ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

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SIMULATION OF THE ELASTIC SCATTERING PROCESS $e^+e^- \to e^+e^-$ WITH RADIATIVE CORRECTIONS

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НОВОСИБИРСК

ABSTRACT

The way of generating soft γ-quanta in the simulation of the process e^+e^- with radiative corrections is suggested when the required accuracy is about 1%.

The process of e^+e^- elastic scattering (Bhabha scattering) is of great importance in the experiments with the electron-positron colliding beams. Its cross section can be calculated with high accuracy in QED. At the energy less than 10 GeV the number of detected events of Bhabha scattering is compatible with that of the other e^+e^- interaction processes, therefore this process is often used as a monitoring one to determine the integrated luminosity. Apparently the simulation accuracy of this process when it is used for monitoring can restrict the accuracy of determination of the cross section of the other processes. Another evident usage of the Bhabha scattering simulation is to obtain the admixture of its events to the experimentally separated events of the other processes (especially those with a small cross section). In the case when the required accuracy of the process under discussion is better than 10%, one should take into account several effects which can change the cross section of the process. The most complete analysis of these effects at the level of the simulation of the scattered particles parameters was carried out in the paper [1]. They include hard bremsstrahlung, hadronic and leptonic vacuum polarization, the effect of virtual Z-boson exchange. Roughly, the algorithm suggested in [1] is the following.

The total cross section $d\sigma$ of Bhabha scattering is divided into two parts: the cross section $d\sigma_1$ of soft bremsstrahlung with the energy of quantum ω below some threshold E_c and the cross section $d\sigma_2$ of hard bremsstrahlung with $\omega > E_c$.

The former includes both one-loop electromagnetic corrections and corrections for the vacuum polarization and Z-boson exchange, while the latter includes all Feynman diagrams of the third order with a γ-quantum in the final state.

When the cross section $d\sigma_1$ with $\omega < E_c$ is simulated, a γ-quantum in the final state is not generated at all, the angular distribution of the scattered electron and positron is determined by the cross section $d\sigma_0$ without radiation.

The choice of the threshold energy E_c is done according to the following considerations. The cross section of soft bremsstrahlung $d\sigma_1 = d\sigma_0(1+\delta)$ where δ is usually referred to as a radiative correcton. The formulae for δ used in [1] have an error of about δ^2, therefore in the case when the simulation accuracy must be about 1% the radiative correction δ should not be higher than 10%. This limits E_c from below. For example, if the energy of initial particles $E_0 = 5000$ MeV, this restriction appears to be rather high: $E_c > 1000$ MeV. The upper boundary for E_c is less definite. The main point can be formulated as follows: in what cases can one neglect the radiated γ-quantum as a separate particle? When the accuracy of 1% is considered, one should take into account not only the changes of the angles and energies of the final electron and positron, but the interaction of the produced γ-quantum with the matter of the detector, the appearance of the «extra» particles in the reconstruction routine and possible rejection of such events as well. It is clear that there does not exist a definite answer to this question, more precisely the answer depends on the detector and experimental separation procedure of the Bhabha scattering events. At first sight it seems to be dangerous to neglect the γ-quantum with the energy of $\sim$100 MeV, when the accuracy of 1% is required.

We obtain the contradictory requirements: on one hand $E_c > 1000$ MeV to provide the accuracy in δ, on the other hand one would prefer to consider the γ-quantum with $\omega > 100$ MeV as the separate particles having the influence on the kinematics of the scattered electron and positron and interacting with the matter of detector.

In the present paper the way of essential decrease of the boundary energy in the bremsstrahlung spectrum in the simulation of the Bhabha scattering is suggested.

The general scheme of the simulation is reserved the same as in [1]. However the energy E_c is considered now as a boundary between the hard photon spectrum and the soft photon one, the algorithms of simulation for them being essentially different. Let us consider separately the algorithms for generating hard and soft photons.

1. SIMULATION OF THE HARD BREMSSTRAHLUNG SPECTRUM

In order to simulate the cross section $d\sigma_2$ of the scattering with the emission of one hard quantum with the energy $\omega > E_c$ one can use the formulae with 10% accuracy, because the integral of the cross section with $\omega > E_c$ itself is about 10% of the total cross section. The simulation program using the algorithm reported in this paper has been written which is included into the UNIMOD code [2] for the simulation of e^+e^- experiments. For the hard part of the γ-quantum spectrum the formula from [3] was used instead of that from [1] (but both formulae provide sufficient accuracy, the choice depends on the convenience only):

$$d\sigma_2 = \frac{4\alpha^3}{(2\pi)^2 E_0^2} F_e \delta(p_1 + p_2 - p_3 - p_4 - k) \frac{d^3\mathbf{p}_3 d^3\mathbf{p}_4 d^3\mathbf{k}}{2E_3 \cdot 2E_4 \cdot 2\omega}$$

$$F_e = W \cdot \frac{ss'(s^2+s'^2) + tt'(t^2+t'^2) + uu'(u^2+u'^2)}{k_1 k_2 k_3 k_4 ss'tt'} - \qquad (1)$$

$$-4m^2 \left[\frac{1}{k_1^2}\left(\frac{s'}{t} + \frac{t}{s'} + 1\right)^2 + \frac{1}{k_2^2}\left(\frac{s'}{t'} + \frac{t'}{s'} + 1\right)^2 + \right.$$

$$\left. + \frac{1}{k_3^2}\left(\frac{s}{t} + \frac{t}{s} + 1\right)^2 + \frac{1}{k_4^2}\left(\frac{s}{t'} + \frac{t'}{s} + 1\right)^2 \right]$$

$$W = u(st + s't') + u'(st' + s't) + 2[(s+s')tt' + (t+t')ss']$$

$$s = (p_1 + p_2)^2; \quad t = (p_2 - p_4)^2; \quad u = (p_2 - p_3)^2;$$

$$s' = (p_3 + p_4)^2; \quad t' = (p_1 - p_3)^2; \quad u' = (p_1 - p_4)^2;$$

$$k_n = 2\,(kp_n)$$

p_1, p_2, p_3, p_4, k are four-momenta of all particles (initial and final).

However, one can take both formulae for the cross section and the algorithm for the hard bremsstrahlung simulation directly from [1].

2. SIMULATION OF THE SOFT BREMSSTRAHLUNG SPECTRUM

In the case when the photon energy is small compared to the charged particles energies the cross section of the process looks like a product of $d\sigma_0$ and the factor of accompanying radiation [4]:

$$d\sigma_1 = d\sigma_0 R \qquad (2)$$

$$R = -\alpha \int\limits_{\omega_0 < \omega < E_c} \frac{d^3\mathbf{k}}{4\pi^2\omega} \left[\frac{p_3}{(p_3k)} - \frac{p_1}{(p_1k)} - \frac{p_4}{(p_4k)} + \frac{p_2}{(p_2k)} \right]^2 \qquad (3)$$

where p_i are four-momenta of the charged particles, k is the four-momentum of the γ-quantum with the energy ω,

$$(p_ik) = E_i\omega - \mathbf{p}_i\mathbf{k}$$

$$\omega_0 < \omega < E_c$$

It seems to be convenient to make the cross section (2) and the total cross section of the soft bremsstrahlung with the quantum energy $\omega < E_c$, which can be taken from [1], equal by a proper choice of ω_0:

$$d\sigma_0 R = d\sigma_0 (1+\delta) \quad (4)$$

$$R = 1+\delta$$

Then all the generated events will have one emitted quantum. The lower boundary of the γ-quantum spectrum as derived according to this recipe appears to be rather small. For example, at the energy of initial particles $E_0 = 5000$ MeV and $E_c = 1000$ MeV ω_0 should be about 1 MeV.

To obtain ω_0 it is necessary to calculate the value R. If an auxiliary function is introduced

$$I(x) = \int\limits_{\omega_0<\omega<E_c} \frac{d^3\mathbf{k}}{4\pi\omega} \cdot \frac{(p_i p_j)}{(p_i k)(p_j k)} \cdot \left[\ln \frac{E_c}{\omega_0}\right]^{-1} \quad (5)$$

where p_i, p_j are the four-momenta of two charged particles with the electron mass, the same energy E_0 and angle x between the momentum directions, integration being over all directions of the photon momentum and its energy within the interval $\omega_0 < \omega < E_c$, then the value R could be written in the following way:

$$R = \frac{4\alpha}{\pi} \ln\left(\frac{E_c}{\omega_0}\right) \{I(\pi) - I(0) + I(\theta) - I(\pi-\theta)\} \quad (6)$$

where θ is an electron scattering angle.

The function $I(x)$ can be derived for any x:

$$I(x) = \frac{m^2 + 2L^2}{LK} \cdot \ln\left[\frac{L+K}{m}\right] \quad (7)$$

where $L = p \sin(x/2)$, $K = \sqrt{m^2 + L^2}$, $I(0) = 1$

The algorithm of generating a γ-quantum with the energy $\omega_0 < \omega < E_c$ can be the following.

A γ-quantum energy is obtained from the formula:

$$\omega = E_c \exp\left[-r \ln \frac{E_c}{\omega_0}\right] \quad (8)$$

where r is a random number within the interval (0, 1).

The exact generation of the direction of the γ-quantum emission can be realized in the most simple way by the importance sampling method:

1) With the probability of 0.5 the distribution axis is chosen from two possible directions: along the initial or final electron.

2) The polar angle θ_γ is generated with respect to the chosen axis:

$$\cos\theta_\gamma = \frac{E_0(M-1)}{p(M+1)} \quad (9)$$

where E_0, p are energy and momentum of the initial electron,

$$M = \exp\left[(2r-1)\ln\left(\frac{E_0+p}{E_0-p}\right)\right]$$

r is a random number within the interval (0, 1).

3) The azimuthal angle φ_γ is chosen randomly within the interval (0, 2π).

4) Now the direction of the γ-quantum is determined with respect to the momenta of the four charged particles (initial and final). The weight function F is now calculated:

$$F = \frac{\omega^2 f_1}{8E_0^2 f_2}$$

$$f_1 = -\left[\frac{p_3}{(p_3 k)} - \frac{p_1}{(p_1 k)} - \frac{p_4}{(p_4 k)} + \frac{p_2}{(p_2 k)}\right]^2 \quad (10)$$

$$f_2 = \frac{1}{m^2 + p^2 \sin^2\theta_i} + \frac{1}{m^2 + p^2 \sin^2\theta_f}$$

p_1, p_2, p_3, p_4, k are four-momenta of the charged particles and γ-quantum,

θ_i is an angle between the γ-quantum and initial electron,

θ_f is an angle between the γ-quantum and scattered electron, $0 < F < 1$

Then the function value F is compared to the next random number r. If $r < F$ then the γ-quantum is accepted, otherwise the procedure starts from the point 1 again.

When the γ-quantum is generated, the particles of the final state do not satisfy the energy and momentum conservation laws. Taking into account that soft γ-quanta are emitted and the kinematics of the final charged particles is changed slightly, one can correct the characteristics of the scattered particles so as to satisfy conservation laws. The following way seems to be the simplest. One can use the formulae for the decay of some auxiliary particle with the total energy of $(2E_0 - \omega)$ and the momentum of $(-\mathbf{k})$ to the electron and positron, the angles of electron and positron in the frame of this particle being the same as before the correction.

Evidently, after some small corrections the described algorithm can be applied to the processes of two-body annihilation of the electron and positron ($ee\to\mu\mu$, $ee\to\pi\pi$ etc.).

3. DISCUSSION OF SOME DISTRIBUTIONS OBTAINED WITH SUGGESTED ALGORITHM

For illustration we present below some distributions obtained by this method in the case of $E_0=4730$ MeV (Υ-meson energy), $E_c=1000$ MeV.

The first question to this algorithm which arises is: how well do the distributions over the energy of the soft and hard photons agree at the point $\omega=E_c$? In fig.1 the joint distribution of events with $\omega<E_c$ and $\omega>E_c$ is shown. The divergence of the spectral density to the right and to the left from $\omega=E_c$ ($\sim 20\%$) is connected with the fact that the cross section of hard bremsstrahlung goes to zero at the hard end of the spectrum, and the formula for accompanying radiation gives the simple dependence $d\sigma\sim d\omega/\omega$.

In this paper the correction to the soft photons spectrum is included as a factor $\frac{1}{1+\omega/\varepsilon}$, where ε is some parameter with a dimension of energy. At $\omega\ll\varepsilon$ the distribution does not differ, and for the continuity at the point $\omega=E_c$ it should be $\varepsilon\approx E_0$. The distribution corrected in this way with $\varepsilon=E_0$ is shown in fig.2. For the generation of the photon energy with $\omega<E_c$ taking into account the correction $\frac{1}{1+\omega/E_0}$ one should replace in (6) $\ln\left(\frac{E_c}{\omega_0}\right)$ by $\ln\left[\frac{E_c(1+\omega_0/E_0)}{\omega_0(1+E_c/E_0)}\right]$ and instead of (8) use:

$$\omega=\frac{E_c\varrho}{1+E_c(1-\varrho)/E_0}$$

$$\varrho=\exp\left\{-r\ln\left[\frac{E_c(1+\omega_0/E_0)}{\omega_0(1+E_c/E_0)}\right]\right\} \qquad (11)$$

r is a random number within the interval (0, 1).

Further we shall compare two variants of simulating the process of Bhabha scattering at the angle more than 30 degrees: the first one-soft γ-quantum radiation with $\omega<E_c=1000$ MeV is not simulated, the second one-soft γ-quantum radiation is simulated in accordance with the foregoing algorithm.

In table 1 the distribution of events over the energy of the scat-

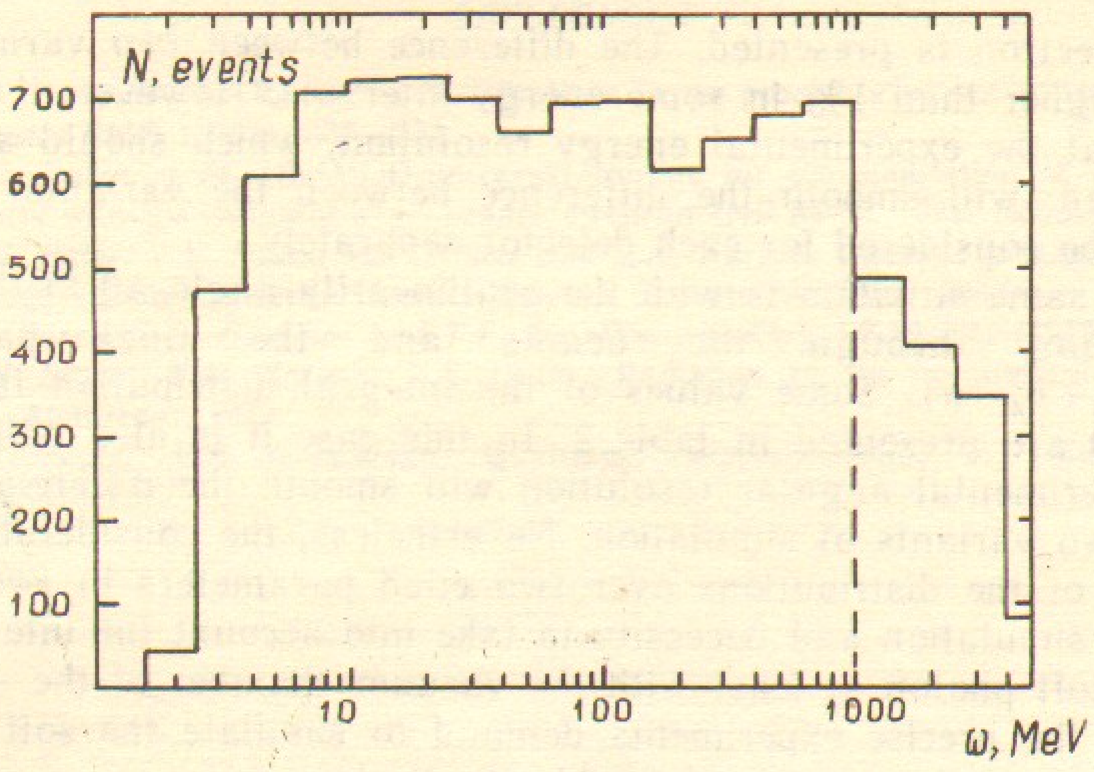

Fig. 1. Energy distribution of the emitted γ-quantum.

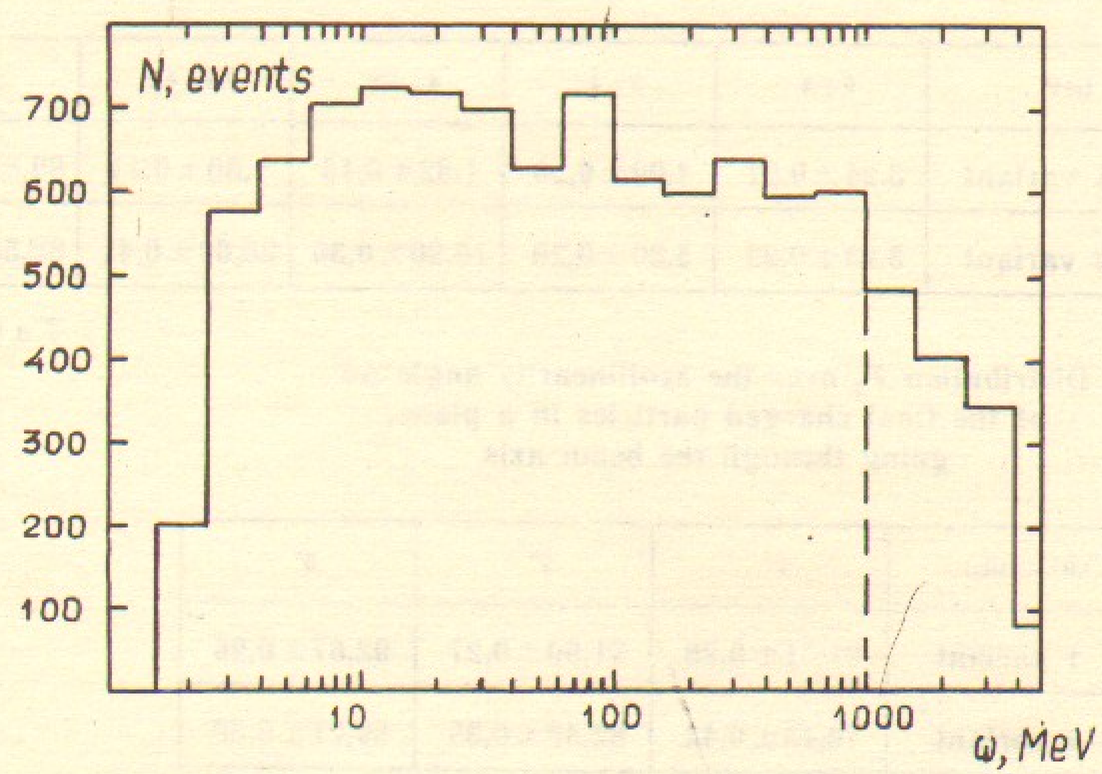

Fig. 2. Energy distribution of the emitted γ-quantum. The cross section of the soft bremsstrahlung is corrected by the factor $\frac{1}{1+\omega/E_0}$.

tered electron is presented. The difference between two variants is much higher than 1% in some energy intervals. However, it is evident that the experimental energy resolution, which should also be simulated, will smooth the difference between the variants, but it should be considered for each detector separately.

The same situation is with the acollinearity angle $\Delta\theta$ in the plane going through the beams and the final particles ($\Delta\theta=\theta_3+\theta_4-\pi$). Some values of the integral distribution function over $\Delta\theta$ are presented in table 2. In this case it is also valid that the experimental angular resolution will smooth the difference between two variants of simulation. Nevertheless, the considerable difference of the distributions over two cited parameters in two variants of simulation and necessity to take into account the interaction of the soft photon at least with the vacuum chamber of the storage ring in the precise experiments demand to simulate the soft bremsstrahlung in the process of Bhabha scattering.

Table 1.

Distribution P_1 of the events over the energy E_3 of scattered electron in two simulation variants

E_3, GeV		$0\div3$	$3\div4$	$4\div4{,}5$	$4{,}5\div4{,}7$	$>4{,}7$
P_1, %	1 variant	$3{,}24\pm0{,}21$	$4{,}09\pm0{,}20$	$1{,}82\pm0{,}13$	$1{,}30\pm0{,}11$	$89{,}55\pm0{,}31$
	2 variant	$3{,}23\pm0{,}22$	$5{,}30\pm0{,}20$	$10{,}20\pm0{,}30$	$20{,}69\pm0{,}41$	$60{,}58\pm0{,}49$

Table 2.

Distribution P_2 over the acollinearity angle $\Delta\theta$ of the final charged particles in a plane, going through the beam axis

$\Delta\theta_0$, ($\lvert\Delta\theta\rvert<\Delta\theta_0$)		1°	2°	5°
P_2, %	1 variant	$91{,}31\pm0{,}28$	$91{,}90\pm0{,}27$	$92{,}67\pm0{,}26$
	2 variant	$76{,}43\pm0{,}42$	$82{,}43\pm0{,}38$	$89{,}71\pm0{,}30$

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