ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

E.V. Shuryak

INSTANTONIC MOLECULES AND THE NONPERTURBATIVE DIVERGENCIES

PREPRINT 87-24

НОВОСИБИРСК

Instantonic Molecules and the Nonperturbative Divergencies

E.V. Shuryak

Institute of Nuclear Physics
630090, Novosibirsk, USSR

ABSTRACT

We show that, unlike single instantons, the small size instantonic molecules become more important with the increase in number of light fermions till, finally, their density becomes ultraviolet divergent. The corresponding critical number is below the point where the asymptotic freedom breaks down.

General role of the light quarks in the QCD vacuum remains a matter of dispute in literature. Some people suggest (and practically apply) the so called «quenching» approximation, neglecting the fermionic determinant in the gauge field weight function. To the contrary, studies of the instanton-related phenomena shows that this determinant strongly affect gauge field configurations, see e. g. our previous work [1].

In this work we present new arguments on the importance of light fermions, considering what happens if one gradually increases their number. We show that the quark-induced interaction among instantons and antiinstantons gradually becomes so strong, that, via a set of phase transitions, this effect probably makes the theory quite meaningless. As far as we know, such phenomenon leading to power ultraviolet divergences of the nonperturbative nature is new for gauge theories, although it is somewhat similar to the well known (logarithmic) divergence of the one-instanton density in the two-dimensional O(3) sigma model .

The instantonic molecules are some gauge field configurations made of an instanton-antiinstanton pair. Similar objects are well known in physics (e. g. the «vortex molecules» in some planar models), their first discussion for $d=4$ gauge theories with light fermions was made in Ref. [2]. More precisely, we are going to discuss «small molecules» which have both the pseudoparticle radii ρ_1, ρ_2 and their separation R small compared to strong coupling scale $\Lambda^{-1}\sim 1$ fm. Therefore, the relevant action is much larger than unity and the relative smallness of quantum effects is garanteed. We also

distinquish «compact» and «dilute» molecules, with $R<\rho$ and $R>\rho$, as well as «symmetric» and «asymmetric» ones, with $\rho_1\sim\rho_2$ and $\rho_1\gg\rho_2$.

As it was discovered by t'Hooft [3], one single instanton cannot exist in the «perturbative» vacuum of the gauge theory with massless quarks. If its mass m is nonzero, or one accounts for the nonzero quark condensate $\langle\bar{\psi}\psi\rangle$ [4], the instanton (space-time) density $dn(\rho, N_f)$ (depending on the radius ρ) is equal to

$$dn(\rho, N_f)\sim(m\rho+\text{const}\,\langle\bar{\psi}\psi\rangle\rho^3)^{N_f}\,dn(\rho, N_f=0) \qquad (1)$$

(As we are interested only in the dependence on the number of light flavors N_f, we do not discuss that for pure gauge theory $dn(\rho, N_f=0)$.) The main lesson is as follows: increasing N_f one finds stronger suppression of small size instantons. As it is well known, such suppression is related to the fermionic zero modes of the topologically nontrivial field configurations. We are going to show, that there is no similar suppression for (topologically trivial) «molecules».

Let us discuss the theoretical status of the molecular-type configurations, which were not much considered in literature. They are not solutions of the Yang—Mills equations for they are not the action minima. Instead, they form a kind of a «valley» , with a nonzero slope along some «longitudinal» coordinate, changing along the valley and being the instanton—antiinstanton separation R. However, if this slope is much smaller than that in all other directions in the configuration space, its separation is meaningful. The so called «streamline» set of configurations (the bottom of the valley) were explicitely found for some toy models [5, 6]. Unfortunately, it is not yet done for gauge theories.

Therefore people use some approximate parametrizations of the configurations in question. The simplest «ansatz» suggested in Ref. [7] is just a sum of $A^a_\mu(x)$ for the instanton and the antiinstanton in the singular gauge (we call it «DP ansatz» below) . Unfortunately, it spoils field properties near the pseudoparticle centers and therefore it is better to use the improved one, e. g. [8]

$$A^a_\mu(x)=\frac{U_I^{ab}\bar{\eta}^b_{\mu\nu}\,y_I^\nu\,\rho_I^2/y_I^4+U_A^{ab}\,\eta^b_{\mu\nu}\,y_A^\nu\rho_A^2/y_A^4}{1+\rho_I^2/y_I^2+\rho_A^2/y_A^2} \qquad (2)$$

Here $\eta^a_{\mu\nu}$ is t'Hooft symbol, $U_I^{ab}(U_A^{ab})$ are some color «orientation» matrixes and $y_I^\mu\,(y_A^{ab})$ are distances from the point x to the instanton

(antiinstanton) centers. In this case there exist two finite maxima of the action density, which may be used for the definition of the ro parameters. At large R the action deviates from $16\pi^2$ only due to the well known «dipole interaction» [2]. At $R\sim\rho$ the results are ansatz-dependent. Numerical studies [8] of various trial functions have shown that for $R>\rho$ (2) is definitely very close to the «streamline». Moreover, the slope along the valley (or the «force», attracting the pseudoparticles) is very small in this region, and even correction to total action is less than 10%. The compact molecules, on the contrary, are not well described by (2). Thus, in order to make our conclusions more convincing, we restrict our discussion only to the case $R>\rho$. As the contributions of «compact» and «dilute» molecules are just additive, it is not important for qualitative conclusion made below.

Let us now discuss the fermionic determinant, starting with the «asymmetric» molecules with, say, $\rho_1\gg\rho_2$. It is quite clear that for smaller instanton one may apply the operator product expansion formalism, as for (1), and obtain the small factor $(\rho_2/\rho_1)^{3N_f}$. Thus, suppression of «asymmetric» molecules is at large N_f evident.

Now we turn to the case of very «dilute» molecules, with $R\gg\rho$. In this case one may separate the «nearly zero modes» from all others, and the small factor due to them in the quark determinant is

$$\det\hat{D}|^{\text{zero modes}}=\left\{\frac{i}{2}\,Tr\,[U_I^+\,U_A\hat{R}]\,F(R^2)\right\}^{2N_f}\equiv\{\cos\Phi R\,F(R^2)\}^{2N_f} \qquad (3)$$

where R_μ is the vector connecting two centers, $\hat{R}=R_\mu\tau_\mu$, τ_m are the Pauli matrices and $\tau_4=i$. (Note that we have introduced here the so called relative orientation angle Φ.) The asymptotics at large R is $f(R^2)=2/R^4$ [9]. Thus, very «dilute» molecules are unimportant too.

(One more important conclusion following from (3) is that whatever is R, the relative orientation of pseudoparticles is nearly fixed. Indeed, the allowed values of Φ group around 0 and π with $\Delta\Phi\sim N_f^{-1/2}$. As the phase space in the SU(2) group is $\sin^2\Phi\,d\Phi$, only its small fraction is in fact populated. This conclusion holds starting from low $N_f=2-3$. Note also, that at this value of Φ the classical «dipole» interaction is most attractive.)

Thus, we have shown that the most important molecules are those with $\rho_1\sim\rho_A\sim R\sim\rho$. If so, their total density in vacuum can be obtained just by simle dimensional arguments

$$dn^{mol}(\rho)=\text{const}\,\frac{d\rho}{\rho^5}(\rho\Lambda)^{2b(1-\delta)} \qquad (4)$$

Here $b=(11/3)N_c-(2/3)N_f$ is the coefficient of Gell-Mann-Low function, while δ corrects for the deviation from twice the instanton action (we again remind that, at least for $R>\rho$, $\delta<0.1$).

If $b(1-\delta)<2$ the density (4) is divergent at small radii ρ: this is the «nonperturbative divergence» mentioned above. Note that this critical value (8 for two and 13 for three colors) is still below the point where the asymptotic freedom is broken. Note also, that because in «compact» molecules the action is reduced stronger, their density may be divergent even earlier. Near this critical point the molecule density (and the nonperturbative vacuum energy density) becomes infinite. Does it spoil the theory completely, or this infinity may be removed by a sort of «renormalization»?

We argue that just in the vicinity of the critical point the latter possibility seems to be the case. Although infinitely numerous, the small-size molecules still occupy negligible fraction of space-time: the integral $\int dn(\rho)\rho^4$ is convergent. Similar situation holds with physical observables. Small molecules affect particle propagation in vacuum, but their contribution to quark or gluon effective mass is of the order of $\int dn(\rho)\rho^2$, also convergent at the critical point mentioned above. This argument shows that there should be two critical points at which $b(1-\delta)$ crosses 2 and 1. We suspect that at the second one the theory indeed becomes meaningless.

Few words about aplications of the phenomenon discussed in this work. Of course, they take place at the unphysical number of light quarks, but they may be used as a test for lattice numerical experiments. Althogh it is not easy to see small-size molecules on the lattice directly, their influence may be detected indirectly, for example by enhenced deviations from the perturbative scaling.

In conclusion, light quark exchanges strongly rearrange the pseudoparticles in the vacuum starting already from few number of flavors. Small-size molecules become more probable at larger N_f and, finally, at some critical point, the vacuum contains infinite number of them.

REFERENCES

1. *Shuryak E.V.* Strong correlation of instantons... Preprint INP 86-136, Novosibirsk 1986, submitted to Phys.Lett.
2. *Callan C.G., R.Dashen and D.J.Gross.* Phys.Rev. D17 (1978) 2717.
3. *t'Hooft G.*, Phys. Rev. 14D (1976) 3432.
4. *Shifman M.A., Vainstein A.I. and Zakharov V.I.* Nucl.Phys. B165(1981)45.
5. *Shuryak E.V.* In: Proceedings of the 1 st conference on numerical experiments in the field theory. Ed. A.A.Migdal. Alma-Ata 1985.(In Russian).
6. *Balitsky Ya.Ya. and A.B. Young.* Phys. Lett. 168B (1986) 113.
7. *Dyakonov D.I. and V.Yu.Petrov*, Nucl.Phys. B245 (1984) 259.
8. *Shuryak E.V.* Phys.Lett. 153B (1985) 162.
9. *Dyakonov D.I. and V.Yu.Petrov.* Nucl.Phys. B272 (1986) 475.

E.V. Shuryak

Instantonic Molecules and the Nonperturbative Divergencies

Э.В. Шуряк

Инстантонные молекулы и непертурбативные расходимости

Ответственный за выпуск С.Г.Попов

Работа поступила 28 января 1987 г.
Подписано в печать 16 февраля 1987 г. МН 08627
Формат бумаги 60×90 1/16 Объем 0,8 печ.л., 0,7 уч.-изд.л.
Тираж 200 экз. Бесплатно. Заказ № 24

Набрано в автоматизированной системе на базе фотонаборного автомата ФА1000 и ЭВМ «Электроника» и отпечатано на ротапринте Института ядерной физики СО АН СССР,
Новосибирск, 630090, пр. академика Лаврентьева, 11.