Euclidean) space-time with the periodic boundary conditions. As each *PP* with the SU(3) color group has such collective coordinates as size, position and 8 orientation angles, we have actually integrated over $13\times N_{PP}$ (208 or 312) variables. Although the weight function of our integral is rather time-consuming, especially the fermionic determinant, we have not used any particular simplifications and have computed it at an «updating» of any variable. The speed of this calculation is the main limitation of the *PP* number under consideration.

Our weight function is not only rather time-consuming, but it is also very complicated in the ordinary sense. Indeed, one should take care of the fact that no pair of *PPs* be too close to each other, otherwise the «classical» interaction is too strong and the contribution of such a configuration is negligible. Also, none of the *PPs* should be too much separated from all others, otherwise the fermionic «bonds» are weak and the fermionic determinant is nearly zero. Thus, operating such a complicated system is a delicate business, and we emphasize necessity to make more extensive numerical experiments with it in the future.

First of all, in order to get some qualitative insight into the properties of the system we have made simulations at various *PP* densities (or the various volume of our box). As such data were not reported in BII, BIII even for simpler cases, we now give them in more details. In Figs 5, 6 we present the distributions over the *PP* radii, both over the «intrinsic» parameter ρ entering the ansatz and over the parameter $\tilde{\rho}$ defined in (6). For small densities these distributions are close to the input semiclassical expression (5). Note that it is peaked at very large $\rho\sim 0.7\Lambda_{PV}^{-1}$, at which this formula

cannot in fact be trusted. However, for dense enough «liquid» the interaction suppresses large instantons and a peak at $\rho\sim(1/3)\Lambda_{PV}^{-1}$ appears. The $\tilde{\rho}$-distribution is less spectacular, but the tendency is similar. The upper scale in Fig. 6 gives the absolute values of the instanton action, and one can see that most of the distribution is indeed at the action of the order of 10, with relatively small admixture of the region where $S_0<5$ where the semiclassical formulae used are not reliable.

The interplay between attraction and repulsion, as well as with the statistical weights of configurations is very nontrivial. Density dependence of *the mean interaction per PP* is shown in the upper half of Fig. 7. It demonstrates that repulsion dominates at high density, while at low ones attraction takes over (may be due to molecule formation). Again, small mean values (compared to the typical $S_0\sim 10$) should not be misleading: the dispersion of its distribution is always of the order of 2, therefore $\exp|S_{int}|$ changes from one configuration to another. by significant factor and this interaction can by no means be neglected (or accounted in the «mean field» approximation).

The *average determinant per particle*, defined as $\langle[\det(i\hat{D})]^{1/N_{PP}}\rangle$ where N_{PP} is the number of *PPs*, is shown versus the *PP* density n_{PP} in the lower part of Fig. 7. It does not decreases much as the «liqiud» becomes dilute, demostrating that this system is very far from the «randomly distributed» gas of *PPs*. We return to this point in the next section.

The density dependence studied so far is, of course, only an academic question because the *PP* density in the QCD vacuum is some fixed quantity. In BII we have madé its measurements for the SU(2) gluodynamics by the explicit calculation of the change in probability caused by imbedding of one extra *PP* pair into the system and demanding that this probability is unchanged (in other terms, that we are at zero chemical potential). Unfortunately, adding a new pair randomly, we have very bad efficiency of the averaging, and should do it many times. With the time-consuming fermionic determinant and with much larger number of color orientation angles we are so far unable to do the same, with the reliable accuracy.

However, we remind the reader that in this series of papers we are going to compare our results not with the lattice data, but with the real experimental numbers. Therefore, even if we have successfully measured our *PP* density in terms of Λ_{PV}, the existing uncertainty in its numerical value will invalidate all these efforts: for $\Lambda_{PV}=150\div250$ MeV the uncertainty in the density $n_{PP}\sim\Lambda_{PV}^4$ are not smaller than one order of magnitude! In view of this we have preferred to fix our «physical unites» from the direct comparison of our results to data, making a kind of «lambda measurements» (similarly to what is done in the lattice-based studies). In the next section we return to this point, using the value of the quark condensate.

5. THE CHIRAL SYMMETRY BREAKING

General introduction into the subject, the history (and necessary references) of the applications of the instanton physics to the description of this important phenomenon can be found in BIII.

Now we start directly with the well known relation between this phenomenon and properties of the eigenvalue spectrum of the Dirac operator. Indeed, the quark propagator is a kind of inverse to this operator (in the massless limit), and therefore its zero eigenvalues makes the propagator to be ill-defined. This fact, if observed, means that one should take one ground state out of some set of possibilities, signalizing the (chiral) *symmetry breaking*.

Moreover, there exist an important relation between *the density of the eigenvalues of the Dirac operator* $i\hat{D}$ at their zero value and *the quark condensate*

$$|\langle\bar{\psi}\psi\rangle|=\sum_n\frac{1}{\lambda_n+im}=\lim_{m\to0}\lim_{V\to\infty}\pi n_{PP}\frac{dN}{d\lambda}\Big|_{\lambda=0}. \qquad (11)$$

As explained in Sect. 2, the projection of the Dirac operator to the «zero mode subspace» is used in our calculations explicitly. The corresponding spectrum of its eigenvalues was measured for the ensemble of the *PP* configurations, and the results are shown in Fig. 8 for several *PP* densities.

First of all, it is seen that the shape of this distribution is changing with density: for the «dense liquid» the distribution is wide and it possesses a maximum at $\lambda=0$, while in the «dilute» regime we have found a dip at $\lambda=0$, similar to that characteristic for the «instantonic molecules» (see Fig. 1). (This fact together with several other indications shows that molecules are indeed an important ingredient at small densities.)

Our second comment to the data of Fig. 8 is as follows: the smallest λ contained in the first bin obviously are out of the trend suggested by other points. This systematic error is presumably a manifestation of *the finite size effects*, known also on the lattice. (Of course, in any finite system the continuous symmetry cannot be spontaneously broken, because its spectrum is never continuous.)

If we disregard the first bin, we may extrapolate to zero using other points and conclude that the density of the eigenvalues at $\lambda=0$ is nonzero, together with the quark condensate. (Only for the smallest *PP* density this conclusions is questionable.)

However, for quantitative measurements of the quark condensate it is better not to use this tricky extrapolation, but to use instead quarks with the finite mass m. Using the spectrum one may study dependence of the quark condensate on the quark mass. Our data (for $n_{PP}=1\Lambda_{PV}^4$ but for two different numbers of pseudoparticles, 16 and 24) are compared in Fig. 9.

The dip at m below $0.1\Lambda_{PV}$ is presumably an artifact due to the finite size effects. Although we were unable to change the size of our system significantly, we do see that for the larger system this effect seems to be indeed smaller. In any case, these data suggest that we should not work with very light quarks in our applications to follow. Thus, in the subsequent works the nonstrange quark masses are taken to be $m_u=m_d=0.1\Lambda_{PV}\simeq20$ MeV. (Note that it essentially smaller than it is typically used on the lattice.)

The data given in Fig. 9 can be used in order to compare the condensate values for different quark flavors. In particular, it follows from them that

$$\frac{\langle\bar{s}s\rangle}{\langle\bar{u}u\rangle}\simeq0.5-0.6. \qquad (12)$$

This may be compared to the numbers extracted by various authors from the data analysis:

$$\frac{\langle\bar{s}s\rangle}{\langle\bar{u}u\rangle}\simeq\begin{cases}0.5 & [9]\\ 0.8-0.9 & [10]\\ 0.81 & [11]\\ 0.8 & [12]\end{cases} \qquad (13)$$

Significant $SU(3)_f$ violation in vacuum (12) has deep roots in our model: it is connected to the fact that the typical matrix elements of the Dirac operator (describing «jumps» from one *PP* to another) are of the order of $0.2-0.6\Lambda_{PV}$, which is not greater than the strange quark mass $m_s \simeq 150$ MeV. Therefore, we claim that the standard chiral perturbation theory should not generally be applicable for the strange quarks. (If this is shown for the QCD vacuum, then the «standard» derivation of the quark mass and the condensate values should be reconsidered.)

Now we briefly consider analogous isospin-breaking parameter

$$\delta = \frac{\langle \bar{u}u \rangle}{\langle \bar{d}d \rangle} = \begin{cases} 0.006 & [9] \\ 0.008 & [12] \end{cases}. \qquad (14)$$

Unfortunately, it is difficult to measure it at present because results depend strongly on the extrapolation to very small quark masses. In particular, the linear fit shown by the solid line in Fig. 9 gives δ value of about 0.02, while that shown by the dashed line leads to much smaller value, of about 0.005.

Generally speaking, this point is potentially interesting, as one of a few observable consequences of the shape of the eigenvalue spectrum in QCD. On general grounds even the sign of the quantity (14) is not obvious. Therefore, it would be nice to reach better understanding of the finite size effects and perform more accurate analysis, both of the «instanton liquid» and the lattice data.

Coming now to the quark condensate value, we show our data on its density dependence in Fig. 10. Its lower part shows that the following dimensionless ratio

$$R_{\psi G} \stackrel{def}{=} |\langle \bar{\psi}\psi \rangle|^{1/3} / \langle (gG^a_{\mu\nu})^2 \rangle^{1/4} \qquad (15)$$

which is remarkably stable over the whole density interval of interest:

$$R_{\psi G} = 0.30 \div 0.32. \qquad (16)$$

It can be compared to the phenomenological values of the «standard» quark and gluonic condensates [6] which give about $R = 0.29 \pm 0.06$. The agreement is good enough, so we may claim that we have reproduced one of the most nontrivial known numbers characterizing the QCD vacuum.

Taking the «experimental» value $\Lambda_{PV} = 220$ MeV and demanding that our measured quark condensate is equal to the standard value $\langle \bar{\psi}\psi \rangle = -(250 \text{ MeV})^3$, one may fix the PP density

$$n_{PP} \simeq 0.7 \ fm^{-4}. \qquad (17)$$

Certainly it is of the reasonable magnitude, slightly smaller than that following from the «standard» gluon condensate, $\simeq 1 \text{ fm}^{-4}$.

However, in what follows we are not going to trick with such numbers very much. The reader (who had followed the lattice-based literature) is probably tired of multiple papers with «real numbers», demonstrating good agreement with phenomenology. The sad point was that the scales used were different in different papers, extracted from conflicting data or from quite different sources (e. g. the string tension, or the rho masses, and so on).

We are not going to claim that our model of the vacuum is able to predict accurate values for all physical quantities. At the moment our aim is to check whether it may provide at least semi-quantitative description of *wide range of hadronic phenomena*, and this will be made in the next papers of this series. Only having at hand rich variety of observables to be compared with, one may seriously speak about such important and delicate problem as «the lambda measurements», fixing the absolute scale in QCD.

6. VEVs OF MORE COMPLICATED OPERATORS

Although in principle we are now able to evaluate instanton-induced contributions *to the vacuum expectation values* (below VEVs) of any relevant operators, we restrict ourselves to only two quantities, the key ones for the understanding of the qualitative distributions of fields in vacuum. The former one is the following ratio

$$R_{4G} = \frac{\langle (gG^a_{\mu\nu})^4 \rangle}{\langle (gG^a_{\mu\nu})^2 \rangle^2}. \qquad (18)$$

As it was shown in AI, it can be estimated for a dilute instanton gas (with fixed instanton radius ρ_c) with the result

$$R_{4G} = \frac{6}{7} \frac{1}{\pi^2 n_{PP} \rho_c^4}. \qquad (19)$$

Thus, in this limit, it is expected to be essentially larger than unity. Our measurements (for $n_{PP} = 1\Lambda^4_{PV}$) gave the following number

$$R_{4G} = 15 \pm 5. \qquad (20)$$

(Calculation of this quantity was made as follows. Naive averaging with random selection of the space-time point has too low efficiency, while evaluation of $G^a_{\mu\nu}$ is based on our rather complicated ansatz for the gauge field and it is rather time-consuming. The measurements were based on generation of the ensemble of points x with the distribution according to the weight function

$$W(x) \sim \sum_{I,A} \frac{1}{[\rho^2_{I,A}+(x-z_{I,A})^2]^8} \qquad (21)$$

with the subsequent averaging of $(G^a_{\mu\nu})^4/W$.

Our result (20) is essentially larger than unity, which means that the gauge fields are distributed very inhomogeneously in space time (the «twincling vacuum», as it was called in AI). Let us remind the reader in this connection, that the so called «factorization hypothesis» suggested by Shifman et al. [6] assumes R_{4G} to be close to unity. This hypothesis is often used in practice, for example in the estimates of high dimension corrections to the QCD sum rules [13]. Thus, we claim that these estimates are wrong at least by one order of magnitude!

Another ratio we are going to consider is the following one

$$R_{ud} = \frac{\langle \bar{u}u\,\bar{d}d\rangle}{\langle \bar{u}u\rangle\langle \bar{d}d\rangle}. \qquad (22)$$

Similarly to R_{4G}, it characterizes the homogeneuity of the quark fields in the QCD vacuum. However, in this case it is not possible to propose some simple estimate of its magnitude for the dilute instanton gas. (If one assumes random distribution of *PPs* over space-time and orientation, he gets

$$\langle \bar{\psi}\psi\rangle \sim n^{1/2}/\rho_c,$$

$$R_{ud} \sim \text{const}\,(n_{PP}\to 0)\,, \qquad (23)$$

but, actually, in the limit $n_{PP}\to 0$ one has a «molecular phase» with zero quark condensate, (see BIII for details).

Our measurements have given the following result

$$R_{ud} \simeq 1.5 \div 2.0 \qquad (24)$$

suggesting that the quark fields, unlike the gluon ones, are distributed in space-time more or less uniformly.

This conclusion is of great importance for the hadronic phenomenology: it means, in other words, that hadrons made of quarks should be lighter than the glueballs! (The general roots of this conclusion follow in our theory from the observation: gluonic fields are concentrated in small spots, the pseudoparticles, but the quarks ones are not: quarks should necessarily «jump» from one *PP* to another.)

7. CONCLUSIONS

1. We have formulated a *definite model* describing ensemble of pseudoparticles in the QCD vacuum, interacting by means of «classical», «quantum» and «fermionic» interactions (derived in BII, BIII and now slightly generalized to the arbitrary color group).

2. We have performed *numerical studies of this model*, taking 16 or 24 pseudoparticles in a 4-dimensional box with the periodic boundary conditions. We have found that, depending on the density (the box size), there are essentially two regimes for the «instantonic liquid». At densities comparable to $1\Lambda^4_{PV}$ repulsive interaction is important, pressing instanton radii distribution the be peaked at sufficiently small value, $\rho \sim \frac{1}{3}\Lambda^{-1}_{PV}$. In more «dilute liquid» the interaction is mostly attractive, the typical radii are shifted toward $1\Lambda^{-1}_{PV}$, and there are multiple signs that the «instantonic molecules» dominates in this case.

3. The qualitative picture of the «dense regime» is in nice agreement with the phenomenologic model suggested in AI. In particular, most of the pseudoparticles have actions S_0 one order of magnitude larger than the Plank constant, justifying our semiclassical approach. The interaction is relatively weak compared to S_0, but significant for ensemble generation.

4. Unfortunately, the absolute normalization of the *PP* density in terms of Λ^4_{PV} was not defined because of the technical reasons. However, as the value of Λ_{PV} is not known with the sufficient accuracy as well, anyway we have to «fix our scale» in MeV by the comparison of our predictions to data. (If it is done for the quark condensate, quite reasonable numbers are obtained.)

5. *The chiral symmetry is broken* at all densities, except probably at very small ones. Dimensionless ratio (15) was found to be nearly constant for all densities. It is also consistent with the corresponding phenomenological value inside the uncertainties.

6. According to our theory, *the strange quark mass is definitely not a small parameter of the vacuum* because it is comparable to the typical matrix elements of the Dirac operator. This statement manifests itself in the significant SU(3) asymmetry of the quark condensate. Note, that recent lattice works have also found strong variation of many parameters at surprisingly small quark masses (e. g. the order of the chiral restoration phase transition) which is in qualitative agreement with this statement. All this makes applications of standard chiral perturbation theory to the strange quarks rather suspicious.

7. Concluding this paper, let me compare our calculations with those made on the lattice from more general point of view. Of course, their virtue is that the integration is made over all possible gauge field configurations, while we restrict ourselves to only their subset. Evaluation of the quark condensate is in both cases made via the eigenvalue spectrum of the Dirac operator. Therefore, the calculation resembles those for, say, the conductivity of a metal (which is proportional to the density of states at the Fermi sphere). The lattice approach starts with the «first principles», and consider *all quark states* (analogous to all electrons of a metal) and deals with huge fermionic operator, while we have selected *specific quark states*, analogous to «valence electrons».

If the «small parameters» claimed in our work are there in QCD, it would be a mistake not to benefit from this significant simplification in such a complicated problem. We emphasize that the number of variables (per unite space-time volume) in our approach is about 4 orders of magnitude smaller than that on the lattice. And moreover, the difference is qualitative: we hope to understand the «instanton liquid», while the understanding of the lattice ensemble seems to be much more difficult.

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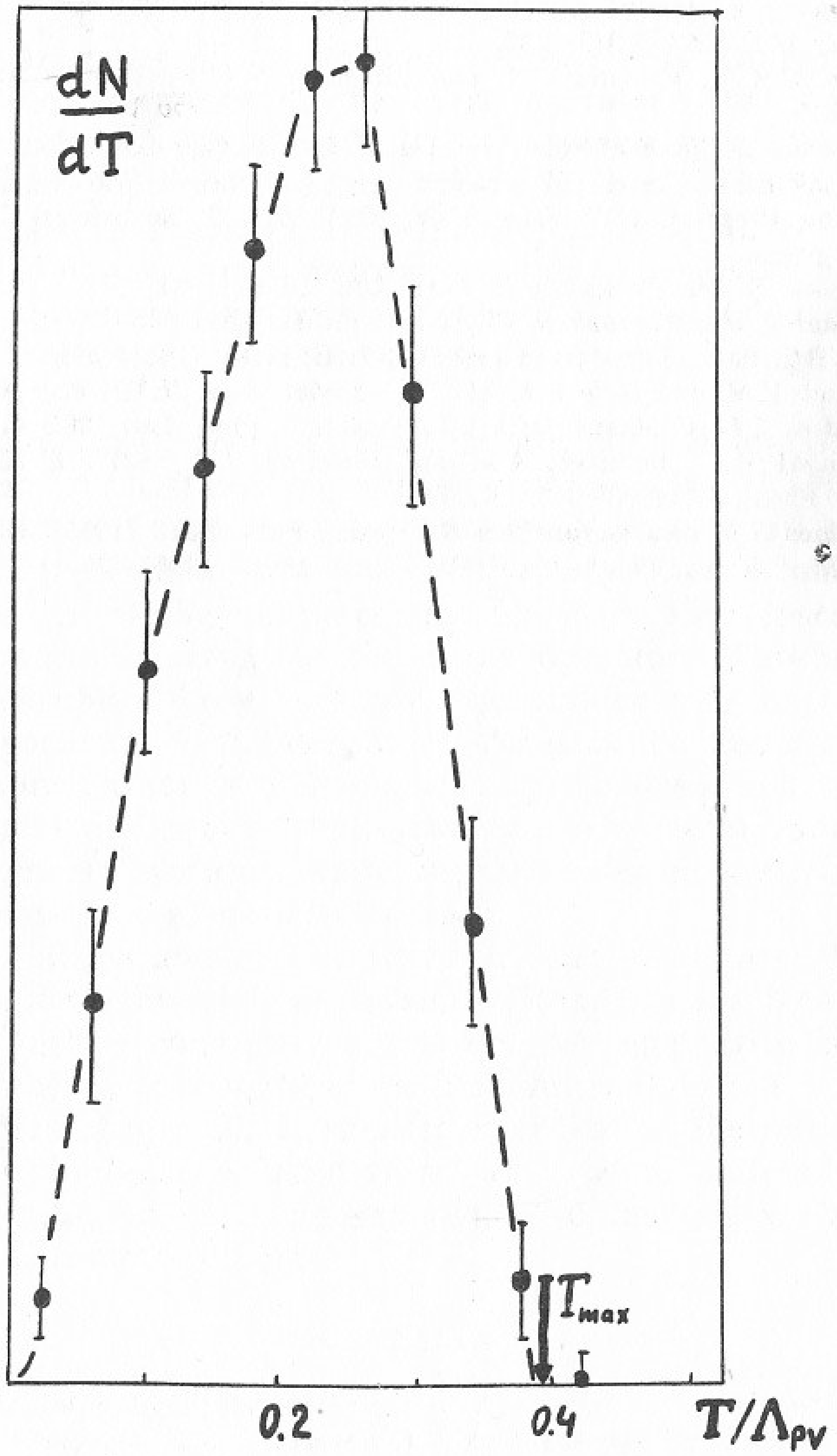

Fig. 1. The distribution over the overlap integral T (measured in Λ_{pv}) for an instanton—anti-instanton molecule. (Here and below the radii of both pseudoparticles are equal to $0.4\Lambda_{PV}^{-1}$).

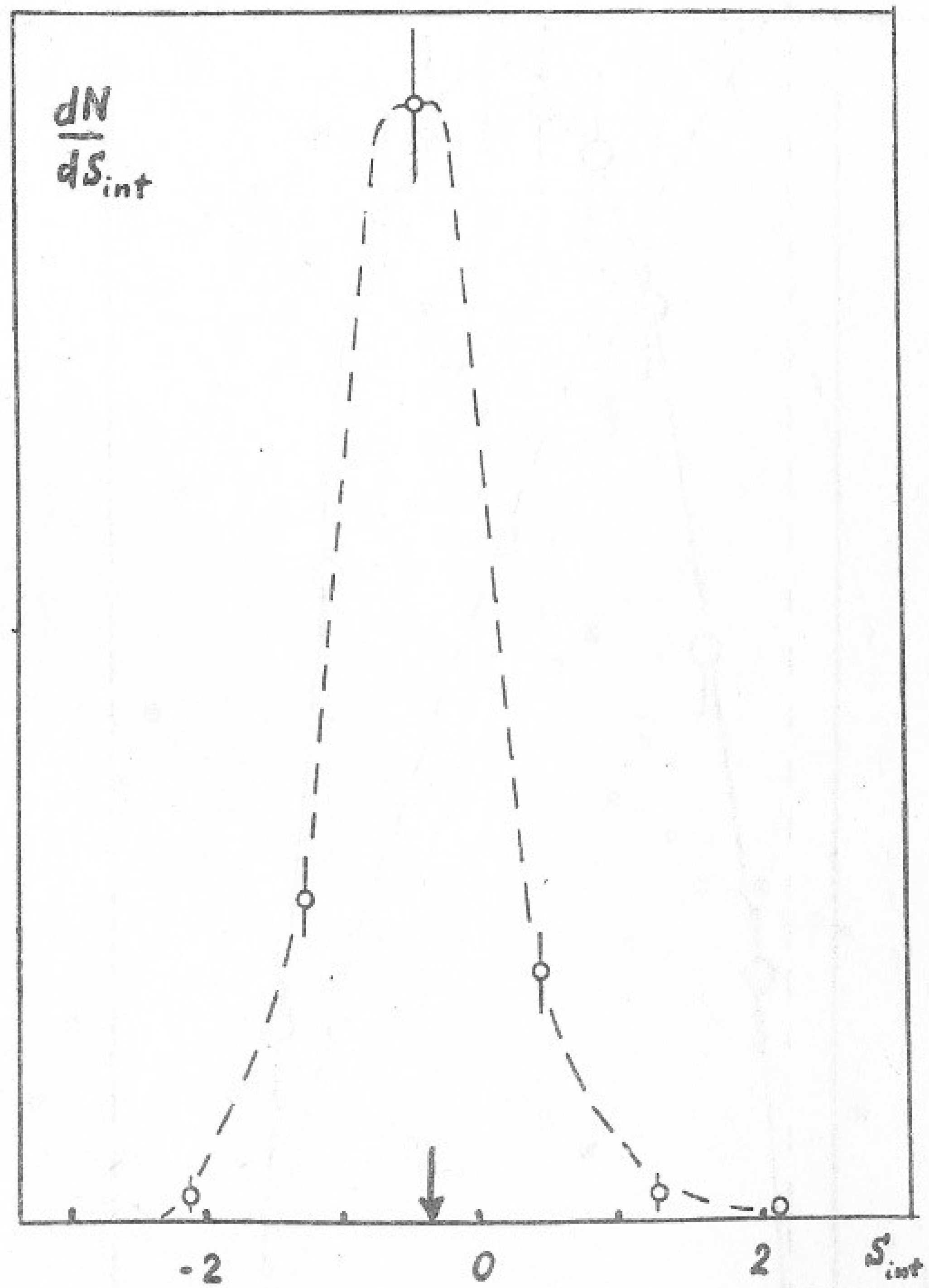

Fig. 2. The distribution over the «classical» interaction for an instanton—anti-instanton molecule. Note for comparison, that the unperturbed action of such pseudoparticle is 8.2 unites, so the mutial interaction is in fact relatively small effect. An arrow shows the mean value.

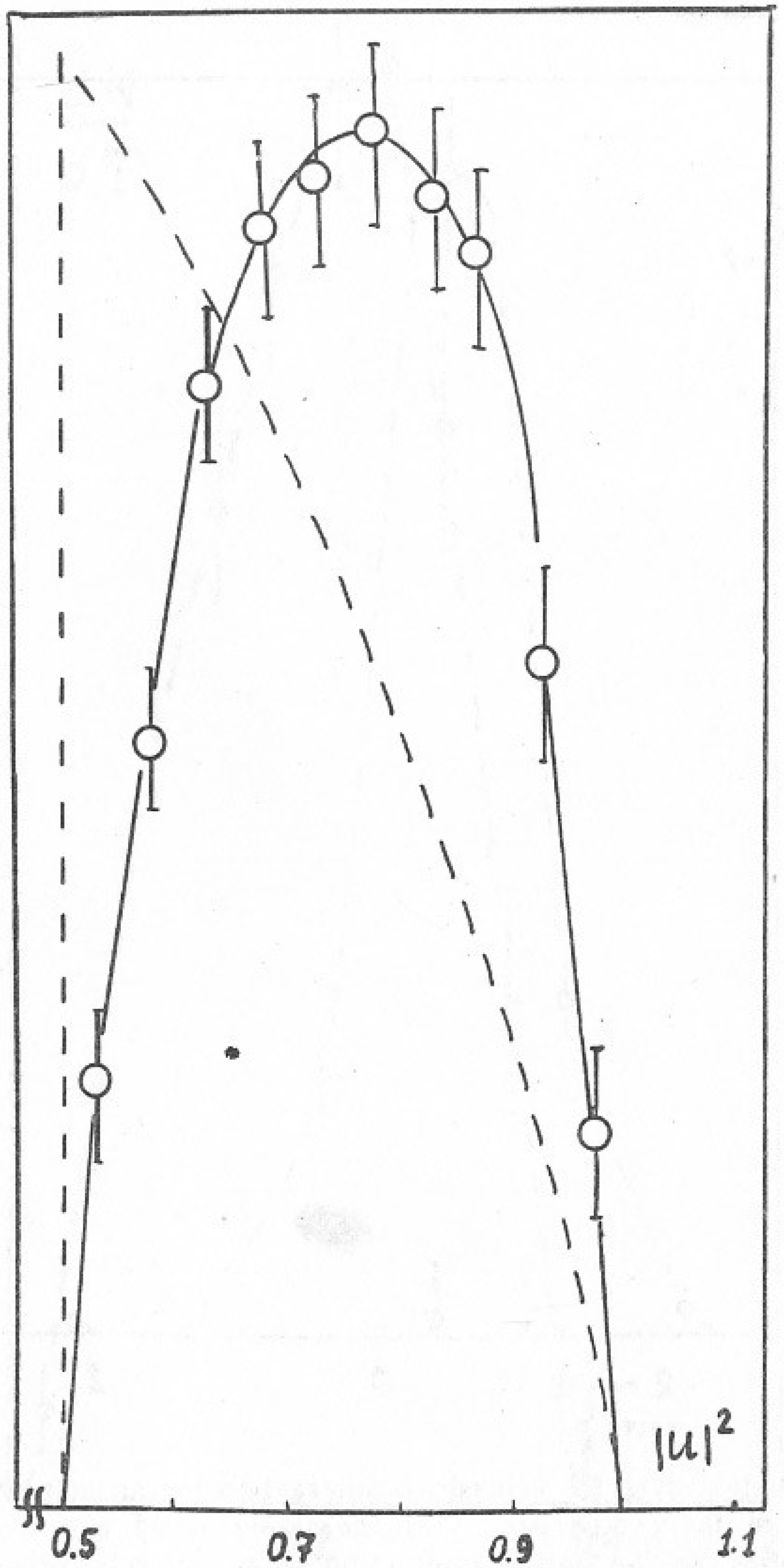

Fig. 3. The distribution over the invariant $|u|^2$, characterizing relative orientation, for an instanton—anti-instanton molecule. Note that the value $|u|^2=1$ means that both pseudoparticles are imbeded into the same SU(2) subgroup. The dashed line (shown for comparison) shows the distribution over $|u|^2$ for the random pairs in the «instanton liquid».

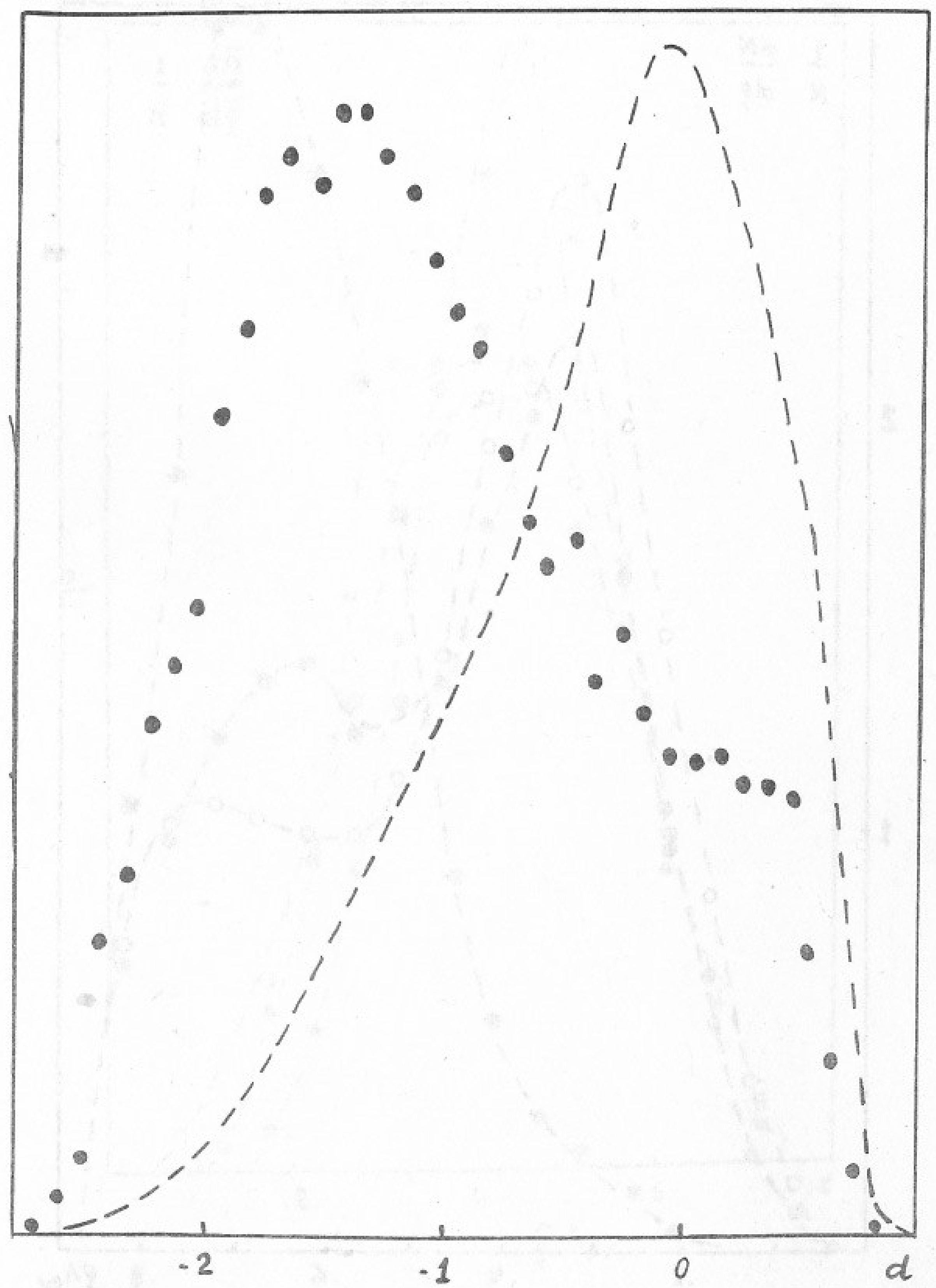

Fig. 4. The same as in Fig. 3, but for the invariant d.

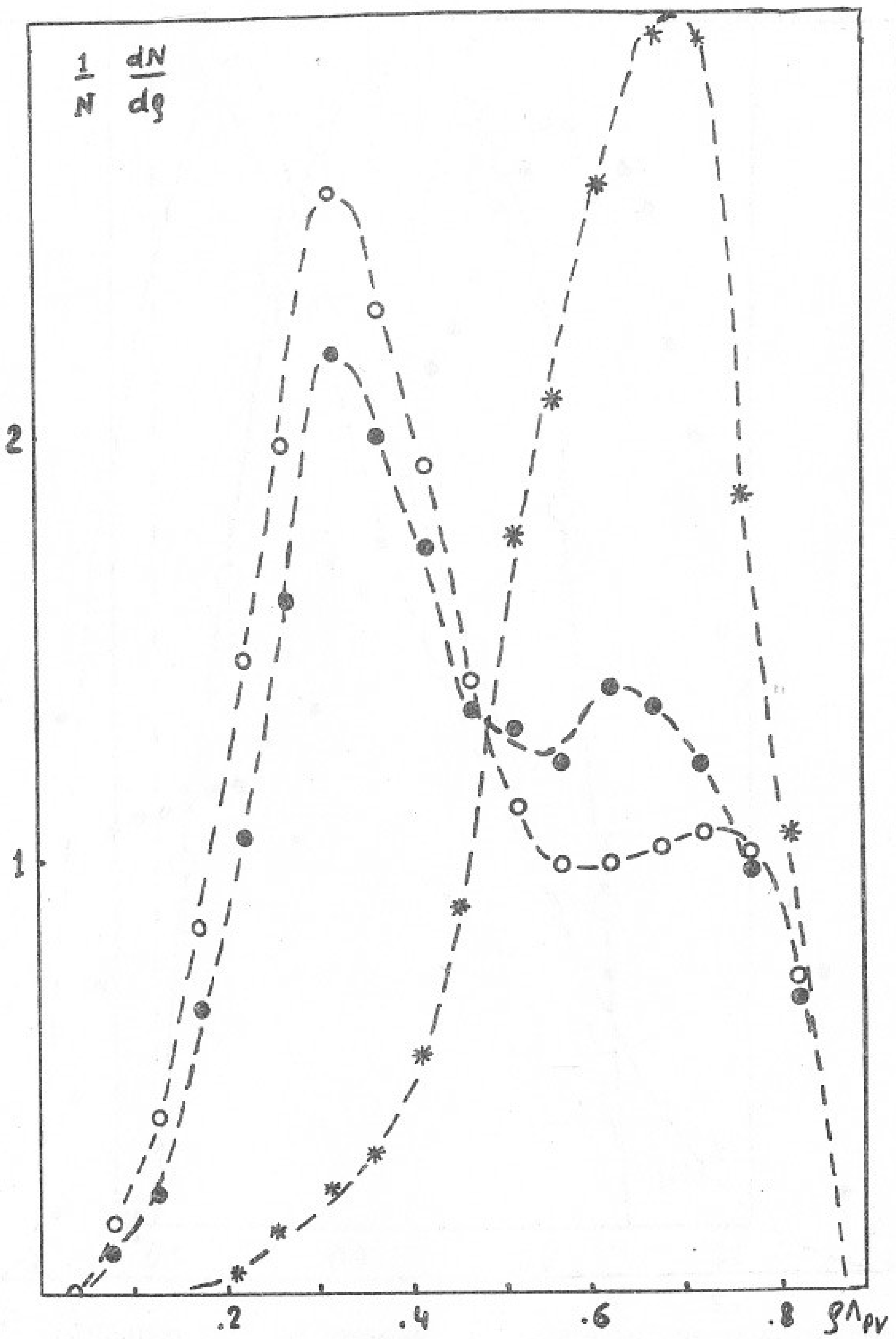

Fig. 5. The distribution over the instanton radii ρ (in Λ_{PV}^{-1}) in the «instanton liquid». Different points are for the *PP* density (the sum for instantons and anti-instantons) equal to $n_{PP}/\Lambda_{PV}^4 = 2.4$ (open points), 1 (solid points) and 0.1 (stars).

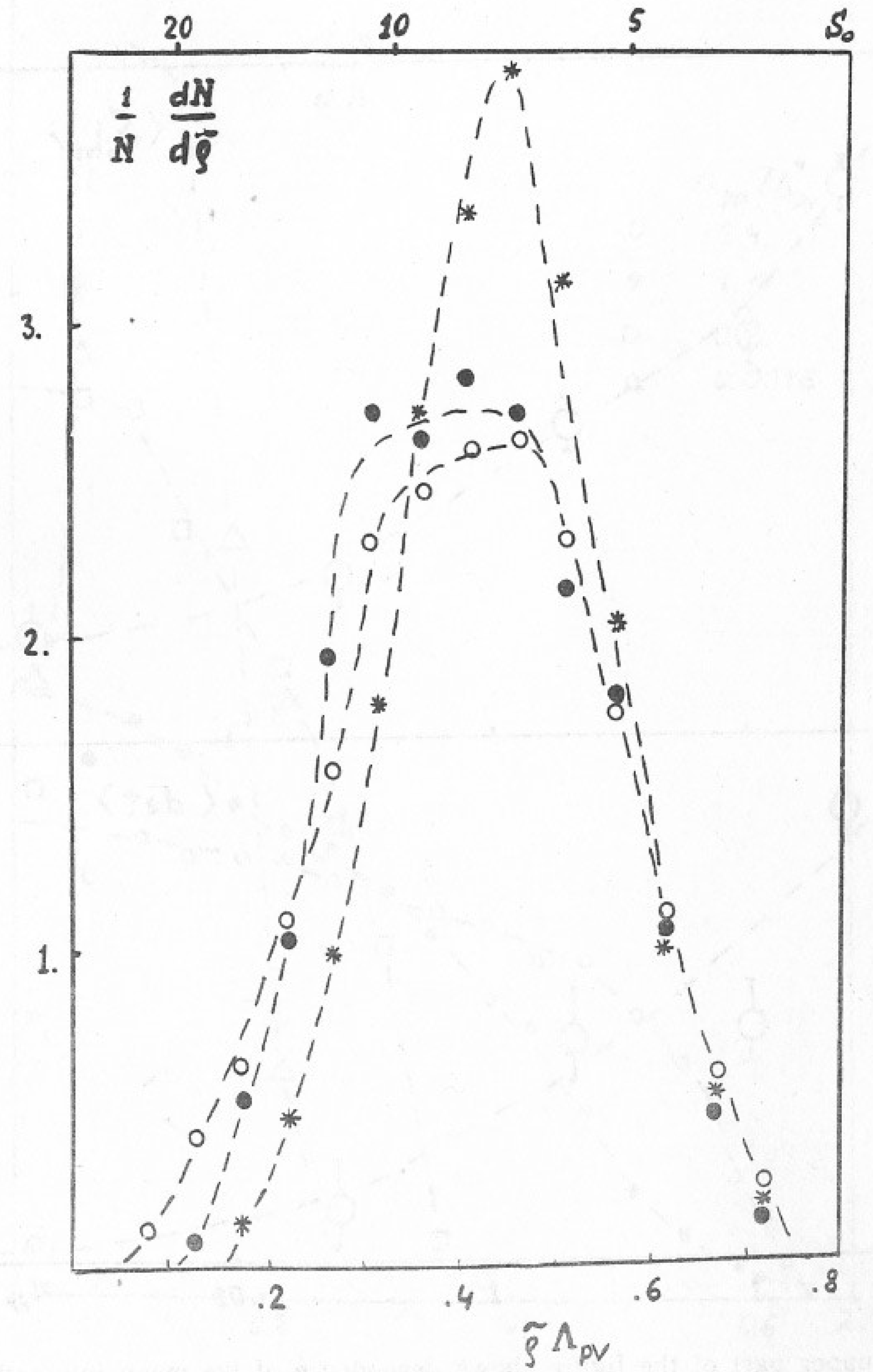

Fig. 6. The same as in Fig. 5 but for the «renormalized» radii $\tilde{\rho}$ defined in the text. The upper scale is for absolute actions per instanton, in unites of the Plank constant.

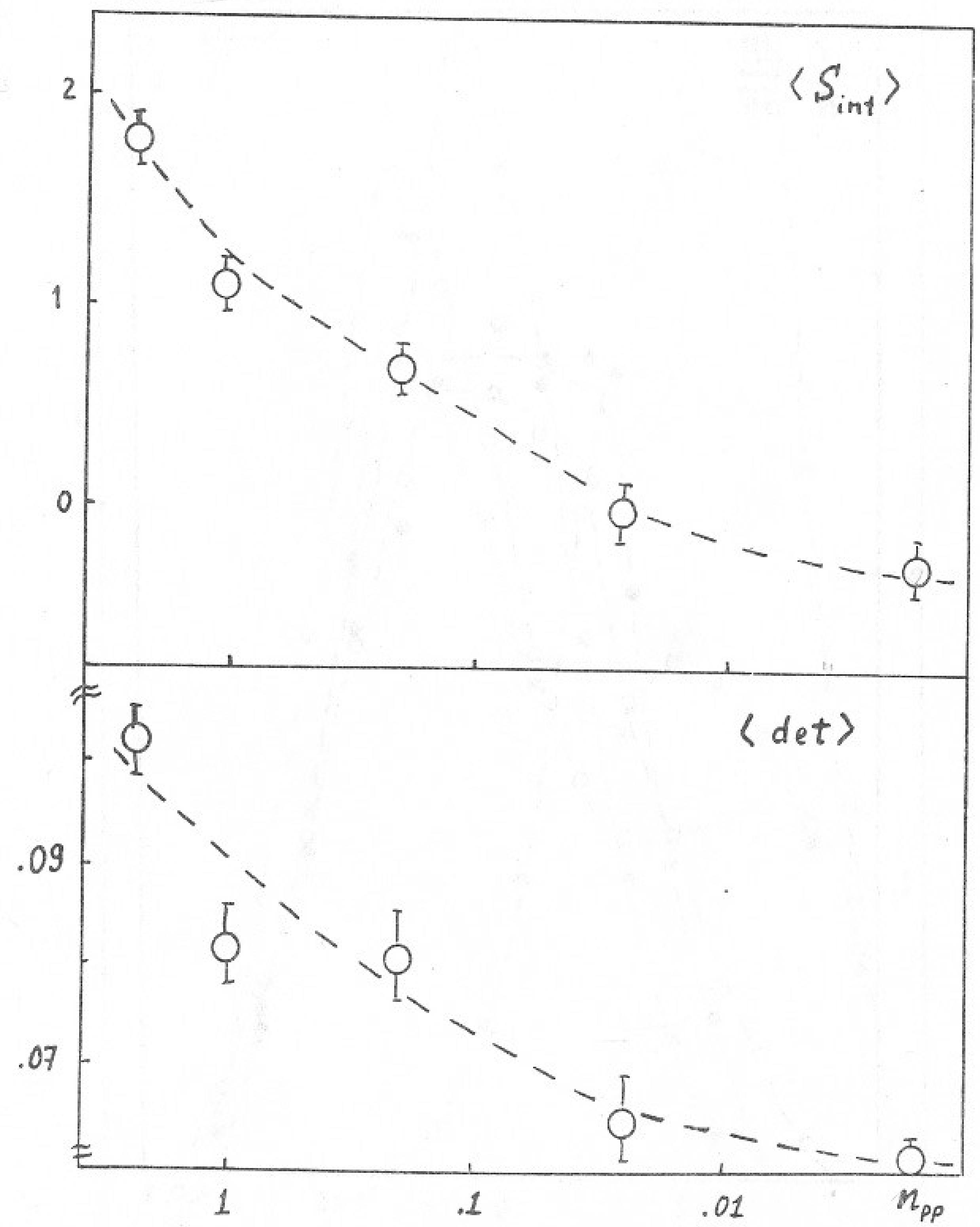

Fig. 7. The upper part of the figure shows dependence of the mean interaction with all neighbours per pseudoparticle (in absolute unites, actually the ratio to the Plank constant) on the *PP* density n_{PP} (in Λ_{PV}^4). The lower part shows similar dependence for mean fermionic determinant per particle defined as $\langle \det \rangle = \langle \det i\hat{D} \rangle^{1/N_{PP}}$.

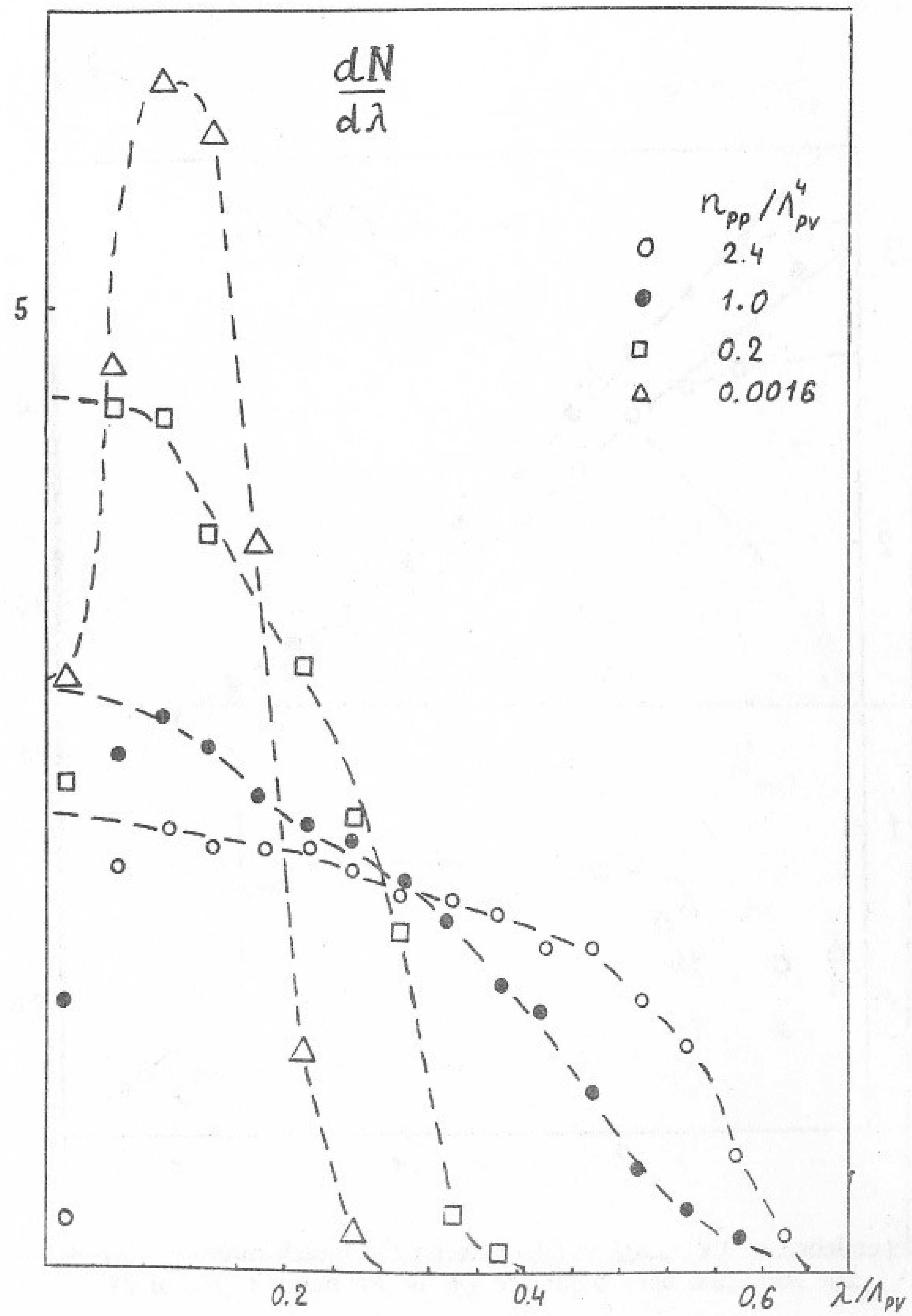

Fig. 8. The spectrum of eigenvalues λ (in unites of Λ_{PV}) of the Dirac operator at se eral densities (indicated in the figure). Note the systematic deviation of the first p at small λ off the general trend: this is presumably a finite size effect. The dashed lines shows some smooth interpolations.

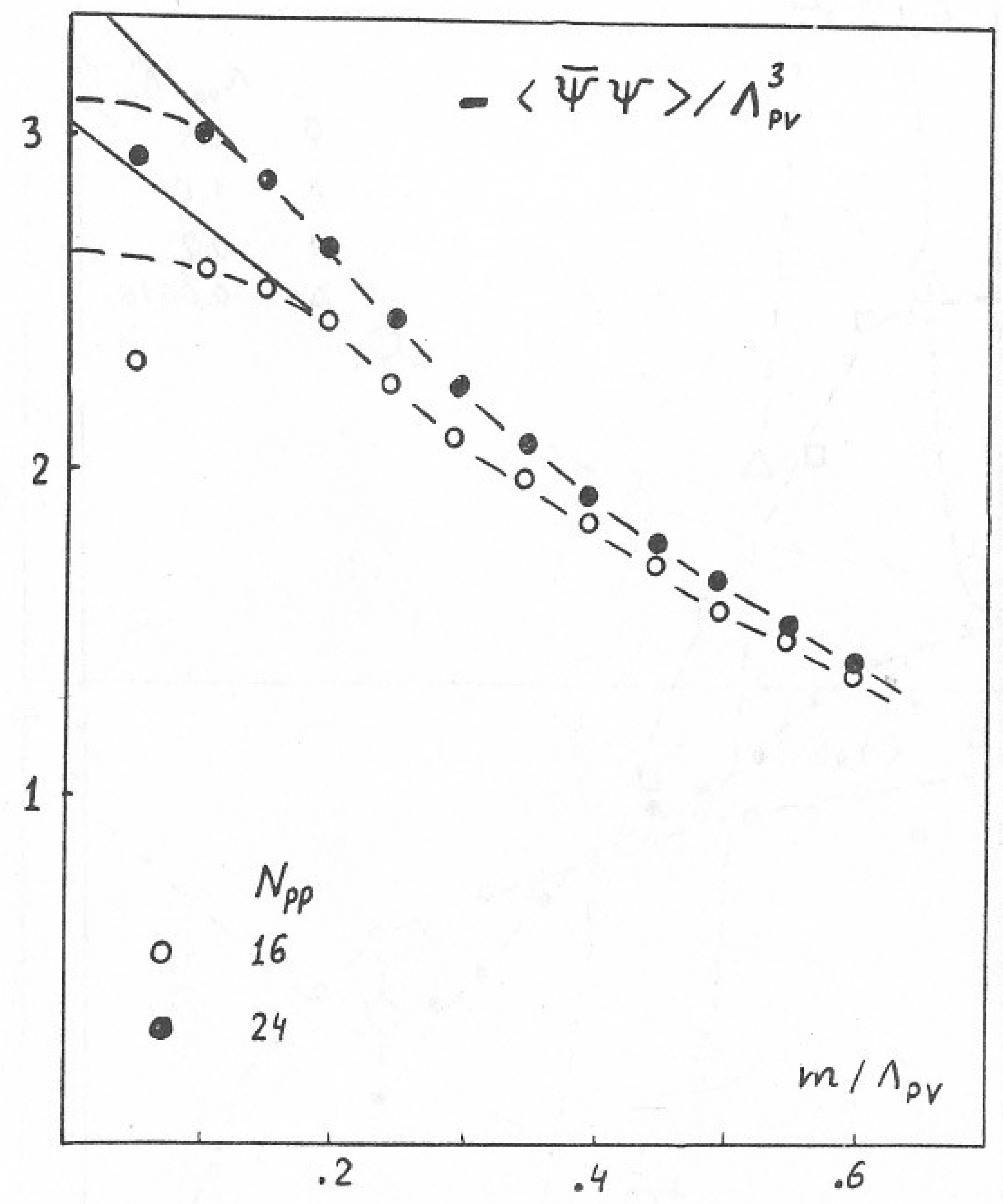

Fig. 9. Dependence of the quark condensate on the quark mass m (for $n_{PP}=1\Lambda^4_{PV}$). The solid and open point are for the *PP* number 16 and 24.

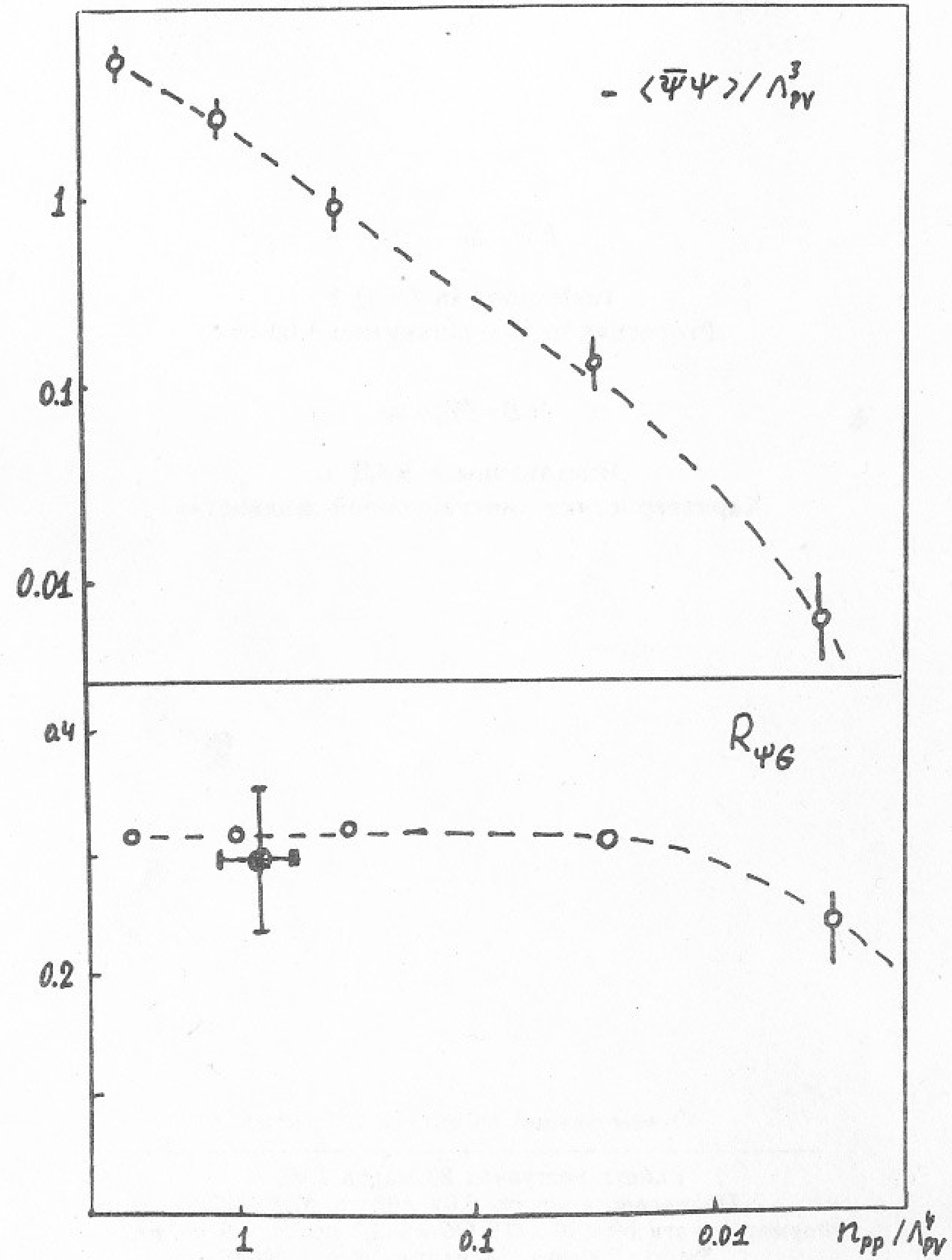

Fig. 10. Dependence of the absolute value of the quark condensate on the *PP* density n_{PP} (in Λ^4_{PV}) is shown in the upper part of the figure, while its lower part represents the analogous dependense of the «scale ratio» $R_{\psi G}$ defined in the text. The «star» with the error bars below corresponds to the phenomenological value corresponding to the «standard» QCD sum rules value.

E.V. Shuryak

Instantons in QCD I.
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Э.В. Шуряк

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