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QUANTUM EFFECTS IN RADIATION EMITTED BY ULTRAHIGH ENERGY ELECTRONS IN ALIGNED CRYSTALLS

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НОВОСИБИРСК

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A b s t r a c t

The energy losses and the radiation spectra of electrons moving in the field of the single crystal's axes have been calculated in the energy range where the radiation in an external field becomes essentially quantum. Comparison with the experimental data has been made.

The radiation mechanism of charged particles moving in the field of the single crystal's axes or planes depends on the energy of these particles. In the range of comparatively low energies the radiation is determined by the transitions between the energy levels in the transverse potential well. When the number of these levels turns out to be large with increasing the energy, the classical description of the motion and radiation become valid. Depending upon the values of the parameter $\varrho_0 = \frac{2V_0\varepsilon}{m^2}$ (V_0 is the characteristic scale of the potential and ε is the particle energy) the radiation is dipole ($\varrho_0 \ll 1$), while at $\varrho_0 \gg 1$ it is of the bremsstrahlung nature (for the frequencies giving the main contribution to the intensity). At high energies the quantum recoil during radiation becomes significant. The character of the radiation is then determined by the parameter $\chi = \frac{V_0\varepsilon}{m^3 a}$ where a is the range of action of the potential (χ is the ratio of the field acting on the particle in its rest frame to the critical field $E_0 = \frac{m^2}{e}$). The classical description is applicable at $\chi \ll 1$. The general formulae describing the radiation with taking the recoil effects into account and valid at any values of the parameters ϱ_0 and χ have been derived in Refs.[1,2].

In the present paper we will consider the radiation under the conditions when the quantum recoil effects are significant. For this purpose, it is convenient to make use of the operator quasiclassical method developed in Ref.[3]. Making allowance for the fact that $\varrho_0 = 2am\chi$ and bearing in mind that $am \gg 1$, we have that $\varrho_0 \gg 1$ in the energy range of our interest, i.e. the characteristics of the radiation depend only on the local value of particle acceleration. Since the acceleration is determined by the gradient of the potential $U(\boldsymbol{\varrho})$, in order to find, for example, the spectral distribution of intensity at a given depth ℓ, it suffices to know only the distribution function over the transverse coordinates of the particles at this depth. Thus, there is no necessity to analyse the particular trajectories of the particles and this considerably simplifies the solution of the problem. The energies at which the recoil effects start to occur are estimated in Ref. /4/. The character of the radiation under these conditions is also discussed in this paper. These problems are analysed in Ref. /5/ as well.

Here we are concerned with the radiation in the field of particular axes. Such an approach holds only at fairly small angles of incidence ϑ_0 (the angle between the direction of the axis and the momentum of the electron incident on the crystal), namely, for $\vartheta_0 \ll V_0/m$ (cf, Ref. /6/). At $\vartheta_0 \gg V_0/m$ the coherent bremsstrahlung theory is valid (see Refs. /7,1/). Note that a similar situation takes place also in the process of pair production by photons in an aligned single crystal (Ref. /6/). The general radiation analysis valid at any ϑ_0 is possible to make according to the same scheme as that in Ref. /6/.

If the particle distribution function $\varphi(\varepsilon, \underset{\sim}{\varrho}, \ell)$ at the depth ℓ is known, the spectral distribution of the radiation intensity of a non-polarized beam of particles at the depth ℓ is of the form:

$$\frac{dI(\ell)}{d\omega} = \int \frac{dI(\underset{\sim}{\varrho}, \omega)}{d\omega} \varphi(\varepsilon, \underset{\sim}{\varrho}, \ell)\, d^2\underset{\sim}{\varrho}\, d\varepsilon \tag{1}$$

where $\frac{dI(\underset{\sim}{\varrho}, \omega)}{d\omega}$ is given by the following expression (see, e.g. formula (10.27) in Ref. /3/):

$$\frac{dI(\underset{\sim}{\varrho}, \omega)}{d\omega} = \frac{\alpha m^2 \omega}{\sqrt{3}\pi \varepsilon^2} \left[\int\limits_{\lambda}^{\infty} dy\, K_{5/3}(y) + \frac{\omega^2}{\varepsilon(\varepsilon-\omega)} K_{2/3}(\lambda) \right] \tag{2}$$

here

$$\lambda = \frac{2\omega m^3}{3\varepsilon'\varepsilon |\underset{\sim}{\nabla} \mathcal{U}(\underset{\sim}{\varrho})|} \equiv \frac{2\omega}{3\varepsilon' \chi(\underset{\sim}{\varrho})}, \quad \varepsilon' = \varepsilon - \omega.$$

Generally speaking, finding the distribution function $\varphi(\varepsilon, \underset{\sim}{\varrho}, \ell)$ in a concrete situation is a quite complicated problem. In this paper we will use an uniform distribution with respect to the transverse coordinate, i.e.

$$\varphi(\varepsilon, \underset{\sim}{\varrho}, \ell) = \frac{1}{S} g(\varepsilon, \ell) \tag{3}$$

where S is the area per one axis. This is valid, in any case, when the interval of the angles of incidence ϑ_0 is not too narrow, $|\vartheta_0| \lesssim \vartheta_c \approx 2\sqrt{\frac{V_0}{\varepsilon}}$. In this case, from the very beginning the above-barrier states for which the distribution is uniform are mainly occupied. For example, under the experimental conditions of Ref./8/ where the angular width of the incident beam was ±30 μrad, we have 80% of the particles in the above-barrier states, and with taking into account as maximum mosaic structure as possible (10μ rad), this number increases to 88%. Hence, even with the kinetics neglected, we have an uniform distribution for most of the electrons under the conditions of the experiment of Ref./8/.

Making allowance for eq.(3), one can perform, in eq.(1), the integration over $d^2\underset{\sim}{\varrho}$ for any given potential $\mathcal{U}(\varrho)$. The concrete calculations have been made for the potential $\mathcal{U} = \mathcal{U}(x)$ ($x = \varrho^2/a_s^2$, a_s is the screening radius) in the form of eq. (29) of Ref./9/; the V_0, η, x_0 and a_s parameters of the potential are listed in the Table. In this case, $\chi(\underset{\sim}{\varrho}) \to \chi(x) = \chi_s \psi(x)$ where $\chi_s = \frac{\varepsilon V_0}{m^3 a_s}$ and $\psi(x) = \frac{2\sqrt{x}}{(x+\eta)(1+x+\eta)}$.

For the variation of the average energy of an electron at the depth ℓ, we have the equation

$$\frac{d\langle\varepsilon(\ell)\rangle}{d\ell} = -\langle Q(\varepsilon)\rangle \tag{4}$$

where

$$Q(\varepsilon) = \int \frac{d^2\rho}{S} I(\chi(\rho)), \quad I(\chi(\rho)) = \int \frac{dI(\rho,\varepsilon)}{d\omega} d\omega$$

is the total radiation intensity; the notation

$$\langle f(\varepsilon)\rangle \equiv \int d\varepsilon\, g(\varepsilon,\ell)\, f(\varepsilon)$$

denotes the averaging over the energy distribution at given depth. In given potential the quantity $Q(\varepsilon)$ is calculated rather easily. It turns out that Eq.(4) may be solved with sufficient accuracy and without the use of the explicit form of the function $g(\varepsilon,\ell)$. The method is based on an approximation of the found function $Q(\varepsilon)$ in the energy interval we are interested in by a simpler function. So, if the average energy of the particles when they leave the crystal is high enough, then $Q(\varepsilon) = a\varepsilon(1 + b\frac{\varepsilon}{\varepsilon_0})$ with a high degree of accuracy (a and b are the fitting parameters). Such a form determined by the properties of the radiation intensity: for $\chi > 1$ the ratio $I(\chi)/\chi$ depends weakly on χ. For example, within the range of the values of χ from 1 to 15 the quantity*) $\frac{I(\chi)}{\chi} \simeq \frac{2}{15}\alpha m^2$ and its relative change is less than 10%. In the appropriate energy interval, the radiation length $L_{ch}^{(0)}$ remains constant with the same accuracy:

$$\frac{1}{L_{ch}^{(0)}} \equiv \frac{I(\varepsilon_0)}{\varepsilon_0} \simeq \frac{\alpha V_0}{2 m a_s x_0}, \tag{5}$$

ε_0 is the initial electron energy.

An increase in the rate of energy losses in comparison with the corresponding amorphous media may be roughly estimated as $z \equiv \frac{L_{rad}}{L_{ch}^{(0)}} \sim \frac{1}{3}\frac{m a_s}{Z\alpha \ln(183 Z^{-1/3})}$; the same estimate has been obta-

*) At larger χ the asymptotic formula for $I(\chi \gg 1)$ is valid.

ined by the authors for the process of pair production by a photon.

For the given approximation $Q(\varepsilon)$, eq.(4) takes the form

$$\frac{1}{\varepsilon_0}\frac{d\langle\varepsilon(\ell)\rangle}{d\ell} = -\frac{a}{\varepsilon_0}\left[\langle\varepsilon(\ell)\rangle + \frac{b}{\varepsilon_0}\langle\varepsilon^2(\ell)\rangle\right] = -a\left[y + b(y^2 + \Delta^2)\right] \tag{6}$$

where $y = \frac{\langle\varepsilon(\ell)\rangle}{\varepsilon_0}$, $\Delta^2(\ell) = \frac{1}{\varepsilon_0^2}\left[\langle\varepsilon^2(\ell)\rangle - \langle\varepsilon(\ell)\rangle^2\right]$ is the dispersion of the distribution with respect to ε, and y(0) = 1. For the monochromatic initial beam of particles, $\Delta^2(0) = 0$, while the quantity $\Delta^2(\ell)$ increases as l grows, and $\Delta^2(\ell) \to 0$ at y(l) → 0. In addition, we would like to remark that for the uniform distribution (this is the maximum estimate) $\Delta^2 = 1/12$. The performed analysis has shown that taking into account the dispersion has little influence on the average relative energy loss $\frac{\Delta\varepsilon(L)}{\varepsilon_0} = 1 - y(\ell)$ which has been calculated according to eq.(6). For example, under the experimental conditions of Ref./8/ the result changes by less than 2-3%. Within this accuracy, Δ^2 is possible to omit in eq.(6) and after that it can be solved simply. Fig. 1 presents the values of $\frac{\Delta\varepsilon(L)}{\varepsilon_0}$ for different crystals, axes and energies vs. the crystal thickness L. These values have been calculated at $\Delta^2 = 0$. In accord with the said above (cf. eq.(5)), the losses depend weakly on the energy (cf. curves 4 and 5) and the difference in energy losses for different media and axes is mainly determined by different values of $L_{ch}^{(0)}$ (see eq.(5)). The temperature dependence proves to be different in the classical ($\chi \ll 1$) and in the quantum ($\chi \gtrsim 1$) regions. This is due to the fact that $I(\chi) \propto \chi^2$ in the classical region, i.e. it is proportional to the squared field; then the regions $\rho \sim u_1$ (u_1 is the thermal vibration amplitude) and $\rho \sim a_s$ contribute approximately equally when integrating over $d^2\varrho$. As has already been mentioned, $I(\chi \gtrsim 1) \infty \chi$ and the contribution of $\varrho \sim u_1$ to $\int d^2\varrho$ turns out to be small. This is illustrated in Fig.1: curves 6 and 7 are closer to each other (larger χ) than curves 3 and 4. Curve 8 has been obtained under the same conditions as curve 4 (the conditions of the experiment in Ref./8/) but according to the classical formula. A sharp distinction between them demonstrates the magnitude of the quantum effects, in this case. On the other hand, curve 3 and 4 are in good agreement with the experiment of Ref./8/ and this indicates the necessity to use quantum electrodynamics when describing the process. As far as we know, this is the first direct observation of quantum effects upon radiation in a strong external field. The agreement confirms also assumption on an uniform distribution of the particle flow. Note that the estimation $\frac{\Delta\varepsilon(L)}{\varepsilon_0} \sim 0.75$, given in Ref./6/, at L = 1.4 mm, in the cooled Ge, with ε = 150 GeV proves to be very close to the experiment of Ref./8/ and to the results of the present paper.

We proceed now to the discussion of the spectral distribution of the radiation intensity $\frac{dI}{d\omega}$. Under the accepted assumptions, averaged over the transverse coordinates, the quantity $\frac{dI}{d\omega}$ is of the form, for a given electron energy,

$$\frac{dI}{d\omega} = \frac{\alpha m^2}{\pi\sqrt{3}} \frac{\omega}{\varepsilon^2 x_0} \int_0^{x_0} dx \left[1 + \frac{\omega^2}{\varepsilon\varepsilon'} + \frac{3x^2 + (1+2\eta)x - \eta(\eta+1)}{3(x+\eta)(1+x+\eta)} \right] K_{2/3}(\lambda), \tag{7}$$

where

$$\lambda = \frac{\omega}{3\varepsilon'\chi_s} \frac{(x+\eta)(1+x+\eta)}{\sqrt{x}}$$

The results of the calculation made according to this formula are presented in Fig.2 (curves 1,4,5, and 6). The calculated average energies of emitted photons $\overline{\omega}$ are well consistent with the estimation $\omega_e \sim \varepsilon \frac{\chi_s}{(2+5\chi_s)}$ given in Ref./6/; as it turned out, $\omega_e > \overline{\omega}$. The energy dependence of the shape of the spectrum is rather weak, as is seen from curves 4 and 5 Fig.2. For them energy differs by a factor of 2.

The energy losses of an electron lead to varying the spectrum $dI/d\omega$. To find it at a given depth, it is necessary to average $dI/d\omega$ (see eq.(7)) with the distribution function $g(\varepsilon, \ell)$ (eq.(3)). As has been mentioned above, taking into account the dispersion of the distribution with respect to ε exerts weak influence on the quantity $\frac{\Delta\varepsilon(\ell)}{\varepsilon_0}$. A similar assertion is not so evident for the spectral distribution. Nevertheless, we make use of it in the present paper as a first (and simple!) approximation. Then, $dI(\ell)/d\omega$ is obtained from eq.(7) by the substitution $\varepsilon \to \langle\varepsilon(\ell)\rangle$ where $\langle\varepsilon(\ell)\rangle$ is the solution of eq.(4). The observed spectrum is the following: $\frac{1}{L}\int_0^L d\ell \frac{dI(\ell)}{d\omega}$. The calculational results for the experimental conditions of Ref./8/ are listed in Fig.2 (curves 2 for L=1.4mm and curve 3 for L = 0.4 mm). It is seen that the spectrum narrows with increasing the thickness and the maximum in distribution shifts to the left (cf. curves 4,3 and 2). The average number of the emitted photons $N(L) \simeq \Delta\varepsilon(L)/\overline{\omega}(L)$ grows as well. Our calculations yields N $\approx$ 3 for L = 0.4 mm and N $\approx$ 10 for L = 1.4 mm. In the experiment of Ref./8/ there are also indications that N > 1. The curves in Fig.2 and 3 in Ref./8/ are the distribution over the energy of electrons passed through the crystal (the substitution $x \to 1 - \frac{\varepsilon}{\varepsilon_0}$ should be made). This is very valuable information but does not allow one to judge the spectrum (N > 1).

Summarize now our analysis:

1. So far the quantum effects in the radiation in external fields have been observed in the build-up of oscillations in accelerators and storage rings (as well as in the process of radiative polarization). In the experiment of Ref./8/ these effects have been observed directly in the energy interval where these are dominating (for example, at ε = 150 GeV the radiation intensity is 7 times lower than that calculated according to the classical theory).

2. In the region $\chi \gtrsim 1$ the relative losses in energy are weakly dependent on the energy (i.e. $L_{ch} \simeq$ const). The rate energy losses is considerably higher than in the appropriate amorphous substance. If the angular width of the incident beam is $\Delta\vartheta_0 \lesssim \vartheta_c$, the radiation is concentrated within the $(1\div 2)\vartheta_c$ angular interval (Ref./4/).

3. The hard photons with $\omega \sim \varepsilon$ are emitted during the process, but one should bear in mind that $\bar{\omega} < \varepsilon/5$ (see Fig.2).

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Table

Crystal	Axis	Tempe-rature	V_0(eV)	a_s(Å)	η	x_0
Ge	⟨110⟩	100	114.5	.302	.063	19.8
Ge	⟨110⟩	280	110	.337	.115	15.8
Si	⟨110⟩	293	70	.324	.145	15.8
Si	⟨111⟩	293	54	.3	.15	15.1
W	⟨111⟩	293	417	.215	.115	39.7
W	⟨111⟩	77	348	.228	.027	35.3

FIGURE CAPTIONS

Fig.1. The average relative energy losses of an electron vs. the crystal thickness L;

1 - Si ⟨111⟩ , T = 293 K, ε_0 = 200 GeV
2 - Si ⟨110⟩ , T = 293 K, ε_0 = 200 GeV
3 - Ge ⟨110⟩ , T = 280 K, ε_0 = 150 GeV
4 - Ge ⟨110⟩ , T = 100 K, ε_0 = 150 GeV

Experimental data of Ref./8/: "a" - at T = 280 K, the remaining ones - at T = 100 K.

5 - Ge ⟨110⟩ , T = 100 K, ε_0 = 200 GeV
6 - W ⟨111⟩ , T = 293 K, ε_0 = 100 GeV
7 - W ⟨111⟩ , T = 77 K, ε_0 = 100 GeV
8 - calculation according to classical formulae for the ⟨110⟩ axis of Ge, T = 100 K, ε_0 = 150 GeV.

Fig.2. The spectrum of radiation intensity at fixed energy;

1 - Si ⟨111⟩ , T = 293 K, ε_0 = 200 GeV
4 - Ge ⟨110⟩ , T = 100 K, ε_0 = 150 GeV
5 - Ge ⟨110⟩ , T = 100 K, ε_0 = 300 GeV
6 - W ⟨111⟩ , T = 293 K, ε_0 = 100 GeV

The intensity spectrum averaged over the crystal thickness with taking into account the energy losses in the ⟨110⟩ axis of Ge, T = 100 K, ε_0 = 150 GeV; 2 - for L = 1.4 mm, 3 - for L = 0.4 mm.

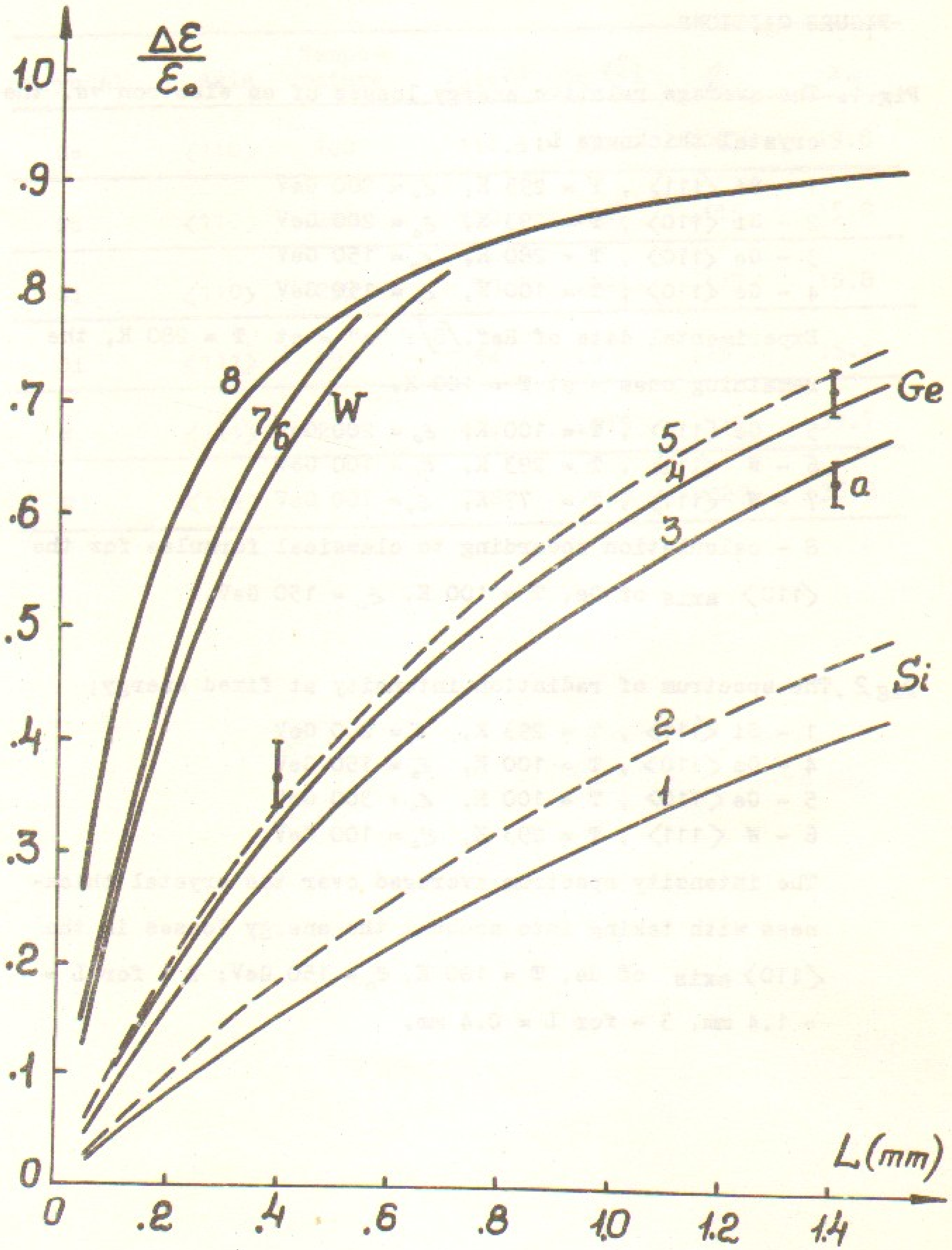
ΔE/E₀
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
8
7
6
W
Ge
5
4
3
a
Si
2
1
L(mm)
0
.2
.4
.6
.8
1.0
1.2
1.4

Fig.1

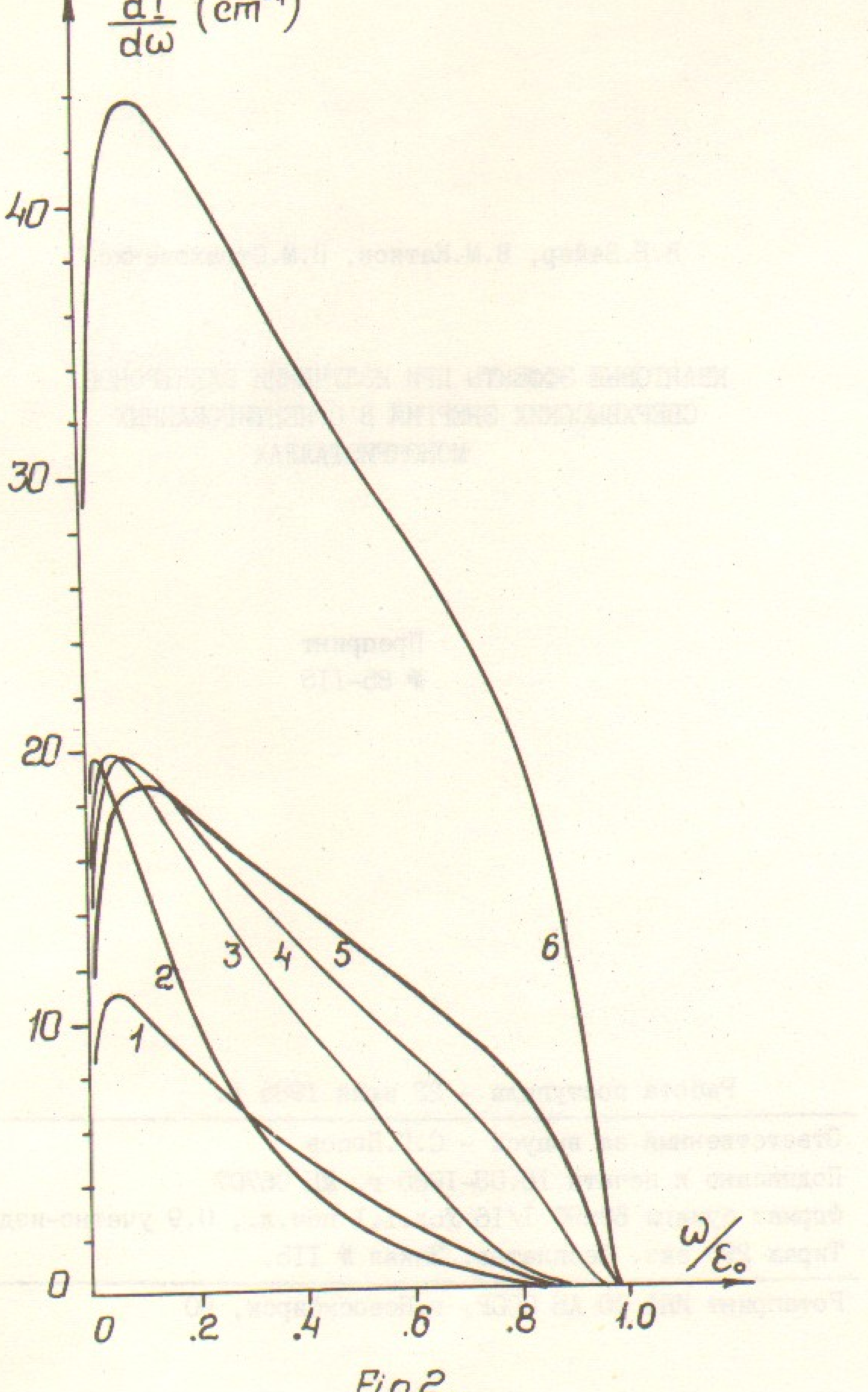
dI/dω (cm⁻¹)
40
30
20
10
0
1
2
3
4
5
6
ω/E₀
0
.2
.4
.6
.8
1.0

Fig.2.

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КВАНТОВЫЕ ЭФФЕКТЫ ПРИ ИЗЛУЧЕНИИ ЭЛЕКТРОНОВ СВЕРХВЫСОКИХ ЭНЕРГИЙ В ОРИЕНТИРОВАННЫХ МОНОКРИСТАЛЛАХ

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