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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

V.M. Khatsymovsky

ON THE DECAY $\Lambda \to n\gamma$

PREPRINT 86-151

НОВОСИБИРСК
1986

ON THE DECAY

V.M.Khatsymovsky

A b s t r a c t

The semiphenomenological analysis of the decay $\Lambda \to n\gamma$ is suggested which uses the QCD sum rule method.

Recently hyperon radiative decays are intensively studied experimentally [1]. In particular, the $\Lambda \to n\gamma$ decay probability was measured [2] . So it is of interest to study the decay $\Lambda \to n\gamma$ theoretically. In [3,4] the unitarity conditions were applied to this decay. In [5] the full phenomenological abalysis based on the dispersion relation was performed. In this note the semiphenomenological analysis of the decay $\Lambda \to n\gamma$ is given which uses the QCD sum rules. The QCD sum rule method was suggested in [6] and developed in [7-13] to find some characteristics of baryons such as masses and formfactors (see also the review [14]). Then the method was used to analyse S-wave non-leptonic [15] and radiative [16] hyperon decays.

The matrix element of a weak radiative decay $B_1 \to B_2\gamma$ is usually parametrized as follows:

$$\int d^4y \, exp(iky) \langle B_2 | T\{J_\lambda(y) H(0)\} | B_1 \rangle = \\ = i G_F m_\pi \bar{u}_2 (a\gamma_5 + b) \sigma_{\lambda\rho} u_1 k^\rho \tag{1}$$

where J_λ is the electromagnetic current, H is the four-quark $\Delta S = -1$ weak Hamiltonian. $Im\,a$, $Im\,b$ can be reliably fixed phenomenologically. $Re\,a$, $Re\,b$ can be divided into two parts which vary rapidly and slowly when virtualities of baryons go from Minkowskian to Euclidean region. The rapidly varying component is analysed phenomenologically. It is the pole contribution from the lowest one-particle states (in b) or from the $B\pi$ states (in a). This contribution is singular as $(\Delta m)^{-1}$ or $ln(\Delta m)^{-1}$ correspondingly (on condition that the baryon mass difference $\Delta m = m_1 - m_2$ is of order of the pion mass m_π). The occurence of the logs in a was discovered by I.B.Khriplovich (see reference in [17]). It is important that the coefficient of the $ln(\Delta m)^{-1}$ is fixed in a modelless way. Finally, the slowly varying component can be derived from the sum rules which can be derived just in the Euclidean region.

Consider the components of a, b for $\Lambda \to n\gamma$ decay in more detail.

1. Imaginary part appears due to the real states $p\pi^-$ in the diagrams shown in fig.1. The calculation of such the

diagrams is considered in detail in [18]. This calculation yields

$$\mathrm{Im}\,a = -\frac{m_\pi}{f_\pi}\frac{g_A}{8\pi}A(\Lambda^\circ_-)\left[\sqrt{1-x^2}+x^2\ln\frac{1+\sqrt{1-x^2}}{x}+\right.$$
$$\left.+\frac{\Delta m}{m}\sqrt{1-x^2}\left(1+\frac{1}{2}x^2\right)\right] = +9{,}1\cdot 10^{-2} \qquad (2)$$

$$\mathrm{Im}\,b = -\frac{m_\pi}{f_\pi}\frac{g_A}{8\pi}B(\Lambda^\circ_-)\frac{\Delta m}{2m}\left[\sqrt{1-x^2}-x^2\ln\frac{1+\sqrt{1-x^2}}{x}+\right.$$
$$\left.+\frac{4}{3}\frac{\Delta m}{m}(1-x^2)^{3/2}\right] = +1{,}0\cdot 10^{-2} \qquad (3)$$

Here $x = m_\pi/\Delta m$, $g_A = 1{,}25$; $A(\Lambda^\circ_-)$ and $B(\Lambda^\circ_-)$ are the S- and P-wave amplitudes of the decay $\Lambda\to p\pi^-$ in the notations of [19]. The contributions of (2) and (3) to $\Lambda\to n\gamma$ decay width agree with those found in [4].

2. Rapidly varying real part of b is

$$b_{pole} = \frac{f_\pi m_\pi}{2m}\left[-\frac{\mu_\Lambda-\mu_n}{m_\Lambda-m_n}A(\Lambda^\circ_-)+\frac{\mu_{\Sigma\Lambda}}{m_\Sigma-m_n}\frac{A(\Sigma^-_-)}{\sqrt{2}}\right] = \qquad (4)$$
$$= +1{,}0\cdot 10^{-2}$$

It appears due to the diagrams of fig. 2. Here μ_Λ, μ_n are the magnetic moments of Λ, n. The matrix elements $\langle B_2|H|B_1\rangle$ are expressed here through the S-waves $A(\Lambda^\circ_-)$, $A(\Sigma^-_-)$ in the soft pion limit; $\mu_{\Sigma\Lambda} = 1{,}82$ [20]. Rapidly varying part of $\mathrm{Re}\,a$ is

$$a_{\log} = -\frac{m_\pi}{f_\pi}\frac{g_A}{8\pi}\frac{A(\Lambda^\circ_-)}{\pi}\ln\frac{m}{\Delta m} = +4{,}3\cdot 10^{-2} \qquad (5)$$

It is given by "uncut" diagram of fig.1a.

3. Slowly varying real part of the amplitude is calculated with the help of QCD sum rules. The method of calculation is that used in [16]. The result is the sum of contributions of the weak interaction from short (s) and large (ℓ) distances:

$$a\gamma_5 + b = (a_s + a_\ell)\gamma_5 + b_s + b_\ell \qquad (6)$$

For the decay $\Lambda\to n\gamma$ we have found

$$a_s = \frac{2}{\sqrt{3}}\frac{csC_-(\sqrt{s_0})}{(6\pi)^2 m_\pi}a\chi\frac{e'}{4\tilde\beta_\Lambda\tilde\beta_n}\left[\Lambda^3-\frac{5}{2}n^3+\right.$$
$$\left.+\frac{3}{2}(\Lambda^2-n^2)-2a^2\chi\right] \qquad (7)$$

$$b_s = -\frac{2}{\sqrt{3}}\frac{csC_-(\sqrt{s_0})}{(6\pi)^2 m_\pi}a\chi\left[\frac{7}{4}+\frac{3}{4}\frac{m}{a\chi}+\frac{1}{3}\frac{a^2}{\tilde\beta^2}\right] \qquad (8)$$

$$a_\ell = \frac{1}{4\sqrt{3}}\chi m_\pi f_\pi A(\Sigma^+_0+\Lambda^\circ_-\sqrt{3}) \qquad (9)$$

$$b_\ell = \frac{5}{3}a_\ell - \frac{4}{9\sqrt{3}}csC_R\frac{a\chi}{\pi^2 m_\pi} \qquad (10)$$

Here $cs = \cos\theta_c\sin\theta_c = 0{,}215$; $C_-(\mu) = (\alpha_s(\mu)/\alpha_s(m_W))^{4/9}$ and $C_R = -C_5-\frac{3}{16}C_6$ are the RG coefficients in H: $C_-(\sqrt{s_0}) = 1{,}5$ at $\sqrt{s_0} = 1{,}5$ GeV (the continuum threshold in the sum rules), $C_R = 0{,}25$ at the low normalization point [19]. Further, $a = -(2\pi)^2\langle\bar q q\rangle = 0{,}546$ GeV^3 [7], χ is the magnetic susceptibility of quark condensate introduced in [10] ; $\chi = 8$ GeV^{-2} from QCD sum rules for the baryon magnetic moments [10] or $\chi = 5{,}7$ GeV^{-2} from the sum rules for the correlator of currents $\bar\psi\sigma_{\mu\nu}\psi$ and J_λ [21] (see also [22]). In the formula (7) $\Lambda \simeq m_\Lambda^2$ and $n \simeq m_n^2$ are the typical scales of Borel parameters in the Λ and n channels whereas the formulas (8) - (10) are obtained by comparing sum rules for a, b in $SU(3)$ limit with those for some known physical quantities and so they are independent of Borel parameters. Finally, $\tilde\beta_\Lambda, \tilde\beta_n$ are the residues of Λ, n states into corresponding currents η_Λ, η_n : $\langle 0|\eta_B|B\rangle = (2\pi)^{-2}\tilde\beta_B\gamma_5 u_B$ where

$$\eta_\Lambda = \sqrt{\tfrac{2}{3}}\,[(su)d - (sd)u], \quad \eta_n = -(dd)u \qquad (11)$$

and $(q_1 q_2) q_3$ means $(q_1^a C\gamma_\mu q_2^b)\gamma^\mu q_3^c \varepsilon^{abc}$ (see [7,8]). We have found $\tilde{\beta}_\Lambda^2 = 1{,}5$ GeV^6, $\tilde{\beta}_n^2 = 1{,}1$ GeV^6 ($= \tilde{\beta}^2$).

Let us sum up the contributions into amplitudes found and calculate branching ratio $Br(\Lambda \to n\gamma)$ and asymmetry parameter $\alpha(\Lambda \to n\gamma)$. At $\chi = 5{,}7$ GeV^{-2}

$$\mathrm{Re}\,a = a_s + a_\ell + a_{log} = (-0{,}22 - 1{,}65 + 4{,}3)\cdot 10^{-2} = +2{,}4\cdot 10^{-2}$$

$$a = (2{,}4 + 9{,}1\cdot i)\cdot 10^{-2}$$

$$\mathrm{Re}\,b = b_s + b_\ell + b_{pole} = (-4{,}9 - 5{,}8 + 1{,}0)\cdot 10^{-2} = -9{,}7\cdot 10^{-2}$$

$$b = (-9{,}7 + 1{,}0\ )\cdot 10^{-2}$$

$$Br\,(\Lambda \to n\gamma) = +2{,}1\cdot 10^{-3}$$

$$\alpha\,(\Lambda \to n\gamma) = 2\,\mathrm{Re}\,\frac{a^* b}{|a|^2 + |b|^2} = -0{,}15 \qquad (12)$$

At $\chi = 8$ GeV^{-2}

$$\mathrm{Re}\,a = (-0{,}41 - 2{,}3 + 4{,}3)\cdot 10^{-2} = +1{,}5\cdot 10^{-2}$$

$$a = (1{,}6 + 9{,}1\ )\cdot 10^{-2}$$

$$\mathrm{Re}\,b = (-6{,}5 - 8{,}1 + 1{,}0)\cdot 10^{-2} = -13{,}6\cdot 10^{-2}$$

$$b = (-13{,}6 + 1{,}0\ )\cdot 10^{-2}$$

$$Br\,(\Lambda \to n\gamma) = 3{,}1\cdot 10^{-3}$$

$$\alpha\,(\Lambda \to n\gamma) = -0{,}09 \qquad (13)$$

The spread between (12) and (13) characterizes the main contribution into the error connected with the uncertainty of input parameters, i.e. $Br\,(\Lambda \to n\gamma) = (2 - 3)\cdot 10^{-3}$, $\alpha\,(\Lambda \to n\gamma) = -(0{,}15 - 0{,}10)$. The experiment gives $Br\,(\Lambda \to n\gamma) = (1{,}02 \pm 0{,}33)\cdot 10^{-3}$ [2].

The reliability of the QCD sum rule calculations is mainly restricted by that the problem in hand is that in the external electromagnetic (and "weak") field with small momentum transfers. At least the electromagnetic interaction at large distances dominates in the considered sum rules. In this case we do not know the double discontinuity $\rho(S_1, S_2)$ of our main correlator of η_n, H and $\bar{\eta}_\Lambda$ in the field J_λ where S_1 and

S_2 are the kinematical variables in the channels separated by J_λ. The function $\rho(S_1, S_2)$ is needed to subtract unambigiously the continuum (i.e. the contribution from higher states n^* and/or Λ^* at large S_2 and/or S_1) from sum rules for n and Λ. Instead, we know the spectral properties of the correlator in variable $S = S_1 = S_2$ only. Therefore the choice of the model for continuum is ambiguous. This ambiguity is essential since the higher states cannot be suppressed enough if one operates in one variable S only [10]. Usual and simplest prescription in this case is the following one: continuum in the correlator in an external field is taken into account so as if the external field were absent. Using this recipe in our case is based on the successful application of the sum rules thus obtained to calculation of a number of hadron formfactors at small momentum transfer (magnetic moment [10,11], axial couplings [12,13], the decays $\mathcal{D}^* \to \mathcal{D}\pi$ and $\mathcal{D}^* \to \mathcal{D}\gamma$ [23], nonleptonic [15] and radiative 16 hyperon decays and so on). In these cases the good agreement with experiment is achieved. However, contrary to the classical (two-point) sum rules 6-9 (which possess no small momentum transfers) the error due to higher states is uncontrollable now and a priori could be larger. If (somewhat arbitrarily) one assumes our sum rules give the slowly varying part of amplitude within a factor of 2, then we get the estimates like $Br\,(\Lambda \to n\gamma) \simeq (1 - 5)\cdot 10^{-3}$, $|\alpha(\Lambda \to n\gamma)| \lesssim 0{,}1$. The value of $Br\,(\Lambda \to n\gamma)$ is close to the experimental one if this slowly varying component is small as compared to $\mathrm{Im}\,a$. In this case $\mathrm{Im}\,a$ dominates in the $\Lambda \to n\gamma$ decay width.

The author is grateful to L.G.Landsberg and V.T.Smolyankin who have drawn our attention to this problem and to I.B.Khriplovich for stimulating interest in the problem and for the discussions.

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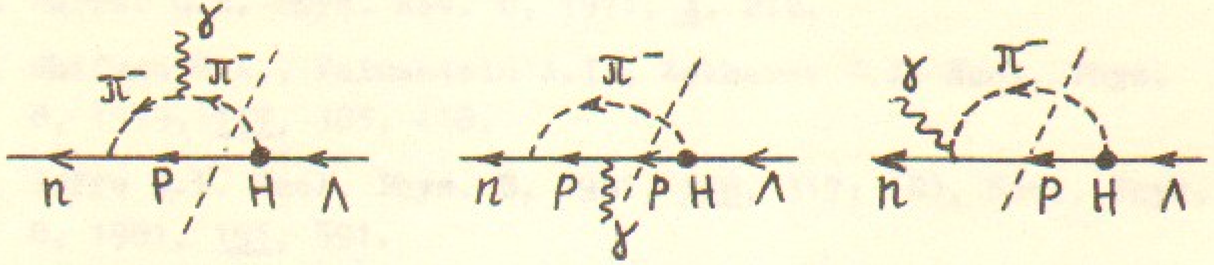
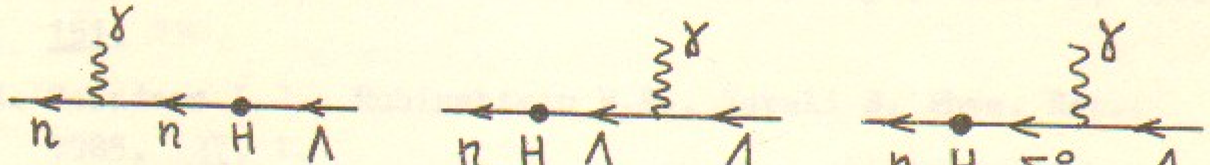
Figures



(a) (b) (c)

Fig. 1.



(a) (b) (c)

Fig. 2

В.М.Хацимовский

О РАСПАДЕ $\Lambda \to n\gamma$

Препринт
86-151

Работа поступила - 20 августа 1986 г.

Ответственный за выпуск - С.Г.Попов
Подписано к печати 10.Х.86г. МН 11825
Формат бумаги 60х90 1/16 Усл.0,8 печ.л., 0,7 учетно-изд.л.
Тираж 180 экз. Бесплатно. Заказ № 151.

Ротапринт ИЯФ СО АН СССР, г.Новосибирск, 90