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НОВОСИБИРСК

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Abstract

An expression for the high-energy Delbrück amplitude at large scattering angles is derived. This expression is exact in the parameter $Z\alpha$. The consideration is based on the use of the relativistic electron Green's function in a Coulomb field.

1. INTRODUCTION

The elastic scattering of a photon in a Coulomb field via virtual electron-positron pairs (Delbrück scattering [1]) is one of the few nonlinear quantum-electrodynamics processes that are directly observable in experiment. The situation when $\omega \gtrsim m$ (ω is the photon frequency, m is the electron mass, and $\hbar = c = 1$) is most favourable (see refs.[2-4]).

At present the amplitude of the process is studied in detail in the lowest-order Born approximation (see refs.[5-8]). The corresponding calculations have been carried out for an arbitrary value of the momentum transfers Δ ($\vec{\Delta} = \vec{k}_2 - \vec{k}_1$, $\vec{k}_1$ and $\vec{k}_2$ are the momenta of the incoming and outgoing photons, respectively; $|\vec{k}_1| = |\vec{k}_2| = \omega$). The amplitude, exact in $Z\alpha$ ($Z|e|$ is the charge of the nucleus, $\alpha = e^2 = 1/137$ is the fine-structure constant, e is the electron charge), has been found by Cheng and Wu (refs.[9-11]) in the limit $\omega/m >> 1$ with $\Delta \ll \omega$. They have solved the problem summing in a definite approximation the Feynman diagrams with an arbitrary number of photon exchange with a Coulomb centre. It appears that the Coulomb corrections at $Z\alpha \sim 1$ drastically change the result as compared to the Born approximation (refs.[9,10]).

The main contribution to the total cross section at $\omega \gg m$ comes from the momentum transfer $\Delta \sim m$, the scattering angle $\theta_0 \sim \Delta/\omega \ll 1$. The characteristic impact parameter $\rho \sim 1/\Delta$. Therefore, the value of the angular momentum $\ell \sim \rho\omega \sim \omega/\Delta$ proves to be large and it is possible to employ the quasiclassical approximation. The corresponding approach intended for description of quantum electrodynamics processes in a Coulomb field at high energies has been developed by V.M.Strakhovenko and one of the authors in refs.[12,13]. In particular, the dependence of the total cross section on the charge of a Coulomb centre has been defined. The consideration has been based on the use of the quasiclassical Green's function obtained from the integral representation for the electron Green's function in a Coulomb field (ref.[14]).

In the last ten years the elastic photon scattering at large angles was being studied intensively in the experiments (see e.g. refs. [15,16] and the survey [4]). The results of these experiments show that the Coulomb corrections to the Delbrück scattering amplitude have to be taken into account.

In the present paper, in order to study the Coulomb corrections the exact Delbrück scattering amplitude is found in the limit $\omega/m \gg 1$, $\Delta/m \gg 1$. At $\Delta \sim \omega$ the scattering angle θ_0 ($\sin(\theta_0/2)=\Delta/2\omega$) is not small and the characteristic angular momentum $l \sim 1$. Therefore, in this case the quasiclassical approach is not valid and one has to develop another approach. At $m \ll \Delta \ll \omega$ the amplitude obtained agrees with the results of refs. [9,13]. It has been shown in ref. [8] that the Delbrück scattering amplitude scales in the form of $f(\theta_0)/\omega$ as $\omega/m \to \infty$ with θ_0 fixed. Our explicit calculation confirms this statement.

2. Calculation of the Delbrück amplitude

Let an incoming photon produce at the point $\vec{r}_1$ a pair of virtual particles that is transformed at the point $\vec{r}_2$ into an outgoing photon. In the Furry representation, the corresponding amplitude is

$$M = 2i\alpha \int d\vec{r}_1 d\vec{r}_2 \, e^{i(\vec{k}_1\vec{r}_1 - \vec{k}_2\vec{r}_2)} \int d\varepsilon_1 d\varepsilon_2 \, \delta(\omega - \varepsilon_1 + \varepsilon_2) \cdot \tag{1}$$

$$\cdot \mathrm{Tr}\, \hat{e}_1 G(\vec{r}_1, \vec{r}_2|\varepsilon_2)\, \hat{e}_2^* G(\vec{r}_2, \vec{r}_1|\varepsilon_1)$$

where $\vec{e}_1$ and $\vec{e}_2$ are the photon polarization vectors, $\hat{e} = e_\mu \gamma^\mu$, γ^μ are Dirac matrices, and $G(\vec{r}_1,\vec{r}_2|\varepsilon)$ is the electron Green's function in a Coulomb field. As known, the function $G(\vec{r}_1,\vec{r}_2|\varepsilon)$ has, in the complex plane ε, cuts along the real axis from $-\infty$ to m and from m to ∞, which correspond to the continuous spectrum. It has also simple poles, corresponding to a discrete spectrum, in the interval $(0, m)$ for an attractive field under consideration. According to the Feynman rules, $G(\varepsilon)$ is equal to $G(\varepsilon + i0)$ at $\varepsilon > 0$ and is equal to $G(\varepsilon - i0)$ at $\varepsilon < 0$. The integral representation for the electron Green's function in a Coulomb field which is valid in the whole complex plane ε has been obtained in ref. [14]. Let us represent the δ-function in

(1) in the form

$$\delta(\omega - \varepsilon_1 + \varepsilon_2) = \frac{i}{2\pi}\left[\frac{1}{\omega - \varepsilon_1 + \varepsilon_2 + i0} - \frac{1}{\omega - \varepsilon_1 + \varepsilon_2 - i0}\right] \tag{2}$$

Using the analytical properties of the function G, it is possible to deform the contour of the integration with respect to ε_1 and ε_2 in (1), in such a way that with the first term in (2) the integrals with respect to ε_1 and ε_2 encircle the right- and left-hand cuts, respectively. With the second term in (2), the contours of integration with respect to ε_1 and ε_2 will encircle respectively the left- and right-hand cuts. The contribution of the discrete spectrum can be neglected at $\omega \gg m$. Performing the stated transformation, we obtain

$$M = \frac{\alpha}{\pi} \int d\vec{r}_1 d\vec{r}_2 \, e^{i(\vec{k}_1\vec{r}_1 - \vec{k}_2\vec{r}_2)} \iint_0^\infty d\varepsilon_1 d\varepsilon_2 \cdot \tag{3}$$

$$\cdot \mathrm{Tr}\left[\frac{\hat{e}_1 \delta G(\vec{r}_1,\vec{r}_2|-\varepsilon_2)\hat{e}_2^* \delta G(\vec{r}_2,\vec{r}_1|\varepsilon_1)}{\omega - \varepsilon_1 - \varepsilon_2 + i0} - \frac{\hat{e}_1 \delta G(\vec{r}_1,\vec{r}_2|\varepsilon_2)\hat{e}_2^* \delta G(\vec{r}_2,\vec{r}_1|-\varepsilon_1)}{\omega + \varepsilon_1 + \varepsilon_2 - i0}\right]$$

where $\delta G(\varepsilon) = G(\varepsilon + i0) - G(\varepsilon - i0)$ is the discontinuity of the Green's function on the cut. Note that each term in the expression (3) corresponds to the contribution of the non-covariant perturbation theory diagram. Using eq. (19-21) of ref. [14], we have for the function δG at $m = 0$

$$\delta G(\vec{r}_2,\vec{r}_1|\pm|\varepsilon|) = -\frac{i}{4\pi r_1 r_2}\sum_{l=1}^{\infty}\int_{-\infty}^{\infty} ds \exp\left\{i\left[\pm 2Z\alpha s + |\varepsilon|(r_1+r_2)\coth s - \pi\nu\right]\right\} \cdot$$

$$\left\{\gamma^0\left[1 + \vec{n}_1\vec{n}_2 + i\vec{\Sigma}[\vec{n}_2\times\vec{n}_1]\right]\cdot\left[\frac{y}{2}J'_{2\nu}(y) \mp iZ\alpha \coth s\, J_{2\nu}(y)\right]B + \gamma^0\left[1 - \vec{n}_1\vec{n}_2 - \right. \right. \tag{4}$$

$$\left. - i\vec{\Sigma}[\vec{n}_2\times\vec{n}_1]\right]A J_{2\nu}(y) \pm \left[\frac{i|\varepsilon|(r_2-r_1)}{\sinh^2 s}(\vec{\gamma}, \vec{n}_1+\vec{n}_2)B - \coth s(\vec{\gamma}, \vec{n}_2-\vec{n}_1)A\right]J_{2\nu}(y)\Big\}$$

where $\nu = \sqrt{l^2 - (Z\alpha)^2}$, $x = \vec{n}_1\vec{n}_2$, $B = \frac{d}{dx}\left(P_l(x) - P_{l-1}(x)\right)$, $A = l\frac{d}{dx}\left(P_l(x) + P_{l-1}(x)\right)$, $y = 2|\varepsilon|\sqrt{r_1 r_2}/\sinh s$, $\vec{n}_{1,2} = \vec{r}_{1,2}/r_{1,2}$.

In formula (4) $J_{2\nu}(y)$ are Bessel functions, $P_\ell(x)$ are Legendre polynomials, $J'_{2\nu}(y) = \frac{d}{dy} J_{2\nu}(y)$. Let us make the following change of variables: $\tau_1 = Rt$, $\tau_2 = R/t$, $\varepsilon_{1,2} R = p_{1,2}$. It is easy to see that the substitution $\vec{\varepsilon}_{1,2} \to -\vec{\varepsilon}_{1,2}$ does not change the trace in (3). Using this fact, one can show that the sum of two terms in (3) is equal to the first term in which the integration over R is extended from $-\infty$ to ∞. In consequence, we have the integral over R

$$\int_{-\infty}^{\infty} \frac{dR \cos(\omega R a)}{\omega R - p_1 - p_2 + i0} = -\frac{i\pi}{\omega} e^{-i|a|(p_1+p_2)} \tag{5}$$

where $a = \vec{\lambda}_1 \vec{n}_1 t - \vec{\lambda}_2 \vec{n}_2 / t$, $\vec{\lambda}_{1,2} = \vec{k}_{1,2}/\omega$. One can see from eqs.(5) and (6) that we have the factorization of the integrals with respect to the variables p_1, p_2, s_1, s_2.

Let us consider a typical integral (other integrals can be calculated quite similarly):

$$N = \int_0^{\infty} dp \int_{-\infty}^{\infty} ds\, e^{i(\varphi - p|a|)} J_{2\nu}\left(\frac{2p}{\operatorname{sh} s}\right) =$$
$$= \int_0^{\infty} dp \int_0^{\infty} ds\, e^{-ip|a|} J_{2\nu}\left(\frac{2p}{\operatorname{sh} s}\right)\left(e^{i\varphi} + e^{-i\varphi}\right), \quad \varphi = 2Z\alpha s + p\left(t + \frac{1}{t}\right)\operatorname{ch} s - \pi\nu \tag{6}$$

where the relation $J_{2\nu}(e^{i\pi}x) = e^{2i\pi\nu} J_{2\nu}(x)$ is used. Then, we change over to the variable $p/\operatorname{sh} s \to p$ and deform the contour of the integration over p in the second term so that the integral is extended from 0 to $-\infty$ $(p \to p \cdot e^{i\pi})$. As a result, we have

$$N = e^{-i\pi\nu} \int_0^{\infty} dp\, J_{2\nu}(2p) \int_{-\infty}^{\infty} ds \cdot \operatorname{sh} s \cdot \exp\left\{ i\left[p\left(t + \frac{1}{t}\right)\operatorname{ch} s + 2Z\alpha s - p|a|\operatorname{sh} s \right]\right\} =$$
$$= \pi \exp\left[\mu(s_0 + i\pi/2) - i\pi\nu\right] \cdot \int_0^{\infty} dp\, J_{2\nu}(2p) \left[\operatorname{sh} s_0\, \dot{H}^{(1)}_{\mu}(p\rho) - \frac{\mu}{p\rho} \operatorname{ch} s_0\, H^{(1)}_{\mu}(p\rho)\right] \tag{7}$$

where $\mu = 2iZ\alpha$, $H^{(1)}_\mu(x)$ is the Hankel function of the first kind, $\dot{H}^{(1)}_\mu(x) = \frac{d}{dx} H^{(1)}_\mu(x)$, $\rho = \sqrt{(t+1/t)^2 - a^2}$, $\operatorname{sh} s_0 = |a|/\rho$, $\operatorname{ch} s_0 = (t+1/t)/\rho$. Calculating the integral over ρ in (7) we have made the change of variable $s \to s + s_0$, and have

used the standard definition of the Hankel functions. Taking the integrals over s_1 and s_2, as in deriving eq.(7), we get for the amplitude M (8):

$$M = -\frac{i\alpha}{2\omega} \sum_{\ell_1,\ell_2=1}^{\infty} \int_0^{\infty} \frac{dt}{t} \int d\vec{n}_1 d\vec{n}_2 \left\{ \frac{1}{2} \Phi_{\nu_1}(\rho) \Phi_{\nu_2}(\rho) B_1 B_2 \left[Z_1 a^2 + Z_4\left(t - \frac{1}{t}\right)^2 \right] + \right.$$
$$+ 2A_1 A_2 \left[\dot{F}_{\nu_1}(\rho) \dot{F}_{\nu_2}(\rho) \left(Z_2 \operatorname{sh}^2 s_0 + Z_5 \operatorname{ch}^2 s_0 \right) - \frac{\mu^2}{\rho^2} F_{\nu_1}(\rho) F_{\nu_2}(\rho) \left(Z_2 \operatorname{ch}^2 s_0 + Z_5 \operatorname{sh}^2 s_0 \right) \right] - \tag{8}$$
$$\left. - 2A_1 B_2 \dot{F}_{\nu_1}(\rho) \Phi_{\nu_2}(\rho) \left[|a| Z_3 \operatorname{sh} s_0 + (t - 1/t) Z_6 \operatorname{ch} s_0 \right] \right\}$$

where

$$\Phi_\nu(\rho) = e^{i\pi(\mu/2 - \nu)} \int_0^{\infty} p\, dp\, J_{2\nu}(2p)\, H^{(1)}_\mu(\rho p)$$
$$F_\nu(\rho) = e^{i\pi(\mu/2 - \nu)} \int_0^{\infty} \frac{dp}{p} J_{2\nu}(2p)\, H^{(1)}_\mu(\rho p) \tag{9}$$

The functions $\Phi_\nu(\rho)$ and $F_\nu(\rho)$ are expressed via the hypergeometric functions (see the Appendix). The subscripts 1, 2 in A_1, B_1, ν_1, A_2, B_2, ν_2 denote the dependence of these quantities on ℓ_1 and ℓ_2, respectively (the definition see after eq.(4)). The coefficients Z_i are

$$Z_1 = (\vec{n}_1 \vec{n}_2)(1 + \vec{n}_1 \vec{n}_2)(\vec{e}_1 \vec{e}_2^{\,*}) + (\vec{e}_1, \vec{n}_1 \times \vec{n}_2)(\vec{e}_2^{\,*}, \vec{n}_1 \times \vec{n}_2) + (1 + \vec{n}_1 \vec{n}_2)[\vec{e}_2^{\,*} \times \vec{e}_1][\vec{n}_2 \times \vec{n}_1],$$
$$Z_4 = (\vec{e}_1, \vec{n}_1 + \vec{n}_2)(\vec{e}_2^{\,*}, \vec{n}_1 + \vec{n}_2) - (\vec{e}_1 \vec{e}_2^{\,*})(1 + \vec{n}_1 \vec{n}_2),$$
$$Z_6 = (\vec{e}_1 \vec{n}_2)(\vec{e}_2^{\,*} \vec{n}_2) - (\vec{e}_2^{\,*} \vec{n}_1)(\vec{e}_1 \vec{n}_1), \tag{10}$$
$$Z_2 = Z_1(\vec{n}_1 \to -\vec{n}_1), \quad Z_3 = \frac{1}{2}(Z_1 + Z_2) - \vec{e}_1 \vec{e}_2^{\,*}, \quad Z_5 = Z_4(\vec{n}_1 \to -\vec{n}_1)$$

These coefficients appear as a re ult of taking the trace.

Let us discuss now the polari ation properties of the amplitude. In terms of linear polar izations, by virtue of parity conservation, the amplitude diifers from zero only if the polarizations of the incoming and outgoing photons lie both in the scattering plane ($M_\parallel$) or are perpendicular to it ($M_\perp$). The corresponding polarization vectors are

$$\vec e_{1\parallel} = (\vec\lambda_2 - \rho\vec\lambda_1)/\sqrt{1-\rho^2}\,, \quad \vec e_{2\parallel} = (\rho\vec\lambda_2 - \vec\lambda_1)/\sqrt{1-\rho^2} \tag{11}$$

$$\vec e_{1\perp} = \vec e_{2\perp} = [\vec\lambda_1\times\vec\lambda_2]/\sqrt{1-\rho^2}\,, \quad \vec e_\perp\vec e_\parallel = 0\,, \quad \vec e_{1\parallel}\vec e_{2\parallel} = \rho$$

where $\rho = \vec\lambda_1\vec\lambda_2$. Therefore, the tensor $T^{ij} = e_1^i e_2^j$ is of the form

$$T_\perp^{ij} = \delta^{ij} + \frac{\rho}{1-\rho^2}(\lambda_1^i\lambda_2^j + \lambda_2^i\lambda_1^j) - \frac{1}{1-\rho^2}(\lambda_1^i\lambda_1^j + \lambda_2^i\lambda_2^j),$$
$$T_\parallel^{ij} = \lambda_1^i\lambda_2^j - \frac{1}{1-\rho^2}(\lambda_1^i\lambda_2^j + \lambda_2^i\lambda_1^j) + \frac{\rho}{1-\rho^2}(\lambda_1^i\lambda_1^j + \lambda_2^i\lambda_2^j) \tag{12}$$

For helical amplitudes, the following relations hold: $M_{++} = M_{--} = (M_\parallel + M_\perp)/2$, $M_{+-} = M_{-+} = (M_\parallel - M_\perp)/2$; the helical polarizations are defined as $\vec e^{\,\pm}_{1,2} = (\vec\xi\times\vec\lambda_{1,2} \pm i\vec\xi)/\sqrt2$, where $\vec\xi = \vec\lambda_1\times\vec\lambda_2/\sqrt{1-\rho^2}$.

Note that the amplitude M is a function of $\vec\lambda_1\vec\lambda_2$, when all the integrals are taken. Therefore, one can use a very convenient trick: let us multiply both sides of eq.(8) by $\delta(\vec\lambda_1\vec\lambda_2 - \rho)/8\pi^2$ and take the integrals over the angles of unit vectors $\vec\lambda_1$ and $\vec\lambda_2$, using the relation $\int\int d\vec\lambda_1 d\vec\lambda_2\,\delta(\vec\lambda_1\vec\lambda_2 - \rho)/8\pi^2 = 1$. After that, the integrand for M in eq.(8) will depend on $\vec n_1$ and $\vec n_2$ only in the combination $\vec n_1\vec n_2$. So, the integration over $\vec n_1$ and $\vec n_2$ reduces to the integration over $x = \vec n_1\vec n_2$

Let us consider now the integral

$$g = \frac{1}{8\pi^2}\int d\vec\lambda_1 d\vec\lambda_2\,\delta(\vec\lambda_1\vec\lambda_2 - \rho)\,e^{i(\vec q_1\vec\lambda_1 - \vec q_2\vec\lambda_2)} \tag{13}$$

Using the well-known expansion of a plane wave in spherical harmonics, we get

$$g = \sum_{\ell=0}^{\infty}(2\ell+1)\,j_\ell(q_1)\,j_\ell(q_2)\,P_\ell(\rho)\,P_\ell(\psi) \tag{14}$$

where $j_\ell(x) = (\pi/2x)^{1/2}\cdot J_{\ell+1/2}(x)$ is the spherical Bessel function, ψ is the angle between vectors $\vec q_1$ and $\vec q_2$. In order to take the integral

$$g^{ij} = \frac{1}{8\pi^2}\int d\vec\lambda_1 d\vec\lambda_2\,\delta(\vec\lambda_1\vec\lambda_2 - \rho)\,e^{i(\vec q_1\vec\lambda_1 - \vec q_2\vec\lambda_2)}\,T^{ij} \tag{15}$$

one can replace $\vec\lambda_1$ by $-i\vec\nabla_{q_1}$ and $\vec\lambda_2$ by $i\vec\nabla_{q_2}$ in T^{ij} (12), and act on g (13) by the operator obtained. We shall illustrate our futher calculations by the consideration of a typical integral:

$$G = \int_0^\infty\frac{dt}{t}\int\frac{d\vec\lambda_1 d\vec\lambda_2}{8\pi^2}\,\delta(\vec\lambda_1\vec\lambda_2 - \rho)\,\mathcal{D}(\varrho) \tag{16}$$

where $\mathcal{D}(\varrho)$ is a function of the variable $\varrho = \sqrt{(t+1/t)^2 - a^2}$, $a = \vec\lambda_1\vec n_1 t - \vec\lambda_2\vec n_2/t$ (see (7)).
Making the identical transformation

$$\mathcal{D}(\varrho) = \int_{-\infty}^{\infty}dx\,\mathcal{D}\left(\sqrt{(t+1/t)^2 - x^2}\right)\int_{-\infty}^{\infty}\frac{d\xi}{2\pi}\exp\left\{-i\xi\left(x + \vec\lambda_1\vec n_1 t - \vec\lambda_2\vec n_2/t\right)\right\}, \tag{17}$$

substituting (17) into (16) and using (13) we get

$$G = 2\sum_{\ell=0}^{\infty}(2\ell+1)\int_0^\infty\frac{dt}{t}\int_0^\infty dx\,\mathcal{D}\left(\sqrt{(t+1/t)^2 - x^2}\right)\int_{-\infty}^{\infty}\frac{d\xi}{2\pi}\cos(\xi x)\,j_\ell(\xi t)\,j_\ell(\xi/t)\,P_\ell(\rho)\,P_\ell(\vec n_1\vec n_2) \tag{18}$$

Let us use now the following formula for a product $j_\ell(x_1)j_\ell(x_2)$ (see [17], p.838):

$$j_\ell(x_1)\,j_\ell(x_2) = \frac12\int_{-1}^{1}dy\,P_\ell(y)\,\frac{\sin\varkappa}{\varkappa} = \frac12\int_{-1}^{1}dy\,P_\ell(y)\int_0^1 dz\,\cos(\varkappa z),$$
$$\varkappa = \sqrt{x_1^2 + x_2^2 - 2x_1x_2y} \tag{19}$$

With (19), we take the integral over ξ and then over x . We obtain

$$G=\frac{1}{2}\sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\beta)P_\ell(\vec{n}_1\vec{n}_2)\int_0^\infty\frac{dt}{t}\int_0^1 dz\int_{-1}^1 dy\, P_\ell(y)\,\mathcal{D}\left(\sqrt{(t+\tfrac{1}{t})^2-z^2(t^2+\tfrac{1}{t^2}-2y)}\right) \quad (20)$$

Then we make the transformation such as in eq.(17):

$$\mathcal{D}=\int_0^\infty \rho\, d\rho\, \mathcal{D}(\rho)\int_{-\infty}^{\infty}\frac{d\varsigma}{2\pi}\exp\left\{i\varsigma\left[(t^2+1/t^2)(1-z^2)/2+yz^2+1-\rho^2/2\right]\right\} \quad (21)$$

Substituting (21) into (20) and making a change of the variable $t=e^{\varphi/2}$, we carry out the integration over φ and y. We have

$$G=\frac{1}{4}\sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\beta)P_\ell(\vec{n}_1\vec{n}_2)\int_0^\infty\rho\, d\rho\,\mathcal{D}(\rho)\int_0^1 dz\int_0^\infty d\varsigma\, j_\ell(\varsigma z^2)\cdot$$
$$\cdot\left[i^{\ell+1}e^{i\varsigma(1-\rho^2/2)}H_0^{(1)}(\varsigma(1-z^2))+\frac{1}{i^{\ell+1}}e^{-i\varsigma(1-\rho^2/2)}H_0^{(2)}(\varsigma(1-z^2))\right] \quad (22)$$

Here $H_0^{(1)}$ and $H_0^{(2)}$ is the Hankel functions of the first and the second kind, respectively. One can take now the integral first over z and then over ς, using the relations (ref.[17], pp.692 and 725)

$$\int_0^1\frac{dz}{z}J_\mu(\varsigma z)J_\nu(\varsigma(1-z))=\frac{J_{\mu+\nu}(\varsigma)}{\mu} \quad (23)$$

$$\int_0^\infty\frac{dx}{\sqrt{x}}e^{ipx}J_{\nu+1/2}(x)=\sqrt{\frac{2}{\pi}}\,e^{i\pi(\nu+1)/2}\,Q_\nu(p+i0)$$

where Q_ν are the Legendre functions. The representation of the Hankel function

$$H_0^{(1,2)}=J_0(x)\pm\frac{2i}{\pi}\tilde{J}_0(x)\,,\quad \tilde{J}_0(x)\equiv\frac{\partial}{\partial\nu}J_\nu(x)\Big|_{\nu=0}$$

has to be used too. Finally, one obtains, for the integral G

$$G=\sum_{\ell=0}^{\infty}P_\ell(\beta)P_\ell(\vec{n}_1\vec{n}_2)\int_0^\infty\rho\, d\rho\,\mathcal{D}(\rho)\,S_1(\rho) \quad (24)$$

where $S_1(\rho)$ is

$$S_1(\rho)=\frac{1}{2}(-1)^{\ell+1}\vartheta(2-\rho)\tilde{P}_\ell(1-\rho^2/2)+\upsilon(\rho-2)\,Q_\ell(\rho^2/2-1) \quad (25)$$

$\tilde{P}_\ell(x)\equiv\frac{\partial}{\partial\nu}P_\nu(x)\Big|_{\nu=\ell}$. Then it is easy to take the integrals over $\vec{n}_1$ and $\vec{n}_2$ in the amplitude M (8). The corresponding integrals are expressed via the Wigner 3-j symbols. In the same manner, one can obtain the expression for the whole amplitude M.

As was mentioned above, there are two amplitudes $M_\parallel$ and $M_\perp$ which differ from zero, in terms of linear polarizations. It is convenient to consider the combination $M_1=(M_\parallel+\beta M_\perp)/2$ and the amplitude $M_2\equiv M_\perp$. Let us start with the evaluation of M_1. After the integration over $\vec{\lambda}_1$ and $\vec{\lambda}_2$, we obtain that the coefficients (10) in eq.(8) for the amplitude M_1, should be replaced by

$$Z_i=\frac{1}{2}\sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\beta)f_i$$

$$f_{1,4}=\pm\left\{(1+x)P_\ell(x)\left[\beta j_\ell(q_1)j_\ell(q_2)+j'_\ell(q_1)j'_\ell(q_2)\pm\ell(\ell+1)\frac{j_\ell(q_1)j_\ell(q_2)}{q_1q_2}\right]-\right.$$
$$\left.-(1-x^2)P'_\ell(x)\frac{j_\ell(q_1)j_\ell(q_2)}{q_1q_2}\right\},$$

$$f_{2,5}=\pm\left\{(1-x)P_\ell(x)\left[\beta j_\ell(q_1)j_\ell(q_2)-j'_\ell(q_1)j'_\ell(q_2)\mp\ell(\ell+1)\frac{j_\ell(q_1)j_\ell(q_2)}{q_1q_2}\right]-\right. \quad (26)$$
$$\left.-(1-x^2)P'_\ell(x)\frac{j_\ell(q_1)j_\ell(q_2)}{q_1q_2}\right\},$$

$$f_3=-(1-x^2)P'_\ell(x)\left[2\frac{j_\ell(q_1)j_\ell(q_2)}{q_1q_2}+j'_\ell(q_1)j_\ell(q_2)/q_2+j'_\ell(q_2)j_\ell(q_1)/q_1\right],$$

$$f_6=-(1-x^2)P'_\ell(x)\left[j'_\ell(q_2)j_\ell(q_1)/q_1-j'_\ell(q_1)j_\ell(q_2)/q_2\right]$$

where $x=\vec{n}_1\vec{n}_2$, $j'_\ell(x)=\frac{d}{dx}j_\ell(x)$, $q_1=\varsigma t$, $q_2=\varsigma/t$. The representations for the products of Bessel functions in (26) are given in the Appendix. Then we take the integrals in the sequence used in deriving eq.(24). We represent our amplitude

$M_{1,2}$ as follows

$$M = -\frac{8i\alpha\pi^2}{\omega}\sum_{\ell_1,\ell_2=1}^{\infty}\sum_{\ell=0}^{\infty}\int_0^{\infty}\rho\,d\rho\Big\langle \Phi_{\nu_1}(\rho)\Phi_{\nu_2}(\rho)\,C_1 -$$

$$-\Big(\dot F_{\nu_1}(\rho)\dot F_{\nu_2}(\rho) + \frac{\mu^2}{\rho^2}F_{\nu_1}(\rho)F_{\nu_2}(\rho)\Big)C_2 - \qquad (27)$$

$$-\Big(\dot F_{\nu_1}(\rho)\dot F_{\nu_2}(\rho) - \frac{\mu^2}{\rho^2}F_{\nu_1}(\rho)F_{\nu_2}(\rho)\Big)C_3 - \frac{4}{\rho}\dot F_{\nu_1}(\rho)\Phi_{\nu_2}(\rho)\,C_4\Big\rangle$$

After cumbersome calculations we obtain for the coefficients C_i in M_1 the following expressions:

$$C_1 = \Big\{\gamma_1\big[\ell(\ell+1)(S_3-S_2)+\sigma(\beta S_1+S_5)-\sigma\gamma_2 S_2\Big\}P_\ell(\beta)/2$$

$$C_2 = \Big\{\gamma_3\big[\ell(\ell+1)S_2+\beta S_1 - S_5\big]-\gamma_4 S_2\Big\}P_\ell(\beta)/2$$

$$C_3 = \frac{2\ell(\ell+1)}{\rho^2}\gamma_3(S_3+\sigma S_2)P_\ell(\beta) \qquad (28)$$

$$C_4 = -\big[3S_3+(2\sigma-1)(S_4+2S_2)+S_6\big]\gamma_5 P_\ell(\beta)/4$$

Here $\sigma = 1-\rho^2/4$, $\beta=\vec\lambda_1\vec\lambda_2=\cos\theta_0$. The coefficients γ_i are the integrals over $x=\vec n_1\vec n_2$:

$$\gamma_1 = \int_{-1}^{1}dx(1+x)P_\ell(x)B_1(x)B_2(x)\ ,\quad \gamma_3=(-1)^{\ell_1+\ell_2+\ell}\,\ell_1\ell_2\,\gamma_1 ,$$

$$\gamma_2 = \int_{-1}^{1}dx(1-x^2)P_\ell'(x)B_1(x)B_2(x)\ ,\quad \gamma_4=(-1)^{\ell_1+\ell_2+\ell+1}\,\ell_1\ell_2\,\gamma_2 \qquad (29)$$

$$\gamma_5 = \int_{-1}^{1}dx(1-x^2)P_\ell'(x)A_1(x)B_2(x)$$

These coefficients are expressed via the Wigner 3-j symbols (see the Appendix). The functions S_i appear as a result of integration over the parameters ξ and z (compare with eqs.(23-25)):

$$S_1 = \frac{(-1)^{\ell+1}}{2}\vartheta(2-\rho)\tilde P_\ell + \vartheta(\rho-2)\,Q_\ell ,$$

$$S_2 = \frac{(-1)^\ell\vartheta(2-\rho)}{2(2\ell+1)}\left[\frac{\tilde P_{\ell-1}}{2\ell-1}+\frac{\tilde P_{\ell+1}}{2\ell+3}+\frac{P_{\ell-1}-P_{\ell+1}}{2\ell+1}\right]+\frac{\vartheta(\rho-2)}{2\ell+1}\left[\frac{Q_{\ell-1}}{2\ell-1}+\frac{Q_{\ell+1}}{2\ell+3}\right],$$

$$S_3 = (-1)^{\ell+1}\vartheta(2-\rho)\left[\frac{\tilde P_\ell}{(2\ell-1)(2\ell+3)}+\frac{P_{\ell-2}-P_\ell}{4(2\ell-1)^2}+\frac{P_\ell-P_{\ell+2}}{4(2\ell+3)^2}\right]+\frac{2\vartheta(\rho-2)Q_\ell}{(2\ell-1)(2\ell+3)} ,$$

$$S_4 = \frac{(-1)^\ell\vartheta(2-\rho)}{2\ell+1}\left[\frac{\ell\tilde P_{\ell-1}}{2\ell-1}-\frac{(\ell+1)\tilde P_{\ell+1}}{2\ell+3}+\frac{P_{\ell+1}-P_{\ell-1}}{2(2\ell+1)}\right]+\frac{2\vartheta(\rho-2)}{2\ell+1}\left[\frac{\ell Q_{\ell-1}}{2\ell-1}-\frac{(\ell+1)Q_{\ell+1}}{2\ell+3}\right], \qquad (30)$$

$$S_5 = \frac{(-1)^\ell}{2}\vartheta(2-\rho)\left[\frac{\ell\tilde P_{\ell-1}}{2\ell-1}+\frac{(\ell+1)\tilde P_{\ell+1}}{2\ell+3}\right]+\vartheta(\rho-2)\left[\frac{\ell Q_{\ell-1}}{2\ell-1}+\frac{(\ell+1)Q_{\ell+1}}{2\ell+3}\right],$$

$$S_6 = \frac{(-1)^\ell}{4}\vartheta(2-\rho)\left[\frac{P_\ell-P_{\ell-2}}{2\ell-1}+\frac{P_\ell-P_{\ell+2}}{2\ell+3}\right]$$

where the Legendre polynomials $\tilde P_\ell$ (recall that $\tilde P_\ell(x) = \frac{d}{d\nu}P_\nu(x)\big|_{\nu=\ell}$) depend on $q=1-\rho^2/2$, and the Legendre functions Q_ℓ depend on $p=\rho^2/2-1$. It is easy to verify that C_i have no singularities at $\rho = 2$. By virtue of the momentum conservation $M(z=0) = 0$ for the case under study $\Delta \neq 0$. It is convenient to subtract, from the integrand of M in (27), its value at $z = 0$. This subtraction removes fictitions divergences, which cancel after the summation over $\ell_{1,2}$ and ℓ. In the following such a subtraction is assumed to be made.

Quite similar are the calculations of the coefficients in M_2 :

$$C_1 = \sigma P_\ell(\beta)\Big\{\gamma_1 S_1+\frac{2}{1-\beta^2}\Big[\beta\gamma_2 S_2+\frac{\gamma_6}{2}(S_3+S_6)-S_3\gamma_7\Big]\Big\}+\gamma_1(S_3-S_2)P_\ell'(\beta) ,$$

$$C_2 = P_\ell(\beta)\Big\{\gamma_3 S_1+\frac{2}{1-\beta^2}\Big[\beta\gamma_4 S_2+\frac{\gamma_8}{2}(S_3+S_6)-S_3\gamma_9\Big]\Big\}+\frac{4}{\rho^2}(1-\sigma)\gamma_3 S_2 P_\ell'(\beta) , \qquad (31)$$

$$C_3 = \frac{4}{\rho^2}\gamma_3\big[\sigma S_2+S_3\big]P_\ell'(\beta) ,$$

$$C_4 = \frac{P_\ell(\rho)}{1-\rho^2}\left\{\rho\gamma_5\left[(2\sigma-1)S_2+S_3\right]-\gamma_{10}\left[(2\sigma-1)S_3+S_2\right]+\right.$$

$$\left.+\frac{\gamma_{11}}{2}\left[(2\sigma-1)(S_3+S_6)+S_4\right]\right\}$$

The coefficients γ_{6-11} are defined by

$$\gamma_6 = \int_{-1}^{1} dx\,(1-x^2)P_\ell(x)B_1(x)B_2(x)\,,\quad \gamma_7 = \int_{-1}^{1} dx\; x(1-x^2)P'_\ell(x)B_1(x)B_2(x)\,,$$

$$\gamma_8 = (-1)^{\ell_1+\ell_2+\ell}\,\ell_1\ell_2\gamma_6\,,\quad \gamma_9 = (-1)^{\ell_1+\ell_2+\ell}\,\ell_1\ell_2\gamma_7\,, \tag{32}$$

$$\gamma_{10} = \int_{-1}^{1} dx\; x(1-x^2)P'_\ell(x)A_1B_2\,,\quad \gamma_{11} = \int_{-1}^{1} dx\,(1-x^2)P_\ell(x)A_1B_2$$

It is seen that the amplitude M (27) has a scaling form $M = f(\theta_0)/\omega$ as $\omega/m \to \infty$ with θ_0 fixed, i.e. is in agreement with the result of ref.[8].

Let us discuss the asymptotic form of M_1 and M_2 at $\theta_0 \ll 1$ ($\Delta \ll \omega$). In this case the main contribution to the amplitudes comes from the region $\ell_1 \sim \ell_2 \sim \ell \sim 1/\theta_0$, $\rho \sim \theta_0$, $1+x \sim \theta_0^2$. So, one can neglect $(Z\alpha)^2$ in the quantities $\nu_{1,2} = \sqrt{\ell_{1,2}^2-(Z\alpha)^2}$. After that one can takes the sum over ℓ_1 and ℓ_2 before the integration over x (29), (32). This summation can be performed, using the formulae in the Appendix of ref.[13]. Then, replacing the Legendre polynomials by its asymptotics we obtain

$$S_1 = -S_5 \simeq \frac{(-1)^\ell}{2\ell}\, y J_1(y)\,,\quad S_3 = 2S_2 \simeq \frac{(-1)^{\ell+1}}{6\ell^5}\, y^3 J_3(y)\,, \tag{33}$$

$$S_4 = S_6 \simeq \frac{(-1)^{\ell+1}}{2\ell^3}\, y^2 J_2(y)$$

where $y = \ell\rho$. We have also $P_\ell(\rho) \simeq J_0(\ell\theta_0)$ and $P_\ell(x) \simeq (-1)^\ell J_0(\ell\theta)$, where $x = -\cos\theta \simeq -1+\theta^2/2$. Substituting these asymptotics into (27) we see that $(-1)^\ell$ disappears and we can replace the summation over ℓ by integration. Performing this integration and then taking the integral over ρ and $P_{1,2}$ (see the definitions of Φ_ν and F_ν (9)), we obtain

ultimately for the small-angle asymptotics:

$$M_1 = -i\,\frac{8\alpha}{3\omega\theta_0^2}\left[\frac{2\pi Z\alpha}{\mathrm{sh}(2\pi Z\alpha)}\left(1-2(Z\alpha)^2\right)-1\right] \tag{34}$$

$$M_2 = M_1 + \frac{i8\alpha}{\omega\theta_0^2}(Z\alpha)^2\left[Z\alpha\,\mathrm{Im}\,\psi'(1-iZ\alpha)-1\right]$$

where $\psi(x) = \frac{d}{dx}\ln\Gamma(x)$, $\psi'(x) = \frac{d}{dx}\psi(x)$. Note that $M_1 \to M_{++}$ as $\rho \to 1$. Our result (34) coincides with the results of refs. [9], [12-13]. The asymptotics (34) are the imaginary quantities, but at $\theta_0 \sim 1$ the real parts of the amplitudes (27) are not equal to zero.

3. DISCUSSION

Eqs.(27), (28) and (31) solve, in general form, the problem of calculation of high-energy Delbrück amplitude at large angles. One should bear in mind that in scattering by atoms the point-charge approximation is valid if $\Delta \ll R^{-1}$, where R is the radius of the nucleus. Then, one has to know the corrections of order $(m/\omega)^2$ to determine ω for which our results are applicable. This problem is very difficult, but it can be solved using the technology of the present paper. We hope that the appropriate photon energy ω is not very high.

The problem of numerical calculations with the use of eq.(27) and comparison with the experimental data is an independent one. Note that the terms with a small $\ell_{1,2}$ and ℓ give the contribution to the amplitude at $\theta_0 \sim 1$. We will discuss this problem elsewhere.

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APPENDIX

In the Appendix we discuss the properties of the functions introduced in the text. Let us consider the functions $\Phi_\nu(\rho)$ and $F_\nu(\rho)$ (see (9)). These functions are expressed via the hypergeometric functions. If $\rho > 2$, we have

$$\Phi_\nu(\rho) = \frac{i}{2\pi}\left(\frac{4}{\rho^2}\right)^{\nu+1} \frac{\Gamma(1+\nu+\mu/2)\,\Gamma(1+\nu-\mu/2)}{\Gamma(2\nu+1)}\, F\left(1+\nu+\mu/2,\, 1+\nu-\frac{\mu}{2};\, 2\nu+1;\, \frac{4}{\rho^2+i0}\right) \tag{A.1}$$

$$F_\nu(\rho) = -\frac{i}{2\pi}\left(\frac{4}{\rho^2}\right)^{\nu} \frac{\Gamma(\nu+\mu/2)\,\Gamma(\nu-\mu/2)}{\Gamma(2\nu+1)}\, F\left(\nu+\mu/2,\, \nu-\mu/2;\, 2\nu+1;\, \frac{4}{\rho^2+i0}\right)$$

If $\rho < 2$, the formulae for Φ_ν and F_ν are the analytical continuation of (A.1). The function $F_\nu(\rho)$ has no singularities at $\rho = 2$: $F_\nu(2) = -i\left[2\pi(\nu^2-\mu^2/4)\right]^{-1} = -i/2\pi \ell^2$ (recall that $\nu^2 = \ell^2 + \mu^2/4$). The function $\Phi_\nu(\rho)$ has a singularity as $\rho \to 2$. However, the coefficients at the divergent terms do not depend on Z:

$$\Phi_\nu(\rho) \approx \frac{i}{2\pi}\left\{ \frac{1}{\rho^2/4 - 1 + i0} - \nu + \ell^2\left[\ln(\rho^2/4 - 1 + i0) + \Psi(1+\nu+\mu/2) + \right.\right.$$
$$\left.\left. + \Psi(1+\nu-\mu/2) - \Psi(1) - \Psi(2)\right]\right\} \tag{A.2}$$

We give now the formulae for the products of spherical Bessel functions, which are needed for the calculations, like in the derivation of eq.(22). Using (19) one can obtain that

$$\frac{j_\ell(q_1)\,j_\ell(q_2)}{q_1 q_2} = \frac{\delta_{\ell 0}}{\xi^2} - \frac{1}{4}\int_{-1}^{1} dy\, P_\ell(y)\, \varkappa^2 \int_0^1 dz\,(1-z)^2 \cos(z\varkappa\xi)$$

$$j'_\ell(q_1)\,j_\ell(q_2)/q_2 + j'_\ell(q_2)\,j_\ell(q_1)/q_1 = -\frac{1}{4}\int_{-1}^{1} dy\, P_\ell(y)\, \varkappa^2 \int_0^1 dz\,(1-z^2)\cos(z\varkappa\xi)$$

$$j'_\ell(q_1)\,j_\ell(q_2)/q_2 - j'_\ell(q_2)\,j_\ell(q_1)/q_1 = -\frac{1}{4}\,(t^2 - 1/t^2)\,\cdot$$
$$\cdot \int_{-1}^{1} dy\, P_\ell(y) \int_0^1 dz\,(1-z^2)\cos(z\varkappa\xi) \tag{A.3}$$

Here $q_1 = \xi t$, $q_2 = \xi/t$, $\varkappa = \sqrt{t^2 + 1/t^2 - 2y}$.
To obtain the formula for the product $j'_\ell(q_1)\,j'_\ell(q_2)$, one has

to perform the integration by parts over ξ (see (18). Then, with the use of the second relation in (A.3) we get that this product may be replaced by

$$j'_\ell(q_1)\,j'_\ell(q_2) \to j_\ell(q_1)\,j_\ell(q_2)\left[\frac{\rho}{2} - 1 - \frac{\ell(\ell+1)}{q_1 q_2}\right] + \frac{j_\ell(q_1)}{q_1}\,j'_\ell(q_2) + \frac{j_\ell(q_2)}{q_2}\,j'_\ell(q_1) \tag{A.4}$$

Let us calculate now the coefficients γ_i (see (29) and (32)). As mentioned above, these quantities are expressed via the Wigner 3-j symbols. We use the recursion relations for the Legendre polynomials (see e.g. [17])

$$(1+x)\frac{d}{dx}\left(P_\ell(x) - P_{\ell-1}(x)\right) = \ell\left(P_\ell(x) + P_{\ell-1}(x)\right),$$

$$\sqrt{1-x^2}\,\frac{d}{dx}\left(P_\ell(x) - P_{\ell-1}(x)\right) = P^1_\ell(x) - P^1_{\ell-1}(x), \tag{A.5}$$

$$P'_\ell(x) = \frac{1}{2}\left[P^2_{\ell+1}(x) + \ell(\ell+1)\,P_{\ell+1}(x)\right]$$

and the following formula (see [18], p.63)

$$\int_{-1}^{1} dx\, P_\ell^{|m|}(x)\, P_{\ell_1}^{|m_1|}(x)\, P_{\ell_2}^{|m_2|}(x) = 2\left(a^{m_1}_{\ell_1} a^{m_2}_{\ell_2} a^{m}_{\ell}\right)^{-\frac{1}{2}} \begin{pmatrix} \ell & \ell_1 & \ell_2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell & \ell_1 & \ell_2 \\ -m & m_1 & m_2 \end{pmatrix} \tag{A.6}$$

where $P^m_\ell(x)$ are the associated Legendre polynomials, $a^m_\ell = (\ell - |m|)!/(\ell + |m|)!$. The signs of m, m_1 and m_2 are chosen so that $m = m_1 + m_2$. Let

$$f_\ell = P_\ell + P_{\ell-1},\quad g_\ell = P_\ell - P_{\ell-1},\quad f^1_\ell = P^1_\ell + P^1_{\ell-1},\quad g^1_\ell = P^1_\ell - P^1_{\ell-1} \tag{A.7}$$

With the help of (A.5) we have

$$\gamma_1 = \frac{1}{2}\int_{-1}^{1} dx\, P_\ell(x)\left[\ell_1 \ell_2\, f_{\ell_1}(x)\, f_{\ell_2}(x) + g^1_{\ell_1}(x)\, g^1_{\ell_2}(x)\right],$$

$$\gamma_2 = \frac{1}{2}\int_{-1}^{1} dx\, g^1_{\ell_1}(x)\, g^1_{\ell_2}(x)\left[P^2_{\ell+1}(x) + \ell(\ell+1)\,P_{\ell+1}(x)\right],$$

$$\gamma_3 = (-1)^{\ell_1+\ell_2+\ell}\,\ell_1\ell_2\,\gamma_1,\quad \gamma_4 = (-1)^{\ell_1+\ell_2+\ell+1}\,\ell_1\ell_2\,\gamma_2, \tag{A.8}$$

$$\gamma_5 = \frac{\ell_1}{2} \int_{-1}^{1} dx\, f^1_{\ell_1}(x)\, g^1_{\ell_2}(x) \left[P^2_{\ell+1}(x) + \ell(\ell+1) P_{\ell+1}(x) \right] ,$$

$$\gamma_6 = \int_{-1}^{1} dx\, P_\ell(x)\, g^1_{\ell_1}(x)\, g^1_{\ell_2}(x) ,$$

$$\gamma_7 = \frac{1}{2} \int_{-1}^{1} dx\, g^1_{\ell_1}(x)\, g^1_{\ell_2}(x) \left[P^2_\ell(x) + \ell(\ell+1) P_\ell(x) \right] ,$$

$$\gamma_8 = \ell_1 \ell_2 (-1)^{\ell_1+\ell_2+\ell} \gamma_6 , \quad \gamma_9 = \ell_1 \ell_2 (-1)^{\ell_1+\ell_2+\ell} \gamma_7 ,$$

$$\gamma_{10} = \frac{\ell_1}{2} \int_{-1}^{1} dx\, f^1_{\ell_1}(x)\, g^1_{\ell_2}(x) \left[\ell(\ell+1) P_\ell(x) + P^2_\ell(x) \right] ,$$

$$\gamma_{11} = \ell_1 \int_{-1}^{1} dx\, f^1_{\ell_1}(x)\, g^1_{\ell_2}(x)\, P_\ell(x)$$

Substituting (A.6) into (A.8), the final result can be easily obtained.

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