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TOWARD THE QUANTITATIVE THEORY OF THE TOPOLOGICAL EFFECTS IN GAUGE FIELD THEORIES II. THE SU(2) GLUODYNAMICS

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Toward the Quantitative Theory of the Topological Effects in Gauge Field Theories II. The SU(2) Gluodynamics

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ABSTRACT

In this paper we construct the trial function for field configurations and study resulting interaction of pseudoparticles. Then we perform numerical simulations of the effective theory formulated in terms of collective coordinates. The main qualitative features of the «instanton liquid» model are confirmed: the instantons possess large enough action to be treated semiclassically and are separated well enough. We have evaluated their density in vacuum, as well as the instantonic contribution to the gluonic condensate, the static potential between quarks and to some correlation functions.

1. INTRODUCTION

The main results of the general semiclassical theory of the topological phenomena in gauge theories, some phenomenological facts (including those coming from recent studies in lattice numerical experiments) and the collective coordinate method used was already discussed in the paper I of this series [1]. In the present work we carry out the main part of the programm outlined in I. Naturally, we start with the simplest SU(2) gauge theory without fermions. The main questions addressed in this work are as follows:

1. How to take the trial function, approximating the «streamline» set of the instanton—anti-instanton configurations well enough? (Sect. 2)
2. How the interaction of the pseudoparticles depends on the collective coordinates? (Sect. 3 and 5.)
3. Are the current-induced corrections to «classical» interaction large or small for various trial functions? (Sect. 4.)
4. Can one describe the pseudoparticle interaction by some binary forces (as it is common in the statistical mechanics of atoms and molecules), or one needs also to take into account the multibody forces? (Sec. 5.)
5. What are the properties of the resulting statistical ensemble of the pseudoparticles, what is their mean size and the total density? Is the mean separation large or small compared to their dimensions? Is it in the ordered (crystall) or the disordered phase? (Sect. 7 and 8.)

6. Do the small-size (strong field) or the large-size (weak field) pseudoparticles play the most important role? Can the theory really be formulated in the semiclassical language? (Sect. 8.)
7. What may be the observable effects of the instanton-induced phenomena? What is the relation between our results and those found on the lattice, say for the static quark potential and for the correlation functions? (Sect. 9.)

The summary of the answers available and their discussion is made in Section 10 at the end of this paper.

2. THE TRIAL FUNCTIONS

Unlike the BPST instanton (and the multi-instanton configurations found later) the superpositions of instantons and anti-instantons we are going to study do not correspond to the action minima and, therefore, do not obey the classical (Yang-Mills) equations.

However, it does not mean that all of them are equally suitable for the construction of the collective coordinates. On the contrary, as it was already discussed in I, there exist a set of configurations which is optimal for this purpose. So to say, there are no global minima, but there exist the conditional ones.

More precizely, the «map» of the action distribution for such configurations discussed in I displays some «valleys», on the bottoms of which the so called «streamline» set of configurations is placed. Unfortunately, it is not easy to find it in practice because decomposition into the «longitudinal» and the «transverse» parts of the vectors in the configuration space can only be made via some complicated integral operators. The situation is even more complicated for the gauge theories, for one should make the additional projection, to the subspace orthogonal to pure gauge transformations. Therefore, it is reasonable to approach this problem first in the explaratory manner, taking various trial functions and then testing whether it is close to the «valley bottom» or not.

The first step into this direction was made by D.I. Dyakonov and V.Yu. Petrov [2], who have used the simplest «sum anzatz» (marked as «S»-ansatz below), being just the sum of potentials for pseudoparticles:

$$A^{(S)}_{a\mu}(x)=\sum_I U^{ab}_I\, a^{(I)}_{b\mu}(y_I)+\sum_A U^{ab}_A\, a^{(A)}_{b\mu}(y_A) \qquad (1)$$

where $a^{(I)}_{a\mu}(x)$ ($a^{(A)}_{a\mu}(x)$) is the standard instanton (anti-instanton) solution in the singular gauge

$$a^{(I)}_{a\mu}(y)=2\bar{\eta}_{a\mu\nu}\, y_\nu\, \rho^2/[y^2(\rho^2+y^2)],$$
$$a^{(A)}_{a\mu}(y)=2\eta_{a\mu\nu}\, y_\nu\, \rho^2/[y^2(\rho^2+y^2)], \qquad (2)$$

rotated by the matrices $U_I(U_A)$ and shifted by the vectors z_I (z_A), the positions of the PP centers: $y_I=x-z_I$, $y_A=x-z_A$. For the SU(2) group there are three Euler angles of the matrix U, four z and the radius ρ: in sum there are 8 parameters per PP. (We remind the reader, that due to some particular properties of instantons one should not rotate them also in space-time: it is in fact identical to the color rotations.)

Unfortunately, expression (1) has two main defects. The most essential one was pointed out in my letter [3]. Consider the structure of the single instanton solution (2) near its center:

$$a^{(I)}_{a\mu}(x)\underset{(x\to 0)}{\longrightarrow}(2\bar{\eta}_{a\mu\nu}\, x_\nu/x^2)\cdot(1-x^2/\rho^2+\ldots). \qquad (3)$$

The former singular term is pure gauge and, by itself, it leads to zero field strength. However, in combination with the second (vanishing at $y\to 0$) term it produces finite field strength. Such interplay does not happen for the «S» ansatz (1), for which the potentials near one of the centers look as follows

$$A^{(S)}_{a\mu}(x)\underset{x\to z_I}{\longrightarrow}2\bar{\eta}_{a\mu\nu}(x-z_I)_\nu/(x-z_I)^2+\text{const} \qquad (4)$$

and, due to the commutator term, the field strength (and the action density $(G^a_{\mu\nu})^2$ is in fact infinite at the center. This singularity is not physically justified, and it also is very inconvenient in practice.

Another problem is related with the instanton—instanton (II) interaction. It is the well known theorem, that exact n-instanton solutions have the action just n times that for the single instanton. Thus, in principle, it is possible to have the trial function which automatically possesses zero II classical interaction. Unfortunately, the known general solution is not sufficiently simple to be used in practice, but nevertheless it is desirable to take an ansatz for which the II classical interaction is at least weaker than that for the instanton—anti-instanton (IA) case. Looking at the ansatz (1) at this angle one finds (see below) that for both II and IA channels it leads to equally strong repulsion at small distances, which looks very suspicious.

Trying to improve this ansatz *I* have looked for other trial functions. It is obvious that any sum of contributions of separate pseudoparticles cannot solve the «singularity» problem. One way out (indicated by the t'Hooft multi-instanton solution) may be a potential in the form of the ratio of the two sums. This idea has lead me to the following trial function (the so called «ratio ansatz», below «*R*»):

$$A^{(R)}_{a\mu}(x)=\frac{\sum_{I} U_I^{ab}\,\bar\eta_{b\mu\nu}\,y_I^{\nu}\,\rho_I^2\, f(y_I)/y_I^4+\sum_{A} U_A^{ab}\,\eta_{b\mu\nu}\,y_A^{\nu}\,\rho_A^2\, f(y_A)/y_A^4}{1+\sum_{I,A} f(y_{I,A})\,\rho_{I,A}^2/y_{I,A}^2}. \qquad (5)$$

One may check that (5) indeed leads to finite action density at the centers, although its value is not exactly the same as that for the separate *PP*. We remind that for one instanton with the radius ρ it is

$$(G^a_{\mu\nu}(0))^2=192/\rho^4. \qquad (6)$$

For reasons to be discussed in Section 6 we prefer to keep to the relation (6) and define the so called «effective» (or «renormalized») radius

$$\tilde\rho_I=\rho_I/\Big[1+\sum_{I'\neq I,A}\rho_{I',A}^2\, f(z_{I',A}-z_I)/(z_{I',A}-z_I)^2\Big]^{1/2}. \qquad (7)$$

In terms of $\tilde\rho$ (7) one has the same expression (6) for the action maxima in any configuration. Note however, that $\tilde\rho$ depends on the positions and the sizes of neighbouring instantons. Therefore, one may indeed speak about the «renormalization» of the instanton size in matter.

We have not yet specified the «shape variation function» $f(x)$ present in (5). First of all, in order not to spoil the field topology at the instanton centers it should obey the condition $f(0)=1$. Second, it should provide sufficient cut off at large distances for the sum convergence, both in the nominator and the denominator of (5).

One physical motivation for a modification of the instanton profile was suggested in Ref. [4], where it was found that the nongaussian quantum fluctuations of the fields around the classical instanton solution modify its large distance «tail» so that the mean $A_{a\mu}(x)$ in fact decays at large x as $\exp(-x^2\cdot\text{const})$. Another physical motivation came from the *DP* work [2], where such cut off was

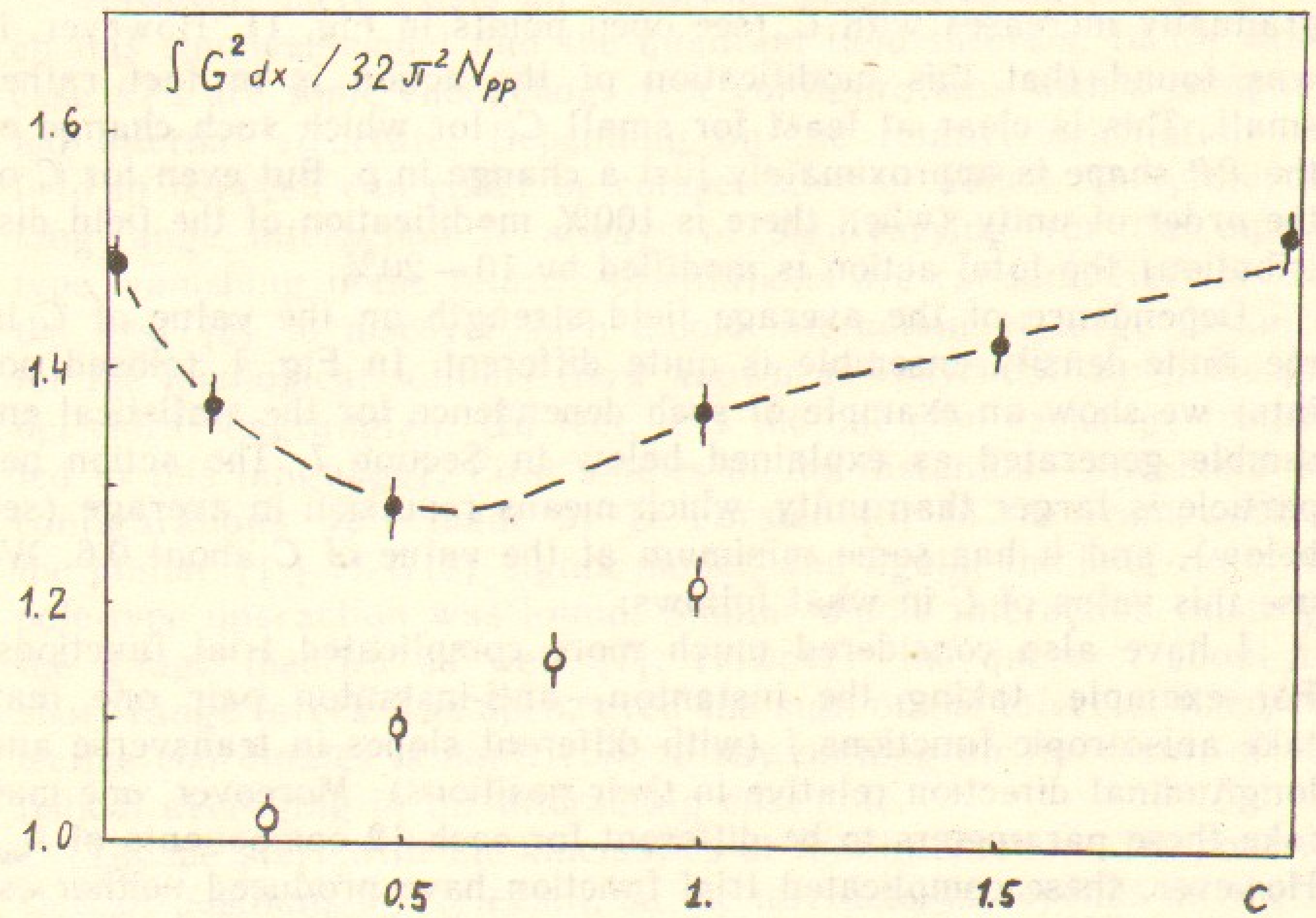

Fig. 1. Modification of the action per instanton as a function of the value of the parameter *C* of the shape function $f(R)$. Open points stand for a single instanton, the closed points correspond to the ensemble to be derived below.

used in order to minimize the repulsion of instantons in matter. (Such modified objects were called «fremons», from «free energy minimization»). For simplicity, we take it to be a function of the distance to ro ratio and of the Gaussian shape

$$f(x)=\exp(-Cx^2/\rho^2). \tag{8}$$

For a pseudoparticle well separated from all others the action gradually increases with C (see open points in Fig. 1). However, it was found that this modification of the action is in fact rather small. This is clear at least for small C, for which such change of the PP shape is approximately just a change in ρ. But even for C of the order of unity (when there is 100% modification of the field distribution) the total action is modified by 10—20%.

Dependence of the average field strength on the value of C in the finite-density ensemble is quite different. In Fig. 1 (closed points) we show an example of such dependence for the statistical ensemble generated as explained below in Section 7. The action per particle is larger than unity, which means repulsion in average (see below), and it has some minimum at the value of C about 0.6. We use this value of C in what follows.

I have also considered much more complicated trial functions. For example, taking the instanton—anti-instanton pair one may take anisotropic functions f (with different slopes in transverse and longitudinal direction relative to their positions). Moreover, one may take these parameters to be different for each 12 components of $A_{a\mu}$. However, these complicated trial function have produced neither essentially different interaction law, nor much smaller currents, and therefore most of this work is based on the «R» ansatz (5).

3. CLASSICAL BINARY INTERACTIONS

By definition, the «classical interaction» is the difference between the classical action for the given field configuration and $8\pi^2 N_{PP}$ where N_{PP} is the number of pseudoparticles under consideration. It is natural to start with the binary interactions ($N_{PP}=2$), for which the number of parameters is not yet very large. As the experience of the atomic and molecular physics shows, in many cases one may understand the properties of a condensed matter accounting only to binary interactions of the constituents.

Let us first comment on some qualitative aspects of the problem.

The simplest toy model usually kept in mind while discussing the instanton physics is the well-known double-well potential, in which, the pseudoparticles are just the tunneling events from one well to another. Obviously, putting forward and backward tunneling events closer in time, one finds smaller action, which means attractive instanton—anti-instanton interaction.

However, there exists an important qualitative difference between this simplest model and the quantum field theories. In the latter case PPs are some «hedgehog»-type configurations with a complicated internal structure. Depending on the relative orientation, the field in between can either be compensated or added. Therefore, the long-range interaction is always of sign-varying (or the dipole) type, vanishing if the relative orientations are random. Skyrme was the first [5] to discover such dipole interaction at large distances for the topological solitons (now known as «skyrmions») of the model chiral Lagrangian. (In fact, it is the one-pion exchange translated to this language). First studies of the instanton—anti-instanton configurations were made by D. Forster [6] in the framework of the planar (1+1) $O(3)$ sigma model, and again, the long-range dipole-type interaction was found. Similar dipole interaction exists for the gauge theories as well [7]. However, the question about the short-range forces was open, even the sign of the effect at fixed collective coordinates is nontrivial, to say nothing on the result of statistical averaging in the interacting ensemble.

Let me start with the kinematics of this interaction. The common shift of positions is irrelevant, so the interactions depends on the relative distance $R^2=(z_1-z_2)^2$. Similarly, common rotation in color and Lorentz spaces are irrelevant for the action, so we may take the PP placed along the 4-th axis and put one of them to the standard orientation ($U_1=1$). The U_2 matrix left describes relative orientation of the pseudoparticles.

Although orientation matrices U_I, U_A are defined in vector representation, we prefer to use the spinor one and define the following «quaternion» u_μ:

$$U^{ab}=\frac{1}{2}\,\mathrm{Tr}\,[\tau^a(u_\mu\tau_\mu^+)\tau^b(u_\nu\tau_\nu^-)],$$

$$\tau_\mu^{\pm}=(\vec{\tau},\pm i) \tag{9}$$

defined on the unite sphere. We did it because in terms of u the SU(2) invariant measure looks very simple.

Returning to the IA pair rotated to the specific position defined above, we define the «relative orientation angle» Φ by $u_4=\cos(\Phi)$. Its invariant definition is as follows

$$\cos\Phi=\frac{i}{2}\,\mathrm{Tr}\,[U_I^+\,U_A\,(z_I-z_A)_\mu\,\tau_\mu^+]. \qquad (10)$$

Dependence on this angle Φ can be expressed in standard angular functions, for example the dipole interaction mentioned above can be written as

$$S=4\rho_I^2\,\rho_A^2\,d/R^4,\quad d=1-4\cos^2\Phi,\quad R^2=(z_I-z_A)^2. \qquad (11)$$

Note that for random orientation $\langle\cos^2\Phi\rangle=1/4$, which indeed leads to $\langle d\rangle=0$.

Dyakonov and Petrov [2] have evaluated the large-distance tail of the binary interaction for the «S» ansatz (1): $(\beta=8\pi^2/g^2)$

$$\Delta S^{IA}/\beta=\frac{4\cdot d\cdot\rho_I^2\,\rho_A^2}{R^4}+\frac{(-15\cdot d+9)\rho_I^2\,\rho_A^2\,(\rho_I^2+\rho_A^2)}{2R^6}+O(R^{-8}),$$

$$\Delta S^{II}/\beta=\frac{9\rho_1^2\,\rho_2^2\,(\rho_1^2+\rho_2^2)}{2R^6}+O(R^{-8}). \qquad (12)$$

Thus, in this case there are the known $O(1/R^4)$ dipole forces and also some $O(1/R^6)$ scalar (Φ independent) repulsive term. The question whether similar results (especially at non-asymptotic distances) can be obtained with other trial functions was later studied by myself [3] (see below). Recently Young [8] has found the ansatz which is parametrically close to the «streamline» at large distances (see Section 6), so that the $O(1/R^6)$ term in the interaction was fixed uniquely. It is of purely dipole type

$$\Delta S^{IA}/\beta=4\cdot d\cdot\rho_A^2\,\rho_I^2/(R^2+\rho_I^2+\rho_A^2)^2)^2. \qquad (13)$$

Unfortunately, the asymptotics mentioned takes place only at large enough R, where the interaction is too small and unimportant.

We have found the binary interaction law at all distances by the numerical integration of the field strength squared over the space-time. But before we present our results, let me explain how this integral was actually calculated. We need accuracy of the order of few per cent and, because $(G^a_{\mu\nu}(x))^2$ at any point is calculated rather slowly and because its distribution in four dimensions is rather inhomogeneous, the ordinary Monte Carlo routines turns to be very

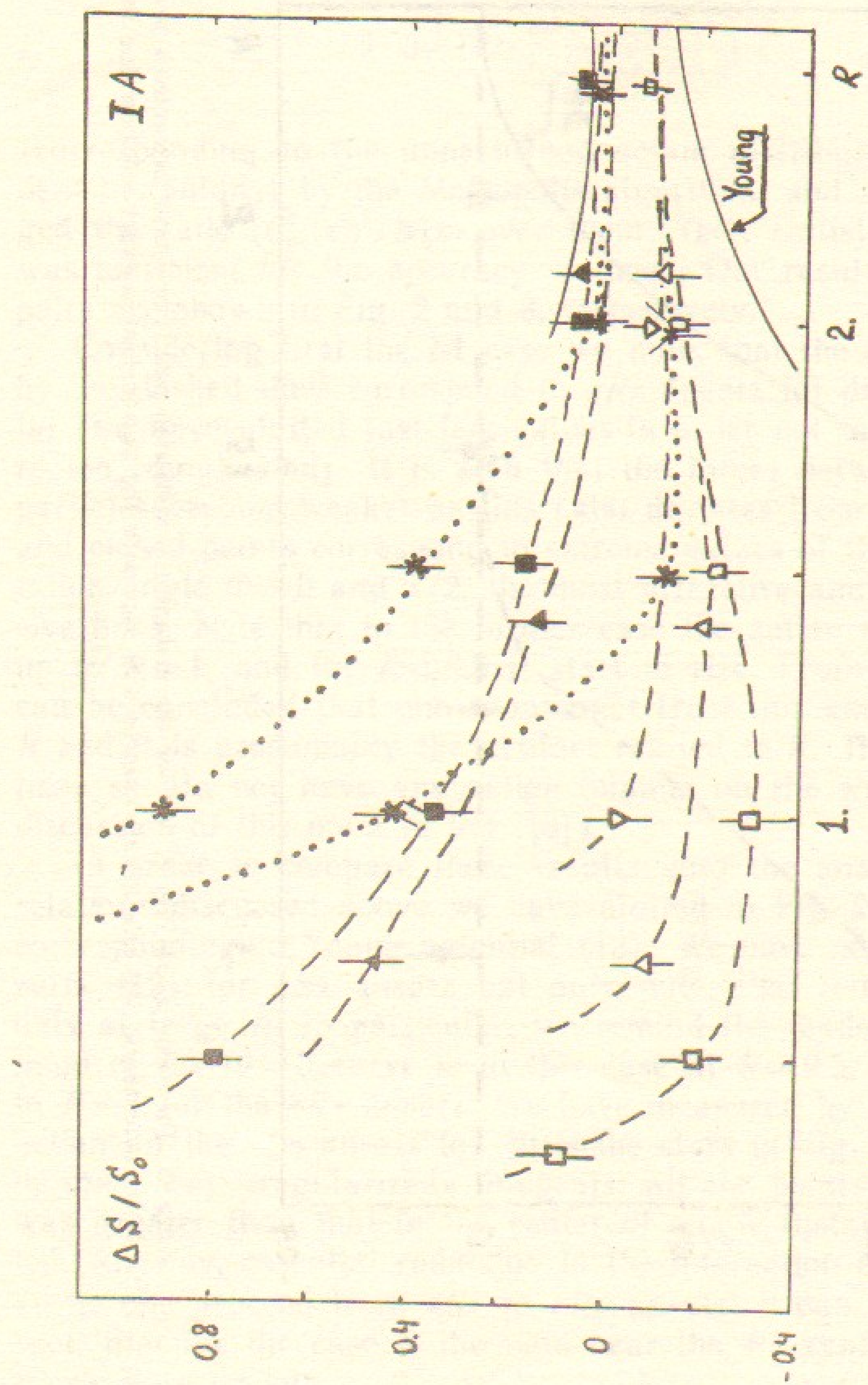

Fig. 2. The action deviation from $2S$ for the instanton—anti-instanton pair versus the distance R between the centers. The open (closed) points stand for the most attractive (most repulsive) relative orientation, they correspond to «R» ansatz. The particular values of the PP radii are as follows: $\square$: $\rho_I=1$, $\rho_A=1$; ∇: $\rho_I=0.7$, $\rho_A=1.4$; Δ: $\rho_I=2.$, $\rho_A=0.5$. The dashed lines are not a fit, they are shown just for guiding the eye. The «stars» $*$ shown for comparison stand for the case $\rho_I=\rho_A=1$ and the «S» ansatz.

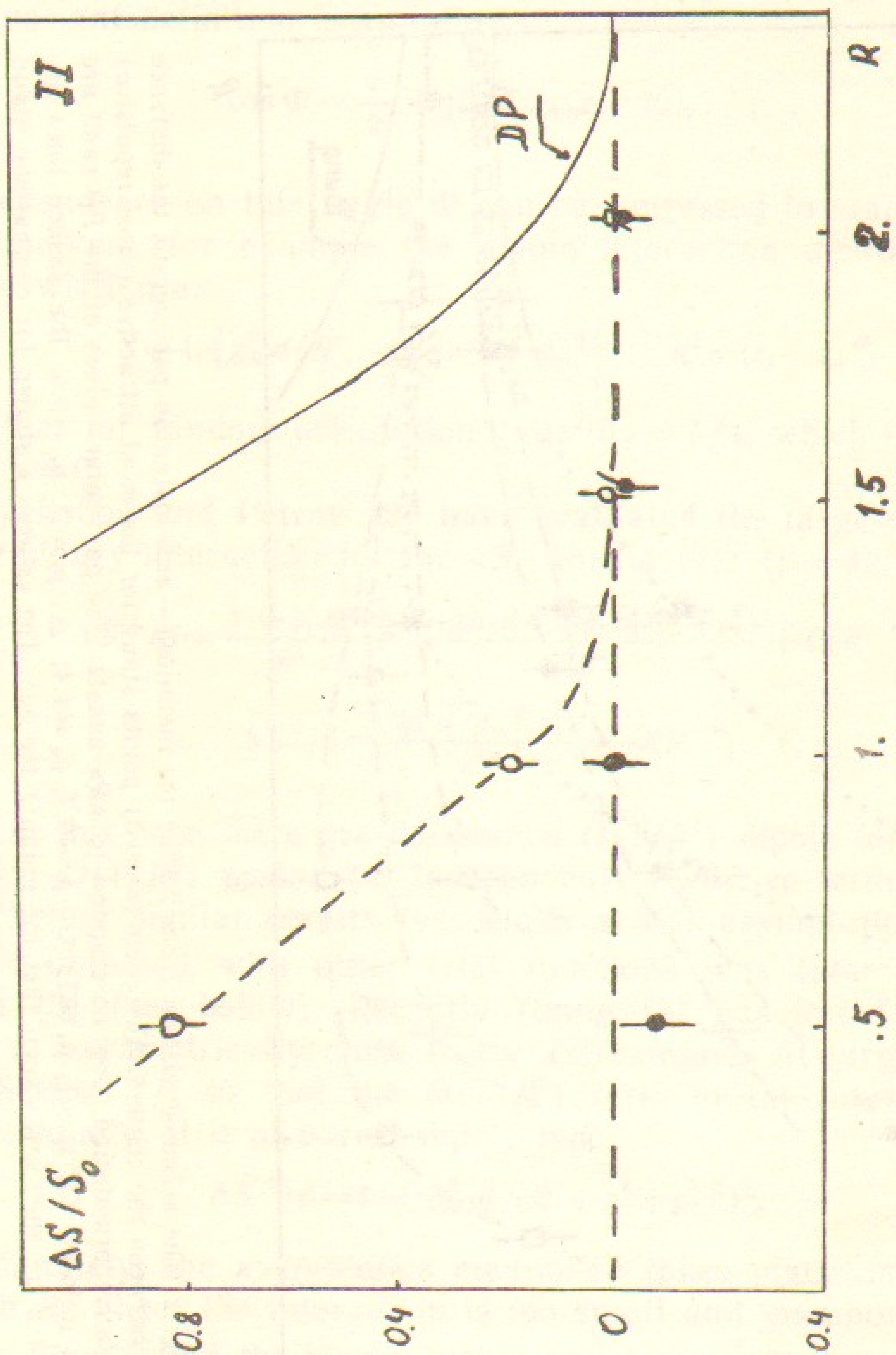

Fig. 3. The same as in Fig. 2, but for the instanton—instanton pair. For «most attractive» relative orientation the «R» ansatz coincides in fact with the two-instanton exact solution due to t'Hooft.

ineffective. Thus we have generated ensemble of points with the weight function $W(x)$

$$W(x)=192\cdot\sum_{I,A}\frac{1}{[(x-z_{I,A})^2+\rho_{I,A}^2]^4}. \qquad (14)$$

(corresponding to the unperturbed action distribution for independent instantons) by the Metropolis algorithm, and then have averaged the ratio $(G^a_{\mu\nu}(x))^2/W(x)$ over them. Then statistics of about 10^4 was sufficient for the accuracy we need. Our results for IA and II pairs are shown in Fig. 2 and 3, respectively.

Considering first the IA case we note, that the points connected by the dashed lines correspond to «R» ansatz for different ρ_1/ρ_2 ratio (we have plotted just few values in order not to make this figure too complicated). It is seen that the forces between the pseudoparticles become weaker as this ratio deviates from unity. The open and closed points correspond to extreme values of the relative orientation angle $\Phi=0$ and $\pi/2$, the most attractive and the most repulsive ones. Note that in the former case the action slowly decreases up to $R=1$, and for $R<0.5$ it start to rize. From what follows it can be concluded that one should not trust this ansatz at so small R and it is presumably the artifact related to it. The true «streamline» should not have any action minima on the way to $R\to 0$ (see discussion of this point in Ref. [8]).

In order to compare these results with the analytic asymptotic relations discussed above we have plotted in Fig. 2 the solid curve corresponding to Young potential (13). We have not plotted the results (12) for «S» ansatz but only note, that it obviously works only at large R. In particular, we remind the reader that the minimum of the $\Phi=0$ curve is in this case at $R=2.5$, to be compared to $R=1$ for the «R» ansatz. We have measured by our routines the action for the «S» ansatz too (it is the stars in Fig. 2), but they are in some way «regularized» integrals: all the points where the field was greater than that in the center of single instanton were rejected. Anyway, essential reduction in the interaction magnitude is seen as one proceed from «S» to «R» ansatz: it can be traced to the fact, that for the case R the field near the PP centers behaviour is much more smooth.

Now we turn to Fig. 3 for the instanton—instanton interaction. The solid curve corresponds to (12) and to the «S» ansatz, while the points are for the «R» case. Note, that for one of the orientations the *II* interaction is absent: it is because in this case the ansatz *R* coincides with the exact t'Hooft solution. (By the way, this is a good test of the algorithm accuracy.) We emphasize, that although *II* interaction is not exactly zero, it is very short-range, and (as it is shown below), it does not play the crucial role in what follows.

For an integration over the collective variables we need some simple parametrization of these «binary potentials». We have used the following one for the ansats «*R*»

$$\Delta S^{IA}/\beta = 4d/(4+R^2/\rho_I\rho_A)^2 + 16/(3+R^2/\rho_I\rho_A)^3,$$

$$\Delta S^{II}/\beta = \left(\frac{3+d}{4}\right) 1.6/(1+R^2/\rho_1\rho_2)^3. \quad (15)$$

4. THE CURRENT-INDUCED CORRECTIONS

As it was already explained in paper I of this series, in addition to the classical interaction considered in the previous section there exist also the current-induced corrections, which are formally of the same magnitude (in terms of the semiclassical parameter β). The physical nature of them is easy to understand: any ansatz used follows the bottom of the «valley» only approximately, and the resulting current (more precizely, its part transverse to the ansatz plane) produce the «linear term» in our gaussian integral which shifts the maximum to the true bottom of the «valley». The sum of the «classical» term and the current-induced correction is the ansatz-independent effective interaction.

The general expression for this current-induced term is as follows

$$\Delta S^{current} = -\frac{1}{2}\int dx\, dy\, j^{\perp}_{a\mu}(x)\, \Box^{\perp}_{a\mu,b\nu}(x,y)\, j^{\perp}_{b\nu}(y) \quad (16)$$

where $\Box^{\perp}(x,y)$ is the part of the gluonic propagator in the ansatz background field, related to the modes transverse to the ansatz plane (and to the gauge transformations), and $j^{\perp}_{a\mu}(x)$ is the transverse part of the current.

It is clear that at large enough distances *R* between the *PPs* this term is negligible. Indeed, for the separated instanton $j_{a\mu}=0$, and for both our trial functions

$$j(R) \sim 1/R^3 \quad (R\to\infty) \quad (17)$$

and therefore

$$\Delta S^{current} \sim (1/R)^6 \quad (R\to\infty). \quad (18)$$

As the «classical» interaction starts with the $O(1/R^4)$ term, we may conclude that it is indeed ansatz-independent. Young's improved ansatz leads to the current which decays more rapidly, as $1/R^4$, so he is able to fix the $O(1/R^6)$ term in the interaction (13) in the ansatz-independent way. Unfortunately, these asymptotic formulae take place at rather large *R*, while in practice it is necessary to evaluate these current-induced correction at $R=1-2$. I have tried various methods, but straightforward calculation turns to be a very difficult business, especially projections in the functional space. In order to do it accurately one has to develop special algorithms and, presumably, to use significantly greater computer resources. Therefore, all we can do now is to present some rough estimates of these corrections, providing at least qualitative understanding of their magnitude and dependence on the distance *R*.

It is not a problem to evaluate numerically the current at any point for a given ansatz: one just needs to perform double differenciation

$$j_{a\nu}(x) = D^{ab}_{\mu}\, G^{b}_{\mu\nu}(x). \quad (19)$$

The next step I made was the «approximate» projection on the mode related to collective coordinate *R*, the interparticle distance. (The word «approximate» stands in parentheses because no projection to non-gauge modes was in fact incerted, and the tangent vector $V(x)$ was evaluated just as a derivative of the ansatz.) It was found that the following «cos θ» combination

$$\cos\theta = \int dx\, j_{a\mu}(x)\, V^{(R)}_{a\mu}(x) / \left[\int dx\, j^2(x) \cdot \int dx\, (V^{(R)}(x))^2\right]^{1/2} \quad (20)$$

is even for «*R*» ansatz only 10—20%. Thus the current is mainly «transverse» and evaluating the current-induced correction it is possible to use the total current instead of its transverse part. (This qualitative conclusion can hardly be altered if the gauge projector be included.)

As for the propagator, it is a difficult problem which may be a subject of separate investigation. In order to get some qualitative insight into the problem, we have considered two simplified versions of this propagator. The simplest one is an attempt to substitute it by some local expression

$$\square^{\perp}_{a\mu,b\nu}(x,y)\to\delta^{ab}\delta_{\mu\nu}L^2\delta(x-y) \qquad (21)$$

where L stands for some constant with the dimension of the length. Then $\Delta S^{current}$ becomes propotional to the current norm:

$$\Delta S^{current}=-\frac{L^2}{2}\int j^2\,dx\,. \qquad (22)$$

Our results for both «S», «R» trial functions are plotted in Fig. 4,a. (This figure corresponds to an instanton—anti-instanton pair with $\rho_I=\rho_A=1$ in the most attractive relative position.) This quantity indeed rapidly decays with distance, but the ansatz «R» leads to $\langle j^2\rangle$ systematically about one order of magnitude smaller. This fact (discovered at the early stage of this work [3]) can be consider as one more argument that this ansatz is better: the current-induced corrections are presumably smaller by a similar factor. I have tried to minimize $\langle j^2\rangle$ with much more complicated trial functions possessing up to 14 parameters (it is described in [3]), but, as seen from this figure, they are unable to improve the situation significantly.

If one makes another guess conserning the propagator, using the free one

$$\square^{\perp}_{a\mu,b\nu}(x,y)\Rightarrow\delta^{ab}\delta_{\mu\nu}/4\pi^2(x-y)^2, \qquad (23)$$

he obtains another estimate of $\Delta S^{current}$. Our results are shown in Fig. 4,b. This integral seems to drop somewhat less rapidly, but the picture is qualitatively similar to the behaviour considered above.

The dashed line in Fig. 4 at the 0.1 level is drown because the typical β (to be found below) is of the order of 10, so at this level the current-induced corrections become of the order of unity and can affect particle distributions. Although our discussion in this section is qualitative at best, due to strong decrease of these quantities with distance R one may conclude from these data that the current-induced corrections may become significant for the «S» and «R» trial functions at $R<2$ and at $R<1$, respectively. It may appear not so great a difference, but it will be shown below that in the «instanton liquid» the typical interparticle distances are 2—3 radii, which makes this point crucial. For exampe, taking roughly $\bar\rho=1/3\Lambda_{PV}^{-1}$ as the typical PP radius and $0.5\Lambda_{PV}^4$ as the typical PP density one finds that (for the homogeneous space-time distribution) the mean number of particles inside the sphere of $R/\rho=1,\ 2,\ 3$ is

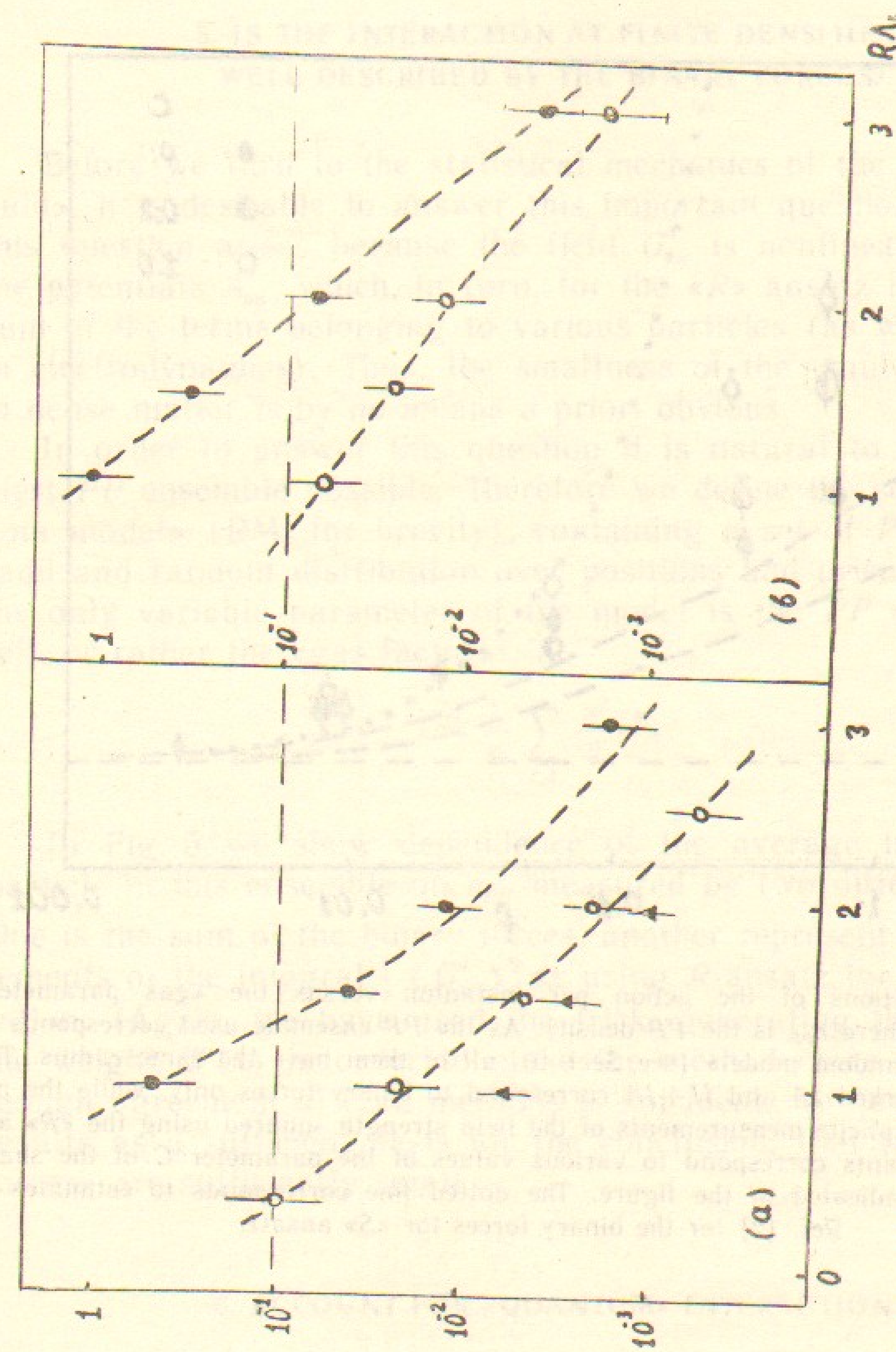

Fig. 4. Two estimates for the current-induced corrections to the interaction of instantons with anti-instantons. In figure (a) we show the case when the gluon propagator is substituted by some delta functions, while in figure (b) it is substituted by the free propagator. Closed and open points correspond to «S» and «R» trial functions, respectively.

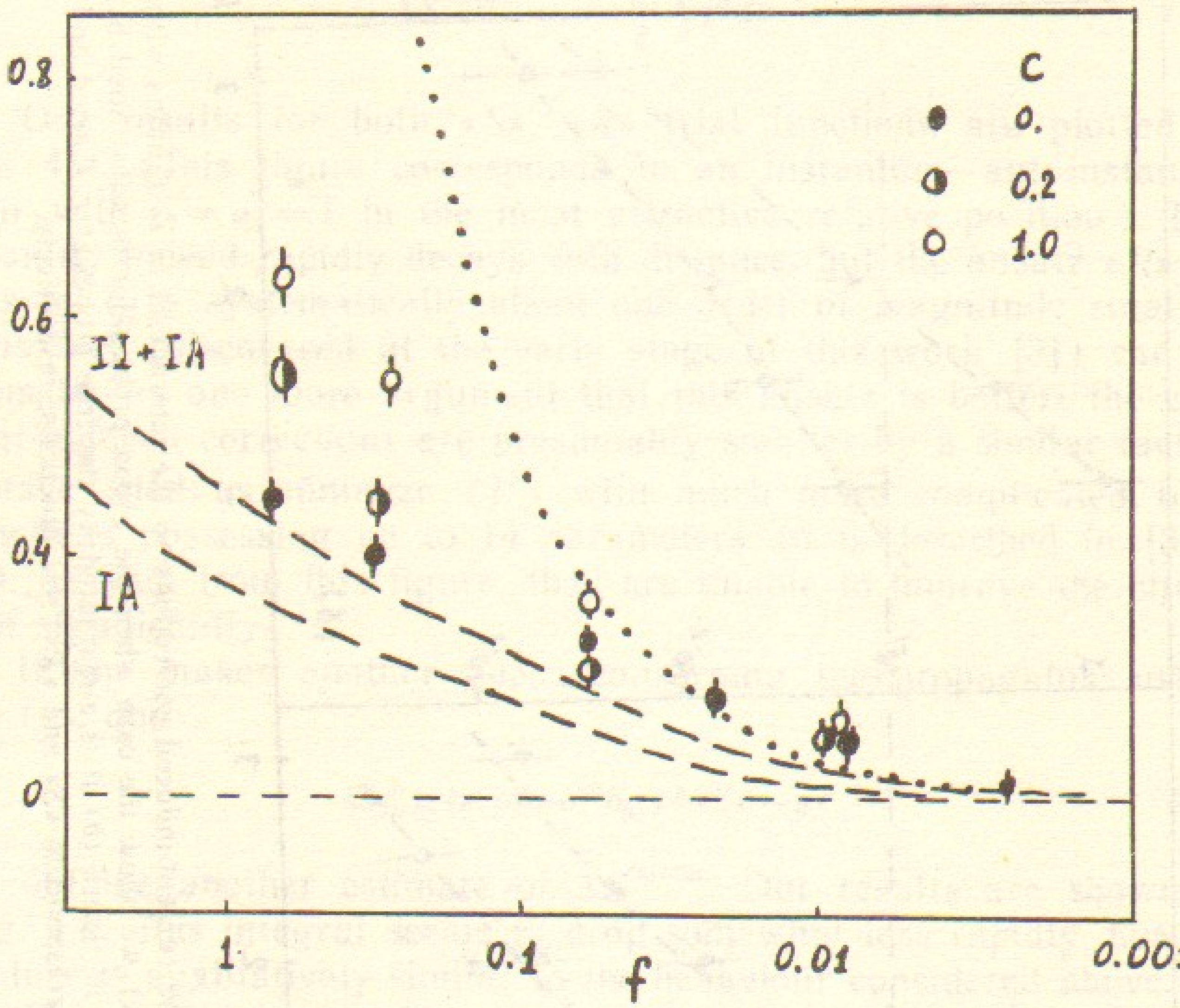

Fig. 5. Modifications of the action per instanton versus the «gas parameter» $f=(\pi^2/2)n_{PP}\rho^4$ where n_{PP} is the *PP* density. As the *PP* ensemble used corresponds to the so called «random model» (see Sect. 6), all of them have the same radius. The dashed lines marked *IA* and *II+IA* correspond to binary forces only, while the points represent explicite measurements of the field strength squared using the «*R*» ansatz. Various points correspond to various values of the parameter *C* of the shape function *f*, as indicated in the figure. The dotted line corresponds to estimates in Ref. [2] for the binary forces for «*S*» ansatz.

0.03, 0.5 and 2.4, respectively. That is why one may well live with the inaccurate potential at $R/\rho<1$, but we definitely have to care about its accuracy at twice larger distances!

5. IS THE INTERACTION AT FINITE DENSITIES WELL DESCRIBED BY THE BINARY FORCES?

Before we turn to the statistical mechanics of the «instanton liquid», it is desirable to answer this important question. In general, this question arises because the field $G^a_{\mu\nu}$ is nonlinear in terms of the potentials $A_{a\mu}$, which, in turn, for the «*R*» ansatz is not just the sum of the terms belonging to various particles (as we are used to in electrodynamics). Thus, the smallness of the «multibody» forces in dense matter is by no means a priori obvious.

In order to answer this question it is natural to take the simplest *PP* ensemble possible. Therefore we define the so called «random model» (RM, for brevity), containing a set of *PPs* with fixed radii and random distribution over positions and orientations. Thus, the only variable parameter of the model is the *PP* density n_{pp} itself, or rather the «gas factor»

$$f \overset{def}{=} \frac{1}{V}\sum_{I,A}\frac{\pi^2}{2}\rho_{I,A}^4. \qquad (24)$$

In Fig. 5 we show dependence of the average interaction per particle in this ensemble on n_{pp} measured by two different methods. One is the sum of the binary forces, another represent direct measurements of the integral $\int \langle G^a_{\mu\nu}\rangle^2 dx$ using *R*-ansatz for each configuration. (Again, we have used the trick, generating the points with the weight $W(x)$ in order to make the method more effective.) Although agreement of both methods is not ideal, we have taken these results as a sufficient basis for the conclusion, that the «multibody forces» are sufficiently small.

6. ACCOUNT FOR «QUANTUM» INTERACTION

So far we have discussed only the classical action for different configurations. Recollecting our picture of «valleys» in configuration space, one may say that so far we have dealed only with the «mapping» of the valley bottoms. Now we proceed to quantum effects, connected with the «widths» of the valleys. Naturally, a quantum system has less chances to penetrate into some narrow valley than into a wide one, even if their bottom are at the same level. In other terms, one should include the energy of the zero-point transverse oscillations in the effective action.

The «width» in Gaussian approximation is represented by the quadratic form of the action expansion in the «transverse» directions. For a single instanton explicite account for all modes of its shape variation (preserving its position, orientation and the radius, being orthogonal to pure gauge transformations and ultraviolet regularized) is an extremely complicated problem, solved by G. t'Hooft. It is clearly impossible to do the same analytically for our complicated configurations, while application of suitable numerical method we hope to report in later papers of this series.

However, at a semi-quantitative level quantum effects may be taken into account quite easily. Indeed, the «quantum action» contains the renormalized charge, depending on the field:

$$S=\frac{1}{g^2(G)}\left(-\frac{1}{4}\int(G^a_{\mu\nu})^2\,dx\right) \tag{25}$$

where G in the coupling constant means some «typical field». The stronger is the color field, the smaller is the charge. The factor $1/g^2(G)$ in this limit better suppress quantum fluctuations. The topic of our discussion now is how to normalize this coupling constant most accurately.

Let me remind the reader that the statistical sum in the dilute-gas approximation (see I) contains the following «beta parameter» as a «one-loop» quantum action of a pseudoparticle

$$\beta=b\log\left(\frac{1}{\rho\Lambda_{PV}}\right);\quad b=\frac{22}{3} \tag{26}$$

(Of course, that statistical sum is not just the order-of-magnitude logarithmic estimate. In particular, one cannot take 2ρ instead of ρ: this would alter the normalization constant in the instanton density by the huge factor 2^b! Similarly, as that formula is written with the two-loop accuracy, the value of the lambda parameter is fixed by the regularization prescription used, the Pauli—Villarse method).

Relying on the fact that «a posteriori» our liquid turns to be

rather dilute, it is reasonable to assume that (apart of the integral over collective variables) the quantum determinants for n PPs is just the product of the single-instanton determinants.

It is tempting to substitute just the parameters $\rho_{I,A}$ of our trial functions into «betas» in the statistical sum. However, our parameters ρ (which we still call the «instanton radii») do not play exactly the same role as that for the separated instantons. It is possible to make better approximation at this point, holding to somewhat more physical characteristics.

It was noted in Section 2 that as two PPs approach each other the field strength distribution is qualitatively changed in such a way, that its peak values increases and at the perifery decreases. (Note, that the action $\int(G^a_{\mu\nu})^2\,dx$ is nearly conserved, and its variations are taken into account as the «classical interaction» considered above.) Such field distribution can be approximated by the single-instanton ones, but with some effective radii ρ_{eff}. Thus the natural and quite practical way to do this is to use in such fit the peak value of the field strength. In other terms, we argue that it is more reasonable to substitute $\tilde\rho$ (7) into the coupling constant, rather than just ρ. (Of course, in principle explicite evaluation of the determinants for the ansatz configurations are needed in order to test real accuracy of this guess.) As $\tilde\rho_{IA}$ depends on the proximity of the surrounding instantons, one may speak about the «quantum interaction» between the pseudoparticles created by this effect. For our «R» ansatz, for which the fields grow as the PPs approach each other, this interaction turns to be repulsive.

(I was often asked at this point why we do not take $\tilde\rho$ as our collective coordinate from the start: this will make this discussion much simpler. The answer is that, fixing positions of all PPs and their ρ one can find $\tilde\rho$, but not vice versal. Also there are no formulae expressing potentials in terms of $\tilde\rho$.)

In order to demonstrate the magnitude of this interaction we use the «random model», introduced in the previous section. Classical action is unchanged if the PP radii and mean separations are changed by the same factor, but the quantum one is changed: it has its own absolute scale, Λ_{PV}. Therefore, in Fig. 6 there are series of curves, corresponding to various PP radii versus the gas factor. The dashed curve corresponds to classical interaction found above. Comparing them one may see, that both effects are of the same sign and of similar magnitude. Below we present results both with and without «quantum interaction» considered in this section.

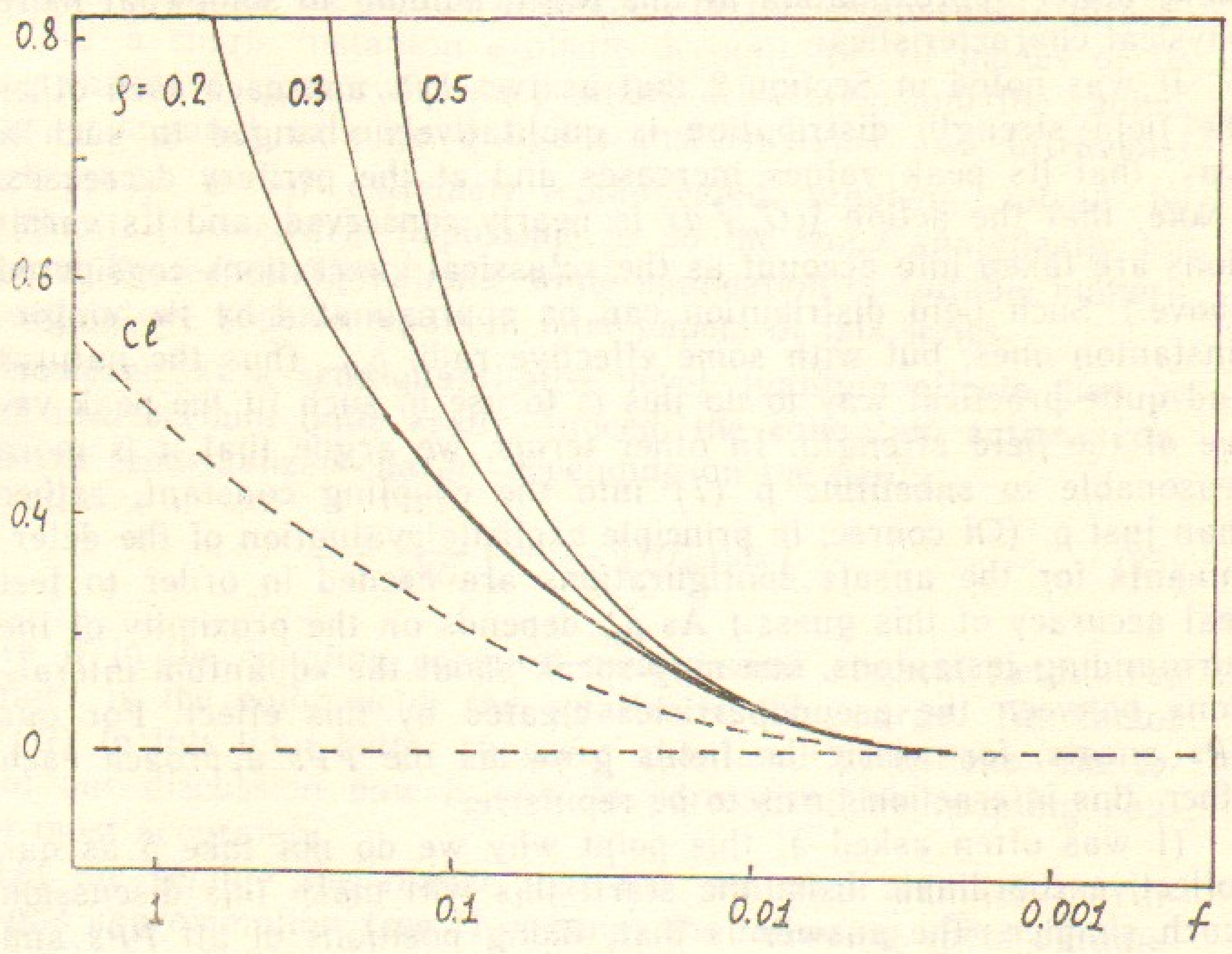

Fig. 6. The same as in Fig. 5, but with the account of the «quantum interaction» as explained in Sect. 6. The dashed curve marked «*Cl*» corresonds to the contiribution of the «classical» binary forces from Fig. 5 The solid lines correspond to the «random model» with the value of the *PP* radii indicated in the figure. It is seen that with the increase in the radii the interaction becomes more repulsive.

7. STATISTICAL MECHANICS OF THE INSTANTON LIQUID

The partition function of our effective theory looks as follows

$$z=\sum_{N_I,N_A}\frac{1}{N_I!\,N_A!}\int\prod_{I,A}d\Omega_{I,A}\exp(-S_{int});$$

$$d\Omega_I=C_{N_c}(d\rho_I\,d^4z_I/\rho_I^5)\cdot\beta_I^{(2N_c-b'/2b)}\left(\frac{b}{2}\right)^{b'/2b}\times$$

$$\times\exp\left[-\beta_I-\left(2N_c-\frac{b'}{2b}\right)\frac{b'}{2b}\ln\beta_I/\beta_I\right]\left(1+O\left(\frac{1}{\beta_I}\right)\right)\qquad(27)$$

where β_I is defined in (26) and the nontrivial element is S_{int}, depending on all collective variables. We take it to be the sum of binary interparticle interactions defined above. We have only to add, that our «binary potentials» have been fixed for the classical (and therefore scale-independent) actions. As it was explained in the preceeding section, betas in (27) are defined by $\bar\rho$ rather than ρ. In the binary interaction term the beta factor is for definiteness determined for the geometric mean value $\rho^2_{mean}=\rho_1\rho_2$.

Evidently, we have to face a very nontrivial computational problem, similar to evaluation of a thermodinamical quantities of some «liquid». Moreover, this system lives in four dimensional space, and its elementary objects have nontrivial interaction, depending on relative distances and orientations. Naturally, the first attempt to solve it [2] was based on some additional approximations. Their method was a kind of the mean field method: the multibody distributions were assumed to be the product of the single-particle ones, and the interaction was then simplified by the orientation and position averaging. The resulting (repulsive) average interaction leads to the following cut-off factor in the ρ distribution

$$dn(\rho)=dn(\rho,\text{ dilute gas})\cdot\exp(-\rho^2\text{ const}),\qquad(28)$$

However, trying to develop a quantitative theory we are not inclined to make any additional approximations. Therefore, we have straightforwardly simulated this statistical system numerically, using the standard Metropolis algorithm. We have typically worked with 32 *PPs* in a box with periodic boundary conditions, making few thousands of iterations.

First of all, we have obtained data for boxes of different volume but the same number of *PPs*, measuring the density dependence of various quantities. However, one value of the density is most interesting, the one corresponding to the maximum of the grand partition function and to the real vacuum of the gauge theory. A number of the methods to fix its value was tried, and finally we have used the following simple idea. One extra particle is added to the ensemble and the following extra factor

$$F_{N+1} = \int d\Omega_{N+1} \exp\left(-S_{int}^{(N+1)}\right)/N \tag{29}$$

appears [illegible] to it in the statistical sum (here S_{int}^{N+1} is the interaction of this extra particle with all others). Its physical meaning is the change in the configuration probability if a particle is added. At the maximum of the partition function F should be equal to unity:

$$F_{N+1} = 1 . \tag{30}$$

(In other terms, it is the condition that our ensemble corresponds to zero chemical potential.) This factor was numerically evaluated and averaged over configurations, the volume was ajusted so that this condition was fullfilled.

8. THE MAIN PROPERTIES OF THE «INSTANTONIC LIQUID»

First of all, we have to discuss the features of this system which are most crucial for the justification of the approximations made above.

In particular, we have used the semiclassical expressions, containing perturbative meaningful only at sufficiently large β values (we remind that it is just the action in unites of the Plank constant.) In Fig. 7 we show the distribution over β in our ensemble. It is peaked at betas of the order ten with the «tail» toward much larger values, while small betas of the order of unity are not very important ingredient of our ensemble. Therefore, although the formulae used fails at this end, it is probably not very important for the conclusions to be drawn below.

Going further we display the distribution over shifts in betas caused by the interaction of one particle with all others, see Fig. 8. First of all, its width is of the order of 2, which is reasonably small compared to mean betas. Thus, our *PPs* are not «melted» by the interaction! Second, repulsion definitely wins over attraction. And third, as the distribution over this quantity is wide enough, $\exp(|\Delta\beta|)$ may well change by one order of magnitude. This shows, that the

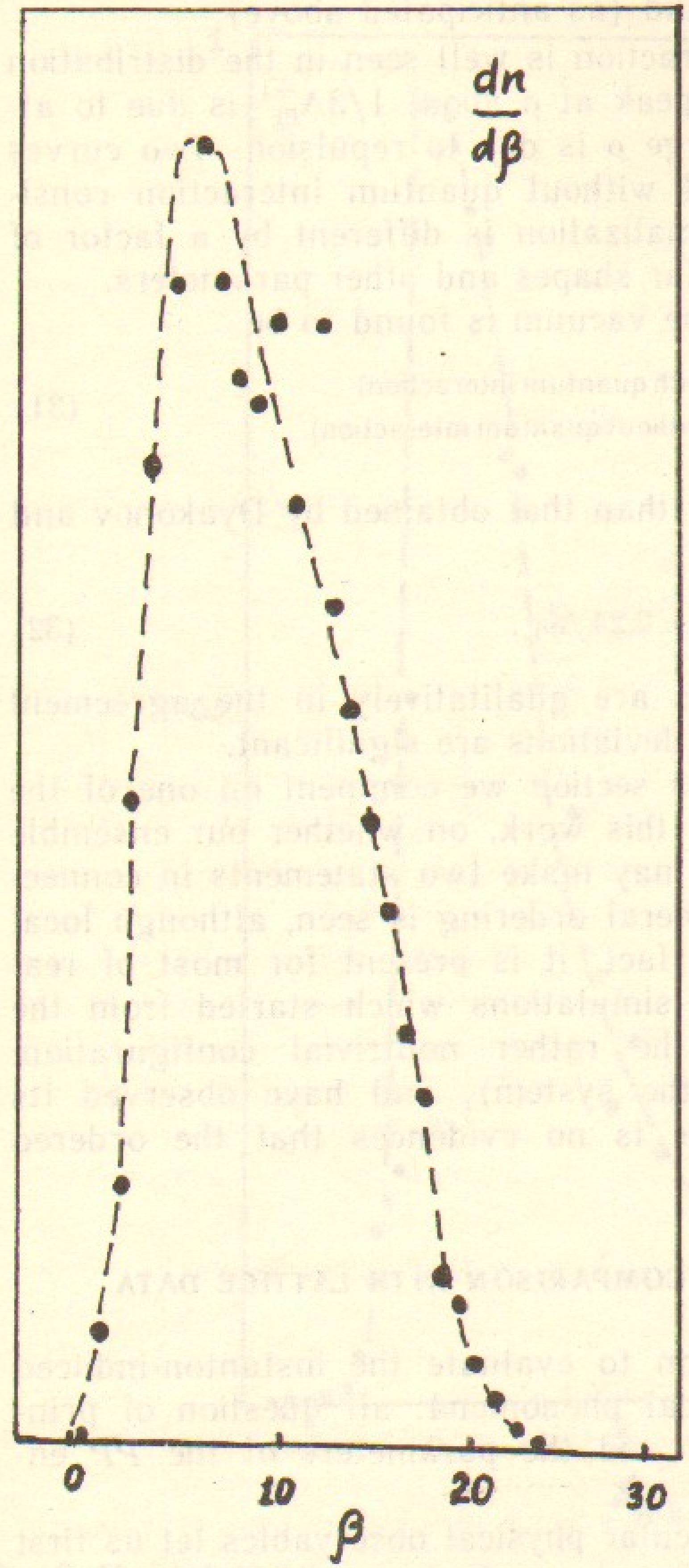

Fig. 7. The distribution over «quantum actions» $\beta = b \log(1/\bar{\rho}\Lambda_{PV})$ in the «instanton liquid» (arbitrary unites). The dashed line is just for guiding the eye. The points are given without the error bars, which may be estimated from the spread of the points.

probability for a particle to be in certain positions strongly depend on its environment. Thus, we do not deal with a dilute gas, but with a strongly interacting liquid (as anticipated above).

The role of the mutial interaction is well seen in the distribution over ρ, shown in Fig. 9. The peak at ρ about $1/3\Lambda_{PV}^{-1}$ is due to attraction, while damping at large ρ is due to repulsion. Two curves are the calculations with and without quantum interaction considered in Section 6. Their normalization is different by a factor of two, but they have rather similar shapes and other parameters.

The global *PP* density in the vacuum is found to be

$$n_{PP} = \begin{cases} 0.7\,\Lambda_{PV}^4 \text{ (with quantum interaction)} \\ 1.2\,\Lambda_{PV}^4 \text{ (without quantum interaction)} \end{cases} \quad (31)$$

This is several times larger than that obtained by Dyakonov and Petrov [2]:

$$n_{PP} \simeq 0.24\,\Lambda_{PV}^4\,. \quad (32)$$

Thus, although our results are qualitatively in the agreement with Ref. [2], the quantitative deviations are significant.

As the final remark in this section we comment on one of the question raised at the start of this work, on whether our ensemble of *PPs* is ordered or not. We may make two statements in connection with this issue: (i) no general ordering is seen, although local order cannot be excluded (in fact, it is present for most of real liquids!); (ii) we have made simulations which started from the Dyakonov—Petrov crystall (the rather nontrivial configuration, being the action minima of the system), and have observed its «melting». Summarising, there is no evidences that the ordered phase does take place.

9. PHYSICAL RESULTS AND COMPARISON WITH LATTICE DATA

Now we are in the position to evaluate the instanton-induced contributions to various physical phenomena: all question of principle are more or less settled and the parameters of the *PP* ensemble are fixed.

But before we turn to particular physical observables let us first comment on the method of comparison of our results with the lattice data. We have already considered phenomenology of the topological fluctuations on the lattice in I and have mentioned there, that

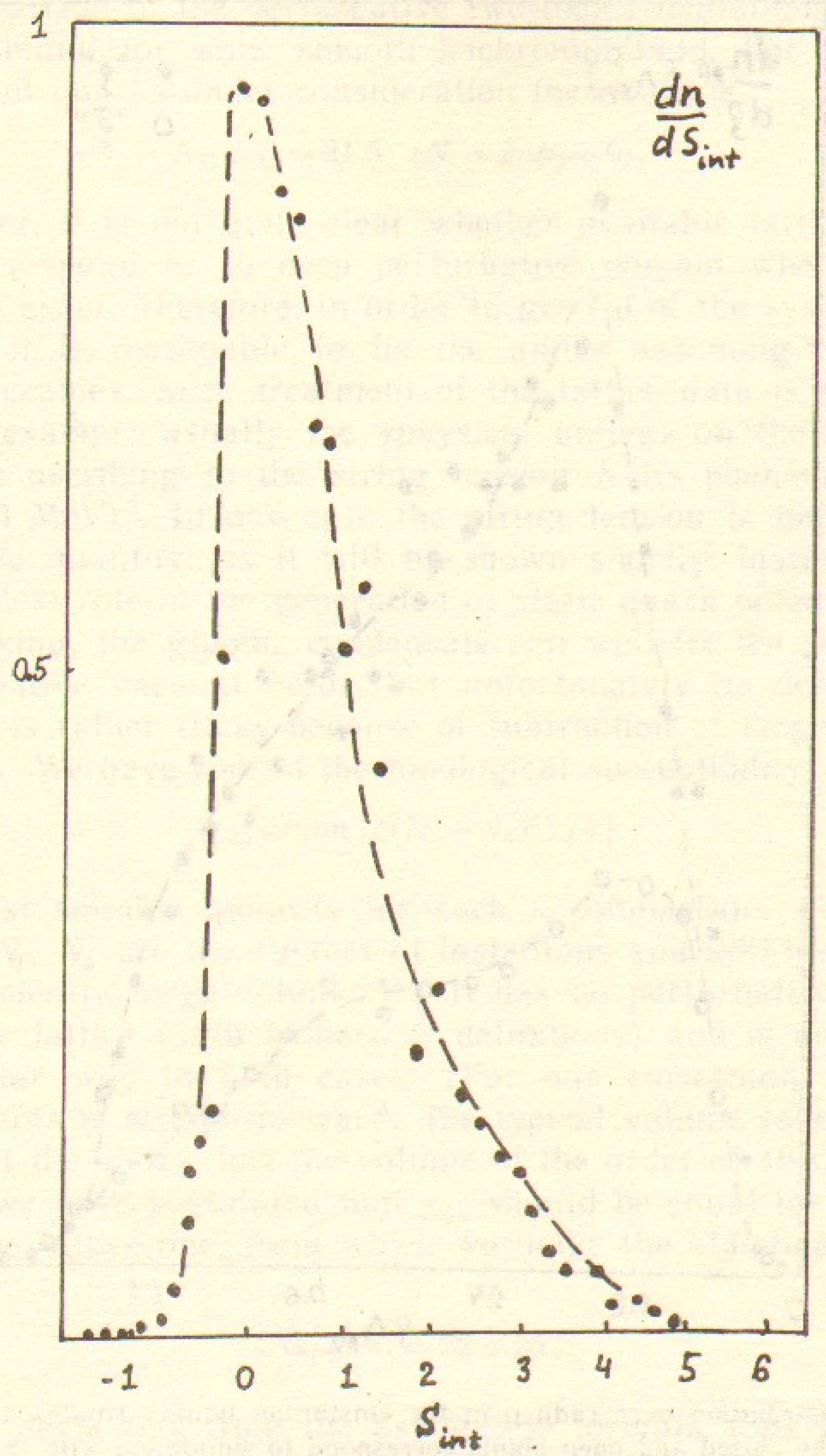

Fig. 8. The distribution over quantum action modification due to the interaction of the pseudoparticles in the «instanton liquid» (arbitrary unites).

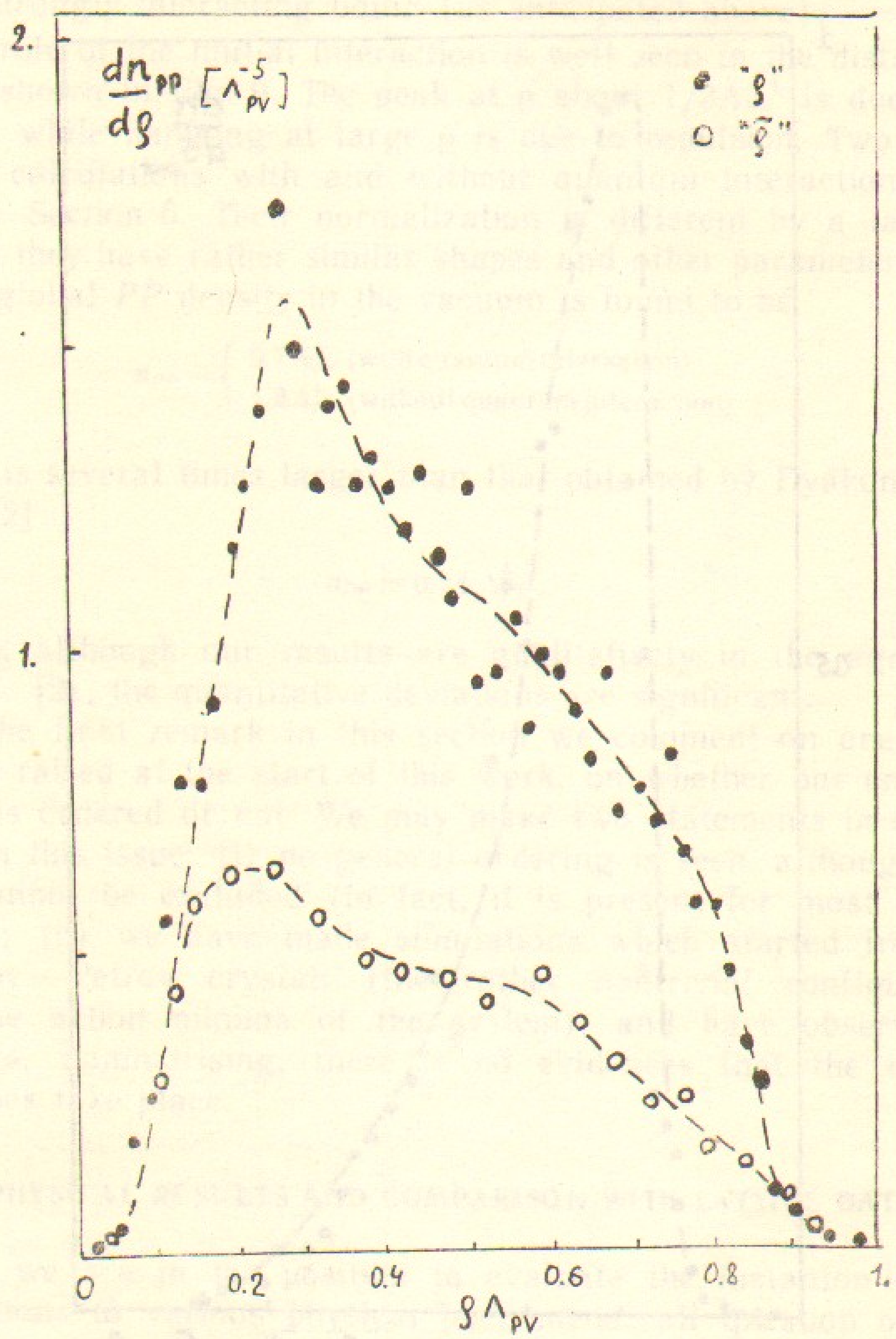

Fig. 9. The distribution over radii ρ in the «instanton liquid» (unites are shown in the figure). The closed and open points correspond to simulation with the «classical» interaction only and that together with the «quantum» one (see Sect. 6).

in the perturbative domain the relation between the unites used in this work (namely, Λ_{PV}) and the lattice unites (Λ_L) can be found from comparison of the perturbative two-loop calculations of the effective potential for some smooth backround field. For the SU(2) case without quarks under consideration the result is

$$\Lambda_{PV}/\Lambda_L = 21.5 \quad (N_c=2,\ N_f=0). \tag{33}$$

However, it is not quite clear whether available lattice data do indeed correspond to so deep perturbative domain where this expression is exact. Therefore, in order to get rid of the systematics of this type, it is reasonable to fix the unites assuming equality of some observables. Such treatment of the lattice data is quite standard, for example usually the «physical unites» on the lattice are defined by ascribing to the string tension K its phenomenological value (420 MeV)^2. In our case the string tension is definitely not the suitable quantity: as it will be shown shortly, instantons play rather modest role in the generation of static quark potential. Generally speaking, the gluonic condensate can well fix the scale of the nonperturbative vacuum fields, but unfortunately its derivation on the lattice is rather tricky because of subtraction of large perturbative effects. We have chosen the topological succeptibility

$$\chi_{top} = \lim_{V\to\infty} [\langle (N_I - N_A)^2 \rangle / V] \tag{34}$$

as the most suitable quantity for such a comparison. (We remind that here N_I, N_A are the number of instantons and anti-instantons in some sufficiently large volume V.) It has no perturbative contribution on the lattice (with modern Q definitions) and is measured in quite similar way in both cases. (For our «instanton liquid» its measurements is straightforward. The typical volume taken, usually just half of the «box», has the volume of the order of $40\Lambda_{PV}^4$.)

Thus, we have postulated that χ_{top} should be equal for our vacuum and the lattice one, from which we make the «lambda measurements»

$$«\Lambda»_{PV}/\Lambda_L = 22 \div 26. \tag{35}$$

Note, that deviations from the asymptotic formula (33) are noticeable, but not very large. They are of reasonable magnitude, similar to that found from «scaling violation» measurements.

As the first example of the comparison we consider a values of the gluonic condensate. Our data for the $\langle G^2 \rangle$ (normalized to the «dilute gas value» $32\pi^2 n_{PP}$) were in fact shown in Fig. 1 as a function of the parameter C. Its minimum is at about 1.3, and multipling the PP density by this factor one obtains the value of this condensate. Comparing it to the lattice data mentioned in I with the scale (35) we have

$$\langle G^2\rangle/32\pi^2\,|_{instanton}=1\div 1.5\,\Lambda_{PV}^4\,,$$
$$\langle G^2\rangle/32\pi^2\,|_{lattice}\simeq 1\div 3«\Lambda»_{PV}^4\,. \qquad (36)$$

Taking into account all uncertainties, the agreement is very good. We may therefore conclude that instantons probably dominate in generation of the gluonic condensate (or at least contribute quite significant fraction of it).

Next we discuss the static potential between heavy charges. This quantity $V(R)$ can be defined as a correlation of two «Polyakov lines» separated by the distance R

$$\langle L^+(R)\,L(0)\rangle=\exp\left[-T\,V(R)\right],$$
$$L=\exp\left[i\int_0^T dx_4 A_4^a\,\tau^a/2\right] \qquad (37)$$

where T is the box length in temporal direction. My measurements were rather straightforward, the path-ordered exponent was calculated by multiplication of color rotation matrix in 10—20 steps along the line. We have tested that «nonunitarity» of the resulting matrix was never larger than few percent, demonstrating that our step was sufficiently small.

Our results are shown in Fig. 10, in the form of the force $F=-dV/dR$ between the charges H. Being compared to the lattice data (we have used the high-statistics data due to Berg and Billoir [9]) the instanton-induced forces turns to be relatively small, constituting only about 15% at $R=0.5\Lambda_{PV}^{-1}$ and being smaller elsewhere. Note that very large force at small distances is just perturbative Coulomb force, while the approximately constant lattice effect is presumably the effect of a «string». Therefore, contrary to some claims in the literature, we conclude that instantons do not contribute much to interquark static potential. Roughly speaking, instantons provide the strongest nonperturbative fields, while in the confinement problem one has to deal rather with the most long-range field fluctuations of the unknown nature.

This problem may also be approached from the other side. In the

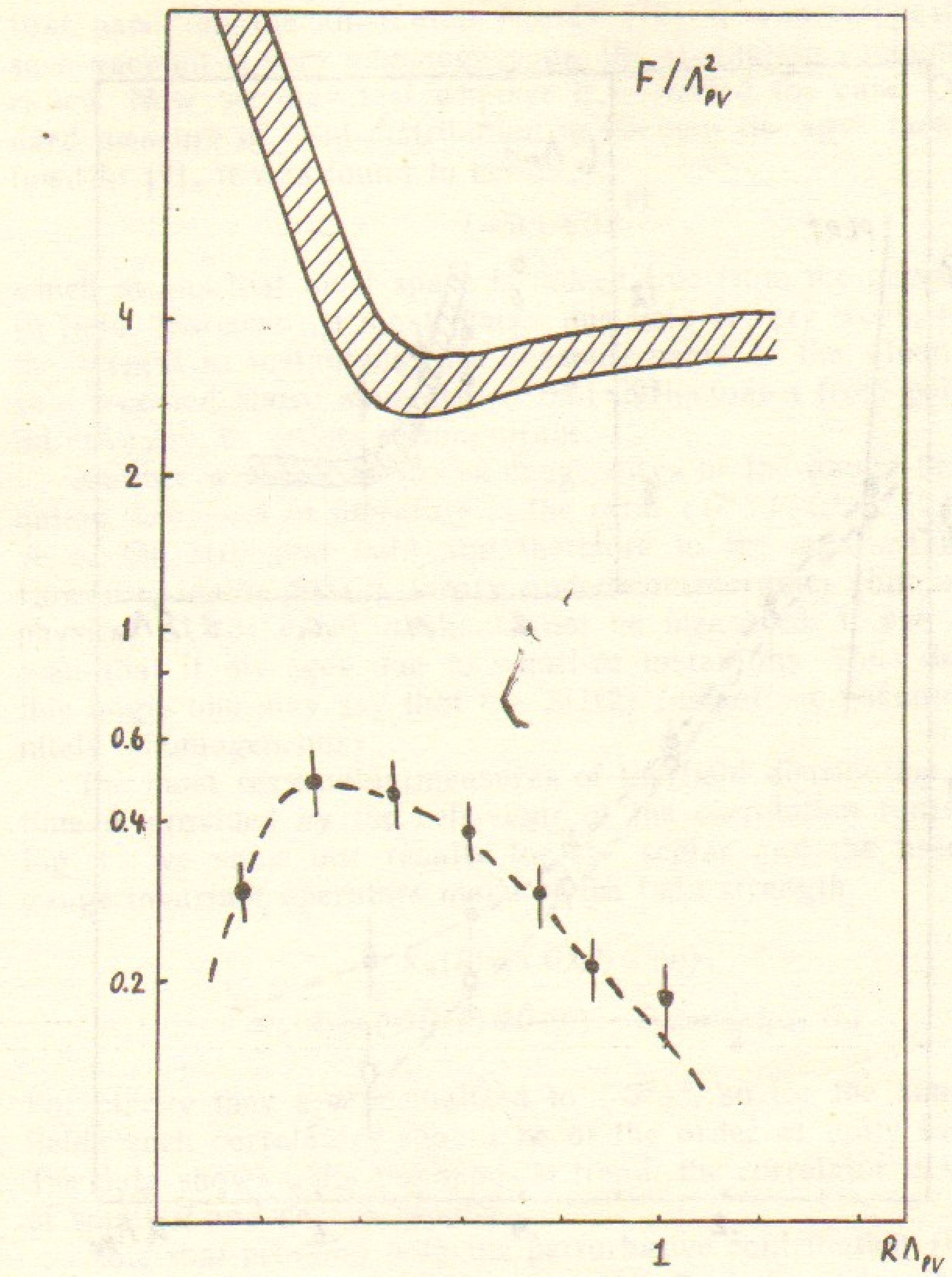

Fig. 10. The force F between two static charges (in Λ_{PV}^2) versus the distance R (in Λ_{PV}^{-1}). The points correspond to the contribution of the pseudoparticles (evaluated in the ensemble with $n_{PP}=0.5\Lambda_{PV}^4$). The shaded region above represents the results of the string tension measurements on the lattice (from [9]), transformed to «Λ_{PV}» unites as explained in the text.

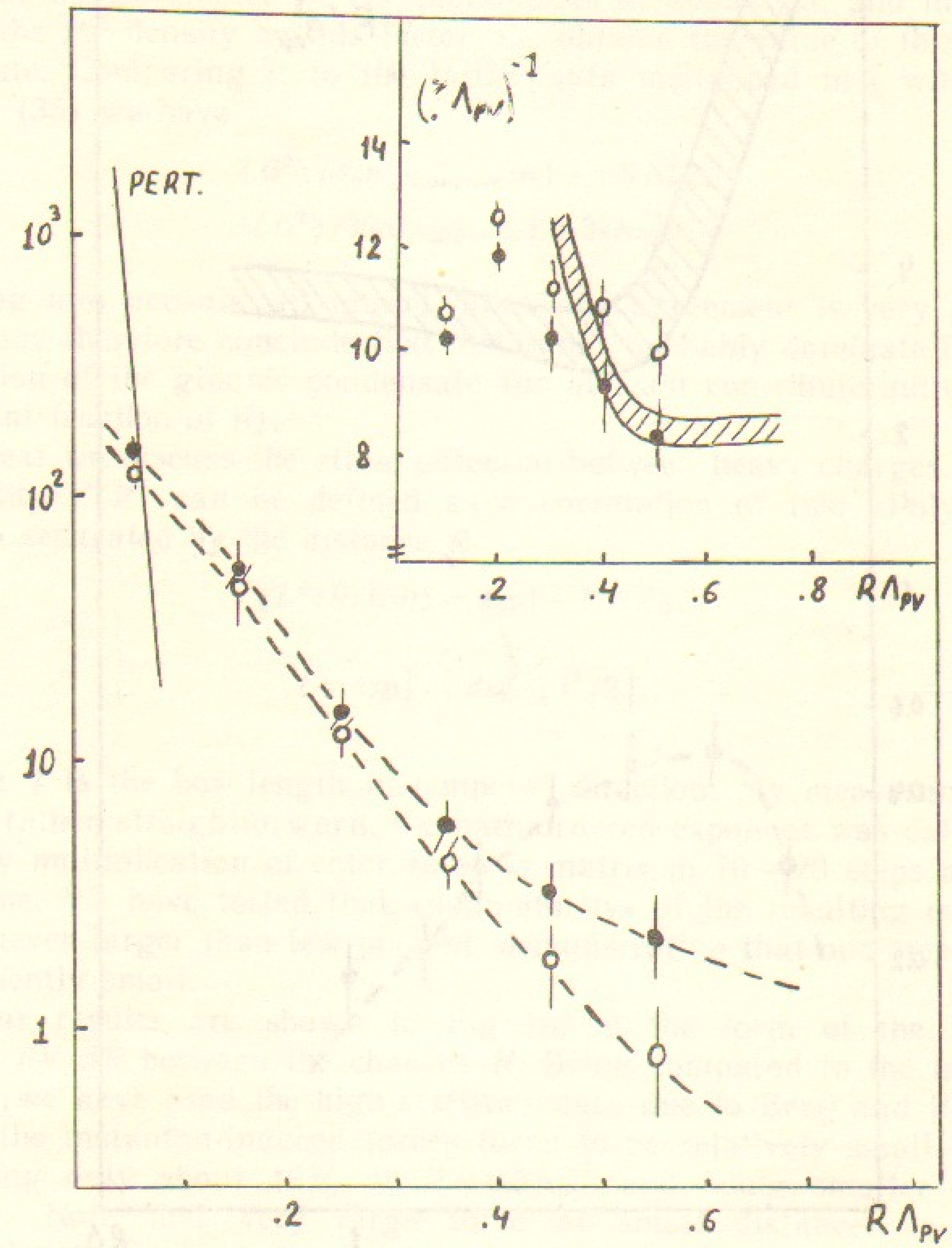

Fig. 11. Dependence of the correlation functions $K_+(R)$ (closed points) and $K_-(R)$ (open points) divided by $\langle G^2(0)\rangle^2$ on the distance R. The solid line marked «PERT» corresponds to the perturbative contribution (free propagation of two gluons): note that it crosses the instanton-induced curve at very small distances. Incertion in the upper right corner shows the logarithmic derivative of the correlators found in the «instanton liquid» model (points) and in the «mass gap measurements» on the lattice [9] (the shaded region). Again, comparison is made using the unites connection discussed in the text.

first paper on the «instanton liquid» [10] it was emphasized that such vacuum is very inhomogeneous, the «twinkling vacuum» it was called. Now we may test whether it is indeed the case. One standard measure of field distribution in vacuum the «gas factor» f defined in [7], it was found to be

$$f = 0.1 \div 0.3 \tag{38}$$

which means that most space is indeed free from the nonperturbative field. Moreover, it most places this field is very weak, related to the largest-ro instantons. The measurements of the gluon condensate reported above have shown that it fluctuates from point to point strongly, by orders of magnitude.

Another measure of the «homogeneity» of the gauge field distribution discussed in literature is the ratio $\langle G^4\rangle/\langle G^2\rangle^2$. It is sensitive to the strongest field and therefore to the smallest intantons. However, in the SU(2) theory under consideration (but not in the physical SU(3) case) it should not be measured: it can easily be seen that it diverges due to small-ro instantons. Thus, looking at this angle one may say that the SU(2) instantonic vacuum is «infinitely inhomogeneous».

The most reasonable measures of the field distribution in space-time is provided by the behaviour of the correlation functions . In Fig. 11 we show our results for the scalar and the pseudoscalar gauge invariant operators made of the field strength:

$$K_+(R) = \langle G^2(R)\, G^2(0)\rangle,$$

$$K_-(R) = \langle G\tilde{G}(R)\, G\tilde{G}(0)\rangle, \quad \tilde{G}_{\mu\nu} = \frac{1}{2} f_{\mu\nu\sigma\lambda}\, G_{\sigma\lambda} \tag{39}$$

For clarity they are normalized to $\langle G^2\rangle^2$, so for the homogeneous fields such correlators should be of the order of unity everywhere. The data shows quite the opposite trend: the correlator is very large at small R and decays rapidly.

Note that crossing with the perturbative contribution (free gluon propagation) takes place at very small R of the order of $1/20\Lambda_{PV}^{-1}$. Only in the vicinity of this point the standard sum rules based on the operator product expansion may have chances to be valid. In I we have already mentioned that due to Novikov et al in these channels there should be violation of the asymptotic freedom at very small distances, at the momentum transfer $Q^2 = 20\ GeV^2$. Now we see that our data for the instantonic liquid do indeed reproduce these observation.

The logarithmic derivative of the correlator may be identified with some correlation length, which turns to be very small:

$$\xi=\left\{ \frac{d}{dR} \log[\langle G^2(R)\, G^2(0)\rangle - \langle G^2\rangle^2] \right\}^{-1} \simeq 0.1\, \Lambda_{PV}^{-1}. \qquad (40)$$

Close values for this length were found on the lattice (see insertion in Fig. 11), the agreement is so good that we may conclude that instantons do explain the data on the correlation function.

In the unites (40) even the small boxes used in lattice studies look large, while the boxes used in this work is even «huge», about $(30\cdot\xi)^4$! However, these arguments are but misleading. Large and strongly decaying correlation function just signalizes that there are very small «spots» of the strong field, the small-size instantons. This observation is very important by itself, but obviously it is not really the true correlation length. At large distances the correlation function should be related with the long-range correlations. In fact we have only few instantons $(32^{1/4})$ along each axis of our box (and on the lattice only few instantons at all), and we do know that they are correlated at distances of the order at least $1\,\Lambda_{PV}^{-1}$.

And nevertheless, the «instantonic liquid» does reproduce the correlation function at some intermediate region, say between $R=1/20$ and $1\,\Lambda_{PV}^{-1}$, in which it drops by about two orders of magnitude. We were thus able to evaluate both the «asymptotic freedom threshold» and the masses of the glueballs.

10. CONCLUSION AND DISCUSSION

The main result of this work is that the qualitative features of the «instantonic liquid» suggested in [10] are confirmed. Many specific approximations and assumptions made in the first variational approach to this problem [2] were improved, but this has not affected the qualitative picture.

The global parameters found (such as the gluonic condensate and the topological succeptibility) are fixed up to the factor of two. At this accuracy level we have found agreement with measurements of these quantities made on the lattice. This fact is highly nontrivial, considering that transition from lattice to our unites have contained huge numerical factor like $(21.5)^4$ etc.

This work also provides much more detailed information on the properties of instantons than one can draw from lattice numerical experiments. We are not restricted by the discrete nature of the lattice and have all the differential distributions (such as the distribution over the radii etc. shown above).

Looking at the problem from more practical angle we emphasize the fact that in our case commitment of the computer power is smaller than in the lattice works mentioned by the enormous factor of the order of 10^5. Indeed, most of the variables used on the lattice describe the short-wavelength gluons which are not interesting for us. Their main role is the generation of the renormalized charge in the effective action. We did evaluated it in some crude approximation, but this point may be improved. Anyway it is doubtful that on the lattice one may have better accuracy, keeping in mind severe technical limitations of this approach.

However, the cost for our successful performance was commitment to only topological fluctuations of quite specific type, while on the lattice one integrate over all configurations. Of course, «instantonic liquid» is not the final picture of the gauge field vacuum: e. g. we have observed that it does not generate sufficiently strong force between static charges even at small distances, to say nothing on the confinement at large distances.

We do not think that such cost is too high because it becomes more and more obvious that instantons dominate in generating the strongest vacuum fields. For example, they were shown to reproduce well lattice data on the correlation functions. We are quite sure that new quantitative theory of instantons will shed light on many phenomena in hadronic physics, and in the subsequent paper (devoted to incorporation of light quarks) we will turn to this.

Completing our comparison with the lattice data, we would like to emphasize one more point. In contrast to that works we are able to understand the nature of all the distributions, for they follow from the features of the binary interactions of the pseudoparticles.

Now, let us turn to the most difficult physical question. It was demonstrated that all nontrivial small parameters of the problem (such as «diluteness», large betas etc.) suggested in [10] are reproduced. Why it happens so, do they have some deeper roots?

The gauge theory under consideration have no free parameters, so all our «small parameters» appear as the interplay of some numbers. Trying to «get statistics» one may try to do the same say for the O(3) sigma model in $1+1$ dimensions: it looks like this trick does not repeat itself in this case. As one of the numerical «large numbers» D.I. Dyakonov have suggested to consider 11/3 in the Gell-Mann — Low function, another one is d = 4, the dimension of the space-time. Their interplay does indeed lead to inhomogeneous fields, peaked at some ro value. Unfortunately we have nothing to add to these interesting but not quite clear arguments at the moment.

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E.V.Shuryak

Towards the Quantitative Theory of the Topological Effects in Gauge Field Theories II. The SU(2) Gluodynamics

Э.В. Шуряк

К количественной теории топологических эффектов в калибровочных теориях поля. 2.SU(2) глюодинамика

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