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НОВОСИБИРСК

QUARK ELECTRIC DIPOLE MOMENT AND INDUCED θ-TERM
IN THE KOBAYASHI-MASKAWA MODEL

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ABSTRACT

Closed expressions are obtained for the quark electric dipole moment and induced θ-term in the Kobayashi-Maskawa model in the approximation $G^2\alpha_s$.

1. The quark electric dipole moment (edm) in the Kobayashi-Maskawa (KM) model was discussed by theorists repeatedly/1-6/. To the lowest, one-loop approximation in the weak interaction the quark edm in the KM model turns to zero trivially. The point is that in this approximation only the moduli squared of the elements of the KM matrix enter the answer for the edm, so this answer cannot contain CP-violating phase. Much less evident is the result of Ref./4/ where the quark edm was shown to vanish also to the two-loop approximation in the weak interaction. Furthermore, there is an assertion/6/ that the quark edm is equal to zero in the two-loop approximation in the weak interaction even if the gluon correction is taken into account, to the order $G^2\alpha_s$. Here G is the Fermi weak interaction constant, α_s is the quantum chromodynamics coupling constant. And finally, it is claimed in Ref./5/ that the induced θ-term in the KM model arises, when the gluon corrections are taken into account, in the order $G^2\alpha_s^3$ only, however an explicit expression for the θ-term was not obtained there.

In such a situation a suspicion arises that there is a general, so to say, kinematic reason of the fact that the quark edm and induced θ-term in the KM model turn to zero, at least in the approximations $G^2\alpha_s$ and $G^2\alpha_s$, $G^2\alpha_s^2$ correspondingly. Just this suspicion was the starting point of the present investigation. But its results described below turn out to be quite different from those expected. Closed non-zero expressions were obtained for the quark edm and θ-term in the order $G^2\alpha_s$ for each one.

2. Consider at first the flavour structure of the corresponding expressions. Here we leave aside for a while gluon corrections and the insertions of external fields, electromagnetic one for edm and gluon one for θ-term. Neither of them influences the flavour structure.

Note first of all that the diagrams with two fermion lines, of the kind 1 for the edm and 2 for the θ-term, can be dropped out immediately. The reason is the same that forbids CP-violation to the lowest order in the weak interaction: each fermion line contains elements $V_{qq'}$ of the KM matrix in the combination

$V_{qq'}V^{+}_{q'q} = |V_{qq'}|^2$ only, so that there is no CP-violating phase.

Consider now the flavour structure for the d-quark edm. We mean the diagrams 3a,b. For them this structure can be presented as

$$\sum_{U_1,U_2,D} V^{+}_{dU_2} V_{U_2D} V^{+}_{DU_1} V_{U_1d}\, U_2 D U_1.$$

Its CP-violating part equals

$$-i\tilde{\delta}\,[u(s-b)c - c(s-b)u + c(s-b)t - t(s-b)c + t(s-b)u - u(s-b)t]. \qquad (1)$$

For the KM matrix the standard parametrization/7/ is used:

$$\tilde{\delta} = \sin\delta\, c_1 c_2 c_3 s_1^2 s_2 s_3; \qquad (1a)$$

the letters u,s,b,c etc. denote here the Green functions of the corresponding quarks: U and D mean the sets of quarks with the charges 2/3 and -1/3 correspondingly: U = (u,c,t), D= (d,s,b). The analogous expression for the u-quark edm looks as follows:

$$i\tilde{\delta}\,[d(c-t)s - s(c-t)d + s(c-t)b - b(c-t)s + b(c-t)d - d(c-t)b]. \qquad (2)$$

And at last, the fermion loop structure (see Fig.4) is

$$\sum_{U_1,U_2,D_1,D_2} V_{U_2D_2} V^{+}_{D_2U_1} V_{U_1D_1} V^{+}_{D_1U_2}\, U_2 D_2 U_1 D_1.$$

Its CP-violating part that determines the θ-term equals

$$2i\tilde{\delta}\,[u(dcs - scd + scb - bcs + bcd - dcb) + c(dts - std + stb - bts + btd - dtb) + t(dus - sud + sub - bus + bud - dub)] \qquad (3)$$

In expression (3) each product of four quark propagators allows naturally for cyclic permutations of the kind

udcs = dcsu = csud = sudc.

Note that already from these purely kinematic relations a curious result follows: in the limit of the usual SU(3) symmetry when the propagators d and s coincide, the u-quark edm and the induced θ-term vanish to the second order in the weak

interaction and to every order in α_s (see (2) and (3)).

3. Pass now to the direct computation of the quark edm. We restrict to the local four-fermion limit for the weak interaction, i.e. we take the W-boson mass tending to infinity at the fixed Fermi constant G. The high degree of cancellation among the contributions of various diagrams, evident from formulae (1) - (3), guarantees the ultraviolet convergence of the result even in this limit.

In the diagrams 5a, "double drop", and 5b, "two drops", arising from 3a,b after shrinking of the W-boson lines, it is convenient to perform the Fierz transformation

$$\bar{\psi}_1\gamma_\mu(1+\gamma_5)\psi_2\,\bar{\psi}_3\gamma_\mu(1+\gamma_5)\psi_4 = \frac{1}{3}\bar{\psi}_1\gamma_\mu(1+\gamma_5)\psi_4\,\bar{\psi}_3\gamma_\mu(1+\gamma_5)\psi_2 + \frac{1}{2}\bar{\psi}_1\lambda^a\gamma_\mu(1+\gamma_5)\psi_4\,\bar{\psi}_3\lambda^a\gamma_\mu(1+\gamma_5)\psi_2. \qquad (4)$$

It allows us to consider below every "drop" at diagrams 5a,b as a closed fermion loop. Here λ^a are the colour SU(3) matrices. Formula (4) is written for anticommuting operators ψ_i, therefore, arising fermion loops enter with the standard factor (-1), it is essential below for the calculation of the induced θ-term.

A simple "drop" (see diagram 5b and the lower block at diagram 5a) turns evidently to zero. The insertion of a photon (the circle at Fig.6) into a simple "drop" still gives zero in the limit of the constant external electromagnetic field $F_{\alpha\beta}$. Indeed, in diagram 6, arising in such a way, the axial current in the weak vertex (it is denoted by a point) does not work. Therefore, the polarization operator is left in fact that corresponds to $\partial_\alpha F_{\alpha\beta}$, but in no way to $F_{\alpha\beta}$. Hence follows, by the way, that the quark edm turns to zero to the approximation G^2 without gluon corrections.

As to the approximation $G^2\alpha_s$ discussed here, it is clear that at diagram 5a a gluon should be emitted from the lower "drop", and at diagram 5b both "drops" should be connected by a gluon, it being denoted by a broken line. In the case of diagram 5a it can be easily seen that the virtual gluon should terminate at the external fermion line. Indeed, otherwise the "double drop" would be connected to this line in one point only.

Even if the external photon goes out of the "double drop", one cannot construct the necessary vector from the only characteristic of this "drop", the constant tensor $F_{\alpha\beta}$. So much the more the diagram with one common point turns to zero if the photon is connected to the external fermion line.

Having traced the fate of the virtual gluon, discuss now what to do with the external photon that is still at our disposal. Turn attention to the middle loop at Fig.5a. It follows from expressions (1), (2) that two lines in this loop correspond to different fermions and that the difference of two such diagrams with transposed particles enters the answer always. One can check easily that this difference is equal to zero. Hence the only way out is to connect the external photon to this middle loop. As a result we come to diagrams of the type 7a. Surely, the photon should be connected to both lines of the middle loop, and the virtual gluon can terminate both at the initial and final part of the external fermion line.

As to the diagrams of the type 5b, it can be seen again from formulae (1), (2) that the answer contains the difference of two such diagrams with transposed "drops". One can see easily that this difference turns to zero if the external photon is not connected to one of the "drops". As a result only the diagrams of the type 7b are left here. Surely the photon should be connected to both parts of each "drop".

Passing to the calculations directly, we note that in each of the fermion loops at diagrams 7a,b the colour currents in the rhs of formula (4) are operative only. One can extract the corresponding factor in the answer for the edm immediately and have no care for the λ -matrices anymore.

We present now some intermediate formulae necessary for the final answer. The expression for the fermion loop 8, the simplest block of diagrams 7a,b, is well-known:

$$\int \frac{d^4q}{(2\pi)^4} \, Tr\, \gamma_\mu (1+\gamma_5) \frac{1}{\hat{q}-m} \gamma_\nu \frac{1}{\hat{q}+\hat{k}-m} = \frac{i}{12\pi^2}(\delta_{\mu\nu}k^2 - k_\mu k_\nu) \begin{cases} \ln m_W^2/k^2, & k^2 \gg m^2; \\ \ln m_W^2/m^2, & k^2 \ll m^2. \end{cases} \tag{5}$$

In fact not only diagram 7a, but 7b as well, depend on the difference of two such blocks with different fermion masses m, the well-known "penguin"/8/, so that the W-boson mass m_W drops out

of the answer.

The next in complexity diagram 9, which enters 7b only, describes the interaction of the axial weak current (the vector current in the weak vertex drops out evidently) with electromagnetic and gluon currents. This diagram contains well-known axial anomaly, although in a somewhat unusual kinematic region where the momentum of one of the vector vertices, the electromagnetic one, is small. The last circumstance can be used effectively to simplify the calculations. Expand the expression for a fermion line with electromagnetic vertex

$$\frac{1}{\hat{q}-\hat{k}/2-m} \gamma_\beta e_\beta \frac{1}{\hat{q}+\hat{k}/2-m},$$

where e_β is the photon polarization vector, in the photon momentum k. The zeroth term of the expansion does not contribute here due to the gauge invariance, and the first one by means of simple transformations is reduced to

$$\frac{1}{2}k_\alpha e_\beta \frac{1}{\hat{q}-m}\left(\gamma_\alpha \frac{1}{\hat{q}-m}\gamma_\beta - \gamma_\beta \frac{1}{\hat{q}-m}\gamma_\alpha\right)\frac{1}{\hat{q}-m} = -\frac{i}{2} F_{\alpha\beta} \frac{\gamma_\alpha(\hat{q}-m)\gamma_\beta}{(q^2-m^2)^2}. \tag{6}$$

One can recognize easily in this expression the first term of the expansion of the well-known fermion Green function in the constant external field $F_{\alpha\beta}$.

Arising in this way expression for the sum of the two loops 9 is transformed to

$$E_{\mu\nu} = -\frac{i}{2}F_{\alpha\beta}\int\frac{d^4q}{(2\pi)^4} Tr\,\gamma_\mu\gamma_5 \frac{\gamma_\alpha(\hat{q}-m)\gamma_\beta\gamma_\nu(\hat{q}-\hat{k}+m) + (\hat{q}-\hat{k}+m)\gamma_\nu\gamma_\alpha(\hat{q}-m)\gamma_\beta}{(q^2-m^2)^2[(q-k)^2-m^2]} = -\frac{i}{4\pi^2}F_{\alpha\beta}\int_0^1 dz\, z(1-z)^2 \times \frac{k^2\varepsilon_{\alpha\beta\mu\nu} + \varepsilon_{\alpha\beta\varkappa\lambda}k_\varkappa(\delta_{\mu\lambda}k_\nu - \delta_{\nu\lambda}k_\mu)}{m^2-k^2z(1-z)}. \tag{7}$$

This result for diagrams 9, that contain the axial anomaly, is defined in such a way that its transversality in ν is guaranteed. In fact the answer for the sum of all the diagrams 7b depends on the part of expression (7) antisymmetric in μ, ν which after going to the euclidean region for k^2 is reduced to

$$E_{\mu\nu} - E_{\nu\mu} = \frac{i}{2\pi^2}F_{\alpha\beta}\varepsilon_{\alpha\beta\mu\nu}E(k^2/m^2),$$

$$E = \int_0^1 \frac{dz\, z(1-z)^2 k^2}{m^2+k^2z(1-z)} = \begin{cases} \frac{1}{2} - \frac{m^2}{k^2}\ln\frac{k^2}{m^2}, & k^2 \gg m^2; \\ \frac{1}{12}\frac{k^2}{m^2}, & k^2 \ll m^2. \end{cases} \tag{8}$$

The naive calculation, without the definition that guarantees the transversality in ν, would lead to the answer for $E_{\mu\nu}-E_{\nu\mu}$ that differs from (8) in one respect only. The invariant function E' obtained in this way does not coincide with the true E by a term independent of the fermion mass: $E' = E - 1/2$. One can check easily that such a term in E does not influence the final answer for the quark edm. Therefore, even the naive calculation of diagrams 9 would lead to the correct edm value.

And finally, the most tedious block is the sum of diagrams 10. It is natural here to use again expression (6) for the fermion Green function in external field. The integral

$$-\frac{i}{2}F_{\alpha\beta}\int\frac{d^4q}{(2\pi)^4}\,\mathrm{Tr}\left\{\frac{\gamma_\mu(1+\gamma_5)\gamma_\alpha(\hat{q}-m_1)\gamma_\beta\gamma_\nu(1+\gamma_5)(\hat{q}-\hat{k}+m_2)}{(q^2-m_1^2)^2[(q-k)^2-m_2^2]}+\right.$$
$$\left.+\frac{\gamma_\mu(1+\gamma_5)(\hat{q}+\hat{k}+m_1)\gamma_\nu(1+\gamma_5)\gamma_\alpha(\hat{q}-m_2)\gamma_\beta}{(q^2-m_2^2)^2[(q+k)^2-m_1^2]}\right\} \qquad (9)$$

arising in this way can be divided into the terms that differ by the structure of weak vertices. Two of them, $V_\mu A_\nu$ and $V_\nu A_\mu$, also contain the anomaly and need, generally speaking, the definition that guarantees in the limit $m_1=m_2$ the transversality in μ and ν correspondingly. However, in the sum $V_\mu A_\nu + V_\nu A_\mu$ such additional terms cancel out, so the answer for this sum is reduced to the naive one. Moreover, after the subtraction of the analogous expression with the transposed m_1 and m_2 (see formulae (1), (2)) the discussed term $V_\mu A_\nu + V_\nu A_\mu$ drops out from the answer at all.

Thus, one has to consider in (9) the term that corresponds to $V_\mu V_\nu + A_\mu A_\nu$. It does not vanish since $m_1 \neq m_2$. The momentum of the external quark at diagrams 7a,b is small as compared with the typical k, so expression (9) can be averaged immediately over the directions of the four-dimensional vector k. (By the way, this procedure also turns to zero $V_\mu A_\nu + V_\nu A_\mu$, even without the antisymmetrization in m_1, m_2.) This averaging allows one to simplify greatly the expression for the block discussed. Subtracting the expression with the transposed m_1 and m_2, and then taking into account the analogous terms with m_2, m_3 and m_3, m_1 (see (1), (2): 1,2,3 denote u,c,t or d,s,b), we arrive to the following formula for the sum of diagrams 10:

$$-\frac{1}{4\pi^2}F_{\mu\nu}\,\Phi(k^2, m_1^2, m_2^2, m_3^2);$$
$$\Phi = \Phi_{12}+\Phi_{23}+\Phi_{31}; \qquad (10)$$
$$\Phi_{ij} = (m_i^2+m_j^2+k^2)\int\frac{d^4q}{\pi^2}\left\{\frac{1}{(q^2+m_i^2)^2[(q-k)^2+m_j^2]}-(i\leftrightarrow j)\right\}.$$

We have passed here to the euclidean metric both for q and k.

According to the true relation among the masses of u,c,t or d,s,b, we shall take $m_1 \ll m_2 \ll m_3$. The forthcoming integration over k^2 will be performed with the logarithmic accuracy, so that the following approximate expression for $\Phi(k^2, m_1^2, m_2^2, m_3^2)$ is sufficient:

$$\Phi = \begin{cases} -\dfrac{2m_2^2}{k^2}\ln\dfrac{k^2}{m_2^2}, & m_2^2 \lesssim k^2 \lesssim m_3^2; \\ O(1/k^4), & k^2 > m_3^2. \end{cases} \qquad (11)$$

The interaction of the fermion edm d with external field $F_{\alpha\beta}$ is described by the Hamiltonian

$$H_d = \frac{d}{2}\bar{\psi}\gamma_5\sigma_{\alpha\beta}\psi F_{\alpha\beta}. \qquad (12)$$

The contribution of the diagrams of the type 7a to the edm is found with formulae (5), (10) and (11) easily and with the logarithmic accuracy constitutes

$$d_d^{(a)}/e \simeq -\frac{G^2m_c^2m_d\alpha_s\tilde{\delta}}{54\pi^5}\int_{m_c^2}^{m_b^2}\frac{dk^2}{k^2}\ln\frac{k^2}{m_c^2}\ln\frac{m_b^2}{k^2}; \qquad (13)$$

$$d_u^{(a)}/e \simeq -\frac{G^2m_s^2m_u^2\alpha_s\tilde{\delta}}{108\pi^5}\left\{\ln\frac{m_b^2}{m_c^2}\int_{m_s^2}^{m_c^2}\frac{dk^2}{k^2}\ln\frac{k^2}{m_s^2}+\int_{m_c^2}^{m_b^2}\frac{dk^2}{k^2}\ln\frac{k^2}{m_s^2}\ln m_b^2/k^2\right\} \qquad (14)$$

for d- and u-quarks correspondingly. The difference by a factor of 2 between the numerical coefficients in (13), (14) reflects the fact that the charge of the quarks in the middle block of diagram 7a in the first case is twice as large as in the second one. It should be stressed that $m_{u,d}$ in formulae (13), (14) is without any doubt the current mass of the quark, and not the constituent one, since it arises in the answer directly from the propagator of the corresponding quark through which a large momentum flows.

The contribution of the diagrams of the type 7b is found easily also by means of formulae (5), (8) . With the logarithmic accuracy it is correspondingly

$$d_d^{(b)}/e = \frac{G^2 m_c^2 m_d \alpha_s \tilde{\delta}}{54\pi^5} \int_{m_c^2}^{m_b^2} \frac{dk^2}{k^2} \ln\frac{k^2}{m_c^2} \ln\frac{m_t^2}{k^2}; \tag{15}$$

$$d_u^{(b)}/e = \frac{G^2 m_s^2 m_u \alpha_s \tilde{\delta}}{108\pi^5} \int_{m_s^2}^{m_b^2} \frac{dk^2}{k^2} \ln\frac{k^2}{m_s^2} \ln\frac{m_b^2}{k^2}. \tag{16}$$

Here the quark mass $m_{d,u}$ arises in a somewhat different way than in (13), (14): after taking into account the equations of motion for the corresponding quark. But still it is a current mass, but not a constituent one. A kind of an aesthetic argument in favour of this assertion is a similar structure of the expressions for $d^{(a)}$ and $d^{(b)}$ and remarkable simplification arising in the sum $d^{(a)}+d^{(b)}$. Adding up (13) and (15), (14) and (16), we get

$$d_d/e = \frac{G^2 m_c^2 m_d \alpha_s \tilde{\delta}}{108\pi^5} \ln\frac{m_t^2}{m_b^2} \ln^2\frac{m_b^2}{m_c^2}; \tag{17}$$

$$d_u/e = -\frac{G^2 m_s^2 m_u \alpha_s \tilde{\delta}}{216\pi^5} \left(\ln\frac{m_t^2}{m_b^2} \ln^2\frac{m_b^2}{m_s^2} + \ln\frac{m_b^2}{m_c^2} \ln^2\frac{m_c^2}{m_s^2}\right). \tag{18}$$

In fact the reliability of the obtained expressions for d- and u-quarks is different. In the case of d-quark the typical momenta contributing to the effect are sufficiently large, $m_c^2 \lesssim k^2 \lesssim m_b^2$ (see (13), (15)), much larger than the momenta at which the perturbation theory in α_s becomes inapplicable. Therefore, the result for d-quark is sufficiently reliable, the radiative corrections to it in α_s are comparatively small. On the other hand, the answer for u-quark is no more than an estimate: it is valid literally only in the completely theoretical limit when m_s is much larger than the momenta at which the magnitude of α_s becomes large.

In the conclusion of this section we note that although in the theoretical limit of degenerate quark masses the quark edm vanishes (see (1), (2)), but in the final answers (17), (18) obtained with the account for the realistic relation between the masses ($m_{u,d}^2 \ll m_s^2 \ll m_c^2 \ll m_b^2 \ll m_t^2$) the traces of such a cancellation are practically absent.

4. We pass now to the computation of the induced θ-term. In

the same local limit diagram 4 goes into the diagram presented at Fig.11, it is natural to call it "cheburashka" (popular children toy with round face and large round ears). A simple analysis, essentially repeating that carried out for the quark edm, shows that the virtual gluon should be hanged on the "cheburashka's" "ears". As for the external gluons (here they are also denoted by circles), one of them should be connected with the central loop, and another with one of the "ears". A typical representative of the diagrams arising in this way is given at Fig.12. These diagrams are divided into two classes, in one of them the central loop consists of u,c,t quarks and the "ears" of d,s,b, in another - vice versa.

Unlike the case of the quark edm, here in the weak vertex the first term, without λ-matrices, in the rhs of the Fierz transformation (4) is essential. The F-coupling in the loops is not operative in this case. And when calculating the contribution of the D-coupling, the following relation for the SU(3) group constants is useful:

$$d^{ace} d^{bce} = \frac{5}{3}\delta^{ab}. \tag{19}$$

The calculation by means of formulae (5), (8), (10), (11) is of no difficulty and leads to the following result:

$$\theta = -\frac{7 G^2 m_c^2 m_s^2 \alpha_s \tilde{\delta}}{288\pi^5} \ln\frac{m_t^2}{m_b^2} \ln^2\frac{m_b^2}{m_c^2}\left(\ln\frac{m_c^2}{m_s^2} + \frac{2}{3}\ln\frac{m_b^2}{m_c^2}\right). \tag{20}$$

As usually, the angle θ is defined as the coefficient at $\frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$ in the Lagrangian induced by radiative corrections: $G^a_{\mu\nu}$ is the gluon field strength, $\tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\varkappa\lambda} G^a_{\varkappa\lambda}$. This result, like the answer for the u-quark edm, is an estimate only since it is obtained under the assumption that m_s is much larger than the momenta at which the magnitude of α_s becomes large.

5. But what is the cause of the disagreement between our results and the previous ones? As to the quark edm, the assertion that in the discussed approximation $G^2\alpha_s$ it turns to zero, made in Ref./6/, is valid only for the specific problem considered in Ref./6/. In it the edm of the u-quark only was analyzed in the model where the electric charges of u,c,t quarks were equal to unity and those of d,s,b were equal to zero. But from our ana-

lysis (see formulae (1), (2) and diagrams 7a,b) it follows immediately that the u-quark edm is proportional to the electric charge of d-quark and vice versa. Therefore, in fact the consideration carried out in Ref./6/ does not contradict our results.

Pass now to the induced θ-term. It was expressed in Ref. /5/ through the quark mass operator. It can be shown easily that in the terms of the gluon polarization operator, used by us, it would correspond to the diagrams where both external gluons are connected to the same fermion line. In our approximation $G^2\alpha_s$ such diagrams indeed do not contribute to the induced θ-term. It is quite natural that in Ref./5/ as well there is no such a contribution in the order $G^2\alpha_s$.

Another point is surprising. It is claimed in Ref./5/ that starting from the quark mass operator one can get the contribution to the θ-term $\sim G^2\alpha$ where $\alpha = 1/137$. Indeed, such a contribution exists. It is described by the diagrams of the same type 12 with the virtual photon substituted for the virtual gluon. The answer differs from (20) by the substitution of the factor $\frac{\alpha}{72}$ for the factor $\frac{7\alpha_s}{288}$. But this correction to θ as well cannot be expressed through the quark mass operator.

6. In the conclusion we shall discuss the real magnitude of the effects considered. As to the θ-term, the estimate according to formula (20) gives

$$\theta \sim 4 \cdot 10^{-19} \tag{21}$$

For $\tilde{\delta} = \sin\delta c_1 c_2 c_3 s_1^2 s_2 s_3$ we assume here and below the value $\tilde{\delta} \sim 5 \cdot 10^{-5}$, for α_s the value ~ 0.2. The number (21) is negligibly small, by 9 - 10 orders of magnitude smaller than the upper bound for θ obtained in Ref./9/ from the experimental bound on the neutron edm.

The quark edm calculated by formulae (17), (18) constitute

$$d_d/e \sim 2 \cdot 10^{-34}\ \text{cm}; \tag{22}$$

$$d_u/e \sim 10^{-35}\ \text{cm}. \tag{23}$$

The small value of d_u as compared with d_d is caused mainly by the smallness of m_s^2 in comparison with m_c^2.

It is natural to think that the contribution of the d-quark to the neutron edm coincides by an order of magnitude with (22):

$$d_n/e \sim 2 \cdot 10^{-34}\ \text{cm}. \tag{24}$$

and the corresponding contribution from the u-quark dipole moment can be neglected. The estimate (24) is much smaller than the result for the neutron edm obtained in Ref./10/ by means of another mechanism in the KM model:

$$d_n/e \sim (2-4) \cdot 10^{-32}\ \text{cm}. \tag{25}$$

(It should be mentioned that the estimate (25) is the only theoretical result for the neutron edm in the KM model that has not been disproved.) Such a gap between (25) and (24) means that the calculation of the quark edm in the KM model is unfortunately of the methodical interest only.

But in fact by means of a small modification of the mechanism discussed above one can get a contribution to the neutron edm larger than (24). Note that the smallness of the found value of the d-quark edm is caused by the small geometric factor $(108\pi^5)^{-1}$ inevitable in the three-loop approximation, as well as by the smallness of the current mass m_d of d-quark arising in the answer since the structure $\gamma_5\sigma_{\mu\nu}$ breaks the chirality. We shall try to soften both suppression factors by passing from 7a to the two-loop diagram 13.

We need now the upper loop in another, as compared to 7a, kinematic region: $k^2 \ll m_s^2$. Here the value of the invariant function Φ (see (10)) is:

$$\Phi = -2. \tag{26}$$

After straightforward calculations we get the following expression for the CP-odd interaction of two quarks with electromagnetic field:

$$\frac{G^2\alpha_s\tilde{\delta}}{18\pi^3}\, eF_{\mu\nu}\Big\{\ln\frac{m_t^2}{m_s^2}\Big[\bar{d}\gamma_\mu(1+\gamma_5)\frac{\lambda^a}{2}d\Big] + \frac{1}{2}\ln\frac{m_t^2}{m_c^2}\Big[\bar{u}\gamma_\mu(1+\gamma_5)\frac{\lambda^a}{2}u\Big]\Big\}\Big(\bar{d}\gamma_\nu\frac{\lambda^a}{2}d + \bar{u}\gamma_\nu\frac{\lambda^a}{2}u\Big). \tag{27}$$

In the nucleon expectation value of this interaction we shall substitute, as a rough estimate, in formula (27) the vacuum expectation value $\langle q\,\bar{q}\rangle$ for the product of two quark operators. Then the CP-odd interaction of a neutron with electromagnetic field is reduced to

$$-\frac{2G^2\alpha_s\tilde{\delta}}{81\pi^3}\langle q\bar{q}\rangle\langle n|\ln\frac{m_t^2}{m_s^2}(\bar{d}\gamma_5\sigma_{\mu\nu}d)+ \tag{28}$$
$$+\frac{1}{2}\ln\frac{m_t^2}{m_c^2}(\bar{u}\gamma_5\sigma_{\mu\nu}u)|n\rangle\frac{e}{2}F_{\mu\nu}.$$

The estimate for this contribution to the neutron edm is

$$d_n/e\sim-\frac{2G^2\alpha_s\tilde{\delta}}{81\pi^3}\langle q\bar{q}\rangle\ln\frac{m_t^2}{m_c^2}\sim 2\cdot 10^{-33}\,cm. \tag{29}$$

Our hope for the effective lifting of the suppression $\sim m_d$ in formula (17) for the d-quark edm has not been justified: at the standard value for the quark condensate $\langle q\,\bar{q}\rangle=(0.25\text{GeV})^3$ the ratio of the dimensional parameters in formulae (17), (29) $\langle q\bar{q}\rangle/m_c^2 m_d$ is close to unity. The enhancement of (29) by about an order of magnitude in comparison with the estimate (24) is due mainly to a larger geometric factor. Unfortunately, even the contribution (29) to the neutron edm is still by about an order of magnitude smaller than the result (25) for this moment obtained previously in Ref./10/.

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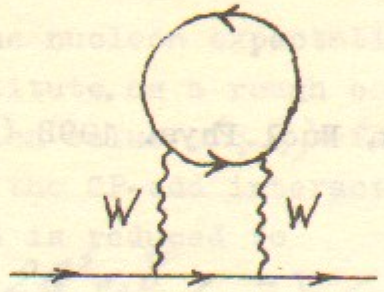


Fig. 1

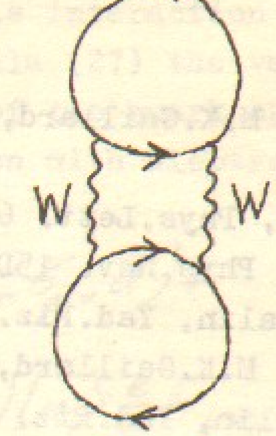
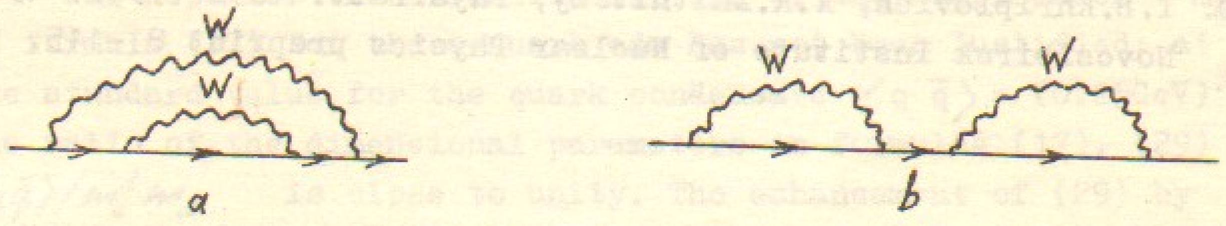


Fig. 2



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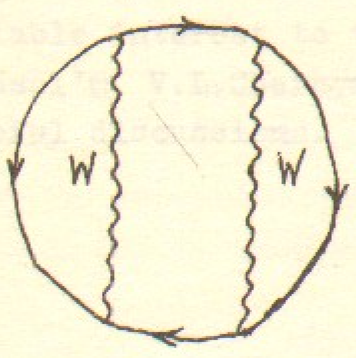

Fig. 4

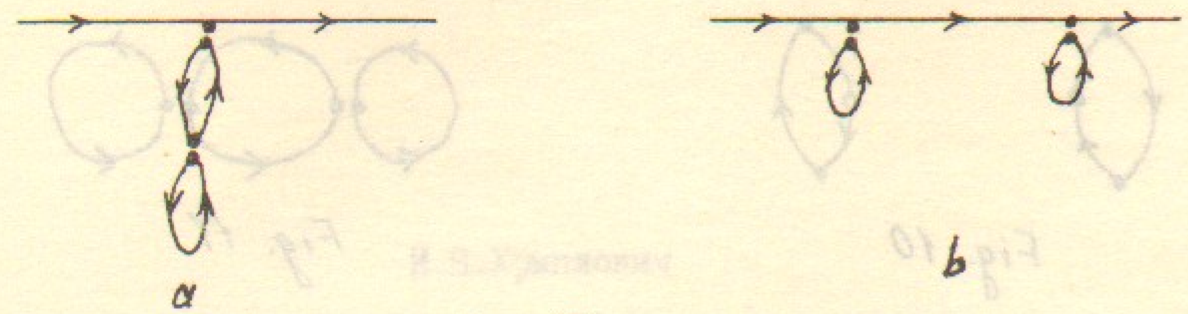

Fig. 5.

Fig. 6

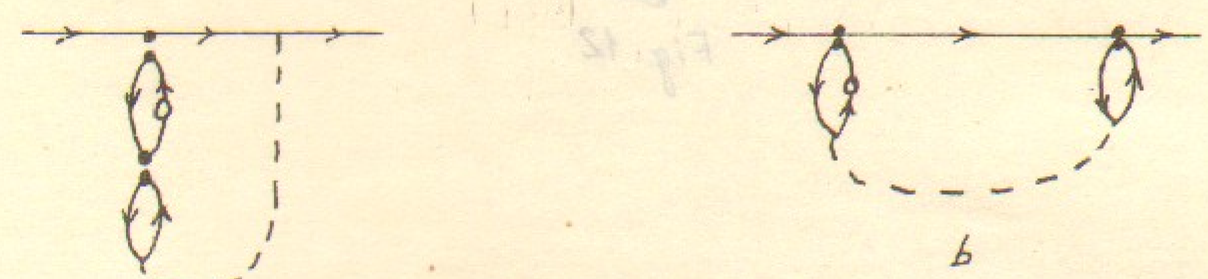

Fig. 7

Fig. 8

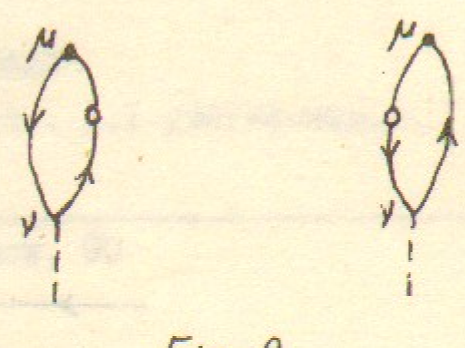

Fig. 9

Fig. 10

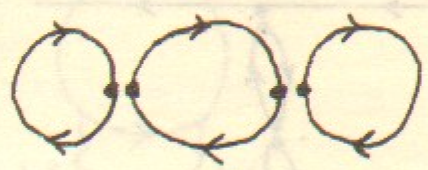

Fig. 11

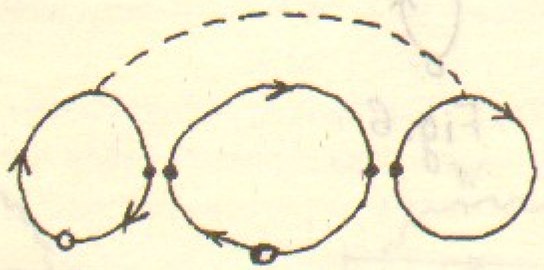

Fig. 12

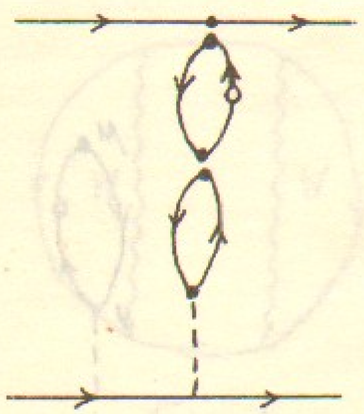

Fig. 13

И.Б.Хриплович

ЭЛЕКТРИЧЕСКИЙ ДИПОЛЬНЫЙ МОМЕНТ КВАРКА И ИНДУЦИРОВАННЫЙ 0-ЧЛЕН В МОДЕЛИ КОБАЯШИ-МАСКАВА

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