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S.I. Eidelman, E.A. Kuraev

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BOUNDS ON THE NEUTRAL PION POLARIZABILITY FROM e^+e^- ANNIHILATION

S.I.Eidelman, E.A.Kuraev

Institute of Nuclear Physics,
630090, Novosibirsk, U S S R

Abstract

The possibility is discussed to extract model independent bounds on the neutral pion polarizability from the processes $e^+e^- \to \pi^0\pi^0,\ \pi^0\pi^0\gamma$ at the c.m. energy less than or about 1 GeV.

1. Introduction

In recent experiments on radiative π^+ -meson scattering on nuclei the charged pion polarizability has been determined for the first time /1/. The polarizability of the neutral pion can be measured in experiments on radiative photoproduction ($\gamma p \to \gamma \pi p$) /2/, as well as in the production of the $\pi\pi$ system on nuclei ($\gamma p \to \pi\pi p$) /3/ or by e^+e^- annihilation /4/. Whereas in Compton scattering experiments the polarizability is directly determined, in the case of $\pi^0\pi^0$ production the combined effect of the polarizability and final state interaction is measured.

In the present paper the general form for the amplitudes of the processes $e^+e^- \to \pi^0\pi^0, \pi^0\pi^0\gamma$ (section 2) and (section 3) has been used to calculate the corresponding cross sections. The expre ssions obtained allow model independent bounds on the value of the neutral pion polarizability at the c.m. energy not far from the threshold of two-pion production.

2. $e^+e^- \to \pi^0\pi^0$

The amplitude of the process $e^+e^- \to \pi^0\pi^0$ (Fig. 1)

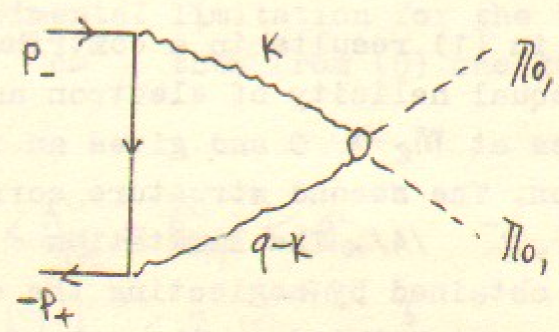

Fig. 1.

includes the amplitude of $\pi^0\pi^0$ production by two virtual γ - quanta $\gamma^*\gamma^* \to \pi^0\pi^0$. The main logarithmic contribution to this amplitude $\sim \ln^2(S/m_e^2)$, $S=4\varepsilon^2$, and can be calculated from dispersion relations. In the calculation of the imaginary part of the amplitude with logarithmic accuracy it is sufficient to consider the intermediate state with two real photons. The general form for the amplitude of $\gamma\gamma\to\pi^0\pi^0$ production by two real photons is

$$T(\gamma(k_1)+\gamma(k_2)\to\pi_0(q_1)+\pi_0(q_2))=\left(B_1\mathcal{L}^{(1)}_{\mu\nu}+B_2\mathcal{L}^{(2)}_{\mu\nu}\right)e^{\mu}(k_2)e^{\nu}(k_1), \tag{1}$$

where tensor structures $\mathcal{L}^{(1,2)}_{\mu\nu}$ are given by

$$\mathcal{L}^{(1)}_{\mu\nu}=(k_1k_2)g_{\mu\nu}-k_{1\mu}k_{2\nu},\qquad \Delta=q_1-q_2,$$

$$\mathcal{L}^{(2)}_{\mu\nu}=(k_1k_2)\Delta_\mu\Delta_\nu-k_{2\nu}\Delta_\mu(k_1\Delta)-k_{1\mu}\Delta_\nu(k_2\Delta)+k_{1\mu}k_{2\nu}\frac{(k_1\Delta)(k_2\Delta)}{(k_1k_2)}, \tag{2}$$

and are gauge-invariant $\mathcal{L}^{(i)}_{\mu\nu}k_1^\nu=\mathcal{L}^{(i)}_{\mu\nu}k_2^\mu=0$. The coefficient B_1 is a smooth function of s at $s\geq 4m_\pi^2$. Similarly to the amplitude of the pion Compton scattering we write it as [1]

$$B_1=8\pi m_\pi\cdot\alpha_0,\quad s\to 4m_\pi^2, \tag{3}$$

where α_0 is the neutral pion polarizability.

The first structure in (1) results in a contribution to helical amplitudes with equal helicity of electron and positron. This contribution vanishes at $m_e\to 0$ and gives an isotropic differential cross section. The second structure corresponds to opposite helicities e^+, e^- /4/. The limitation for the cross section can be obtained by neglecting the contribution of the second structure not related to the polarizability.

The imaginary part of the amplitude corresponding to the first structure is

$$2\,\mathrm{Im}\,M(s)=\frac{4\pi\alpha}{16\pi^4}\cdot 4\pi^2\cdot 8m_\pi\cdot\pi\alpha_0\int\frac{d^4k\,\bar V(p_+)\gamma_\mu(\hat p_--\hat k+m_e)\gamma_\nu U(p_-)}{(p_--k)^2-m_e^2}\cdot$$

$$\cdot\,\delta(k^2)\,\delta((q-k)^2)\left[k(q-k)g^{\mu\nu}-k^\mu(q-k)^\nu\right]. \tag{4}$$

The numerator of the integrand in (4) can be transformed to

$$\bar V(p_+)\left[-s(\hat p_--\hat k)+2sm_e+\hat k(\hat p_--\hat k+m_e)(\hat q-\hat k)\right]U(p_-)=s\,m_e\bar V(p_+)U(p_-)$$

where terms $\sim p_-k$ not resulting in a logarithmic contribution have been omitted. Integration gives

$$\int\frac{d^4k\,\delta(k^2)\delta((q-k)^2)}{p_-k}=\frac{2\pi}{8}\int_{-1}^{1}\frac{2dc}{s(1-\beta c)}=\frac{\pi L}{2s},\quad L=\ln s/m_e^2.$$

The real part of the amplitude can be obtained from dispersion relations. As a result the limitation for the theoretical cross section is

$$d\sigma_{th}>\frac{|2\alpha\alpha_0 m_\pi m_e\cdot\varepsilon|^2}{32\pi^2 s}\cdot\beta\cdot\frac{dO_1}{2!}\cdot L^4=d\sigma_{\text{изотр.}}, \tag{5}$$

where the factor (1/2) takes into account identical pions and integration over the total phase space of the pion is assumed. If an experimental limitation for the cross section at 1 GeV is $\sigma_{exp}<\sigma_0=10^{-34}$ cm^2, then from (5) one gets a bound on the polarizability

$$\sigma_{\text{изотр.}}<\sigma_{th}=\sigma_{exp.}<\sigma_0$$

$$|\alpha_0|<3\cdot10^{-40}\ \text{cm}^3 \tag{6}$$

3. $e^+e^-\to\pi^0\pi^0\gamma$

Consider first production of the $\pi^0\pi^0\gamma$ system by a virtual photon γ^*:

$$\gamma^*(k_1)\to\pi_0(q_1)+\pi_0(q_2)+\gamma(k_2).$$

Using the technique allowing construction of tensor bases free of kinematical singularities and zeros in invariant amplitudes /5/, one can write the general expression for the amplitude (7):

$$T^{\gamma^* \to \pi^0\pi^0\gamma} = 8\pi m_\pi e^{\mu}(k_1)\, e^{*\nu}(k_2)\left[a_1 L^{(1)}_{\mu\nu} + a_2 L^{(2)}_{\mu\nu} + a_3 L^{(3)}_{\mu\nu}\right] \tag{8}$$

where the tensors

$$L^{(i)}_{\mu\nu} k_2^{\nu} = 0\,, \quad L^{(i)}_{\mu\nu} k_1^{\mu} = 0, \quad L^{(1)}_{\mu\nu} = (k_1 k_2)\, g_{\mu\nu} - k_{1\nu} k_{2\mu}\,,$$

$$L^{(2)}_{\mu\nu} = (k_1 k_2) Q_\mu Q_\nu - (QK)(Q_\mu k_{1\nu} + Q_\nu k_{2\mu}) + (QK)^2 g_{\mu\nu}\,, \tag{9}$$

$$L^{(3)}_{\mu\nu} = (QK)(k_1^2 g_{\mu\nu} - k_{1\mu} k_{2\nu}) + Q_\nu (k_1 k_2 \cdot k_{1\mu} - k_1^2 \cdot k_{2\mu}),\quad Q = \frac{q_1 - q_2}{2}\,,\ K = \frac{k_1 + k_2}{2}\,,$$

are gauge-invariant. Note that two other existing structures $L^{(4)}_{\mu\nu} = k_{2\nu}(k_1 k_2 \cdot k_{1\mu} - k_1^2 k_{2\mu})$ and $L^{(5)}_{\mu\nu} = k_{2\nu}(k_1 k_2 \cdot Q_\mu - QK \cdot k_{2\mu})$ do not contribute to the cross section. The numerical factor in (8) is chosen so that the coefficient in the first structure corresponds to the polarizability (cf. (2)). The matrix element for the process $e^+e^- \to \pi^0\pi^0\gamma$ has the form

$$M^{e^+e^- \to \pi^0\pi^0\gamma} = 8\pi m_\pi \sqrt{4\pi\alpha}\, \frac{1}{s} \bar{v}(p_+)\gamma_\mu u(p_-) e^*_\nu(k_2)\left[a_1 L^{(1)}_{\mu\nu} + a_2 L^{(2)}_{\mu\nu} + a_3 L^{(3)}_{\mu\nu}\right]. \tag{10}$$

The cross section is given by

$$d\sigma^{e^+e^- \to \pi^0\pi^0\gamma} = \frac{\alpha}{2!\,\pi^2 m_\pi^4} \frac{d^3k\, d^3q_1\, d^3q_2}{\omega \varepsilon_1 \varepsilon_2} \delta^{(4)}(p_+ + p_- - k - q_1 - q_2) \{C_{11}|\alpha_1|^2 +$$

$$+ C_{22}|\alpha_2|^2 + C_{33}|\alpha_3|^2 + 2C_{12}\,\mathrm{Re}(\alpha_1\alpha_2^*) + 2C_{13}\,\mathrm{Re}(\alpha_1\alpha_3^*) + 2C_{23}\,\mathrm{Re}(\alpha_2\alpha_3^*)\}, \tag{11}$$

where dimensionless quantities C_{ij} are combinations of four-momenta of the final particles:

$$C_{11} = \frac{1}{\varepsilon^2}(2\omega^2 - \bar{k}^2),\quad C_{22} = (\varepsilon^2 m_\pi^4)^{-1}\left[8\varepsilon^2 Q_0^4 + 2\omega Q_0 \cdot Q^2 \cdot \bar{k}\bar{Q} - \omega^2 Q^2 \bar{Q}^2 - 4\varepsilon^2 Q_0^2 \bar{Q}^2\right.$$
$$\left. - Q_0^2 Q^2 k^2\right],\quad C_{33} = 4(m_\pi^4)^{-1}\left[8\varepsilon^2 Q_0^2 - Q^2\bar{k}^2 + 4\varepsilon Q_0 \cdot \bar{k}\bar{Q}\right],\quad C_{13} = (\varepsilon^2 m_\pi^2)^{-1}\left[2\varepsilon\omega \cdot \bar{k}\bar{Q}\right.$$
$$\left. + 8\varepsilon^2\omega Q_0 - 2\varepsilon Q_0 \bar{k}^2\right],\quad C_{12} = (\varepsilon^2 m_\pi^2)^{-1}\left[-\omega^2\bar{Q}^2 + 2\omega Q_0 \cdot \bar{k}\bar{Q} + 4\omega\varepsilon Q_0^2\right. \tag{12}$$
$$\left. - 2\varepsilon Q_0 \cdot \bar{k}\bar{Q} - Q_0^2\bar{k}^2\right],\quad C_{23} = (\varepsilon^2 m_\pi^4)^{-1}\left[16\varepsilon^3 Q_0^3 + 2\varepsilon\omega \cdot Q^2 \cdot \bar{k}\bar{Q} - 4\varepsilon^2 Q_0 \omega \cdot \bar{Q}^2\right.$$
$$\left. + 4\varepsilon^2 Q_0^2 \cdot \bar{k}\bar{Q} - 2\varepsilon Q_0 \cdot Q^2 \cdot \bar{k}^2\right].$$

and

$$Q = \frac{1}{2}(q_1 - q_2),\quad Q_0 = \frac{1}{2}(\varepsilon_1 - \varepsilon_2),\quad q_1^2 = q_2^2 = m_\pi^2,\quad Q^2 = \frac{1}{2}(m_\pi^2 - q_1 q_2),$$
$$\bar{k}^2 = k_x^2 + k_y^2,\quad \bar{Q}^2 = Q_x^2 + Q_y^2,$$
$$\bar{Q}\bar{k} = Q_x k_x + Q_y k_y. \tag{13}$$

Here $p_{x,y}$ are components of the four-momentum perpendicular to the beam axis. The parameter α_1 is related to the polarizability through

$$\alpha_1 = m_\pi^3 a_1 = m_\pi^3 \alpha_0\,, \tag{14}$$

while the parameters

$$\alpha_2 = m_\pi^5 a_2\,,\quad \alpha_3 = m_\pi^5 a_3 \tag{15}$$

are other characteristics of the neutral pion. As one of the photons is highly virtual, α_1 is the generalized pion polarizability. One can expect that the quantities $|\alpha_1|, |\alpha_2|, |\alpha_3|$ are of the same order of magnitude. Then all structures in (12) besides C_{11} are suppressed by a factor of about 10 , since they contain particle momenta in higher powers than C_{11} . Additional suppression comes from the more complicated angular dependence ($\sim \sin^m\theta_i,\ m \geq 4$). Neglecting all the terms but $\sim C_{11}$ and performing integration in (11), one obtains the following limitation for the cross section:

$$\sigma^{e^+e^- \to \pi^0\pi^0\gamma} \simeq \frac{16\alpha}{3m_\pi^2}\left(\alpha_0 m_\pi^2 \varepsilon\right)^2 \cdot I(s), \quad s = 4\varepsilon^2, \tag{16}$$

where the function $I(s)$ is given by

$$I(s) = \int_0^{1-\frac{4m_\pi^2}{s}} dx \cdot x^3 \left[1 - \frac{4m_\pi^2}{s(1-x)}\right]^{1/2} \tag{17}$$

and $I(s) = 0.14$ at $2\varepsilon = 1$ GeV. Then an experimental limitation of $\sigma < 4 \cdot 10^{-34}\ \text{cm}^2$ at 1 GeV corresponds to a bound on the polarizability

$$|\alpha_0| < 15 \cdot 10^{-43}\ \text{cm}^3. \tag{18}$$

One can see that this bound is much lower than that of (6). In general, the process $e^+e^- \to \pi^0\pi^0\gamma$ is much more favourable than $e^+e^- \to \pi^0\pi^0$ for studying polarizability effects, since the cross section of the latter contains a suppressing factor $(m_e/\varepsilon)^2$. Finally, one should remember that we have assumed α_i to be energy independent. Therefore the expressions for the cross sections obtained above are valid at the c.m. energy not far from the threshold of two-pion production, i.e. $2\varepsilon < 1$ GeV.

Note that different theoretical models give dramatically differing predictions for this quantity (see, for example, /6/).

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Э.А.Кураев, С.И.Эйдельман

ОГРАНИЧЕНИЯ НА ПОЛЯРИЗУЕМОСТЬ НЕЙТРАЛЬНОГО ПИОНА ИЗ ЭКСПЕРИМЕНТОВ ПО e^+e^- АННИГИЛЯЦИИ

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