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# STUDY OF COULOMB-TYPE INTERACTION OF b QUARKS

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ABSTRACT

We discuss QCD sum rules for upsilon system and consider information on short-range interaction of $b$ quarks which follows from available experimental data on $e^+e^-\to b\bar{b}$. Constraints on $b$ quark mass and parameter $\Lambda$ are found.



## 1. INTRODUCTION

Predictive power of various applications of QCD is nowdays strongly restricted by the fact, that the numerical value of its fundamental $\Lambda$ parameter remains so far rather poorly known. The problem is to find an effect, which, at one hand, is purely perturbative, and, at another hand, can be accurately measured. In the present work such effect is the Coulomb-type interaction between quarks at sufficiently small distances.

In the limit of very large quark mass quarkonium states are essentially different from common hadrons, because the separation of quarks in them is much smaller than 1 fm, the typical confinement length. So, from the first days of QCD it was argued [1] that such particles are bound essentially by perturbative Coulomb-type forces, with small and calculable correstions. Unfortunately, the size of charmonium and upsilon mesons is not sufficiently small, and non-perturbative effects are quite significant. This can be seen from the phenomenological potentials, extracted from data analysis [2]. In order to emphasize an ambiguity of the potential at small distances, we remind that among successful potentials there are those without any Coulomb-type term at all. Thus, the Coulomb forces among quarks have not been yet clearly observed.

In this work we discuss whether the data on $e^+e^-\to b\bar{b}$ indicate the presence of Coulomb-type potential between $b$ quarks at small distances and obtain some estimates of its strength. Unlike the previous works dealing with phenomenological potentials, we do not consider spectroscopy of stationary states. Instead, we study some virtual «wave packets», constructed out of stationary states and being much more compact than any of them. It is obvious that its properties are affected by Coulomb forces much stronger, while ambiguities related to nonperturbative effects are significantly reduced.

Investigations of such wave packets are the key element of the so called QCD sum rules, suggested by Shifman, Vainshtein and Zakharov [3]. First considerations of upsilon system in this framework were made by Voloshin [4]. Among his conclusions there was the statement that Coulomb-type potential with «running coupling constant» (due to asymptotic freedom) is incompatible with experimental data. Voloshin has suggested to «freeze» the colour coupling at $\alpha_s \simeq 0.3$.

The technical problem here is to evaluate the propagation amplitude, taking into account both quark-antiquark Coulomb-type interaction and nonperturbative vacuum fields. Using the method based on numerical evaluation of the relevant path integral one of the authors has attempted such calculations for few models of QCD vacuum structure [5], and no contradiction between the data and the modified Coulomb law has been found. However, in this work the region of small distances was not studied in much details, in particular, no attempts to fix the lambda parameter and the $b$ quark mass were made.

This problem was also addressed by Baier and Pinelis [6, 7]. In the former work the perturbative part of the Green function was taken for pure Coulomb force, with the coupling constant taken at some typical distance. In the latter work some more elaborate approach was suggested. These authors claim that the value of relevant parameters, $m_b$ and $\Lambda$, can be rather accurately fixed. However, as we show below, these results are obtained in the region where the perturbative potential used is meaningless. Moreover, the method for evaluation of the Green function fails by itself in this region. Thus, our opinion is that these results are not well grounded.

The work is structured as follows. In section 2 we outline the sum rules used, and in section 3 we describe the method used for the evaluation of the Green functions, while the properties of potential under study are considered in section 4. Results of the calculations and data analysis are discussed in section 5, and our conclusions are summarized in section 6.



## 2. SUM RULES

The standard starting point of any sum rules is the dispersion relation for real and imaginary parts of the polarization operator

$$\mathrm{Re}\,\Pi(Q^2)=\frac{1}{\pi}\int ds\,\frac{\mathrm{Im}\,\Pi(s)}{s+Q^2},$$

$$\Pi(Q^2)\,(q_\mu q_\nu-g_{\mu\nu}q^2)=\int d^4x\,e^{iqx}\,K_{\mu\nu}(x),\qquad Q^2=-q^2>0, \tag{1}$$

$$K_{\mu\nu}(x)=\langle 0|\,T\{\bar q(x)\gamma_\mu q(x)\,\bar q(0)\gamma_\nu q(0)\}|\,0\rangle,$$

where the r.h.s. is known from experimental data on $e^+e^-$ annihilation into $b$ quarks

$$\mathrm{Im}\,\Pi(s)=\frac{s}{16\pi^2\alpha^2 e_b^2}\,\sigma(e^+e^-\to b\bar b, s) \tag{2}$$

(here $\alpha=1/137$ is the fine structure constant and $e_b=-1/3$ is the $b$ quark charge). It is traditional to follow ref. [3] and to perform Borel transformation, but we prefer more transparent possibility suggested in refs [4, 5] and perform Fourier transformation, returning us from momentum to coordinate space

$$K(\tau)\equiv K_{\mu\mu}(x)|_{x^2=\tau^2}=$$

$$=\frac{3}{16\pi^3\alpha^2 e_b^2}\int ds\,s^2\,\sigma(e^+e^-\to b\bar b, s)\,D(s,x^2=\tau^2). \tag{3}$$

Here $\tau$ is the so called «Euclidean time» (or distance) between two points at which the external electromagnetic currents affect the QCD vacuum. The function $D(s,x^2)$ is defined as

$$D(s,x^2)=\int\frac{d^4p}{(2\pi)^4}\,\frac{e^{-ipx}}{p^2+s}=\frac{1}{4\pi}\left(\frac{s}{x^2}\right)^{1/2}K_1((x^2s)^{1/2}) \tag{4}$$

($K_1$ is the modified Bessel function) and it is just the amplitude of propagation of the particle of mass $s^{1/2}$ from one space-time point to another. So, the physical meaning of relation (3) is selfevident.

The correlation function $K(\tau)$ is the main quantity we deal with, but it is inconvenient to plot directly this functions because due to obvious reasons it is very strongly varying over the region of $\tau$ under investigation. Therefore, we introduce its logarithmic derivative

$$E(\tau) = -\frac{d \ln K(\tau)}{d\tau}. \qquad (5)$$

Note, that this quantity has simple physical meaning, representing some average energy of the virtual wave packet with the lifetime $\tau$. In particular, at large $\tau$ this quantity tends to the mass of the upsilon meson. In order to get rid of trivial kinematical contributions we additionally subtract from (5) the logarithmic derivative of free quark propagator squared (with the mass $m_b = 4.9$ GeV):

$$F(\tau) = E(\tau) - \left[ -\frac{d}{d\tau} \ln K^{free}(\tau) \right]. \qquad (6)$$

Thus, this quantity is nonzero only either due to interaction between quarks, or to deviations of their masses from this reference value. Roughly speaking, we have subtracted from the measured energy of the packet the part, corresponding to quark kinetic energy.

At this point it is meaningful to consider the magnitude of relativistic corrections. There are nontrivial effects like the spin forces, retarding potentials etc. which it is very difficult to take into account. Kinematical relativistic effects are much simpler, and we may evaluate their magnitude by the following simple trick. We may subtract the logarithmic derivative corresponding either to relativistic free propagators or to its nonrelativistic version. The results are presented in Fig. 1 by the shaded region and the dashed line, respectively. The conclusion, following from this exercise, is that we cannot trust nonrelativistic calculations at $\tau$ less than about 1 GeV$^{-1}$.

The parametrization used for resonance and continium structure is as follows

$$\sigma(e^+e^- \to b\bar{b}) = \sum_{i=0}^{3} \frac{12\pi^2 \Gamma_{li} \delta(s - M_i^2)}{M_i} + \frac{4\pi\alpha^2}{s} R_b \cdot \theta(s - s_0),$$

where leptonic widths $\Gamma_{li}$ and masses $M_i$ of upsilons are taken from Particle Data [8], while for continuum we use $R_b = 0.31 + 0.06$ [9] and assume that it starts just from the mass of $\Upsilon'''$ state. In order to estimate the errors we have simulated spread in data and have measured the corresponding effect on the correlation function and its derivative.





We have to calculate the propagation amplitude of two quarks, coming from one point to another at distance $\tau$. One of our main approximations is the nonrelativistic approach used, assuming that the interaction is described by some potential $V(x)$. We use well known Dirac—Feynman representation for the propagation amplitude in terms of path integral

$$\frac{K(\tau)}{K^{free}(\tau)} = \frac{\int Dx[t] \exp\left\{ -\int_0^\tau dt \left[ \frac{m\dot{x}_1^2}{2} + \frac{m\dot{x}_2^2}{2} + V(x_1 - x_2) \right] \right\}}{\int Dx[t] \exp\left\{ -\int_0^\tau dt \left[ \frac{m\dot{x}_1^2}{2} + \frac{m\dot{x}_2^2}{2} \right] \right\}} =$$

$$= \langle \exp\{ -\int_0^\tau dt\, V(x_1 - x_2) \} \rangle_{free\ paths}, \qquad (7)$$

where $\langle \ldots \rangle_{free\ paths}$ means the average over path ensemble of free (noninteracting) quarks.

Methods for numerical evaluation of the path integrals are usually based on such Monte-Carlo methods as Metropolis algorithm. However, we have prefered to generate directly the paths for free propagation and then average out the factor related with mutual interaction. It was done by means of the following Fourier representation of the path

$$x(t) = \sum_k c_k \sin\left( \frac{\pi k t}{\tau} \right). \qquad (8)$$

The coefficients $c_k$ may be generated independently with Gaussian weight. As we show below, this simple method provides sufficiently accurate results, which was tested e. g. for the case of pure Coulomb forces, for which the analytic results are known. We have also applyed this method to potentials discussed in literature and have compared our results with those found in original papers. No significant deviations were found. Because this method provides reliable results for the Green functions in any potential, it is reasonable to check various approximations used in the previous works. But before we come to this issue, we outline the region of parameters where effects of perturbative potential dominate.

## 4. POTENTIALS

At small distances one-gluon exchange and the asymptotic freedom leads to the following «running coupling» potential

$$V(r)=\frac{4}{3}\frac{2\pi}{b_0\ln(\Lambda r)}=-\frac{4}{3}\frac{\alpha_s(r)}{r}. \qquad (9)$$

The two-loop calculations made in refs [10, 11] relate our parameter lambda to more standard definitions, e. g. $\Lambda=2.63\Lambda_{\overline{ms}}$. (We have disregarded the term $\log(\log(r\Lambda))$, also appearing in two-loop approximation, which is not important at distances under consideration.)

It is tempting to neglect the first log too, and substitute this potential by pure Coulomb with some fixed coupling. However, in spite of the fact that the deviations from pure Coulomb potential are slowly varying (see Fig. 2), the Green function depends on the potential exponentially, thus the effect of this log is quite noticeable.

The main problem we address now is to fix the applicability limits of this potential for $b$ quarks. As soon as one approach the distance $r=1/(\Lambda e)$ ($e=2.71928...$) the potential (9) has a maximum, and at larger distances attraction between the quark and the antiquark changes to repulsion. Obviously, this is an artifact, and some physical effect should modify the interaction earlier.

Some simple-minded ways to «cure» this expression are shown in Fig. 2 by dashed lines: ($a$) correspond to condition, that the force is equal to standard string tension, while ($b$) assume for potential to be constant to the right from the maximum. Theoretically, the problem is how to take into account the nonperturbative effects. As noted by Voloshin [4], at small enough distances their effect is essentially non-potential (see below).

Indeed, the potential is meaningful if the typical periods of quark motion are large compared to correlation time of the nonperturbative fields. If vacuum is mainly populated by small-size vacuum fluctuations (e. g. the instantons) this approach can be meaningful. In the instanton case the potential was calculated in ref. [12] and substituting here numbers from phenomenological analysis [13] we have the following corrections to potential

$$\Delta V(r)=11.27r^2\int\frac{d\varrho}{\varrho^4}D(\varrho)\simeq 3.6\cdot 10^{-2}r^2\langle G^2_{\mu\nu}\rangle\,\varrho_0\simeq 5.8\cdot 10^{-2}r^2,$$



$$\varrho_0\sim 1.6\ \text{GeV}^{-1},\qquad \langle g^2G^2_{\mu\nu}\rangle\sim 1\,(\text{GeV})^4. \qquad (10)$$

The sum of (9) and (10) is shown in Fig. 2 by dotted curve.

The question we discuss now is where the ambiguity of the potential is relevant. In Fig. 3 the results of calculations of $F(\tau)$ for these potentials are shown. We also have displayed by dot-dashed curve a non-potential correction found by Voloshin, which corresponds to vacuum model with very long-range nonperturbative field.

$$\Delta F(\tau)\simeq\frac{3}{4}\frac{\tau^2}{m_b}e^{-0.8\gamma}\left(1-\frac{2}{15}\gamma\right)\cdot\eta,$$

$$\gamma\equiv\frac{2}{3}\alpha_s\cdot(m\tau)^{1/2},\qquad \eta\equiv\left\langle\frac{\alpha_s}{2\pi}G^2_{\mu\nu}\right\rangle. \qquad (11)$$

These considerations allow us to fix the bounds of the «window» to be used in our analysis. (Note, that with an increase in lambda this window disappears.)

Voloshin [5] had used the Green functions corresponding to pure Coulomb potential, with $\alpha_s$ being fixed at 0.3. Baier and Pinelis [6] have used the same Coulomb formulae, but substituting $\alpha_s$ by its value corresponding to some typical distance $\alpha_s=\alpha_s(r_0)$ with $r_0$ depending on $\tau$. Later [7] they have improved this approximation, introducing more elaborated definition of $r_0(\tau)$. It corresponds to approximate potential of the form

$$V(r)\simeq V_0(r)+\Delta V(r)=-\frac{4}{3}\frac{\alpha_s(r_0)}{r}\left(1+\frac{\ln(r/r_0)}{\ln(\Lambda r_0)}\right), \qquad (12)$$

where $r_0(\tau)$ is taken from the condition that first order correction in $\Delta V$ to the Green function vanishes. So, the neglected terms are of the order $(\Delta V)^2$.

In Fig. 4 we plot the Green functions calculated in these approximations and for the original «running coupling» one (9). It is seen, that at small $\tau$ the results are consistent, while for $\tau\gtrsim 1-2\ \text{GeV}^{-1}$ deviations are larger than the factor of two. The reason for such bad accuracy is connected with the fact, that, with large value of $\Lambda\sim 0.5$ GeV used in [7], at such distances one comes into the strong coupling region, $\alpha_s\sim 1$, where all perturbative expansions are meaningless.

## 5. RESULTS

We start with consideration of the phenomenological potentials suggested for charmonium and upsilon systems. As some limiting cases we have chosen the so called Martin potential [14]

$$V(r) = -8.064 + 6.8694 \cdot r^{0.1} \qquad (13)$$

and Cornell potential [15]

$$V(r) = -\frac{0.52}{r} + 0.18 \cdot r. \qquad (14)$$

The former does not posess any Coulomb term, while for the latter it is rather large.

Making comparison with the data one should take into account the following general fact. The small shift of the potential $V(r)$ by some constant $\delta V$ is practically undistinguishable from the same shift in the quark mass $\delta m_b = -\delta V/2$. In other terms, we know the total energy of our virtual packet, but do not know how to split it into the quark mass and the potential energy.

Experimentally allowed region is shown in Fig. 5 by the shaded band, while the two curves correspond to potentials (13) and (14). It is seen that Martin potential agrees with data, while for Cornell one a shift in the quark mass by about 70 MeV (compared to that indicated in the original work) also leads to good description of the data. Note however, that the slope of these two curves at small times is quite different. Therefore, with better data on $e^+e^-$ annihilation into beauty above the threshold one may hope to obtain purely experimental contstraints on the shape of the phenomenological potential at small distances.

The same problem can be considered from another, more theoretical side. We know that QCD prescribes the potential to be equal to «running coupling» potential (9) at small distances. Assuming this, can we fix the parameter lambda? The fit should be done in the «window», in which one can neglect both relativistic and nonperturbative effects, and we have already discussed its bounds above.

Therefore, we can at least fix the sum of the quark mass and the potential depth. At Fig. 3 our results for the logarithmic derivative $F(\tau)$ for various lambdas are compared with data. Making the vertical shift (mass adjustment) one can fit the data inside the



«window». Accuracy is of the order of 100 MeV, both for data themselves and for the fit. This fit is then translated to lambda value, and the final results for the $b$ quark mass and lambda parameter are given in Fig. 6. Our strip ends at large enough lambda (the dashed region), where the window disappears. The point shown in this plot is taken from ref. [7]. Let us note, that in refs [6, 7] the correlator has been fitted to data for the region of $\tau = 2-5$ GeV$^{-1}$. For a rather large value of $\Lambda \sim 0.5$ GeV obtained in the fit this region lies obviously outside the allowed «window». Moreover, as it was shown above, the analitical approach for correlator developed in ref. [7] fails in this region as well.

With better data and account for relativistic corrections one may hope to make better fit and extend the «window», fixing not only the absolute magnitude of $F(\tau)$ in it, but its slope too. Only in this case, one may obtain stronger restrictions on parameters under investigation, in particular, extract $m_b$ and lambda separately.

## 5. CONCLUSIONS

The main result of our investigations of the sum rules for upsilon system is that, unlike for all other known hadrons, there exist a possibility to consider the wave packet which is neither too large, in order to be affected by nonperturbative effects, nor too small, so that nonrelativistic approach is meaningful. Moreover, available experimental data fix the total energy of such packet with the accuracy of the order of 100 MeV.

This observation put important constraint on both the quark mass and the magnitude of Coulomb-type potential. Unfortunately, experimental accuracy is insufficient to fix these fundamental parameters of strong interaction theory independently. By the same reason, it turns out that such sum rules cannot decrease existing ambiguity in the shape of phenomenological potentials at small distances. Nevertheless, with progress in data quality one may hope to do this, because the corresponding curves are in fact rather different.

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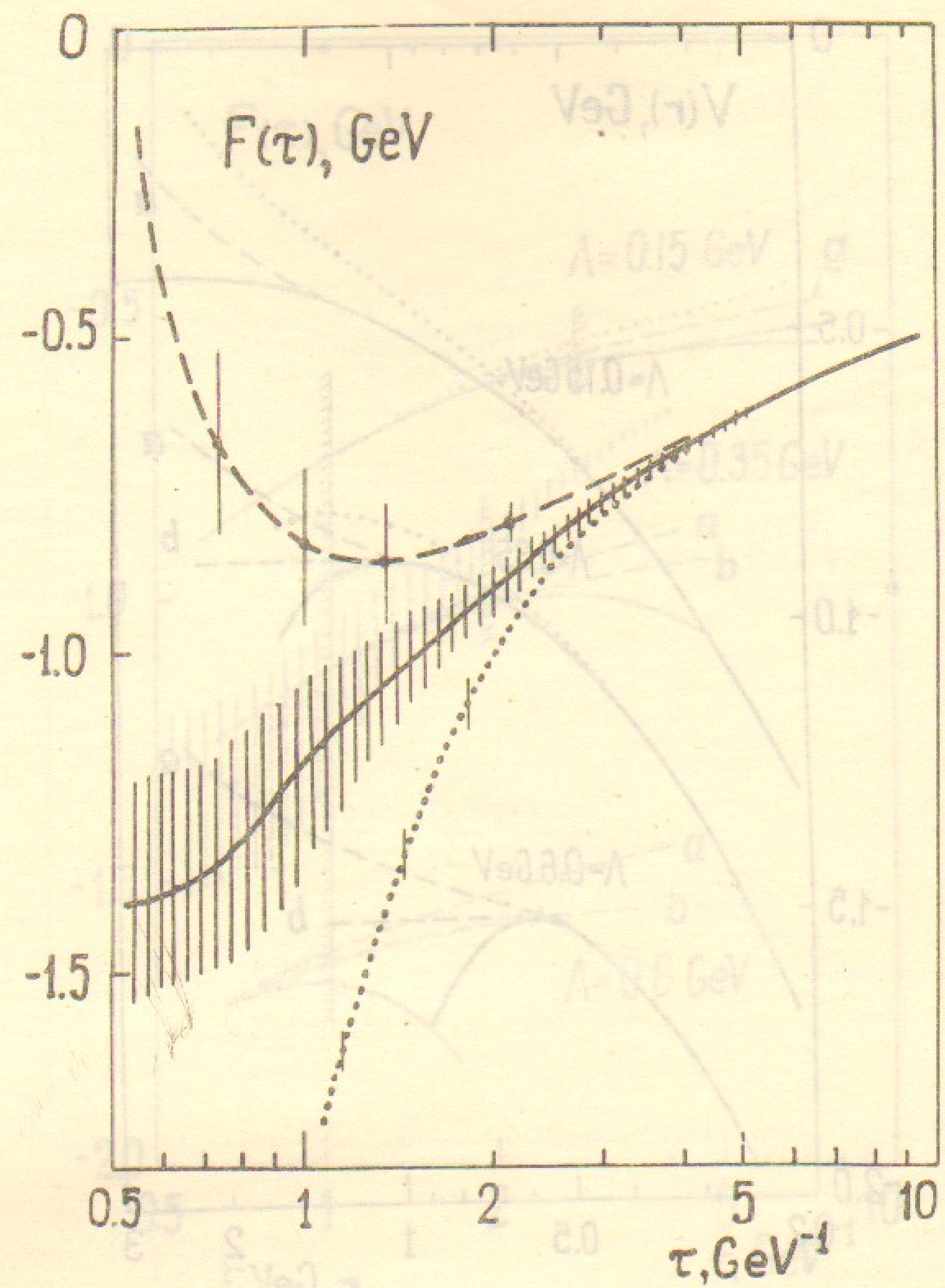


*Fig. 1.* The logarithmic derivative of the correlation function $F(\tau) = -d/d\tau\,(\log K(\tau))$ minus that for the propagation of two noninteracting quarks with the mass $m_b = 4.9$ GeV. The shaded regions show the experimental errors. The dashed curve shows what happens if the quark propagators are taken in nonrelativistic approximation. The dotted curve shows what happens if one ignores the contribution of states in continuum energy region.

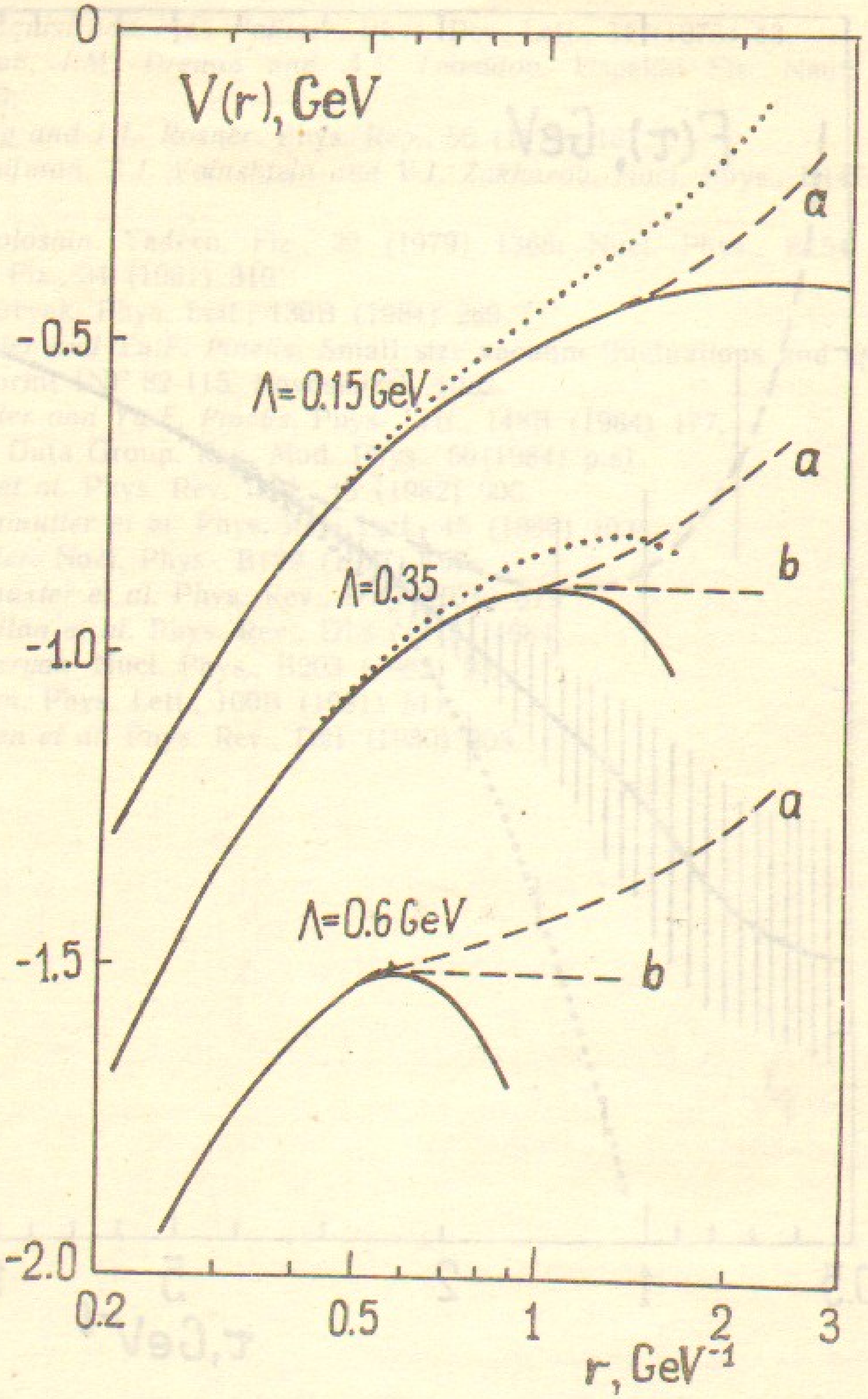


*Fig. 2.* The set of potentials between *b* quarks under consideration. Solid curves show the «running coupling» potential given by eq. (9), while dashed and dotted curves correspond to its different redefinition near the infrared pole (see text).



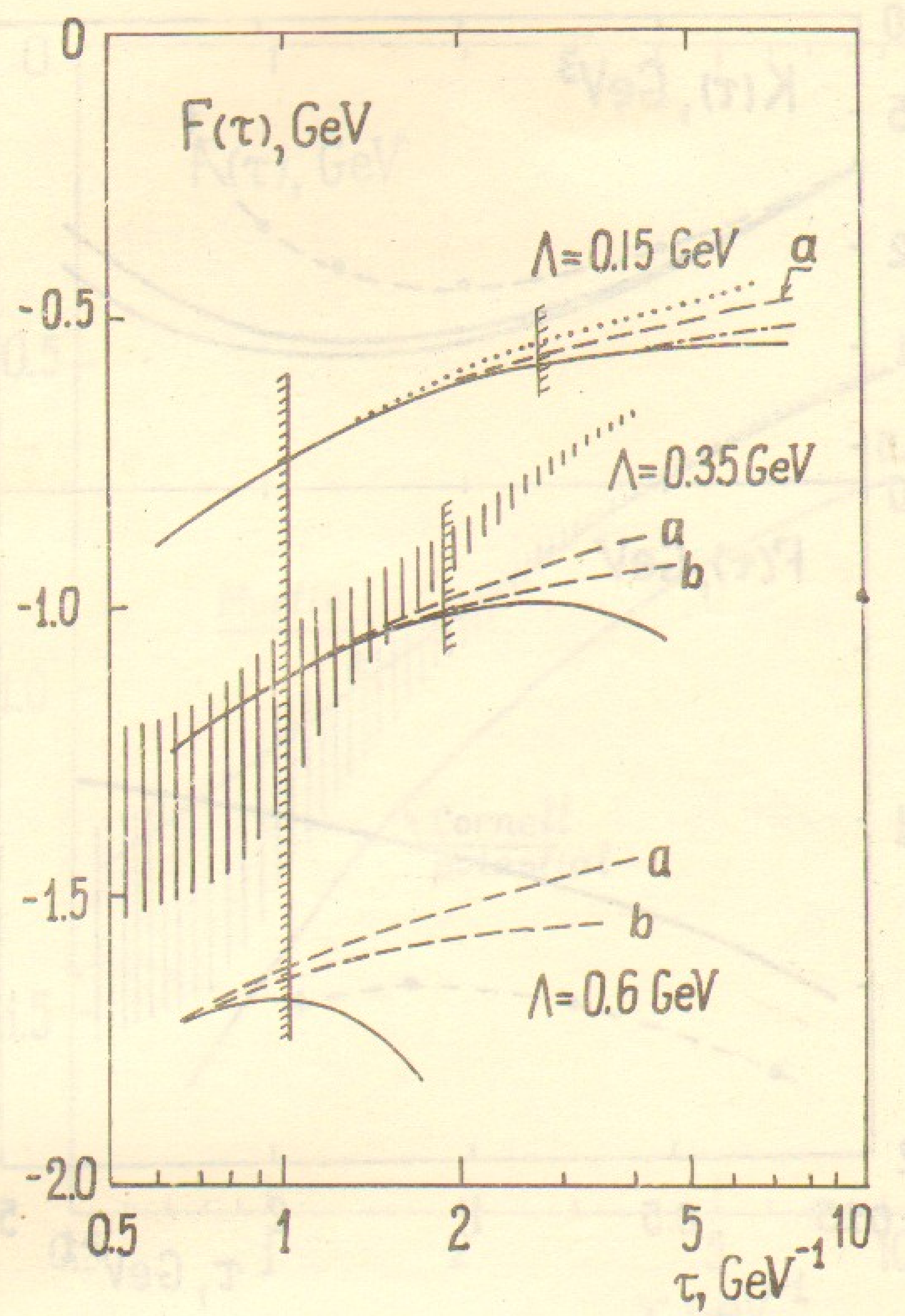


*Fig. 3.* The same quantity as in Fig. 1. The curves are calculated for the set of potentials shown in Fig. 2. The dot-dashed curve shows the magnitude of nonpotential contribution given by eq. (11). The left bound ($\tau \sim 1$ GeV$^{-1}$) arises because of relativistic effects and right bounds indicate places, where nonperturbative effects seem to be dominant.

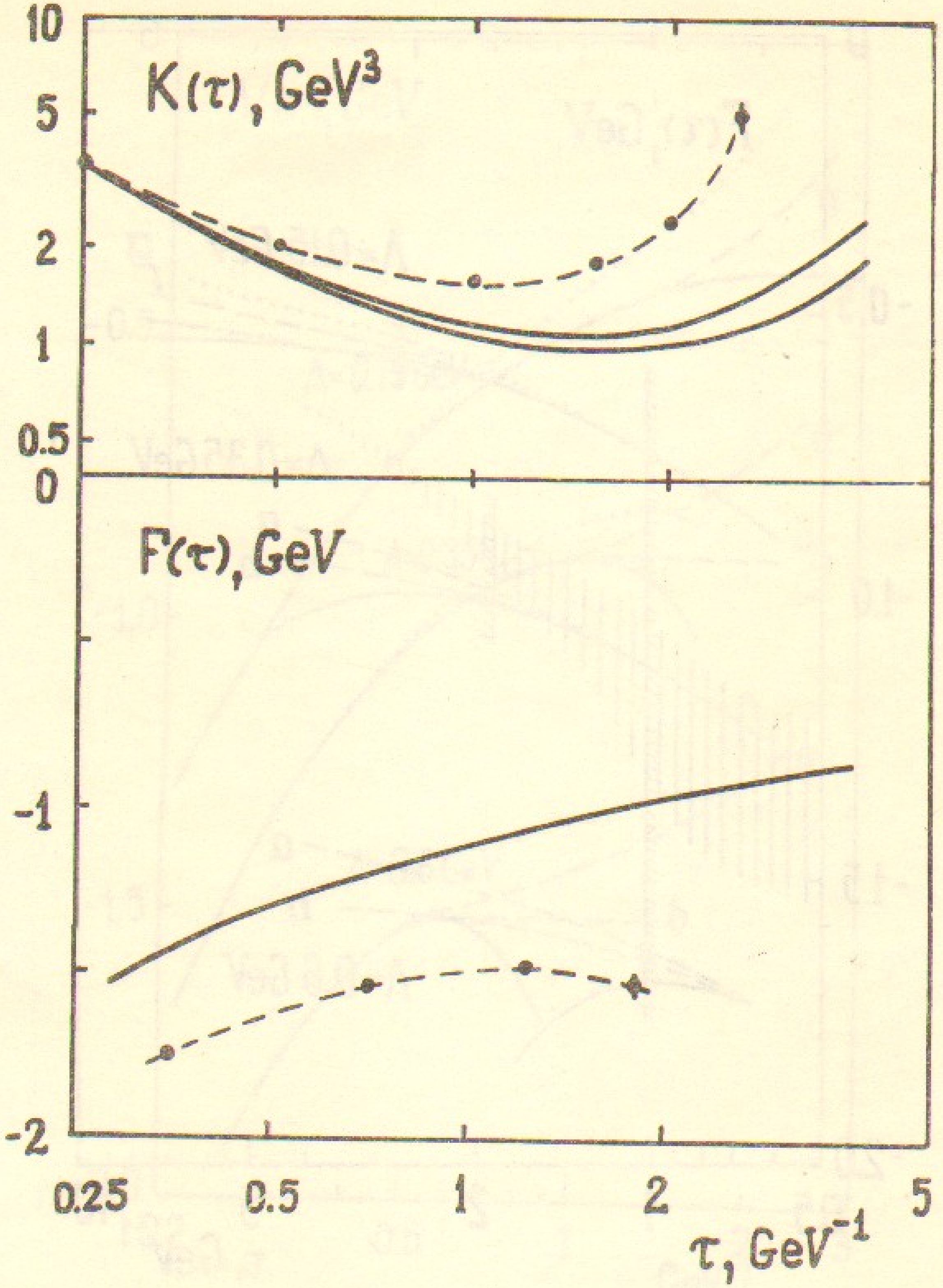


*Fig. 4.* Correlator $K(\tau)$ and its logarithmical derivative $F(\tau)$. Points show our numerical results, while solid curves are calculated by analitical formula taken from refs [6, 7].



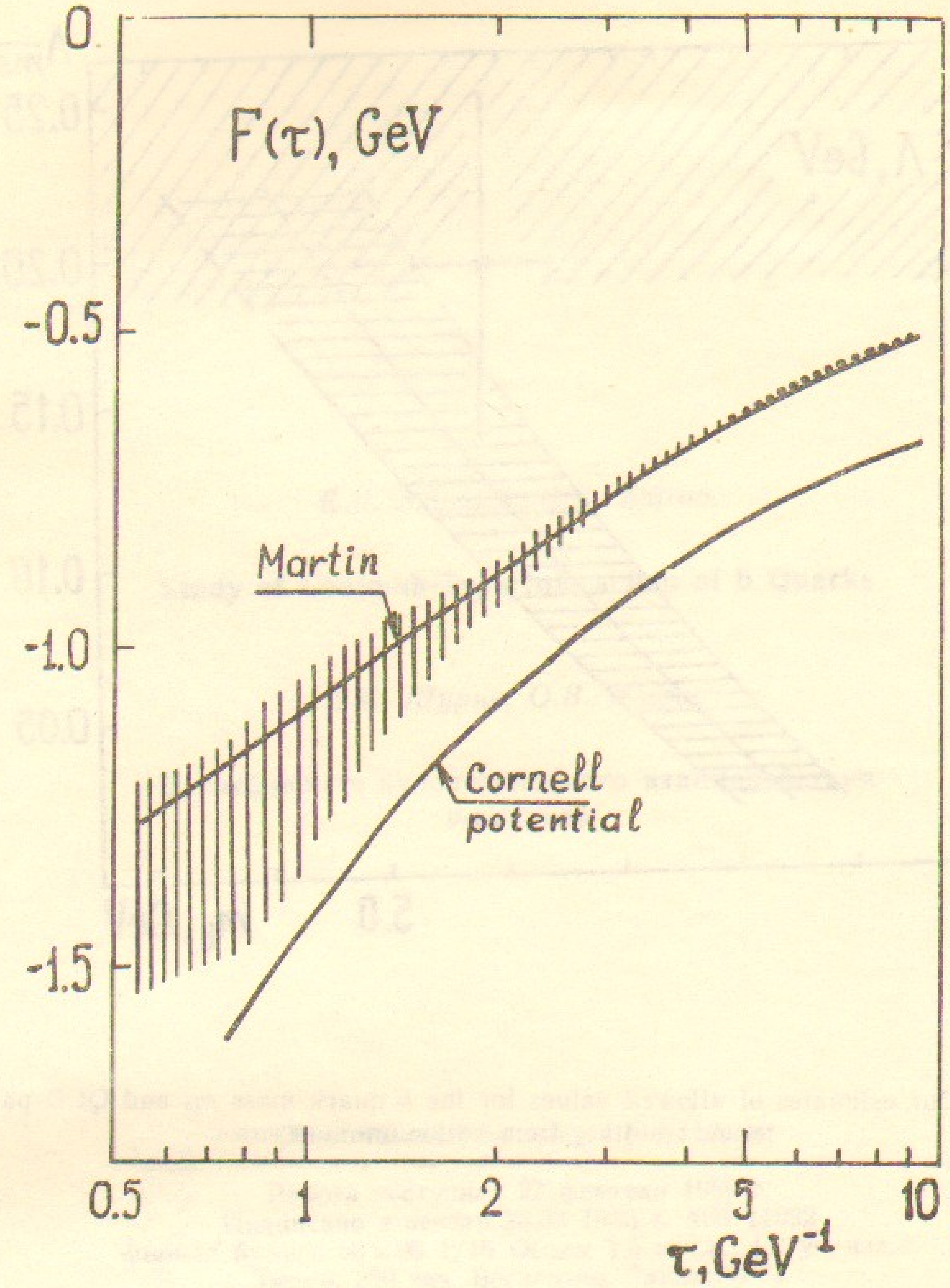


*Fig. 5.* The same quantity as in Fig. 1. The curves correspond to Martin's and Cornell potentials (see text).

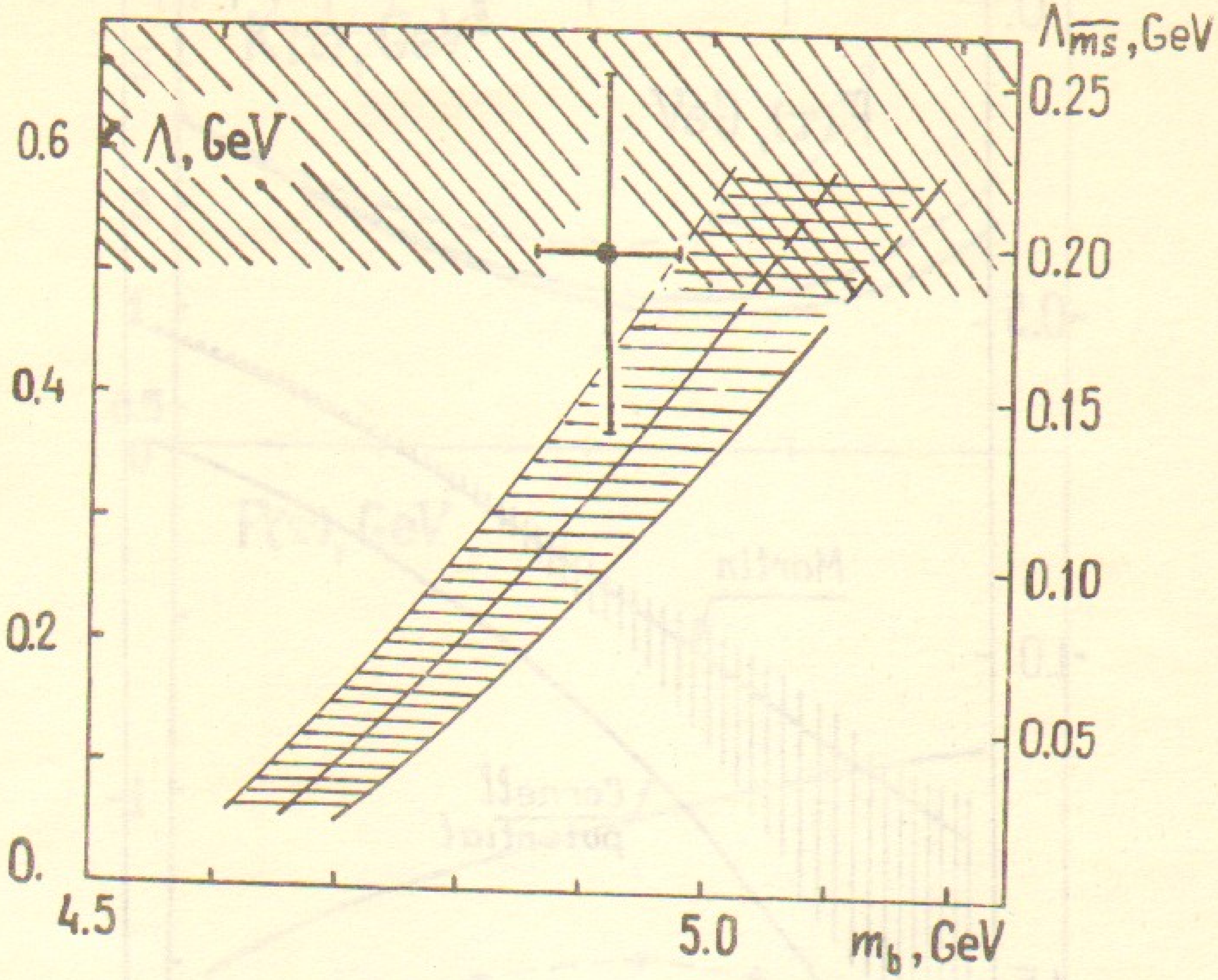


*Fig. 6.* Our estimates of allowed values for the $b$ quark mass $m_b$ and QCD parameter Λ, resulting from bottonium sum rules.



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**Study of Coulomb-Type Interaction of b Quarks**

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**Исследование кулоноподобного взаимодействия $b$-кварков**

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