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BOSON MODELS OF LOW-LYING VIBRATIONS IN SPHERICAL NUCLEI

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Boson Models of Low-Lying Vibrations in Spherical Nuclei*)

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ABSTRACT

A new type of symmetry (O(5)-invariance with strongly nonlinear dynamics) is proposed to describe adiabatic collective excitations in even-even soft spherical nuclei. Predictions for physical quantities are made; when compared with experimental data they lead to a fairly good agdeement.

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Collective spectra of low-lying excited states in complex atomic nuclei are not completely understood [1]. Magic nuclei (those with occupied nucleon shells) have a stable spherical shape and multipole vibrations around it with small amplitudes and rather high frequencies ($\omega \approx 1.5-2$ MeV). Ground states of all even-even nuclei have quantum numbers $J^{\pi}=0^{+}$ of the angular momentum J and parity π. This is connected with the condensate of Cooper pairs due to strong superconducting pair correlations and does not imply the sphericity of the nuclear shape. It is well known that the interaction between valence nucleons and the polarization of the magic core lead to the instability of the spherical shape. As a result, nuclei with many nucleons in partially occupied outer shells have a stable quadrupole deformation. The pattern of the low-lying spectrum of a well deformed nucleus is again relatively simple being that of distinctly separated rotational bands built on various intrinsic configurations. The most difficult problem is that of description of pretransitional soft nuclei which are intermediate between magic and deformed ones. For definiteness, below we limit ourselves to even-even nuclei.

Experimental data manifest clearly [2] the collective character of low-lying states in nuclei under consideration. Almost all observed levels can be regularly classified according to the multiplet scheme corresponding to irreducible reprsentations of the SU(5) group. This group is generated by the set of operators $d^{+}_{\mu}d_{\mu'}$ where d^{+}_{μ} and $d_{\mu'}$ are operators of creation and annihilation of a quadrupole quantum with the angular momentum $l=2$ and its projection μ. In contrast to magic or near-magic nuclei, typical energy intervals ω between the multiplets are small here ($\omega \sim 0.5$ MeV) compared with breaking energies $2E \sim 2$ MeV of Cooper pairs. Among the electric quadrupole transitions between multiplets one can find significantly enhanced ones their probabilities being one or two orders of magnitude higher than it follows from single-particle estimates.

Thus, this part of spectrum is dominated by the soft quadrupole collective mode. At small ω, the vibration amplitude is large $(\sim 1/\sqrt{\omega})$ so that other (noncollective) degrees of freedom become aware of the slowly changing field of quadrupole symmetry. Therefore nonlinear phenomena are essential resulting in the effective strong anharmonicity. Along with the virtual deformation, the possibility of the collective rotation around the axis perpendicular to that of quadrupole motion appears. This is the origin of the difficulties of the development of the consistent microscopic theory of soft spherical nuclei.

Phenomenological approaches to the problem can be divided into two types. The classical Bohr-Mottelson description [1] assumes that the collective Hamiltonian H_c exists covering the whole subspace of states generated by the quadrupole collective motion. This Hamiltonian can be expressed in terms of coordinates

$$\alpha_\mu = \frac{1}{\sqrt{2\omega}}[d_\mu + (-1)^\mu d^+_{-\mu}] \equiv \frac{1}{\sqrt{2\omega}} d^{(+)}$$

and momenta

$$\pi_\mu = -i\sqrt{\frac{\omega}{2}}[d_\mu - (-1)^\mu d^+_{-\mu}] = -i\sqrt{\frac{\omega}{2}} d^{(-)}$$

of quadrupole phonons. Since the motion is adiabatic one should expect that the terms of higher order with respect to π_μ are small so that

$$H_c = U(\alpha) + \frac{1}{4}\sum_{\mu\mu'} [B_{\mu\mu'}(\alpha), \pi_\mu \pi_{\mu'}]_+ \qquad (1)$$

where the potential energy $U(\alpha)$ and the inertial tensor $B_{\mu\mu'}(\alpha)$ depend on the coordinates α_μ only. The general Hamiltonian (1) contains seven independent tensor structures which are arbitrary functions of the rotational invariants $(\alpha^2)_{00}$ and $(\alpha^3)_{00}$. Here $(\ldots)_{LM}$ means the vector coupling to the total angular momentum L with the projection M. Actually one truncates in some way the expansions of $U(\alpha)$ and $B_{\mu\mu'}(\alpha)$ over these invariants. Then it is necessary to diagonalize the Hamiltonian containing many fitted parameters [3]. Since the anharmonicity is not weak, the number $N_d = \sum_\mu d^+_\mu d_\mu$ of bare quanta is not conserved.

In the more recent alternative approaches (the interacting boson approximation, IBA [4, 5]) bosons are identified with the images of the fermion pairs in the limited subspace of collective states. Then the total boson number N_B is determined uniquely by the nucleon number in the outer shells and should be conserved. In practical calculations one takes into account usually s- and d-bosons with the angular momentum $l=0$ and $l=2$ respectively. At fixed $N_B = N_s + N_d$ such a model corresponds to the SU(6) group. The effective nonconservation of N_d arises due to the excitation $s \rightarrow d$ of the condensate s-bosons into the d-state.

The phenomenological schemes of both types give in general the reasonable description of the data. But the abundance of free parameters together with the lack of the selection principles leads to the feeling of dissatisfaction. The attempts to obtain the collective Hamiltonian (1) microscopically from the nucleon interaction turns on the extremely complicated calculations [6, 7] using the boson expansion of fermion operators [6, 8]. The boson expansion procedures are rapidly convergent at the weak anharmonicity. This is not case in real nuclei. Apart from that, it is necessary to take into account the coherent responce of noncollective degrees of freedom [9, 10] particularly the virtual rotation [11]. As for the different versions of IBA the serious shortcomings of its microscopic justification are not overcome [2, 12]. Specific predictions of this model (for example, the cut-off of rotational bands) have no experimental support.

The analysis [13, 14] with the aid of the microscopic estimates of the main features of the quadrupole motion (collectivity and adiabaticity) makes possible to establish the most important contributions to the collective Hamiltonian, namely the quartic anharmonicity and the virtual rotation. Hence, we are able to formulate the simple phenomenological scheme taking into account the quartic anharmonicity and angular momentum effects (QAAM [13]).

It can be shown that the QAAM scheme gives a reasonable agreement with the vast set of data for the typical soft spherical nuclei. The attractive advantages of the method are the small number of free parameters and the simplisity of the computational work (the essential part of it can be carried out analitically with the use of the group algebra).

Thus, we find that the new type of symmetry is realized approximately in the soft nuclei: the symmetry of the quartic five-dimensional oscillator. As it was shown earlier [14] the ground band of

^{100}Pd gives a good example of such a symmetry. The angular momentum effects as well as minor corrections due to other anharmonic terms are superimposed on the main symmetry.

In accordance with the microscopic analysis and with the experimental data, the square of the five-dimensional angular momentum, i. e. the Casimir operator of the O(5) group

$$C[O(5)]=\frac{1}{2}\sum_{\mu\mu'}[d^+_\mu d_{\mu'}-(-1)^{\mu+\mu'}d^+_{-\mu'}d_{-\mu}][d^+_{\mu'}d_\mu-(-1)^{\mu+\mu'}d^+_{-\mu}d_{-\mu'}], \quad (2)$$

is an approximate constant of motion in spite of the strong anharmonicity. This operator has quantized eigenvalues according to

$$C[O(5)]=v(v+3), \quad (3)$$

where the integer number v is so called seniority number of nonpaired bosons. The O(5) symmetry is confirmed by the multiplet structure as well as the selection rule $|\Delta v|=1$ for the enhanced collective E2-transitions. In our scheme this symmetry follows naturally from the dominance of the quartic anharmonicity. Let us consider an arbitrary quartic phonon Hamiltonian $H^{(4)}$ with the only restriction of the time reversal invariance:

$$H^{(4)}=\sum_{L=0,2,4}\{\gamma_L((\alpha^2)_L(\alpha^2)_L)_{00}+\sigma_L([(\alpha^2)_L,(\pi^2)_L]_+)_{00}+\gamma'_L((\pi^2)_L(\pi^2)_L)_{00}\} \quad (4)$$

This Hamiltonian is prooved to be rotationally invariant in the five-dimensional space.

The proof is based on the isolation of phonon pairs coupled to the three-dimensional angular momentum $L=0$ (the pair number is $n=(N_d-v)/2$) and the construction of O(5) — invariant operators P^+ and P creating and annihilating these condensate pairs without changing the seniority v:

$$P^+=\frac{1}{2}\sum_\mu(-1)^\mu d^+_\mu d^+_{-\mu}, \quad P=\frac{1}{2}\sum_\mu(-1)^\mu d_\mu d_{-\mu}. \quad (5a)$$

Together with the operator

$$P_0=\frac{1}{2}\left(N_d+\frac{5}{2}\right) \quad (5b)$$

the operators (5a) generate the noncompact SU(1,1) group with the Casimir operator $C[SU(1,1)]$ connected with that of O(5) group (3),

$$C[SU(1,1)]=P_0^2-\frac{1}{2}(PP^++P^+P)=\frac{1}{4}C[O(5)]+\frac{5}{16}. \quad (6)$$

Now we can transform the Hamiltonian (4) to the explicitly O(5)-invariant form

$$H^{(4)}=\lambda(2P_0+P+P^+)^2+\lambda'(2P_0-P-P^+)^2+\alpha[2P_0-P-P^+,2P_0+ +P+P^+]_++\beta C[SU(1,1)]+\gamma J(J+1)+\zeta \quad (7)$$

where J stands for the O(3) angular momentum quantum number and designations of coefficients are introduced as

$$\lambda=\frac{1}{5}\sum_{L=0,2,4}\sqrt{2L+1}\,\varphi_L\gamma_L, \quad \lambda'=\frac{1}{5}\sum_{L=0,2,4}\sqrt{2L+1}\,\varphi_L\gamma'_L, \quad (8a)$$

$$\alpha=\frac{1}{5}\sum_{L=0,2,4}\sqrt{2L+1}\,\varphi_L\sigma_L, \quad (8b)$$

$$\beta=\frac{8}{7}\left(\sigma_4+\frac{4}{\sqrt{5}}\sigma_2\right), \quad \gamma=\frac{2}{7}\left(\frac{\sigma_2}{\sqrt{5}}-\frac{\sigma_4}{\sqrt{3}}\right),$$

$$\zeta=-\frac{5}{14}\left(11\sigma_4+\frac{8}{\sqrt{5}}\sigma_2\right), \quad (8c)$$

$$\varphi_0=1, \quad \varphi_2=\varphi_4=2/7. \quad (8d)$$

The Hamiltonian (7) together with the harmonic term $H^{(2)}$ containing a bare phonon frequency Ω can be solved analytically. The appropriate method of solution is the v-dependent canonical transformation [15] which chooses the optimum parameters of the boson pair condensate and of the renormalized phonon frequency ω_v for each subspace with v fixed. For the low-lying states, such approximate procedure guarantees the high precision of results avoiding in many cases the numerical diagonalization. The stationary states can be labelled with the number $\tilde{N}=2\tilde{n}+v$ of new (renormalized) quanta and with the exact constants of motion v, J and M.

The most interesting case is that of the soft collective mode (the adiabatic limit). Here the main terms in (7) are the first one and the quasirotational term $\gamma J(J+1)$. The physical meaning of these terms was discussed above. Other terms can be readily taken into account as corrections. Note that the quasirotational correction should be co isidered for all states except the single-phonon one ($J=2$, $\tilde{n}=0$, $v=1$). By definition, this is pure collective state which serves as a reference point for the calculation of the response of noncollective degrees of freedom.

In the limiting adiabatic situation one can neglect the harmonic term $H^{(2)}$ $(\Omega\to 0)$ so the model has only one parameter $\gamma J(J+1)$ to calculate energy ratios. Transition probabilities for the enhanced (allowed in the harmonic approximation [1]) E2-transitions with $|\Delta N_d| = |\Delta v| = 1$ are determined by the operator

$$T^{(E2)}_{\mu} = d^{(+)}_{\mu} + \varkappa (d^{(+)2})_0 d^{(+)}_{\mu}. \qquad (10)$$

Here the second term arises from the boson expansion of the fermion quadrupole operator when the dominant role of the quartic anharmonicity is taken into account properly. We have neglected in (10) small terms $\sim(d^2)$ responsible for the weak (forbidden in the harmonic approximation) transitions and for the quadrupole moments of excited states.

Intraband E2—transitions inside the yrast band (quasirotational band of states with aligned phonons, $J=2v$, $\tilde{n}=0$) and interband transitions from β-band (one boson pair, $\tilde{n}=1$) to the yrast band have the probabilities

$$B(E2; v+1, \tilde{n}=1, J=2v+2 \to v, \tilde{n}=0, J=2v) = \frac{v+1}{\omega_v} K_v^{2v+7} A_v(\varkappa), \qquad (11a)$$

$$K_v = \frac{2\sqrt{\omega_v \omega_{v+1}}}{\omega_v + \omega_{v+1}}, \quad A_v(\varkappa) = 1 + \frac{2\varkappa}{\sqrt{5}} \frac{2v+7}{\omega_v + \omega_{v+1}}, \qquad (11b)$$

$$B(E2;\ v,\ \tilde{n}=1,\ J=2v \to v+1,\ \tilde{n}=0,\ J=2v) =$$
$$= \frac{(4v+5)(v+1)}{(4v+1)(v+5/2)} \frac{1}{\omega_{v+1}} K_v^{2v+9} B_v(\varkappa), \qquad (12a)$$

$$B_v(\varkappa) = 1 - \left(v + \frac{5}{2}\right) \frac{\omega_{v+1} - \omega_v}{2\omega_v} + \frac{4\varkappa}{\sqrt{5}} \frac{2v+7}{\omega_v + \omega_{v+1}} \times$$
$$\times \left[1 - \left(v + \frac{5}{2}\right) \frac{\omega_{v+1} - \omega_v}{4\omega_v}\right]. \qquad (12b)$$

As an illustrative example we consider the typical soft spherical nuclei ^{104}Ru. Fig. 1 shows the comparison of calculations (11) for $\varkappa = -0.22$ (solid line) with the experimental data [16]. Dashed lines correspond to predictions of the complicated IBA versions (IBA (2) takes into account *s*- and *d*-bosons of two kinds, «proton» and «neutron» ones, whereas IBA+*g* adds *g*-bosons with $l=4$).

In table 1 we have collected the reduced probabilities [16—18] of allowed E2-transitions in ^{104}Ru (column 2; the experimental errors of the last digits are indicated in parentheses). Column 3 show the results of present one-parameter model (10) for $\varkappa = -0.22$.

Energy levels (in units of the energy $E(2_1^+)$ of yrast states are compared in Fig. 2 with the calculations of QAAM model (the simplest adiabatic limit with one parameter $\gamma = 0.026$) and many-parameter models IBA—(2) and IBA+*g* [16, 17]. Similarly, energies of side bands $(J<2v)$ are given in Fig. 3. The number of fitted parameters for IBA calculations in indicated.

We see that the exposure of principal anharmonic effects makes possible to formulate the simple phenomenological approach reproducing main features of the real picture with the minimum number of free parameters. The scheme can be specified further introducing new parameters the total number of them being still essentially less than in traditional models. In such a way one obtains a good agreement for quantities forbidden in the harmonic approximation (expectation values of quadrupole moments and transition probabilities with $|\Delta v| \neq 1$). Moreover, the approach is applicable at not very large negative values of Ω^2 where the standard methods are highly unstable. In some cases one can obtain the better agreement introducing $\Omega^2 < 0$ as an additional parameter (see the last column of Fig. 3) whereas results for the yrast band (Fig. 2) don't change.

The main conclusion of the analysis is that low-lying collective states of soft spherical nuclei manifest the new dynamic symmetry: the five-dimensional isotropic oscillator with the quartic anharmonicity.

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Table 1.

Transition	$\frac{B(E2)}{B(E2;\, 2_1\to 0_1)}$ (Exp)	$\frac{B(E2)}{B(E2;\, 2_1\to 0_1)}$ (Theor)
$2_1 \to 0_1$	1.00	1.00
$4_1 \to 2_1$	1.44 (23)	1.57
$6_1 \to 4_1$	2.03 (28)	1.90
$8_1 \to 6_1$	2.20 (27)	2.05
$10_1 \to 8_1$	2.00 (31)	2.08
$12_1 \to 10_1$	2.09	2.02
$2_2 \to 2_1$	1.0 (2)	1.57
$3_1 \to 2_2$	1.28 (27)	1.36
$3_1 \to 4_1$	0.28 (15)	0.54
$4_2 \to 2_2$	0.54 (16)	0.99
$4_2 \to 4_1$	0.33 (10)	0.91
$5_1 \to 3_1$	0.79 (60)*	1.07
$6_2 \to 4_2$	1.24 (24)	1.40
$8_2 \to 6_2$	1.48 (73)	1.58
$0_2 \to 2_1$	0.41 (6)	0.47
$2_3 \to 0_2$	0.66 (17)	1.22
$2_3 \to 4_1$	0.28 (38)	0.11
$2_3 \to 2_2$	0.06–0.12	0.06
$2_3 \to 0_1$	0.012 (11)	0.018
$5_1 \to 6_1$	0.27 (17)*	0.48
$5_1 \to 4_2$	0.44 (18)*	0.49
$6_2 \to 6_1$	0.38 (17)	0.65
$7_1 \to 8_1$	0.26 (21)*	0.40
$7_1 \to 5_1$	1.30 (67)*	1.40
$7_1 \to 6_2$	0.26 (15)*	0.25
$4_3 \to 6_1$	0.08*	0.04

* estimates from ref. (17).

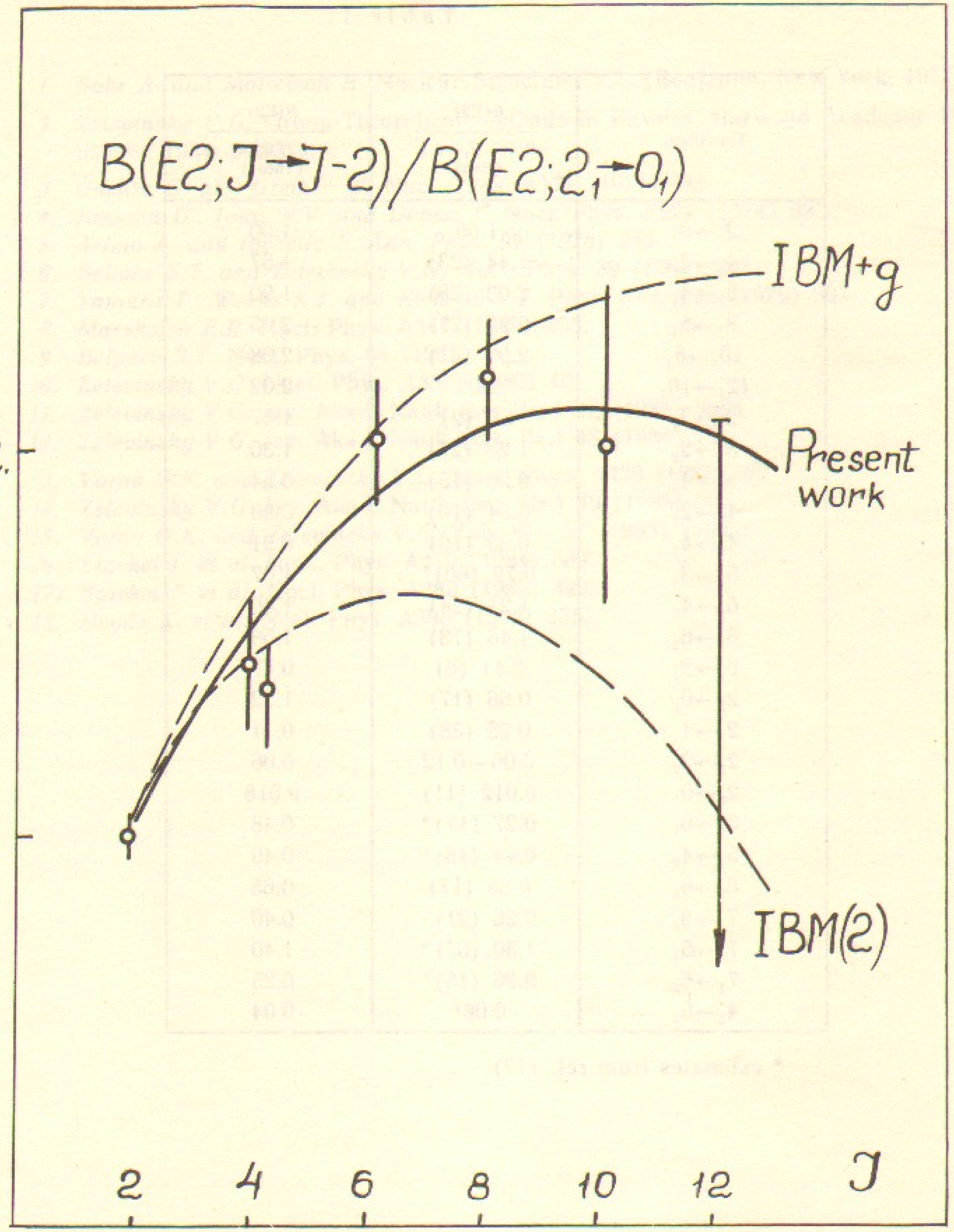
B(E2;J→J-2)/B(E2;2₁→0₁)
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Fig. 1

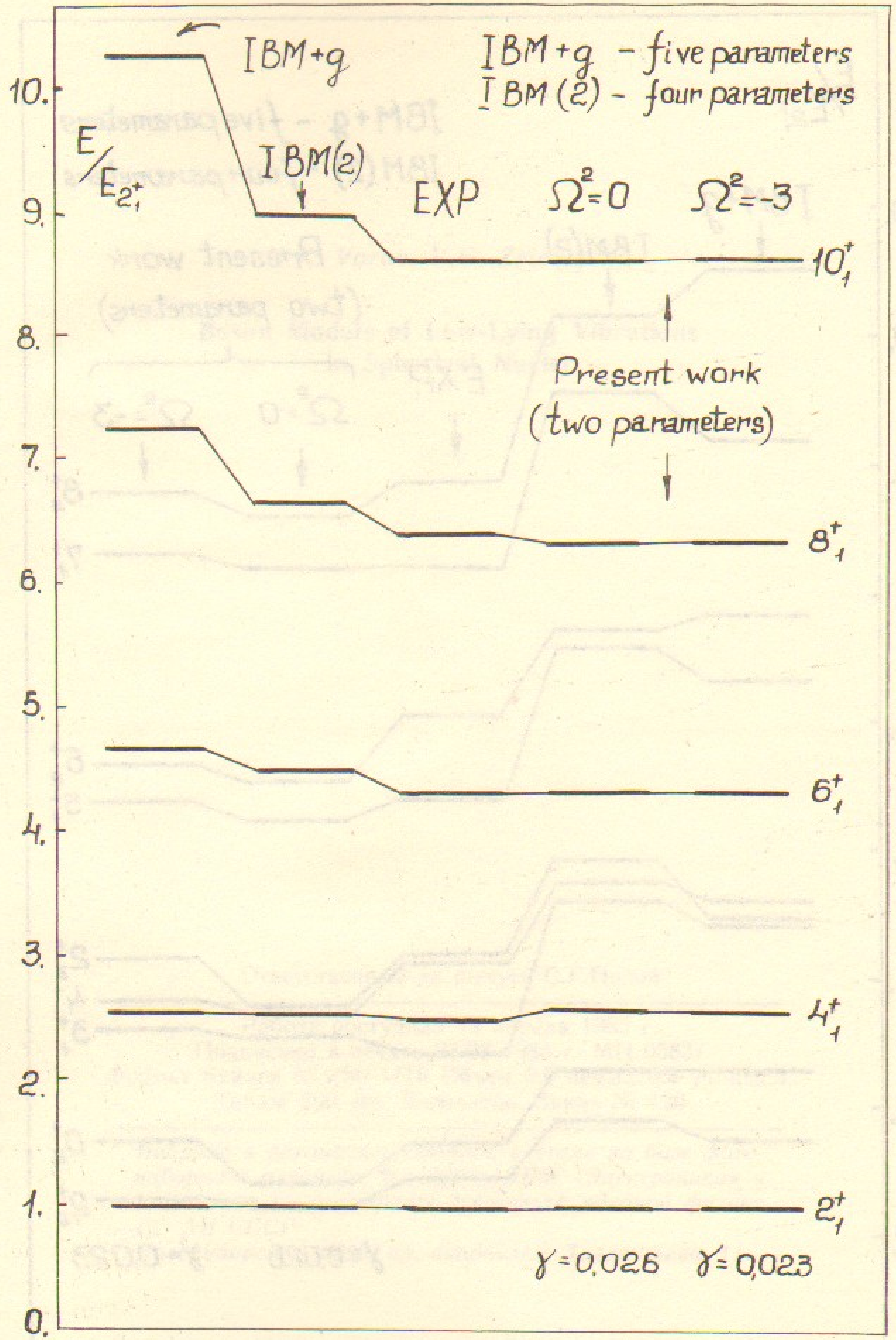
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Fig. 2

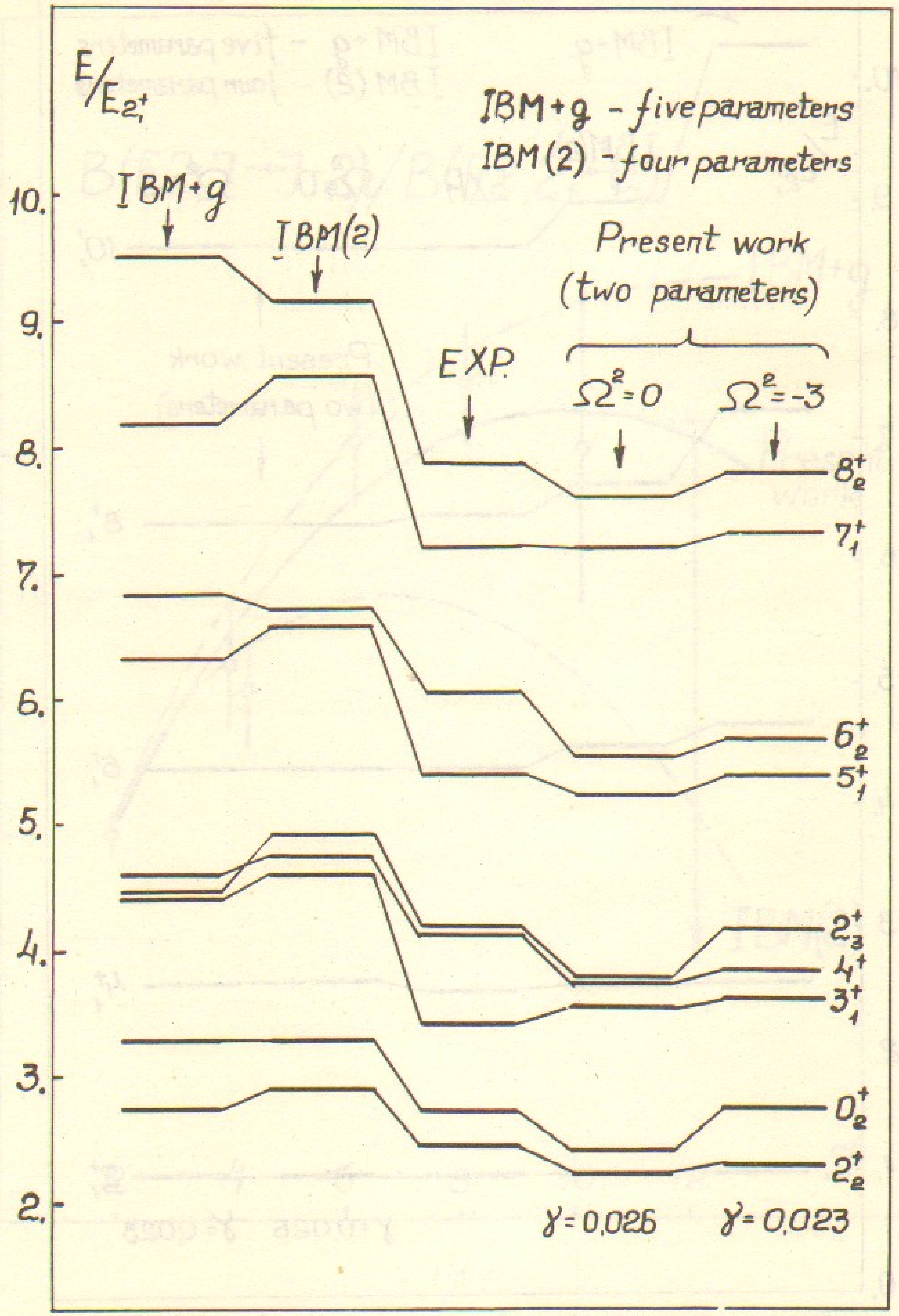

Fig. 3

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