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ABSTRACT

The axial current of photons, $K^{\mu} = -\varepsilon^{\mu\nu\varkappa\lambda} A_{\nu} \partial_{\varkappa} A_{\lambda}$, in an external gravitational field is shown to possess a triangle anomaly analogous to that of the axial fermionic current.

The discovery of the famous triangle anomaly for the axial-vector current in an external electromagnetic field [1] triggered the search for a similar anomaly in case of external gravitational field [2]. The anomaly has been found, so that the total divergence of the current equals to [2]

$$\partial_{\mu} a^{\mu} = \frac{\alpha}{2\pi} F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{192\pi^2} R_{\mu\nu\alpha\beta} \tilde{R}^{\mu\nu\alpha\beta}, \qquad (1)$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor, $R_{\mu\nu\alpha\beta}$ is the Riemann tensor, $\tilde{F}^{\mu\nu}$, $\tilde{R}^{\mu\nu\alpha\beta}$ are the tensors dual to $F_{\mu\nu}$ and $R_{\mu\nu\alpha\beta}$ respectively:

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}, \qquad \tilde{R}^{\mu\nu\alpha\beta} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} R_{\rho\sigma}{}^{\alpha\beta}.$$

First of all we would like to note that the gravitational anomaly is in fact a specific spin effect. Indeed, let $R\tilde{R}$ be nonvanishing in some region of space. Then according to eq. (1) there exists a flow of leptonic charge from this region since the axial current a_{μ} coincides with the leptonic charge current in case of the Weyl neutrinos. Since, on the other hand, leptonic charge does not interact with gravity, the prediction looks somewhat unexpected. To our mind, the resolution of the paradox is that for massless neutrinos the leptonic charge is in one-to-one correspondence with the chirality of the particle. Then eq. (1) is naturally interpreted as a manifestation of spin polarizability of the vacuum. Namely, because of the spin interaction with gravitational field particles of a certain chirality can be attracted to the source of the field while the particles of the opposite chirality are pushed away.

If this interpretation is true, a similar anomaly must exist for bosonic fields as well. Indeed, the gravitational spin interaction is universal and so should be the anomaly.

Our claim is that such a bosonic anomaly does exist and in case of photons reads as

$$\partial_\mu K^\mu = -\frac{1}{96\pi^2} R_{\mu\nu\alpha\beta} \tilde{R}^{\mu\nu\alpha\beta}, \tag{2}$$

where $K^\mu = -\varepsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta$ and A_ν is the vector-potential of the electromagnetic field. Because of the operator identity $\partial_\mu K^\mu = -\frac{1}{2} F\tilde{F}$ eq. (2) can alternatively be written as

$$F\tilde{F} = \frac{1}{48\pi^2} R\tilde{R}. \tag{3}$$

Let us elucidate now why eq. (2) is indeed a direct generalization of eq. (1) to the photonic case. The very choice of the K^μ as the axial-vector photonic current might call for a substantiation. To this end let us start with an infinitesimal but nonvanishing photonic mass m_γ. Such procedure is anyhow convenient when considering anomalies. Then both K^μ and a^μ coincide with the Pauli—Lubanski vector [3] for the photonic and fermionic fields, respectively. Moreover, in the limit of m tending to zero the mean value of the operator $\int K^0 d^3x$ equals to $+1$ for the left-handed photons and to -1 for the right-handed photons provided that the wave functions are normalized in the standard way. Thus, the «charge» $\int K^0 d^3x$ measures the difference between the number of left- and right-handed photons just in the same way as $Q = \int a^0 d^3x$ does in case of neutrinos.

The current K^μ is not gauge invariant. However, the charge $\int K^0 d^3x$ is known to be invariant under gauge transformations so that it can be used to classify the states.

As the next step consider matrix elements induced by the currents considered and describing production of two photons or two gravitons. Each matrix element is determined then by a single form-factor:

$$\langle 0 | a_\mu | 2\gamma \rangle = f_1(q^2) q_\mu F\tilde{F},$$

$$\langle 0 | a_\mu | 2g \rangle = f_2(q^2) q_\mu R\tilde{R}, \tag{4}$$

$$\langle 0 | K_\mu | 2g \rangle = f_3(q^2) q_\mu F\tilde{F},$$

where q is the momentum carried by the current. Naively the imaginary parts of the form-factors f vanish. In particular, the imaginary part $\mathrm{Im} f_3(q^2)$ vanishes by virtue of the conservation of chirality of photons in a background gravitational field. The manifestation of anomalies in this language is the emergence of δ-functions, $\delta(q^2)$ in the imaginary parts considered. To detect such terms introduce first finite photonic (fermionic) mass and then turn it to zero. One can verify that the dispersion integral of $\mathrm{Im} f_{1-3}$ remains finite in the limit of the vanishing mass. Such a treatment of the triangle anomaly in external electromagnetic field was introduced first in Ref. [4]. Here we generalize this approach to the case of the chiral anomaly in the gravitational background.

The straightforward calculations result in the following:

$$\mathrm{Im} f_1(q^2) = -\frac{\alpha}{4q^2}(1-v^2) \ln\frac{1+v}{1-v},$$

$$\mathrm{Im} f_2(q^2) = \frac{1}{128\pi} \frac{1}{q^2}(1-v^2)^2 \ln\frac{1+v}{1-v}, \tag{5}$$

$$\mathrm{Im} f_3 = \frac{1}{32\pi} \frac{1}{q^2}(1-v^2) v^2 \ln\frac{1+v}{1-v},$$

where v is the center-of-mass velocity. Furthermore, invoking dispersion relations to evaluate the real parts of the form-factors f_{1-3} we come to eqs (2, 3).

It is worth noting that some bosonic chiral anomalies have been already discussed in literature. These are the anomaly for the antisymmetric potential $A_{\mu\nu}$ in the gravitational background [5] and the anomaly for the gluonic current K_μ in the external Yang—Mills field [6]. All the known chiral anomalies can be viewed as anomalies of the Pauli—Lubanski spin current. It is only natural to expect therefore that a similar anomaly holds in case of gravitonic analog of K_μ in a gravitational background. Moreover, one might speculate that the coefficient in front of the $R\tilde{R}$ in the r.h.s. of the anomaly equation is proportional to the square of the chirality of the massless particles involved (i. e. the ratios are 1:4:16 for the Weyl spinor, photon and gravitons, respectively).

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