ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

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# MHD STABLE AXISYMMETRIC MIRROR

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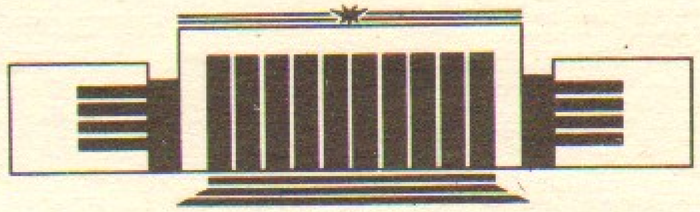

НОВОСИБИРСК

One of the main difficulties in search for MHD stable axisymmetric mirror configurations is associated with the stabilization of the outer plasma boundary on which plasma pressure vanishes. As it is well known [1], in a paraxial axisymmetric mirror cell this boundary is unstable against interchange modes*). In this paper we show that going beyond the limits of paraxial fields one can make an axisymmetric configuration in which plasma (without sloshing ions) would be stable against flutes.

For the sake of simplicity, let us consider stability of a sharp plasma boundary supposing that at some field line plasma pressure drops from a finite value to zero. Assuming $\beta$ to be small, this boundary is stable if the integral

$$V=\int \frac{\varkappa(p_{\parallel}+p_{\perp})}{rB^2}\, ds \tag{1}$$

is positive [1]. In eq. (1) $\varkappa$ is the curvature, $B$ is the field strength, $r$ is the distance from the axis to the boundary field line; the integration is extended from one mirror point to the other. The value (and the sign) of $V$ depends essentially on the pressure distribution along the field line. To begin with, let us take it that the sum $p_{\parallel}+p_{\perp}$ is constant between the points of the maximum magnetic field and sharply falls to zero at these points. Then $V=(p_{\parallel}+p_{\perp})I$, where

$$I=\int \frac{\varkappa\, ds}{rB^2}, \tag{2}$$

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*) This is not true in the case of sloshing ions [2, 3] which we are not considering here.

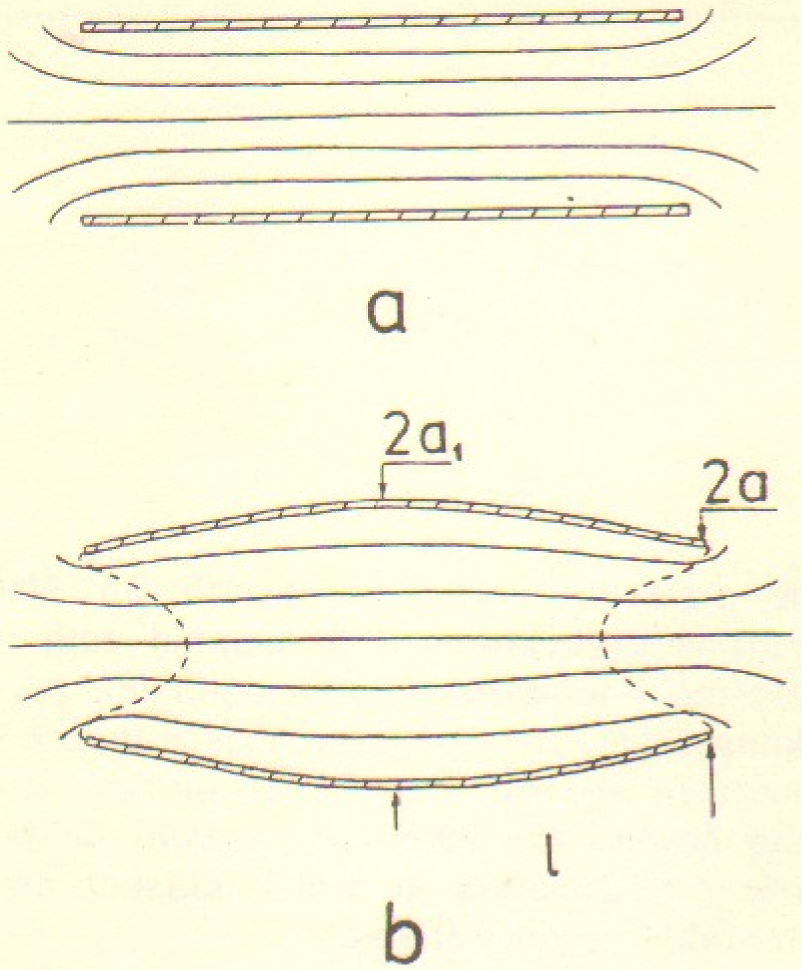

*Fig. 1.* Magnetic field lines in the case of a long ideally conducting straight cilinder (*a*) and the cilinder swelled in the central part (*b*). In figure *b*, dashed lines are lines of mirror points.

and stability corresponds to $I > 0$. The integration in (2) again is extended between the mirror points.

Recently, it has been shown [4] that the inequality $I > 0$ can be met in the limit of very small mirror ratios $k$, $k-1 \ll 1$. In this paper we present an example of a mirror field for which $I > 0$ for any $k$, no matter how much it would be.

To this aim, consider first magnetic field generated by a long ideally conducting thin cilinder shell inclosing the flux $\psi_0$ (see Fig. 1,*a*). The cilinder length $l$ is assumed to be much larger than its radius $a$. Consider how $B$ changes on a field line which is characterized by the flux $\psi$. Deeply inside the cilinder, at large distance from its ends, magnetic field is homogeneous. Near the axis, for $\psi$ smaller than some critical value $\psi_*$, $B$ monotonically decreases when approaching the end. The calculations give $\psi_* = 0.39\psi_0$. But for $\psi > \psi_*$, magnetic field grows from the center to the ends, reaches maximum and then begins to fall outside the cilinder. These lines are able to confine plasma particles trapped between two

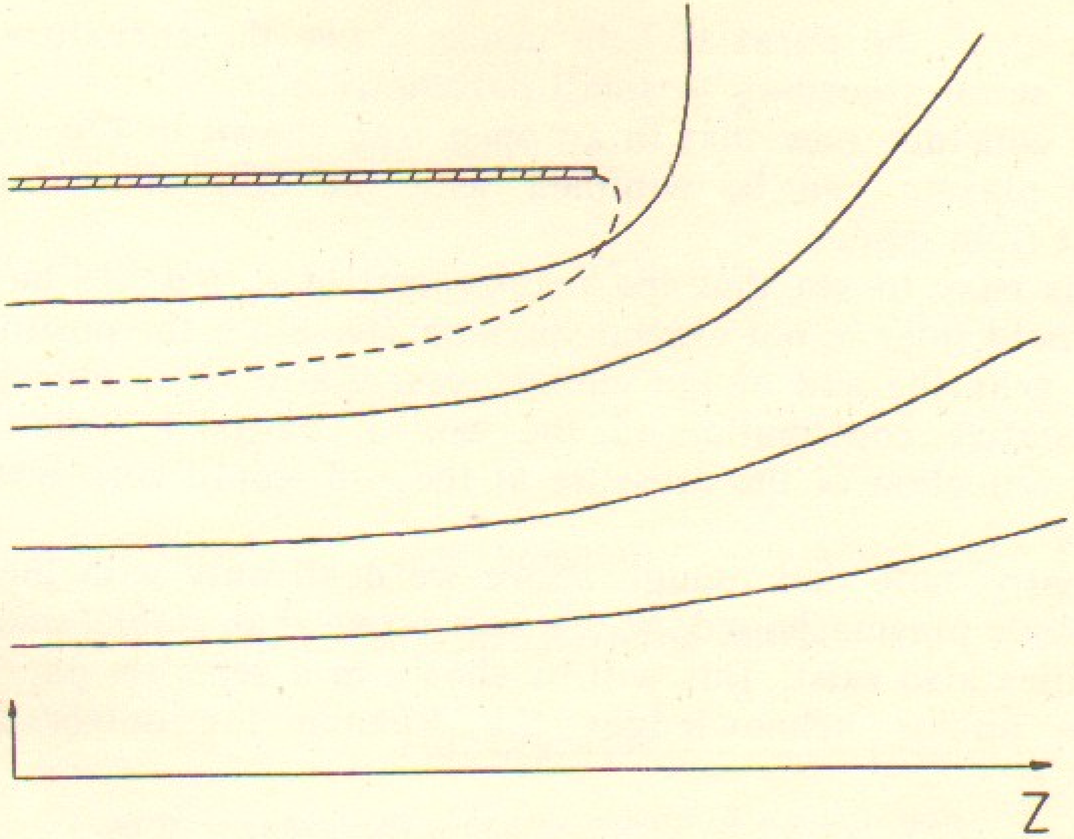

*Fig. 2.* Field lines on the plane $r$, $z$ near the end of the cilinder. The dashed line consists of the points of the maximum $B$.

maxima of $B$ inside the cilinder. It is remarkable, that the curvature of the field lines in the system considered is everywhere favourable. This is clearly seen in Fig. 2 showing the field lines near the end of the cilinder. Hence, integral (2) is positive for any $\psi (\psi > \psi_*)$ and the plasma outer boundary will be stable.

An evident drawback of the scheme described above is the lack of magnetic field maxima near the axis which does not allow confinement of plasmas without hollow in the trap. This difficulty can be easily overcome if one «blows» the central part of the cilinder as is shown in Fig. 1,*b*. As a result, a paraxial magnetic field will be formed inside the cilinder (we assume $l \gg a_1$, $a$) and mirror points will occur at the field lines $\psi < \psi_*$ with the mirror ratio $a_1^2/a^2$. One can show that the distance from these points to the ends of the cilinder is of the order of $a \ln (l/a)$. Since these points are deep enough in the cilinder, the only contribution to the integral (2) comes from the paraxial magnetic field and for $\psi < \psi_*$, in general, $I < 0$. But in the region $\psi > \psi_*$, the transformation from Fig. 1,*a* to 1,*b* makes only slight changes near the end of the cilinder so that the positions of mirror points here and positiveness of $I$ will retain. The value of $I$ at $\psi > \psi_*$ will change a little because a negative contribution to it coming from the paraxial field inside where the curvature $\varkappa\sim a_1/l^2$ will be small according to small parameter $a_1/l$.

We conclude now that in an open trap shown in Fig. 1,*b* a non-hollow plasma can be confined with the outer boundary stable against flute modes.

It is easy to see that the requirement of $p_\parallel+p_\perp$ to be constant along field lines is not crucial for stability. Since the positive contribution near the end of the cilinder essentially ($l/a_1$ times) exceeds the negative contribution of the central paraxial region, even a strong reduction of the pressure at the end would keep integral (1) positive.

Finally, note that though above we dealt only with the stability of a sharp plasma boundary one can prove that stable smooth radial profiles also exist. This will be shown in a separate paper.

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**MHD Stable Axisymmetric Mirror**

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**МГД устойчивый осесимметричный пробкотрон**

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