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PHYSICAL EFFECTS IN THE MODEL OF CP-VIOLATION WITH COLOURED SCALARS

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PHYSICAL EFFECTS IN THE MODEL OF
C P - VIOLATION WITH COLOURED SCALARS

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Abstract

The neutron electric dipole moment, nucleon-nucleon CP-odd interaction and B^o-$\bar{B}^o$ mixing are considered in the model of CP-violation with coloured scalars.

1. CP-odd effects in the standard model are due to non-zero phase in the quark mixing matrix [1]. The ratio ε'/ε which characterizes a deviation from the superweak mechanism predictions [2] in K-meson decays was estimated in the standard model to be $\sim$ 1% [3]. The results of the last measurements

$$\frac{\varepsilon'}{\varepsilon}=\begin{cases}(-4.6\pm5.3\pm2.4)\cdot10^{-3} & (4)\\ (1.7\pm8.2)\cdot10^{-3} & (5)\end{cases}$$

are close to contradiction with the standard model's predictions. This fact has inspired great amount of alternative models of CP-violation. One of them has been proposed in Ref. [6]. It's main idea is as follows. CP-symmetry is violated spontaneously in the sector of scalar particles. Due to the exactness of the colour symmetry coloured scalar fields cannot develop non-zero vevs and lead to a complex quark mixing matrix. Hence CP-odd effects in quarks interactions are the result of the coloured scalars exchange. These effects are suppressed by the large mass of the coloured scalars and the smallness of quarks-coloured scalars interaction constants.

In Ref. [1] the induced θ-term is estimated in the model with coloured scalars. The present paper is devoted to the analysis of CP-odd effects not connected with the θ-term.

2. Before passing the calculations, we present the short description of the model and necessary parameters estimates. The particle content of the standard model with the real Kobayashi-Maskawa matrix is enlarged with new scalars. Some of them are transformed as the 6 representation of $SU(3)_c$ group. The rest are singlets which develop non-zero vacuum expectation values. As a result, an interaction of the $A_{ABCD}6^{*A}6^{B}1^{C*}1^{D}$ kind makes a coloured scalar propagator CP-violating.

For simplicity consider the case of only two coloured scalars, ϕ and ϕ', with bare masses M and M' correspondingly. Due to the interaction with vacuum condensate the physical scalars ϕ_1 and ϕ_2 are the linear combinations of ϕ and ϕ' :

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos\xi & -e^{i\theta}\sin\xi \\ e^{-i\theta}\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} \phi \\ \phi' \end{pmatrix}$$

Here $\theta = \arg\left(A_{CD}\langle 1^C\rangle^{*}\langle 1^D\rangle\right)$, $\operatorname{tg} 2\xi = \dfrac{2\,|A_{CD}\langle 1^C\rangle^{*}\langle 1^D\rangle|}{M'^2 - M^2}$.

Their masses M_1 and M_2 are quite large (in the TeV region). Hence, when discussing low-energy phenomena we can treat the coloured scalars exchange in the local limit.

Begin with the Lagrangian

$$\mathcal{L}_0 = \bar{q}_R\left(\phi h + \phi' h'\right) C \bar{q}_R^T + \text{H.c.}$$

where $q_R = \frac{1}{2}(1-\gamma_5)q$, q is the column of quarks in the generation space, h and h' are real symmetric matrices in the same space, C is the charge conjugation operator. As ϕ and ϕ' are the singlets of the weak isospin group, they interact with the right-handed quarks only. Suppose that the electric charge of coloured scalars is $-\frac{2}{3}|e|$ and hence they interact with down quarks only. This restriction is sufficient to describe CP-odd effects in the neutral kaon system, the only CP-odd effects experimentally seen up to date.

The effective CP-odd four-quark Lagrangian is

$$\mathcal{L}_{eff} = -4i\frac{\Delta}{M^2}\left(h_{ac}h'_{bd} - h'_{ac}h_{bd}\right)\left(\bar{a}_{R_i} C \bar{c}^T_{R_j}\right)\left(b^{T\,k}_R C^{-1} d^l_R\right)\left(\delta^i_k\delta^j_l + \delta^j_k\delta^i_l\right), \quad (1)$$

where $\Delta = |\operatorname{Im} A_{CD}\langle 1^C\rangle^{*}\langle 1^D\rangle|/M^2$, $M^2 = M_1 M_2$, and summation over down quark flavors a,b,c,d and colour indices i, j, k, l is implied. Sometimes we shall use the other form of this Lagrangian:

$$\mathcal{L}_{eff} = 8i\frac{\Delta}{M^2}\left(h_{ac}h'_{bd} - h'_{ac}h_{bd}\right)\Big[\frac{1}{3}(\bar{a}_R\gamma_\mu b_R)(\bar{c}_R\gamma_\mu d_R) +$$

$$+ \frac{1}{2}(\bar{a}_R\gamma_\mu t^a b_R)(\bar{c}_R\gamma_\mu t^a d_R)\Big] \quad (1')$$

(t^a's are the colour group generators, normalized by the condition $\operatorname{Tr} t^a t^b = \frac{1}{2}\delta^{ab}$). It can be derived from (1) with the help of the Fierz transformations.

To estimate a value of M consider CP-odd mixing of neutral kaons [6]. It follows from (1) that $|\Delta S| = 2$ transition $K^0 \to \bar{K}^0$ takes place in the first order of the perturbation theory. Parameter ε of superweak mixing can be estimated using the formula [2]:

$$|\varepsilon| \approx \frac{|\langle \bar{K}^0|\mathcal{L}_{eff}|K^0\rangle|}{\sqrt{2}\,|m_L - m_S|}.$$

Vacuum insertion method gives

$$\langle \bar{K}^0|\frac{1}{3}(\bar{s}_R\gamma_\mu d_R)^2 + \frac{1}{2}(\bar{s}_R\gamma_\mu t^a d_R)^2|K^0\rangle \approx \frac{1}{6} f_K^2 m_K$$

So,

$$|\varepsilon| \approx \frac{4}{3}\frac{\Delta}{M^2} f_K^2 \left|h_{ss}h'_{dd} - h'_{ss}h_{dd}\right| \frac{m_K}{\sqrt{2}\,|m_L - m_S|} \quad (2)$$

Assume that parameter Δ is of the order 10^{-1}. There is no sense in making this parameter very small. Small Δ leads to small scalar masses and large CP-even effects which contradict experimental data. Choose h and h' to be of the same order as the Yukawa couplings of the standard model: $h_{ab} \sim h'_{ab} \sim \sqrt{m_a m_b}/V$, $V \simeq 250$ GeV. Substituting in (2) experimental values for the other parameters, one obtains the estimate for scalars masses:

$$M \sim 1 \div 1.5 \text{ TeV}$$

It's order of magnitude coincides with that in Ref. [6]. Uncertainty of the estimate results from the rough estimate of the matrix element.

Note that due to the fact that the model under consideration treats CP-odd effects with $|\Delta S| = 2$ and $|\Delta S| = 1$ in the same order of the perturbation theory the ratio ε'/ε is many times smaller than in the standard model.

3. Pass now to the calculation of the neutron electric dipole moment (EDM). First of all, consider d-quark EDM (that of u-quark is negligibly small because up quarks do not interact with the coloured scalars).

The interaction of a fermion EDM d with external field $F_{\mu\nu}$ is described by the Lagrangian

$$\mathcal{L}_d = -\frac{d}{2}\bar{\psi} F_{\mu\nu}\sigma_{\mu\nu}\gamma_5\psi \tag{3}$$

It is easily seen from (1) that in the model a quark EDM cannot appear on a one-loop level due to the hermiticity of a Lagrangian. The only way to get the CP-odd effect is to change one of the virtual quarks flavor with the help of the standard weak interaction (which can also be considered in the local limit). Corresponding diagrams are depicted at Fig. 1. Here a quark propagator in the external electromagnetic field is denoted by a solid line, $D, D' = d, s, b$; $U = u, c, t$ and CP-violating vertex is denoted by a point. Easily seen, however, that these diagrams cannot generate the quark EDM. As a matter of fact, a one-vertex loop is equal to zero. If we connect the external field photon to the lower loop at Fig. 1b, we'll get the polarization operator which is zero for a constant field.

Thus, we need to take into account strong interactions. The situation is similar to that in the standard model [7]. To escape turning to zero the one-vertex loop a gluon should be emitted from it. At diagram 1b it should terminate at the external fermion line, at diagram 1a virtual gluon connects two quark loops (Fig. 2; dashed line denotes a gluon propagator).

Crucial difference between the standard model and that under consideration is in the fact that in the latter the d-quark EDM has a non-zero value even in the chiral limit $m_d = 0$. Indeed, consider diagram 2a. Central quark line here begins in a vertex with, say, right-chirality currents and ends in the vertex with currents of left chirality. Hence, only $m/(p^2-m^2)$ part works in the central quark propagator. If characteristic momenta p exceed all down-quark masses, diagram 2a with b-quark in the central line gives the main contribution to the

EDM. As for the diagram 2b, it's contribution is proportional to m_d and we shall not consider it in what follows.

Trace now, what quarks (D' and U) give the main contribution when calculating quark loops. Due to assumed hierarchy of h constants ($h_{ab} \sim \sqrt{m_a m_b}/v$) b-quark contribution dominates in calculation of the loop with CP-violating vertex. In the rest loop it's enough to restrict oneself with c- and t-quark contributions because, as we know from experiment, $|K_{ud} K_{ub}| \ll |K_{cd} K_{cb}| \approx |K_{td} K_{tb}|$ (K is the Kobayashi-Maskawa matrix).

The external field photon can be attached either to the central quark line, or to one of the quark loops. To calculate corresponding contributions to the EDM we should know expressions for the subdiagrams (Fig. 3 and 4). The polarization operator (Fig. 3) in the euclidean region for k^2 is

$$\int\frac{d^4q}{(2\pi)^4}\,\mathrm{Tr}\,\gamma_\mu\frac{1}{\hat{k}+\hat{q}-m}\gamma_\nu\frac{1}{\hat{q}-m} =$$

$$= \frac{i}{2\pi^2}\left(\delta_{\mu\nu}k^2 - k_\mu k_\nu\right)\left[\frac{1}{6}\ln\frac{\mu^2}{k^2} - \frac{1}{12} - I\left(\frac{k^2}{m^2}\right)\right], \tag{4}$$

$$I(\eta) = \int_0^1 dx\, x(1-x)\ln\left[x(1-x)+\frac{1}{\eta}\right] = \begin{cases} -\frac{1}{6}\ln\eta, & \eta \ll 1 \\ -\frac{5}{18}+\frac{1}{\eta}, & \eta \gg 1 \end{cases}$$

where μ is the mass of a heavy particle (coloured scalar or W-boson), which provides an ultraviolet cut-off.

The expression for the quark loop in the external field (Fig. 4) have been obtained in Ref. 7 :

$$\int\frac{d^4q}{(2\pi)^4}\,\mathrm{Tr}\left(\gamma_\mu\gamma_5\frac{1}{\hat{q}+\hat{k}+\hat{l}-m}\hat{e}\frac{1}{\hat{q}+\hat{k}-m}\gamma_\nu\frac{1}{\hat{q}-m} + \begin{pmatrix} k \rightleftarrows l \\ \gamma_\nu \rightleftarrows \hat{e}\end{pmatrix}\right) =$$

$$= \frac{i}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}E\left(\frac{k^2}{m^2}\right), \tag{5}$$

$$E(\eta)=\int_0^1 \frac{dx\, x\,(1-x)^2\eta}{1+\eta x(1-x)}=\begin{cases}\frac{1}{12}\eta\,, & \eta\ll 1\\[2mm] \frac{1}{2}-\frac{1}{\eta}\ln\eta\,, & \eta\gg 1\end{cases}$$

Here e_α is the external field polarization vector. Simple estimate with the help of (4) and (5) shows that typical momenta of integration lie in the region $m_b^2 \ll k^2 \ll m_W^2$. So, when the external field photon is attached to the central quark line, we have

$$\frac{d^{(a)}}{e}\approx\frac{1}{48\pi^4}\Delta\frac{hh'}{M^2}\frac{G}{\sqrt{2}}\frac{\alpha_s}{\pi}m_b K_{td}K_{tb}^{*}\times\int_{m_b^2}^{m_W^2}dk^2\ln\frac{M^2}{k^2}\left(I\left(\frac{k^2}{m_t^2}\right)-I\left(\frac{k^2}{m_c^2}\right)\right).$$

Here G is the Fermi constant, $hh'=h_{db}h'_{bb}-h'_{db}h_{bb}$, $\alpha_s=g^2/4\pi$, g is the strong interaction constant.

Now let the external field interact with one of the quark loops. If the loop is that of the b-quark, corresponding contribution to the EDM is

$$\frac{d^{(b)}}{e}\approx-\frac{3}{64\pi^4}\Delta\frac{hh'}{M^2}\frac{G}{\sqrt{2}}\frac{\alpha_s}{\pi}m_b K_{td}K_{tb}^{*}\times\int_{m_b^2}^{m_W^2}dk^2\left(I\left(\frac{k^2}{m_t^2}\right)-I\left(\frac{k^2}{m_c^2}\right)\right)$$

Finally, if the photon is attached to up-quark loop, the total contribution from t-and c-quakrs is

$$\frac{d^{(c)}}{e}\approx\frac{1}{32\pi^4}\Delta\frac{hh'}{M^2}\frac{G}{\sqrt{2}}\frac{\alpha_s}{\pi}m_b K_{td}K_{tb}^{*}\times\int_{m_b^2}^{m_W^2}dk^2\ln\frac{M^2}{k^2}\left(E\left(\frac{k^2}{m_t^2}\right)-E\left(\frac{k^2}{m_c^2}\right)\right).$$

It's easy to see that $d^{(b)}$ is quite small as compared with $d^{(a)}$ and $d^{(c)}$ because it does not contain the large logarithm. With logarithmic accuracy

$$\frac{d}{e}\sim\frac{1}{36\pi^4}\Delta\frac{hh'}{M^2}\frac{G}{\sqrt{2}}\frac{\alpha_s}{\pi}m_b m_t^2 K_{td}K_{tb}\ln\frac{M^2}{m_t^2}J,$$

$$J=\int_{m_b^2/m_t^2}^{m_W^2/m_t^2}d\frac{k^2}{m_t^2}\left[\frac{2}{3}\left(I\left(\frac{k^2}{m_t^2}\right)-I\left(\frac{k^2}{m_c^2}\right)\right)+E\left(\frac{k^2}{m_t^2}\right)-E\left(\frac{k^2}{m_c^2}\right)\right]$$

The value of J can be calculated numerically. It equals to -0.85. So, we find

$$d\sim-3\cdot10^{-34}\ e\cdot cm \qquad (6)$$

For numerical estimate we assume for α_s the value ~ 0.1 and for $|K_{td}K_{tb}|$ the value $\sim 10^{-2}$.

Thus, in spite of "superweakness" of the CP-violating interaction ($\frac{\Delta}{M^2}hh'\sim(m_\pi G/\sqrt{2})^2$), d-quark EDM is of the same order as in the standard model [7]. This is, in fact, the consequence of the two features of the model. First of all, the right-chirality currents take off chiral suppression in CP--odd effects (this gives factor m_b/m_d as compared to the standard model). And secondly, abscence of the GIM mechanism leads to integration momenta, which exceed the masses of all down-quarks (corresponding enhancement factor is m_t^2/m_c^2).

In the simplest quark model d-quark EDM contributes to the neutron EDM as follows:

$$D_n(d) = \frac{4}{3} d.$$

Considered mechanism gives other contributions of the same order as (6). Indeed, placing diagram 2a in the constant gluonic field, one can find the colour dipole moment (CDM) of d-quark:

$$\frac{d_c}{g} \sim -\frac{1}{144\pi^4}\Delta \frac{hh'}{M^2}\frac{G}{\sqrt{2}} K_{tb} K_{td} \frac{\alpha_s}{\pi} m_b m_t^2 \ln\frac{M^2}{m_t^2} L,$$

$$L=\int_{m_b^2/m_t^2}^{m_W^2/m_t^2} d\frac{k^2}{m_t^2}\left(10\left[I\left(\frac{k^2}{m_t^2}\right)-I\left(\frac{k^2}{m_c^2}\right)\right]+\frac{3}{4}\left[E\left(\frac{k^2}{m_t^2}\right)-E\left(\frac{k^2}{m_c^2}\right)\right]\right)\approx 9.4$$

Quark CDM contribution to the neutron EDM in the nonrelativistic quark model was found in Ref. [8]:

$$D_n(d_c) = \frac{4}{9} e \frac{d_c}{g}.$$

If we change external d-quarks to s-quarks in diagram 2a, placed in the external gluonic field, we'll get the interaction, described by the Lagrangian

$$\mathcal{L}_s = -\frac{d_c(s)}{2} \bar{s} G^a_{\mu\nu} t^a \sigma_{\mu\nu}\gamma_5 s$$

where $d_c(s)$ is the s-quark CDM. This interaction leads to the contribution to the neutron EDM due to the diagram depicted at Fig. 5 [8]:

$$D_n(s) \approx \frac{(2\alpha-1)\sqrt{2}\, g_{\pi NN}\, m^*(m_\Delta - m_N)}{12\pi^2 m_N f_K}\, e\, \frac{d_c(s)}{g} \ln\frac{m_N^2}{m_\Sigma^2 - m_N^2}$$

Here $\alpha = \frac{2}{3}$ is relative weight of the D-coupling, $m^* \sim$ $\sim$ 300 MeV is the quark effective mass and $g_{\pi NN} \approx 13.5$ is the nucleon-π-meson coupling constant.

Summarizing all the contributions, we arrive to the estimate for the neutron EDM in the model with coloured scalars:

$$D_n = D_n(d) + D_n(d_c) + D_n(s) \sim -7\cdot 10^{-34}\ e\cdot cm$$

It is approximately an order smaller than in the standard model [9].

4. Having the value of the quark CDM at hands, we can estimate the constant of CP-odd nucleon-nucleon interaction in the model of nonrelativistic quarks [8]. Assuming that the interaction $ig\bar{u} G^a_{\mu\nu} t^a \sigma_{\mu\nu} d$ is responsible for the nucleon-isobar mass splitting, we get

$$\tilde{g}_{\pi NN} \approx \sqrt{2}\,\frac{m^*(m_\Delta - m_N)}{f_\pi}\frac{d_c}{g} \sim -6\cdot 10^{-20}$$

CP-odd nucleon-nucleon interaction constant is defined by the formula

$$\eta = \frac{g_{\pi NN}\tilde{g}_{\pi NN}}{m_\pi^2}\frac{\sqrt{2}}{G}$$

and constitutes numerically

$$\eta \sim 6\cdot 10^{-12}$$

5. Relatively large constant of b-quark interaction with coloured scalars enables to presume a noticeable CP-odd mixing of neutral B-mesons. Indeed, similar to the K-meson case,

$$\mathrm{Im}\, M_{B\bar{B}} \approx -\frac{4}{3}\Delta\frac{hh'}{M^2} f_B^2 m_B \qquad (7)$$

Here $hh' = h_{bb} h'_{qq} - h'_{bb} h_{qq}$ for B_q-mesons. To estimate the CP-odd mixing parameter

$$\mathrm{Re}\,\bar{\varepsilon} = -\frac{1}{4}\frac{\Gamma_{B\bar{B}}\,\mathrm{Im}\, M_{B\bar{B}}}{|M_{B\bar{B}}|^2} \qquad (8)$$

we shall use (7) and the formulae

$$\mathrm{Re}\, M_{B\bar{B}} \approx 0.23\, \frac{G^2 f_B^2 m_B m_W^2}{12\pi^2} \left(K_{tb} K_{tq}\right)^2$$

$$\Gamma_{B\bar{B}} \approx -\frac{G^2 f_B^2 m_B m_b^2}{8\pi} \left(K_{tb} K_{tq}\right)^2$$

from the Ref. [10]. Neglecting $|\mathrm{Im}\, M_{B\bar{B}}|^2$ as compared to $|\mathrm{Re}\, M_{B\bar{B}}|^2$ in the denominator of (8), we find

$$\mathrm{Re}\, \bar{\varepsilon}_q \approx -\frac{6\pi^3 m_b^2 \Delta \frac{hh'}{M^2}}{(0.23 K_{tb} K_{tq} G m_W^2)^2}$$

Or, numerically, $|\mathrm{Re}\, \bar{\varepsilon}_d| \sim |\mathrm{Re}\, \bar{\varepsilon}_s| \sim 10^{-2}$ unlike the standard model, where $|\mathrm{Re}\, \bar{\varepsilon}_s| \sim 10^{-4}$.

Summing up our results, we can say, that whereas the predictions of the model for neutron EDM and CP-odd nucleon-nucleon interaction constant are very far from the experimental upper limits, an observation of CP-odd effects in the $B^0 - \bar{B}^0$ system would be a crucial check-up for the model with coloured scalars.

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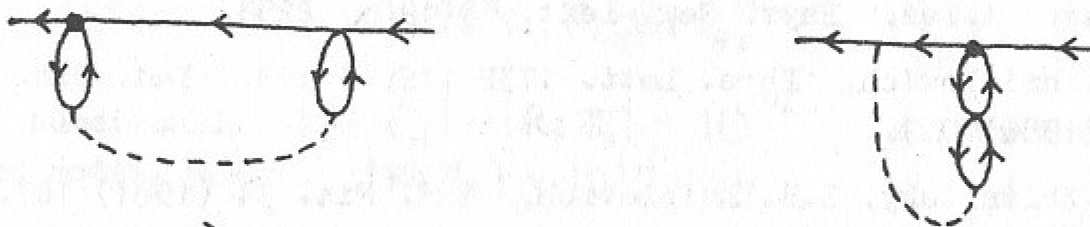
a)

b)

Fig.1



a)

b)

Fig.2

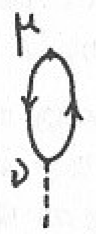


Fig. 3

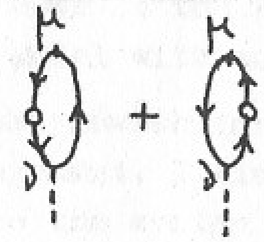


Fig.4

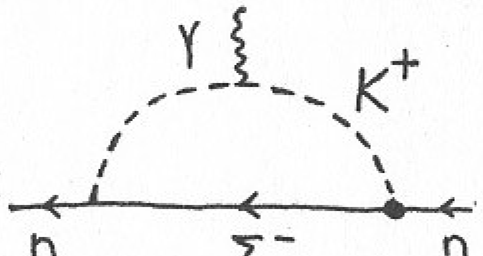


Fig.5.

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