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BERRY'S PHASE AND QUANTIZATION OF COLLECTIVE MODES IN MANY-BODY SYSTEM

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Berry's Phase and Quantization of Collective Modes in Many-Body System

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ABSTRACT

It is shown that so-called Berry's phase (BP) must be taken into account when the mean field adiabatic fluctuations (collective modes) are being quantized. For the many-fermion system, the BP-counterterm in effective action is shown to be equivalent to that of kinetic energy of mean field fluctuations.

The phenomenon of Berry's phase i. e. «quantum adiabatic holonomy», known from Refs [1, 2] where the well-known quantum mechanical adiabatic theorem [3] has been reconsidered is formulated as follows. Let the Hamiltonian of the system, h, (with its eigenvalues E_k) is defined as a function of several parameters q_μ: $h=h(q_\mu)$. Then, if the system evolves adiabatically along the closed curve C in the parameter space, the eigenvector (EV) $|k\rangle$ acquires a phase gain, $\exp(i\Gamma(C))$, BP, per a period T, in addition to the convential dynamical phase:

$$|k(t+T)\rangle=\exp(i\Gamma_k(C))\exp\left[-i\int_0^T E_k(q(t))\,dt\right]|k(t)\rangle\,. \tag{1}$$

Obviously, the BP will be substantial whenever a functional determinant of the operator $i\partial_t-h(t)$ (i. e. inversed Green function) for the system in periodic time-varying external field is being calculated.*)

One fased with this problem when investigating the many-body system in terms of functional integral having in mind to construct an effective low-energy dynamics [5] connected with the collective degrees of freedom of the system. In that case, the effective mean-field (background field) created by the particles themselves plays a role of the adiabatically changing external field. Now, we address the question what is the effect of the BP and what is its

*) The corresponding contribution to an effective action of the Fermi-system interacting with the external bosefield has been considered in Ref. [4], but the authors were concentrated mainly on the topological features of the BP.

physical meaning? The goal of this work is to manifest the simple physical sense of the BP, that is, the BP corresponds to the kinetic energy of the mean field fluctuations, therefore, taking into account the BP itself makes us able to quantize the collective excitations of the system in the functional framework.

Let us consider a non-relativistic system of fermions occupying the energy levels e_i and coupled by the two-body interaction (for example, nucleons in valence shell) described by the propagator $G(T)=\mathrm{Tr}\,[\exp(-iHT)]$, which we shall express in terms of a path integral:

$$G(T)=\int D\{\Psi\}\, D\{\Psi^+\} \exp\left\{ i\int_0^T dt\,[\,\Psi_i^+ (i\partial_t - e_i)\,\Psi_i - \Psi_i^+ \Psi_k V_{ik,i'k'} \Psi_{i'}^+ \Psi_{k'}]\right\},$$

where the Ψ, Ψ^+ means Grassman elements [6]. Throughout the paper, we agree that repeated indices are summed over. Let us introduce the Bose-type field $\sigma_{ik}(t)=-V_{ik,i'k'}\Psi_{i'}^+\Psi_{k'}$ making use of the properties of Gaussian integration, to linearize the fermion action:

$$G(T)=\int D\{\sigma\}\, D\{\Psi\}\, D\{\Psi^+\} \exp\left\{ \int_0^T dt\,[\,\sigma_{ik}(t)\, V^{-1}_{ik,i'k'}\, \sigma_{i'k'}(t) + \Psi_i^+ (i\partial_t\delta_{ik} - h_{ik}(t))\,\Psi_k]\right\}.$$

Here we introduced the inversed interaction kern, V^{-1}, as well as the single-particle Hamiltonian of the fermion in external field, $h_{ik}(t)=e_i\delta_{ik}-\sigma_{ik}(t)$. Executing the integration over the Grassmanians Ψ, Ψ^+, we obtain the effective theory for the fields $\sigma_{ik}(t)$:

$$G(T)=\int D\{\sigma\} \exp\left\{ i\int_0^T dt\,\sigma_{ik}(t)\, V^{-1}_{ik,i'k'}\, \sigma(t) + \log\det[\,i\partial_t - h(t)]\right\} =$$

$$=\int D\{\sigma\} \sum_{\{\nu_k\}} \exp\left\{ i\int_0^T dt\,\sigma_{ik}(t)\, V^{-1}_{ik,i'k'}\, \sigma_{i'k'}(t) - \sum_k \left(\nu_k - \frac{1}{2}\right)\alpha_k[\sigma]\right\},$$

where the last expression is rewritten in the form of the «fermion statistical sum», i. e. sum under all the configurations of the single-particle occupying numbers, $\nu_k=0.1$ [4, 6]*) with the

*) Note, that the «occupying numbers» ν_k are leaving conserved as adiabatic invarianta under the slow evolution of the system.

single-particle energies E_k replaced by the Floquet indices, α_k, which are non-local functionals of the $\sigma(t)$. The spectrum of the α_k is determined by the solutions to the equation

$$[\,i\partial_t - h(t)]\,|k(t))=0, \tag{2}$$

according to $\alpha_k=-\arg\,[\,|k(t+T))/|k(t))]$. Under the adiabatic approximation, the evaluation for the α_k follows from eq. (1), and the effective action S_{eff} in the $G(T)=\int D\{\sigma\}\sum_{\{\nu_k\}} e^{iS_{eff}}$ takes the form [4]:

$$S_{eff}=\int_0^T dt\,\sigma_{ik}(t)\, V^{-1}_{ik,i'k'}\, \sigma_{i'k'}(t) - \sum_k \left(\nu_k - \frac{1}{2}\right)\left[\int_0^T dt\, E_k(\sigma(t)) - \Gamma_k\right], \tag{3}$$

where $E(\sigma(t))$ stands for the solutions of the «snapshot» eigenvalue problem

$$h(\sigma(t))\,|k(\sigma(t))\rangle = E_k(\sigma(t))\,|k(\sigma(t))\rangle.$$

When neglecting the BP, we obtain temporarily local action describing a static field, which corresponds to the Hartree—Fock approximation.*) As for the fluctuations of the field σ, these are described by the BP counterterm in the action which contains time-derivatives of σ, and we are going to calculate this contribution now. Let us search for the $|k(t))$, satisfying eq. (2), in the form $U(\sigma(t),\sigma(0))\,|\tilde{k}(t))$, where $|\tilde{k}(t))$ is an unknown vector and

$$U(\sigma(t),\sigma(0))=\exp\left[(\sigma_{ik}(t)-\sigma_{ik}(0))\,\delta/\delta\sigma_{ik}\right]$$

is the «shifting operator», which transforms the «snapshot» basis of EV for h at $t=0$ to one at an arbitrary t, $|k(t)\rangle = U(\sigma,\sigma^{(0)})\,|k(0))$. Substituting the anzatz into eq. (2), we obtain

$$i\partial_t|\tilde{k}(t))=[\,\tilde{h}(t) - U^{-1}(\sigma,\sigma^{(0)})\, i(\partial_t U(\sigma,\sigma^{(0)}))]\,|\tilde{k}(t)),$$

where the $\tilde{h}(t)=U^{-1}(\sigma(t),\sigma^{(0)})\,i\partial_t U(\sigma(t),\sigma^{(0)})$ is the diagonal operator in the basis $\{|k(0)\rangle\}$ with the eigenvalues $E_k(t)=E_k(\sigma(t))$. Note, that the «Coriolis term», $U^{-1}i\partial_t U$, is non-diagonal in that basis. Starting with the instantaneous EV for h at $t=0$, $|k(t=0))=|\tilde{k}(t=0))=|k(0)\rangle$, one have the solution to eq. (2) in

*) Rigorously speaking, one deals with the Hartree approximation when neglecting the exchange term.

the form of chronological product:

$$|k(t)) = U(\sigma(t), \sigma(0))\, T\Big\{ \exp\Big[-i\int_0^t dt'(\tilde{h}(t') - iU^{-1}(\partial_t U))\Big]\Big\}\, |k(0)\rangle .$$

For the case of adiabaticity, when transitions may be neglected [3], we have

$$|k(t)) = U(\sigma(t), \sigma(0)) \exp\Big[-i\int_0^t E_k(t')\, dt'\Big] \times$$

$$\times |k(0)\rangle\langle k(0)|\, T\exp\Big[i\int_0^t U^{-1} i\partial_{t'} U\, dt'\Big]\, |k(0)\rangle, \qquad (4)$$

where for $t=T$ the matrix element coincides with the one of equivalent definitions of the BP, $e^{i\Gamma_k}$ (for ex. [4]). On the other hand, in adiabatic regime, one may solve the eq. (2), treating the term $U^{-1}i\partial_t U$ as a perturbation. Proceeding the calculations up to second order of $U^{-1}i\partial_t U$ and comparing the result with eq. (4), one have for the BP:

$$\Gamma_k = \sum_{k'\neq k} \int_0^T dt \frac{\langle k|\, U^{-1}i\partial_t U\, |k'\rangle\langle k'|\, U^{-1}i\partial_t U\, |k\rangle}{E_k(\sigma) - E_{k'}(\sigma)} . \qquad (5)$$

Interesting the collective effects at low energies, we consider the configurations which are the nearest ones to the «classical» path, i. e. ones lying on the bottom of the «valley» of the Eucleadean action surface [5]. Such configurations, i. e. ones which the collective trajectories of the system (dominating the path integral in the $G(T)$) pass through, may be ascribed, as usual, by means of several parameters q_μ. The latter ones called collective coordinates being undetermined a priori, mostly may be choosen, more or less successfully, by use of specific physical considerations [7]. For simplicity, propose the consistency requirements arising in this framework to be satisfied, so the most important configurations $\sigma(t)$ i. e. ones corresponding to the large fluctuations of the effective field are parametrized successfully by means of some ansatz $\sigma = \tilde{\sigma}(q_\mu)$.*) Then we have for the Γ_k

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*) Speaking the language of the Faddeev-Popov procedure of introducing the collective coordinates [5], it means that the contributions of the transverse modes determinant as well as the transition Jacobian are neglected.

$$\Gamma_k = \sum_{k'\neq k} \int_0^T dt\, \Pi^{\mu\nu}_{kk'}(\tilde{\sigma}(q))\, \dot{q}_\mu \dot{q}_\nu ,$$

where $\dot{q} \equiv \partial_t q$ and

$$\Pi^{\mu\nu}_{kk'}(\tilde{\sigma}(q)) = \langle k(\tilde{\sigma})|\frac{\partial}{\partial q_\mu}|k'(\tilde{\sigma})\rangle\langle k'(\tilde{\sigma})|\frac{\partial}{\partial q_\nu}|k(\tilde{\sigma})\rangle / (E_k(\tilde{\sigma}) - E_{k'}(\tilde{\sigma})) .$$

Correspondingly, S_{eff} (3) takes the form

$$S_{eff} = \int_0^T dt \Big[\tilde{\sigma}_{ik}(q)\, V^{-1}_{ik,i'k'}\, \tilde{\sigma}_{i'k'}(q) + \sum_k \Big(\frac{1}{2} - \nu_k\Big) E_k(\tilde{\sigma}(q)) + B_{\mu\nu}(q)\, \dot{q}_\mu \dot{q}_\nu\Big] ,$$

where the last term corresponds to the kinetic energy of the fluctuations of σ, and the mass tensor B is given by the formula

$$B_{\mu\nu}(q) = \sum_{k,k'\neq k} \nu_k\, \Pi^{\mu\nu}_{kk'}(\tilde{\sigma}(q))$$

in the absolutely agreement with the result of the cranking model in the theory of the collective excitations in nuclei [7]. The existence of a set of «collective trajectories», yielding approximately the same action and, consequently, entering to the $G(T)$ with the near weights, is the manifestation of an initial symmetry the system possess. In fact, for the case of an exact symmetry (nuclear rotation or collective motion of an instanton in QCD), the action is really a constant when going from one collective path to another. The corresponding collective coordinates q_ϑ appear to be *angular* ones the collective action being indepenedent on them. Hence, the equations $\ddot{q}_\vartheta = 0$ split from the whole set of the Lagrange ones yielding the conserved moments. It provides one with an opportunity to specify the collective trajectories according to this quantum number. Then, the stationary phase quantization [6] being applied yields the selection rule for those moments. Therefore, the resulting effective action from which these angular degrees of freedom are excluded, acquires the dependence on the quantized values — «topological charges» (for ex. eigenvalues of Casimir operator for corresponding group of transformations).

For the case of the rigid rotation of the system the Euler angles play a role of those *angular* variables. The angular momentum plays a role of «topological charge»; in this case, the zero mode (Goldstone's mode) corresponds to exact initial symmetry. Moreover, there are some other smooth modes in the systems like a superfluid nucleus, allowing an approximate classification of the collective states (such as O(5)-symmetry [8]). In this case, the semiclassical approximation, starting from the nontrivial classical solutions with the pronounced symmetries might be a powerful method of the investigating the microscopic structure of the low-lying excitations.

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REFERENCES

1. *Berry M.V.* Proc. R. Soc., 1984, v.A392, p.45.
2. *Aharonov Y, Anandan J.* Phys. Rev. Lett., 1987, v.58, p.1593.
3. *Messiah A.* Quantum Mechanics. v.2 (North-Holland, Amsterdam, 1970).
4. *Kuratsuji H, Iida S.* Prog. Theor. Phys., 1985, v.74, p.439,
   Kaneko K. Phys. Lett., 1987, v.119A, p.329.
5. *Shuryak E.V.* Preprint INP-87-54, Novosibirsk, 1987.
6. *Rajaraman R.* Solitons and Instantons. (North-Holland, Amsterdam,1982).
7. *Zelevinsky V.G.* Nucl. Phys. 1980, v.A337, p.40.
8. *Vorov O.K., Zelevinsky V.G.* Proceedings of the XXI Winter School LINP. Leningrad, 1986.

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