20

ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

A.D. Dolgov and I.B. Khriplovich

DOES STATIC SOLUTION EXIST FOR A GRAVITATING FLAT WALL?

PREPRINT 88-35

НОВОСИБИРСК

Does Static Solution Exist
for a Gravitating Flat Wall?

A.D. Dolgov and I.B. Khriplovich

Institute of Nuclear Physics
630090, Novosibirsk, USSR

ABSTRACT

It is shown that under very general assumptions no static nonsingular solution of the General Relativity equations exists for the gravitational field of an uniformly flat matter distribution.

© Институт ядерной физики СО АН СССР

The question which is put in the title arose mainly in connection with the investigation of the gravitational field of the domain wall separating two regions with opposite vacuum expectation values of the Higgs field, $\varphi(x\to\pm\infty)\to\pm\varphi_0$. This problem was considered in Refs [1—5] and the authors in majority agree that the Einstein equations for the gravitational field of the flat wall have no static solutions.

In this paper we shall prove this statement for almost **any** bounded in transverse direction x and uniform in y and z distribution of matter. The assumptions about the energy-momentum tensor of matter are only the positive definiteness of energy density, $T_{00}\geqslant 0$, and sufficiently regular matter distribution such that any curvature invariants, such as R, $R^{\mu\nu}R_{\mu\nu}$, $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ etc., are nonsingular. Except for the unavoidable transversality property, $T^{\mu}_{\nu;\mu}=0$, the energy-momentum tensor of the gravitating matter is otherwise arbitrary.

Let assume that the static solution exists. In this case the metric could be written as

$$ds^2=g_{00}(x)\,dt^2-dx^2-g_{22}(x)\,(dy^2+dz^2)\,. \qquad (1)$$

From the usual condition $\det g_{\mu\nu}\leqslant 0$ it follows that $g_{00}(x)\geqslant 0$. Hence, not only

$$T_{00}(x)\geqslant 0\,. \qquad (2)$$

but also

$$T^0_0(x)\geqslant 0\,. \qquad (2a)$$

Uniform matter distribution in yz-plane implies that only diagonal components of energy-momentum tensor are nonvanishing and

$$T^2_2(x) = T^3_3(x) . \qquad (3)$$

Let introduce instead of $g_{00}(x)$ and $g_{22}(x)$ the new functions u and v:

$$g_{00} = v^2 u^{-2/3}, \quad g_{22} = u^{4/3}. \qquad (4)$$

In terms of these functions the Einstein equations are written as

$$u'' + 6\pi k T^0_0 u = 0 , \qquad (5)$$

$$v'' + 2\pi k (4T^2_2 - T^0_0)\, v = 0 , \qquad (6)$$

$$u'v' + 6\pi k T^1_1 uv = 0 . \qquad (7)$$

Of course, T^μ_ν could depend on metric functions u and v if one starts from, say, a Lagrangian theory of some matter fields. In what folows we use a different approach, however. We assume that T^μ_ν are arbitrary functions of x restricted only by the conditions $T^0_0 \geqslant 0$ and

$$T^1_{1,x} + (g'_{00}/2g_{00})(T^1_1 - T^0_0) + (g'_{22}/2g_{22})(2T^1_1 - T^2_2 - T^3_3) = 0 .$$

In this sence eqs (5) and (6) can be considered as linear differential equations for u and v. Note that for the particular case of the Higgs field domain wall $T^2_2 = T^0_0$ and eqs (5) and (6) coinside. Eqs (5) — (7) are mutually compatible as usually because of covariant conservation of T^μ_ν. One sees from eq. (7) that for any nontrivial metric

$$T^1_1 \neq 0 . \qquad (8)$$

Nonvanishing of T^1_1 follows from the condition $T^\mu_{\nu;\mu} = 0$ in curved space-time.

To ensure the same metric on both sides of the wall functions u and v must have definite parity, i. e. $u(-x) = \pm u(x)$ and $v(-x) = \pm v(x)$. Hence, as it follows from eq. (7) at least one of the functions $u(x)$ or $v(x)$ tends to a constant when $x \to \pm\infty$ if T^α_β vanishes faster than x^{-2}. In quantum mechanical language this means that at least one of the «potentials» $(-6\pi k T^0_0)$ or $(-2\pi k)(4T^2_2 - T^0_0)$ in Schrödinger type eqs (5) or (6) has a zero-energy level. Even this condition provides a rather strong and nontrivial restriction on the energy-momentum tensor.

More important for our consideration is the condition that u''/u is nonpositive. It follows from eq. (5) and positive semidefiniteness of T^0_0 (2a). Hence, function $u(x)$ with the continuous first derivative must vanish somewhere in the interval $(-\infty, +\infty)$. Discontinuity in u' would lead to δ-function singularity in u'' and by virtue of eq. (5) in T^0_0.

Note that in the attractive potential $(-6\pi k T^0_0)$ of the finite depth there always exists at least one level with negative energy. Hence, the zero-energy eigenstate is not the ground state and the corresponding wave function must have nodes.

Now, let us consider the quantity

$$I = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} \qquad (9)$$

which in the frame-independent way characterizes the properties of space-time. For metric (1) the only nonvanishing components of the Riemann tensor $R_{\mu\nu\alpha\beta}$ are those with $\mu = \alpha$ and $\nu = \beta$ (and of course the ones obtained by the permutations $\mu \leftrightarrow \nu$ and $\alpha \leftrightarrow \beta$). The contribution of each of these components into I is nonnegative. In particular,

$$R_{2323} R^{2323} = \frac{16}{81} \left(\frac{u'}{u} \right)^4 . \qquad (10)$$

This quantity is singular at the point where $u(x)$ vanishes. If u' vanished at the same point, eq. (5) would imply that for nonsingular T^0_0 u vanishes identically.

Thus, the assumption that the gravitational field of the plane wall is static leads to the conclusion that there are naked singularities in the space-time. The solution of this type and the corresponding matter distribution can be easily found.

In conclusion let us discuss briefly the possibility of nonsingular one-dimensional static solution of the Einstein equations with nonvanishing cosmological constant Λ (compare with Ref. [4]). Substitution (4) leads now to the equations which differ from (5) — (7) only by the shift $2\pi k T^\mu_\nu \to 2\pi k T^\mu_\nu - \frac{1}{4} \Lambda \delta^\mu_\nu$. The restriction $u''/u \leqslant 0$ can be relaxed and the previous arguments do not hold. In other words, the Schrödinger type equation for u should be solved for nonzero energy which corresponds to nonzero Λ. Such an equation can possess the solution which does not have nodes. This is the ground state wave function in the attractive potential $(-6\pi k T^0_0)$. So a specific eigenvalue problem should be solved. Correspondingly the static solution is possible only for fine-tuned Λ-term and energy-momentum tensor.

We are grateful to N.S. Kardashev, Ya.I. Kogan and I.V. Kolokolov for helpful discussions.

REFERENCES

1. *Vilenkin A.* Phys. Lett., 1983, B133, 177.
2. *Ipser J. and Sikivie P.* Phys. Rev., 1984, D30, 712.
3. *Tomita K.* Phys. Lett., 1985, B162, 287.
4. *Linet B.* Int. J. Theor. Phys., 1985, 24, 1159.
5. *Roychaudhuri A.K. and Mukherjee G.* Phys. Rev. Lett., 1987, 59, 1504.

A.D. Dolgov and I.B. Khriplovich

Does Static Solution Exist for a Gravitating Flat Wall?

А.Д. Долгов, И.Б. Хриплович

Существует ли решение для гравитирующей плоской стенки?

Ответственный за выпуск С.Г.Попов

Работа поступила 21 января 1988 г.
Подписано в печать 15 февраля 1988 г. МН 08104
Формат бумаги 60×90 1/16 Объем 0,7 печ.л., 0,6 уч.-изд.л.
Тираж 200 экз. Бесплатно. Заказ № 35

Набрано в автоматизированной системе на базе фотонаборного автомата ФА1000 и ЭВМ «Электроника» и отпечатано на ротапринте Института ядерной физики СО АН СССР,
Новосибирск, 630090, пр. академика Лаврентьева, 11.