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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

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STRONG CORRELATIONS OF INSTANTONS IN THE QCD VACUUM DUE TO THE LIGHT QUARK EXCHANGE

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НОВОСИБИРСК

ABSTRACT

First results of numerical studies of the instanton interactions mediated by light quarks are reported. At any instanton density the correlations in their positions and orientations are found to be very strong. We also study the transition from the «polymer-type» phase, possessing the quark condensate, to the chirally symmetric «molecular» phase.

G. t'Hooft [1] has found ten years ago that in the gauge theory with massless fermions the topological fluctuations of the fields (the pseudoparticles [2], *PP*s, or instantons and antiinstantons) may happen only if some additional transitions in the fermionic sector are made. One may imagine *PP*s as some effective vertexes (see Fig. 1*a*) emitting (or absorbing) one quark-antiquark pairs of each massless flavor. This effective interaction is known to violate the U(1) chiral symmetry [1]. In this work we concentrate on other manifestations of these effects, in particular, we study transition from the «polymer-type» vacuum (examplified in Fig. 1*c*) to the «molecular» one (Fig. 1*d*) at smaller *PP* density. This transition is related to the spontaneous breaking of the SU(N) chiral symmetry (for brevity, SBCS) in the QCD vacuum and its restoration in strongly excited matter.

In order to explain the general relation between the SBCS and instantons let me remind the reader the following well-known signal for SBCS. Consider eigenvalues λ of the Dirac operator $i\hat{D}(A)=(i\partial_\mu+gA_\mu^a t^a/2)\gamma_\mu$ and ask whether they have a finite density around zero, $dn/d\lambda(0)\neq 0$. If it is the case, this symmetry is indeed broken and the quark condensate value is equal to

$$|\langle\bar{\psi}\psi\rangle| = \pi dn/d\lambda(0)/V_4$$

where V_4 is the 4-dimensional volume. The individual *PP* possesses one zero mode $\psi_0(x)$, $\hat{D}\psi_0(x)=0$ related (by the Attyah-Singer theorem [3]) to its unite topological charge. Consider some field configuration made of many *PP*s with zero total topological charge. The-

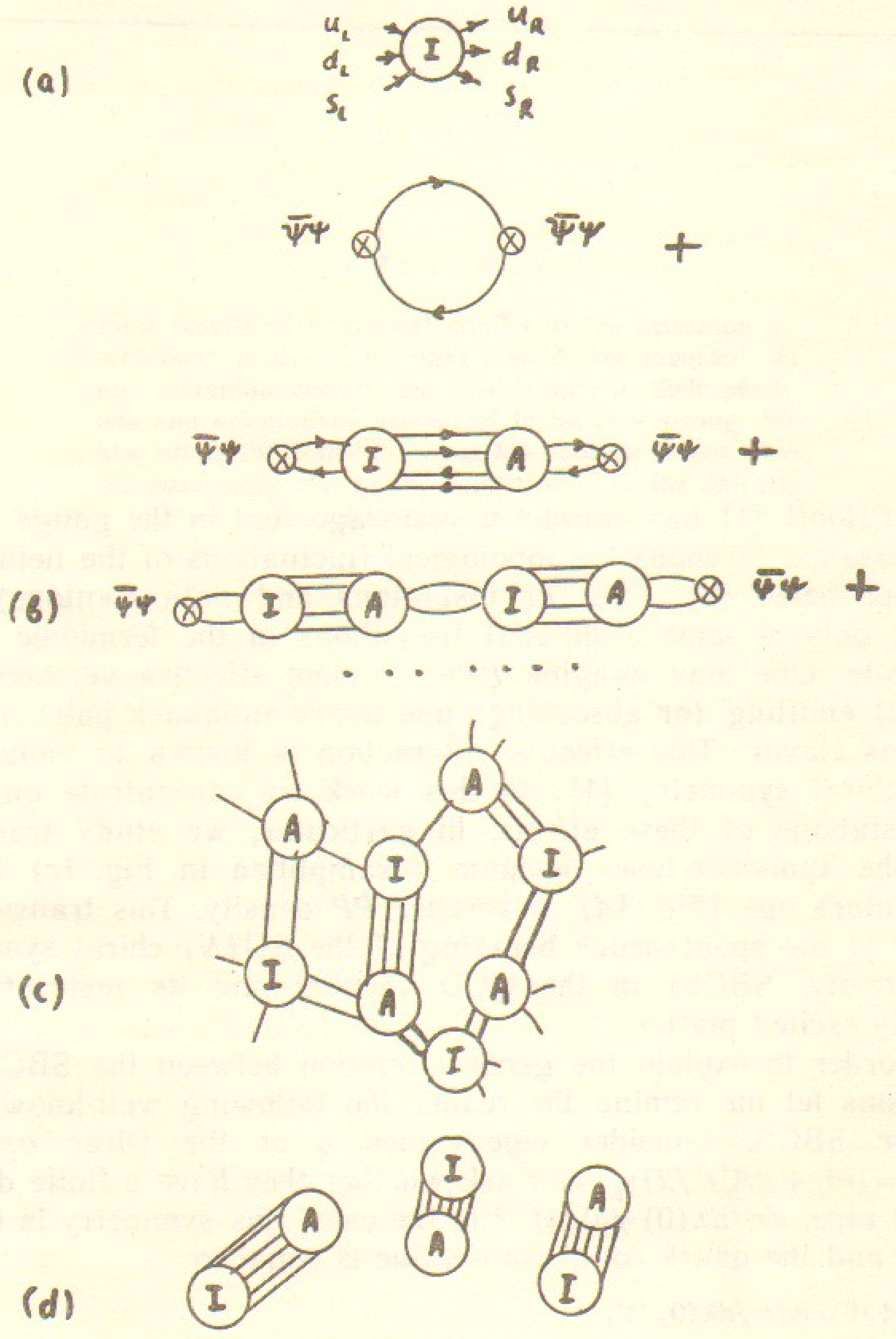

Fig. 1. (*a*) The instanton-generated effective interaction of the light quarks. (*b*) A set of ladder-type diagrams contributing to the correlation function of two $\bar\psi\psi$ operators at different points. (*c*) Schematic picture of a «polymer»-type vacuum, to be compared to the «molecular» one (*d*).

re is no reason to have the zero modes, but if *PP*s overlap relatively weakly then some combinations of their zero modes $\psi_0^{(I)}(x)$ may form the quark wave functions with the eigenvalues very close to zero. Thus, being interested in $dn/d\lambda(0)$, we study quark states formed in this way.

The first attempt to point out whether *PP*s really lead to SBCS was made by Caldi [4], who has selected the ladder-type diagrams (see Fig. 1*b*), summed by the Bethe-Solpeter equation, and has formulated the sufficient condition for the stability of the chirally symmetric vacuum. Later this problem was studied by the mean-field approximation for the quark condensate by Carlitz and Creamer [5], Callan, Dashen and Gross [6] and myself [7]. All these approaches may prove SBCS, provided the *PP*s density in vacuum is sufficiently high. Analysis of many phenomenological facts [7] have lead to the «instanton liquid» model of the QCD vacuum, in which the typical instanton radius $\bar\varrho$ and the *PP* density was found to be about

$$\bar\varrho = 0.3\ \text{fm}, \qquad n_{PP} = 1\ \text{fm}^{-4}. \tag{2}$$

If so, the conditions mentioned above are satisfied. Nevertheless, the question is far from being settled and one definitely needs more quantitative understanding of these effects. In particular, one may wonder what happens in the excited matter, in which the *PP*s are suppressed [8—10], so that their density is smaller than that in vacuum.

Recently discusion of this topic was revived by Dyakonov and Petrov [11]. They have shown that if the positions and orientations of the *PP*s are random, then there is the nonzero $dn/d\lambda(0)$ *at any finite PP density*. They have emphasized the analogy between this problem and the conductivity of semiconductors: whatever small is the impurity density, the «jump conductivity» is nonzero. However, we show that the chiral symmetry is restored at finite density. The loophole in the reasoning just mentioned is the randomness hypothesis: in contrast to the donor atoms in semiconductors, the *PP*s in vacuum are strongly correlated due to the quark exchange interaction. Formally this effect follows from the general fact: integrating quarks away one has in the gauge field partition sum the so called Mattew-Salam determinant

$$Z = \int \det\left(\hat{D}(A^a_\mu)\right) \exp\left(-S(A^a_\mu)\right) DA^a_\mu \tag{3}$$

which in general is extremely nontrivial functional of $A_\mu^a(x)$. Below we discuss it for the field configurations being some superposition of (well separated) *PP*s. For example, one may use the simplest «sum ansatz» [12]:

$$A_\mu^a(x) = \sum_{instantons} U_{(I)}^{ab} a_\mu^b (x - z^{(I)}) + \sum_{antiinst.} \bar{U}_{(I)}^{ab} \bar{a}_\mu^b (x - \bar{z}^{(I)}) \tag{4}$$

where a_μ^a ($\bar{a}_\mu^a$) is the standard instanton (antiinstanton) solution in singular gauge. $U_{(I)}^{ab}, \bar{U}_{(I)}^{ab}$ are the unitary SU(2) color matrixes, their parameters and the *PP*s positions $z^{(I)}, \bar{z}^{(I)}$ are the variables describing our ensemble. Further, we neglect the nonzero modes, considering only linear combination of the zero ones. Finally, we assume all *PP*s to have the same radius $\bar{\varrho}$ (which is taken as the length unite, $\bar{\varrho}=1$), so that all quantities are the function of the dimensionaless *PP* density n_{pp} The question to be discussed are how *PP*s are distributed in space-time and color space, if the only statistical weight is the fermionic determinant (3).

For N instantons and N antiinstantons the «overlap» $(2N)\times(2N)$ matrix representing the Dirac operator has the following structure [11]

$$\overbrace{}^{\text{instantons}} \quad \overbrace{\phantom{T_{11} \ldots T_{1N}}}^{\text{antiinstantons}}$$

$$\begin{bmatrix} 0 & \ldots & 0 & T_{11} & \ldots & T_{1N} \\ & \ldots & & & \ldots & \\ 0 & \ldots & 0 & T_{N1} & \ldots & T_{NN} \\ T_{11} & \ldots & T_{N1} & 0 & \ldots & 0 \\ & \ldots & & & \ldots & \\ T_{1N} & \ldots & T_{NN} & 0 & \ldots & 0 \end{bmatrix} \quad \begin{aligned} T_{IJ} &= i\int d^4x\, \Psi_0^+(x - z^{(I)})\, \hat{\partial}\, \Psi_0(x - \bar{z}^J) = \\ &= i \cdot \mathrm{tr}[U_{(I)}^+ \bar{U}_{(J)} (z_\mu^{(I)} - \bar{z}_\mu^{(I)})\, \tau_\mu^+] f((z^{(I)} - \bar{z}^{(I)})^2) \\ &(\tau_4^+ = i) \\ &f(r) \approx 2./(2.58 + r^2)^2 \end{aligned} \tag{5}$$

($f(r)$ was calculated numerically). It is easy to show that T_{IJ} is real and the whole determinant (up to the general factor $(-)^N$) is positive: all eigenvalues are doubled, $\pm\lambda$.

In any statistical problem there is some trade off between the energy and the entropy. The fermionic determinant is maximal for some rather complicated «crystall» (first considered in ref [12] in another context). It has 16 *PP*s with nontrivial orientations as the elementary unite, and the overlap integrals for all neighbours are

maximal. The maximal entropy is realized in the «random model» (RM), discussed in [11]. The simplest configuration is of course the «molecule», made of one *PP* pair. Note, that in the realistic case (the number of flavors $N_f=3$) the overlap integral T enters in the 6-th power, therefore both the relative distance and relative orientation are nearly fixed. The statistical distribution of the eigenvalues for one molecule is shown in Fig. 2. Of course, it has zero $dn/d\lambda(0)$, and therefore «molecular» vacuum is chirally symmetric.

We have made numerical simulation for the ensemble of *PP*s (which we have no place to describe in details here), governed by the determinant under consideration. In Fig. 3 we have plotted some results for the «geometric mean» value of eigenvalues $\det m = (\det\hat{D})^{(1/2NN_f)}$, called the «determinantal mass», and $dn/d\lambda(0)$ versus density. One may see that the «random model» is wrong in all cases, which means that the correlations induced by quarks are quite significant at any density. In order to see how the «depolymerization» at low density takes place one may use Fig. 2, displaying the eigenvalue distributions at few densities.

In qualitative agreement to what was concluded previously on the basis of the mean field approximation [7], restoration of the chiral symmetry in this model is the clear first order transition, with the «polymer» and «molecular» vacuum coexisting in the mixed phase at densities region about 0.02—0.002. We have also observed the long-lived metastable states outside this density region. The following simple argument may explane why the «molecular» phase has density so much smaller than the «polymer» one: as mentioned above, only tiny fraction of all phase space for the relative orientation matrix U is allowed for molecules, while in the «polymer» phase it is increased at least by the factor proportional to the number of neigbours. In order to compensate for smallness of this phase space, one should allow for a volume per particle increased by the similar factor. Such additional diluteness of the molecular phase means, that the nonperturbative effects in the «quark-gluon plasma» phase are nearly negligible even just above the transition.

Completing this work let me comment that it is devoted to more or less qualitative studies of the problem. Apart of simplifications mentioned above, we have so far ignored the *PP* interaction due to gluonic field effects (discussed in refs [12, 13]). In addition, we typically work with about 20 *PP*s in a box, which is not so large number in 4 dimensions. (Nevertheless, it is already essentially larger than the number of instantons seen in current lattice experi-

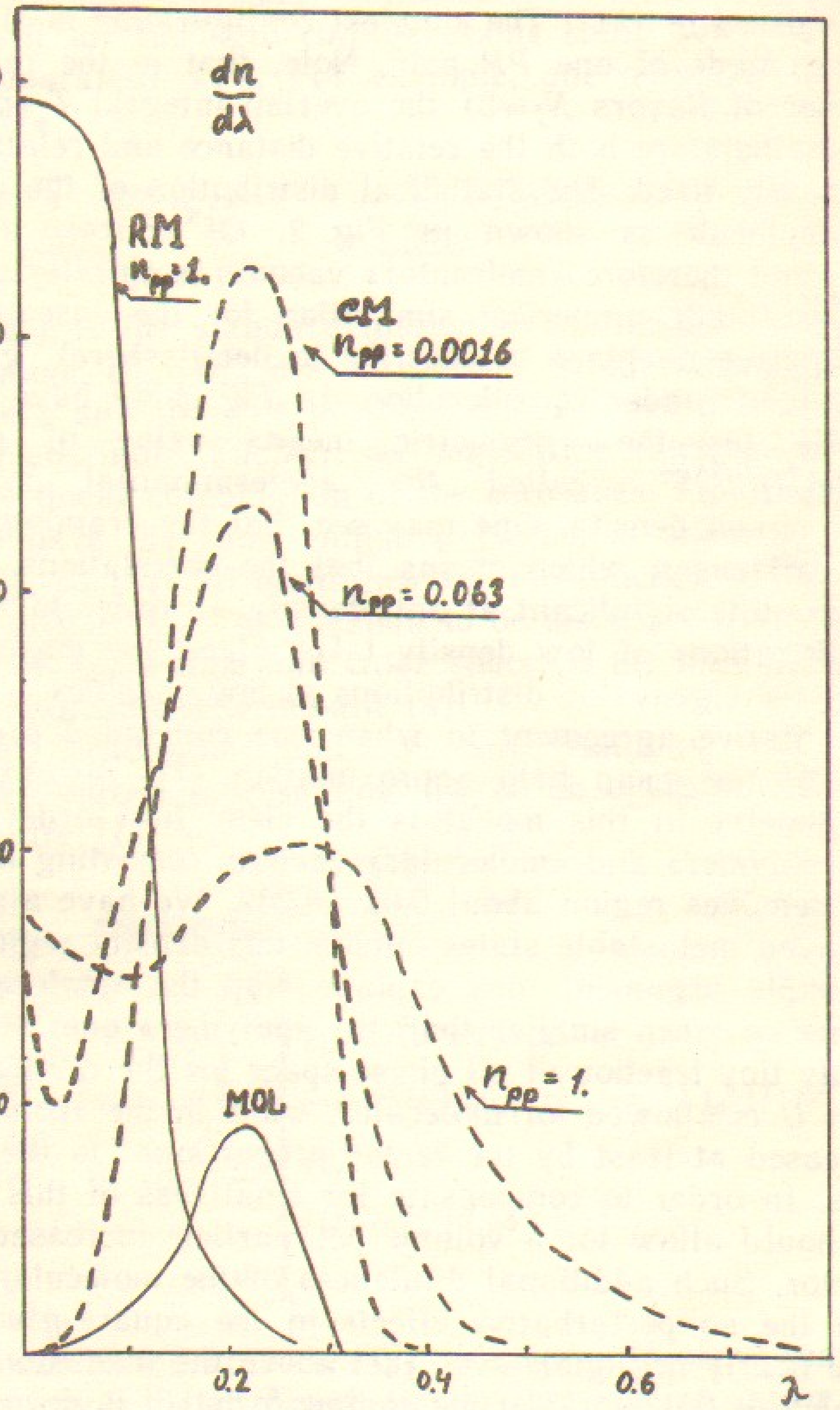

Fig. 2. The density of the eigenvalues of the overlap matrix $dn/d\lambda$ at different pseudoparticle densities n_{pp} The solid curves marked MOL stands for one instanton-ani-instanton molecule, that marked RM stands for «random model» with random positions and orientations. The dashed lines marked CM are for the «correlated madel», which the ensemble of pseudoparticle with properties governed by the fermionic determinant as explained in the text.

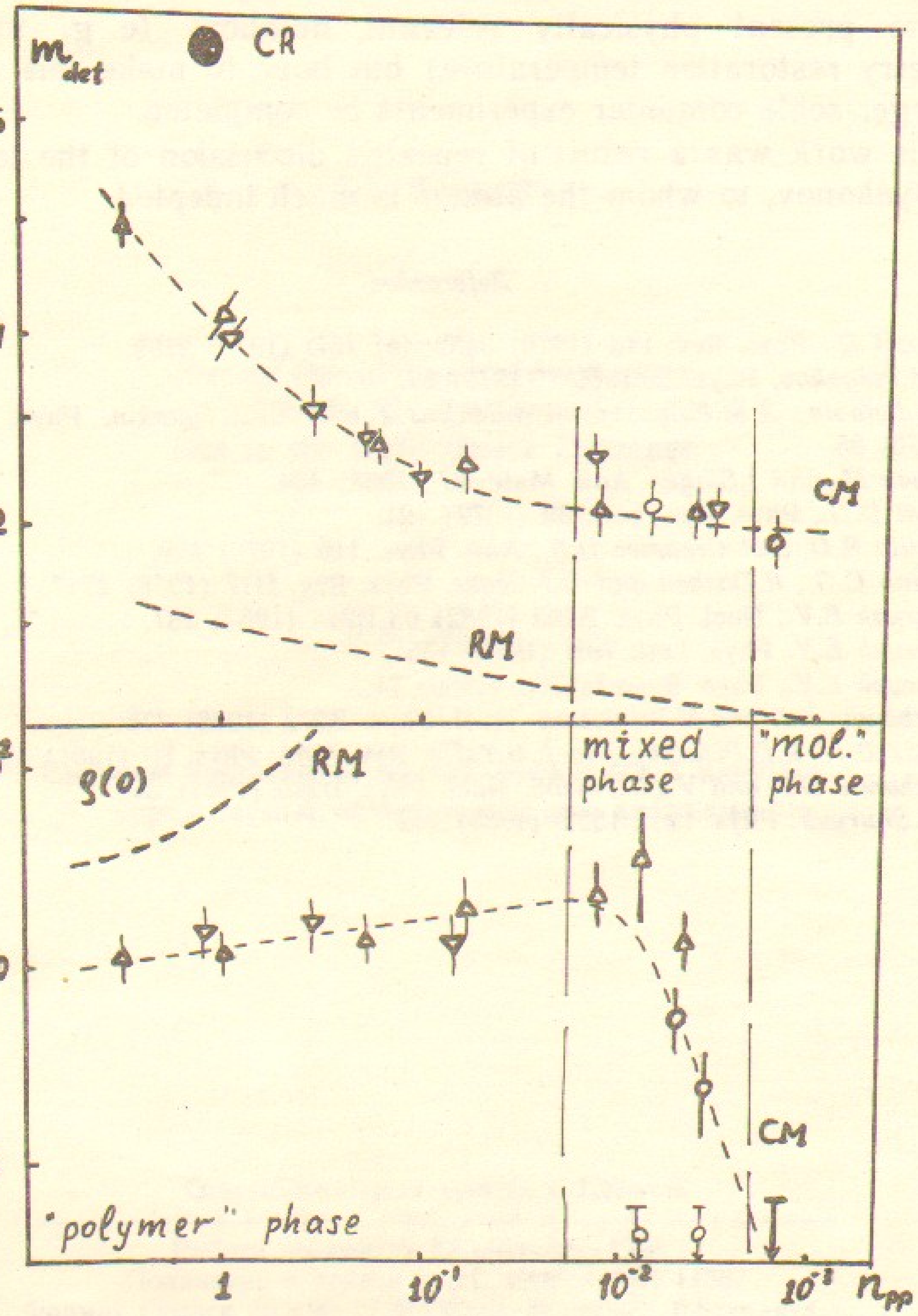

Fig. 3. The upper part of the picture give the so called «determinantal mass» $m_{\rm det}$ (the geometric mean of the eigenvalues of the overlap matrix) versus the pseudoparticle density n_{pp} , while the lower one give the dependence of the eigenvalue density at zero. As above, RM and CM mean the random and the correlated model, respectively, while CR near the star in the upper left corner stands for the instanton-antiinstanton «crystal». Two type of the triangles mean measurements made in expansion and compression experiments, while the open points are for some special large-statistics runs in the phase transition region.

ments, which have observed few *PP*s if any.) Thus, we are not yet able to present physically relevant numbers (e. g. the chiral symmetry restoration temperature) but hope to make this when some larger-scale computer experiments be completed.

This work was a result of repeated discussion of the topic with D.I. Dyakonov, to whom the author is much indepted.

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Сильные корреляции инстантонов в КХД вакууме, порождаемые обменом легкими кварками

Ответственный за выпуск С.Г.Попов

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