47

ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

A. R. Zhitnitsky

ON CHIRAL SYMMETRY BREAKING IN QCD_2 $(N_c \to \infty)$

PREPRINT 85-78

НОВОСИБИРСК

ON CHIRAL SYMMETRY BREAKING IN $QCD_2(N_c \to \infty)$

A.R.Zhitnitsky

Institute of Nuclear Physics,

630090, Novosibirsk, U S S R

ABSTRACT

Different phases of the $QCD_2(N_c)$ are discussed. In the weak coupling regime ($N_c \to \infty$, $g^2 N_c$ = const, $g \ll m_q$) the chiral symmetry breaking takes place and $\langle \bar\psi \psi \rangle = -N_c \sqrt{\frac{g^2 N_c}{12\pi}}$. The another important value characterizing the vacuum of QCD_2 is the gluonic condensate. It is shown that $\langle G^2 \rangle = -2\pi \langle \bar\psi\psi \rangle^2 = -\frac{1}{6} N_c^2 \left(\frac{g^2 N_c}{\pi} \right)$.

1. In this note we discuss the properties of QCD vacuum. This information will be obtained from the well-known spectrum of the theory. We hope that knowledge of vacuum condensates $\langle \bar\psi\psi \rangle$, $\langle G^2 \rangle$ will be useful for investigation of the vacuum structures. This knowledge allows one to approbate different approximate methods for studying of the real world.

The main purpose of this note is to get some information about vacuum of $QCD_2(N_c)$. First, let's discuss main results connected with the spectrum of the theory. QCD_2 was considered firstly by 'T Hooft [1]. The corresponding Bethe-Salpeter equation was solved in the $N_c = \infty$ limit and approximately linear spectrum of states was found:

$$m_n^2 \sim \pi^2 m_0^2 n \quad , \quad m_0^2 = \frac{g^2 N_c}{\pi} \qquad (1)$$

Let's note that the mass of the lowest state tends to zero when m_q (quark mass) $\to 0$.

The results of ref. [1] are reproduced both the functional [2] and operator [3] methods. The lattice calculation is consistent with the results of ref. [1].

We emphasize that the 'T Hooft's solution (1) takes place for the following relations between parameters of theory [5,6]*:

$$N_c \to \infty \ , \quad g^2 N_c = \text{const}, \quad m_q \gg g \sim \frac{1}{\sqrt{N_c}} \qquad (2)$$

As it is seen from (2), the chiral limit $(m_q \to 0)$ in this regime can be reached only after taking the limit $N_c \to \infty$. Thus, the spectrum (1) corresponds to the weak coupling regime (2): $m_q \gg g$. In the strong coupling regime ($m_q \ll g$) the spectrum is quite different [5-10]. Just in the limit $m_q \to 0$ the Goldstone boson is absent, the massless baryons appear instead of it.

Let's try to understand these results qualitatively, ha-

*The unequality $m_q \gg g$ (2) can be obtained in another way. Namely, the nonplanar $\pi\pi$-contribution to the correlator $\int dx \langle 0 | T \{ \bar\psi\psi(x), \bar\psi\psi(0) \} | 0 \rangle$ is order $\frac{1}{g}$ and should be mush smaller than planar one $(\sim N_c)$

ving in view the rigorous statement due to Coleman [11], concerning the impossibility of the spontaneous breakdown of a continuous symmetry in two dimension theories*. The answer is that in regime (2) the Berezinski, Kosterlitz and Thouless effect [15] takes place, and chiral symmetry is "almost" spontaneously broken. Well-known analogous phenomen is observed in the $SU(N\to\infty)$ Thirring model, where chiral symmetry is spontaneously broken (SBCS) [12,13]. The spontaneous breaking in QCD is consistent with the spectrum found in ref. [1] i.e. the states with different P-parities are not degenerate in masses. Below the value of the condensate will be obtained explicitly.

Let's discuss now what happens with the theory when N_c decreases. In this case the condensate and the Goldstone boson must disappear according to the Coleman [11] theorem. Therefore, we expect that the correlator $\langle 0|T\{\bar{\psi}_L\psi_L(x),\bar{\psi}_L\psi_R(0)\}$ $\sim x^{-1/N_c}$, analogously to what is found in ref. [13]. Besides, the mass of Goldstone boson have the additional factor $\sim 1/N_c$:

$$m_\pi^2 \sim m_q + 1/N_c$$

and m_q, m_π^2 never vanishes for finite N_c in regime (2). Thus, all the results obtained in the weak coupling regime don't contradict to Coleman theorem .

To understand qualitatively the spectrum of the low-lying states, let's discuss 'T Hooft's anomaly condition [17]. The statement is that either chiral symmetry must be broken, or

* Discussion of this question by 'T Hooft and Coleman see in ref. [14].

massless (for $m_q\to 0$) fermions must appear*. Evidently, it is the first possibility which is realized in the regime (2). For finite N_c the SBCS is not possible, and massless baryons must exist. They were really discovered [5,7,9,10].

The last point, which I would like to discuss is the following one. For large N_c and small m_q the results depend essentially on the order of limits [18]:

$$\begin{aligned}&1.\ N_c\to\infty \text{ firstly and } m_q\to 0 \text{ after that}\\ &2.\ m_q\to 0 \text{ and than } N_c\to\infty .\end{aligned} \qquad (3)$$

The explanation is that these two cases correspond to different phases. Really, in case 1 we always have $g \ll m_q$ and this unequality corresponds to the weak-coupling regime. In case 2 we always have $g \gg m_q$ which corresponds to the strong coupling regime.

Let's remind that in the Shwinger's model the results are the same, either m_q put equal zero from the beginning or in final formulas [19]. This occurs because the phase transition in the massive Shwinger model can arise only for large $m_q \gtrsim g/\sqrt{\pi}$. Indeed, effective potential $V(\varphi)$ of the equivalent-boson theory is [19]:

$$V(\varphi)=\frac{1}{2}\frac{g^2}{\pi}\varphi^2-m_q g\cos(2\sqrt{\pi}\varphi) \qquad (4)$$

It is seen that the phase transition takes place only for sufficiently large $m_q \gtrsim g/\sqrt{\pi}$, and for small m_q it is not possible.

In the case of QCD_2 ($N_c\to\infty$) the situation is quite different. Since the coupling constant tends to zero $g\sim 1/\sqrt{N_c}$,

* The 'T Hooft's anomaly condition in QCD_2 is discussed in more detail in ref. [9,10].

the regions of phase transition $(m_q \sim g)$ and of chiral limit $(m_q \to 0)$ overlap. Thus, in the cases 1 and 2 above (see eq. (3)) we approach the point $m_q = 0,\ N_c = \infty$ of the phase diagram from different directions.

The following discussion deals with 'T Hooft's phase (2) only, for which the SBCS takes place.

2. The very simple method for calculation of the condensate density is the following one. Let's consider the correlator (at $-q^2 \to \infty$):

$$i\int dx\, e^{iqx} \langle 0 | T\{\bar\psi \gamma_\mu \gamma_5 \psi(x),\ \bar\psi \gamma_5 \psi(0)\} | 0 \rangle = q_\mu \Pi(q^2) \tag{5}$$

The right hand side of eq. (5) can be calculated with the help of operator expansion. It equals to (see fig. 1):

$$\Pi(q^2) = \frac{2\langle \bar\psi\psi \rangle}{q^2} \tag{6}$$

The left hand side of eq. (5) is saturated by Goldstone boson (let's call it π -meson):

$$\langle 0 | \bar\psi \gamma_\mu \gamma_5 \psi | \pi \rangle = i q_\mu A\ , \quad \langle 0 | \bar\psi \gamma_5 \psi | \pi \rangle = i B \tag{7}$$
$$\langle \bar\psi\psi \rangle = \frac{1}{2} A \cdot B$$

First, let's calculate the value of A. Note that in the chiral limit $m_q \to 0$, the $\bar\psi \gamma_\mu \gamma_5 \psi$, $\bar\psi \gamma_\mu \psi$ - currents are connected with the massless pseudoscalar state only. This fact was obtained from the 'T Hooft's equation in ref. [20]. We can prove this statement by the following simple method. Consider the general form of the matrix element:

$$\langle 0 | \bar\psi \gamma_\nu \gamma_5 \psi | n \rangle = i q_\nu A_n + i \epsilon_{\nu\mu} q_\mu C_n \tag{8}$$

It follows from the relation $\partial_\mu \bar\psi \gamma_\mu \gamma_5 \psi \sim m_q \bar\psi \gamma_5 \psi$ and from (8) that A $\sim$ 1 only for π -meson. Besides using the identity $\epsilon_{\mu\nu}\gamma_\nu = -\gamma_\mu \gamma_5$ the expression (8) can be rewritten as follow:

$$\langle 0 | \bar\psi \gamma_\nu \psi | n \rangle = i q_\nu C_n + i \epsilon_{\nu\mu} q_\mu A_n \tag{9}$$

The conservation of vector current of massive (m_q) quark fields

tells us that $C_n = 0$ in any case, excluding probably the states with $m_n^2 = 0$. But the massless bound state can not exist in vectorlike theories with massive quarks [21]. Therefore, $C_n = 0$ always and $\bar\psi \gamma_\mu \gamma_5 \psi$, $\bar\psi \gamma_\mu \psi$ - currents are connected with π -meson only. Taking this fact into account, let's find the value of A from the duality relation. Consider with this purpose the correlator (at $-q^2 \to \infty$):

$$i\int dx\, e^{iqx} \langle 0 | T\{\bar\psi \gamma_\mu \gamma_5 \psi(x),\ \bar\psi \gamma_\nu \gamma_5 \psi(0)\} | 0 \rangle = q_\mu q_\nu \Pi_1 + g_{\mu\nu} \Pi_2 \tag{10}$$

The result is saturated by the fig. 2 diagrams only. It equals to $\Pi_1 = \frac{N_c}{\pi}\left(-\frac{1}{q^2}\right)$. Besides, the absorptive part of (10) is determined by π -meson contribution only, i.e. by the value of A. Therefore $A = \sqrt{\frac{N_c}{\pi}}$ - in agreement with the direct calculation [20]:

$$A = \sqrt{\frac{N_c}{\pi}} \int_0^1 \varphi_\pi(x)\, dx = \sqrt{\frac{N_c}{\pi}} \tag{11}$$

Here $\varphi_\pi(x)$ is the wave function of the massless state [1,20]:

$$\varphi_\pi(x) = 1\ ,\ 0 < x < 1\ ,\quad \varphi_\pi(x) \sim [x(1-x)]^\beta,\ x \to 0, x \to 1 \tag{12}$$
$$\pi\beta \operatorname{ctg} \pi\beta = 1 - \frac{m_q^2 \pi}{g^2 N_c}$$

Let's calculate now the value of B.

$$B = -\frac{m_\pi^2}{2 m_q} A = -\frac{m_\pi^2}{2 m_q} \sqrt{\frac{N_c}{\pi}} \int_0^1 \varphi_\pi(x)\, dx =$$
$$= -\frac{m_q}{2} \sqrt{\frac{N_c}{\pi}} \int_0^1 \varphi_\pi(x) \left[\frac{1}{x(1-x)}\right] \tag{13}$$

Here the first equality is the consequence of the equations of motion; the second one is the consequence of eq. (11). The last equality in eq. (13) is due to the 'T Hooft' equation: $m_n^2 \int \varphi_n(x)\, dx = m_q^2 \int dx\, \varphi_n(x) [x(1-x)]^{-1}$ [1,20]. The value of β from eq. (12) and of B from eq. (13) are equal to:

$$\beta = \sqrt{\frac{3m_q^2}{g^2 N_c \pi}}\ ; \quad B = -m_q \sqrt{\frac{N_c}{\pi}} \int \frac{dx}{x}\, x^{\sqrt{\frac{3m_q^2}{g^2 N_c \pi}}} = -\sqrt{\frac{N_c}{\pi}} \sqrt{\frac{g^2 N_c \pi}{3}} \qquad (14)$$

for sufficiently small m_q.

I would like to make two remarks concerning the eq. (14). Firstly, if we put $m_q = 0$ from the beginning, then we obtain $\langle \bar\psi\psi \rangle \sim B \sim m_q \to 0$. This result corresponds to the strong coupling regime ($m_q \ll g$). Secondly, the absolute value of B is saturated mainly by infrared region of integration, i.e. $x \sim 0$, $x \sim 1$, as it is seen from eq. (14). Remind that x has the meaning of fraction of momentum carried by quark in meson. Thus, the regions $x \sim 0$, $x \sim 1$ correspond to the case when the whole meson momentum is carried by only one of the quarks, while another one is "wee". From eq. (7),(11),(14) we have:

$$\langle \bar\psi\psi \rangle = -\frac{N_c}{\sqrt{12}}\, m_0 \quad , \quad m_0 \equiv \frac{g^2 N_c}{\pi} \qquad (15)$$

We see that $\langle \bar\psi\psi \rangle \sim N_c$, as it was expected.

3. Let's check selfconsistency of the result (15). We will show that the spectrum found in the ref. [1] ($m_n^2 \sim \pi^2 m_0^2 n$) provides the nonzero value of condensate (15). The idea is the following one. On the one hand we know exactly the spectrum and different matrix elements. Consequently, we know the spectral density of any correlator. On the other hand, we can calculate this correlator at $-q^2 \to \infty$ using operator expansion. The dispersion relation connects these two quantities. This idea was used early for the calculation of $\langle \bar\psi\psi \rangle$ (see eq. (5)). But in eq. (5) the spectral density is contributed by π-meson only. In what follows the whole spectrum will be essential.

Let's denote by $|n\rangle$ the "n"-th 'T Hooft's state with the mass $m_n^2 \propto \pi^2 m_0^2 n$. The corresponding matrix element is defined as follows:

$$\langle 0 | \bar\psi\psi | n \rangle = \sqrt{\frac{N_c}{\pi}}\, f_n \qquad (16)$$

Here the values f_n can be expressed via integrals of the wave functions of the states $|n\rangle$ [20] and, in principle, they can be calculated. However, we do not need an explicit form for

these quantities. Let's calculate the following matrix element:

$$\langle 0 | \bar\psi g G^a_{\mu\nu} \gamma_\nu \lambda^a \psi | n \rangle = i q_\mu a_n \qquad (17)$$

Here a_n is expressed via f_n as follows:

$$a_n = \frac{2g^2 \langle \bar\psi\psi \rangle}{m_n^2} \sqrt{\frac{N_c}{\pi}}\, f_n \qquad (18)$$

To derive the ratio (18) one needs to use both the equation of motion $D_\mu G^a_{\mu\nu} = -g \bar\psi \gamma_\nu \lambda^a \psi$ and the factorization property for large N_c.

Let's consider the following correlator at $Q^2 \to \infty$.

$$i \int dx\, e^{iqx} \langle 0 | T\{ \bar\psi g G^a_{\mu\nu} \gamma_\nu \lambda^a \psi(x), \bar\psi\psi(0) \} | 0 \rangle = q_\mu \Pi(q^2). \qquad (19)$$

On the one hand the $\Pi^{th.}$ ($-q^2 \to \infty$) is given by diagram at fig. 3 and it is equal to:

$$\Pi^{th.}(Q^2 \to \infty) = \frac{i g^2 N_c}{\pi} \langle \bar\psi\psi \rangle \frac{\ln Q^2}{Q^2} \qquad (20)$$

On the other hand the $\Pi^{exp.}$ can be expressed through the spectral density. Using the relations (16), (17), (18) we get:

$$\Pi^{exp.}(Q^2) = \sum \frac{i a_n f_n \sqrt{\frac{N_c}{\pi}}}{Q^2 + m_n^2} = 2 i g^2 \langle \bar\psi\psi \rangle \frac{N_c}{\pi} \sum \frac{f_n^2}{m_n^2 (Q^2 + m_n^2)}. \qquad (21)$$

The sum from (21) can be easily calculated with the logarithmic accuracy. Taking into account that $m_n^2 \sim n$, $f_n^2 \sim 1$ [20], we have:

$$\sum \frac{f_n^2}{m_n^2 (Q^2 + m_n^2)} \xrightarrow[Q^2 \to \infty]{} \frac{1}{Q^2} \sum_{n=1}^{n \sim Q^2} \frac{f_n^2}{m_n^2} \simeq \frac{1}{2} \frac{1}{Q^2} \ln Q^2 / m_0^2 \qquad (22)$$

Here the coefficient $1/2$ has appeared because only one half of states with even P-parity contributes to the correlator (19). Substituting the eq. (22) to eq. (21) we obtain $\Pi^{exp.}(Q^2 \to \infty)$ which exactly coinsides with $\Pi^{th.}$ from (20). From the derivation of this relation the nontrivial character of this result is seen, especially because the matrix elements (17) are proportional to condensate $\langle\bar{\psi}\psi\rangle$.

Let's check the ratio (15) in another way. To do this consider the following matrix elements:

$$\langle 0|\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi|n\rangle = a_n\left(\frac{2q_\mu q_\nu}{m_n^2} - g_{\mu\nu}\right), \qquad \overleftrightarrow{D}_\nu = \overrightarrow{D}_\nu - \overleftarrow{D}_\nu \tag{23}$$

Here a_n are the same as in (18). The eq. (23) is derived from the equation of motion $\partial_\mu(\bar{\psi}\, i\overleftrightarrow{D}_\mu \gamma_\nu) = -g\bar{\psi}\, G_{\nu\lambda}\gamma_\lambda\psi$. Then eq. (23) reduces to eq. (17).

Now, consider the correlator:

$$i\int dx\, e^{iqx}\langle 0|T\{\bar{\psi}\, i\overleftrightarrow{D}_\mu\gamma_\nu\psi(x),\ \bar{\psi}\, i\overleftrightarrow{D}_\lambda\gamma_\sigma\psi(0)\}|0\rangle = q_\mu q_\nu q_\lambda q_\sigma \Pi \tag{24}$$

(at $Q^2\to\infty$), separating the following kinematical structure: $q_\mu q_\nu q_\lambda q_\sigma$. The leading contribution (at $Q^2\to\infty$) is given by the diagram of fig. 2 and is equal to $\Pi^{th.}(Q^2\to\infty) = \frac{N_c}{3\pi}\frac{1}{Q^2}$. On the other hand, the value $\Pi^{exp.}(Q^2)$ can be calculated with the help of dispersion relation because we know all the matrix elements (23). Therefore, $\Pi^{exp}(Q^2\to\infty) = 1/Q^2\, \Sigma\left(\frac{2a_n}{m_n}\right)^2$. Here $a_n \sim \langle\bar{\psi}\psi\rangle$, see eq. (18). So, nonzero value of $\Pi^{th.}(Q^2)$ implies that $\langle\bar{\psi}\psi\rangle$ is also nonzero.

In conclusion we would like to note the gluonic condensate $\langle G^2\rangle$ can be calculated in analogous way*.

$$\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle = -2\langle G^2_{01}\rangle = -2\pi\langle\bar{\psi}\psi\rangle^2 = -\frac{\pi}{6}m_0^2 N_c^2 \tag{25}$$

Moreover, the exact low-energy theorems show that the values $\langle\bar{\psi}\psi\rangle$, $\langle G^2\rangle$ are nonzero and equal to values (15),(25) correspondingly. These results ensure that any other states not coinciding with the those found in ref. [1] or absent in regime (2), or, their matrix elements are suppressed by factor of $1/N_c$.

* The question, concerning the normalization point of operator G^2 is not essential because the QCD_2 is the superrenormalizable theory.

In summary, we have shown that the vacuum in QCD_2 has nontrivial structure in regime (2). But we don't know what kind of fluctuations saturates $\langle\bar{\psi}\psi\rangle$, $\langle G^2\rangle$. It is so because we don't see how to construct the topologically nontrivial field configuration and colorless topological charge (the value $\int d^2x\, g\,\epsilon_{\mu\nu}G^a_{\mu\nu}$ - is the color object).

We know how does the vacuum expectation values vary when the parameters do (for example, the absolute value of $\langle G^2\rangle$ decreases when the quark masses increase). But we don't know what is the value of $\langle G^2\rangle$ in strong coupling regime at finite N_c (the condensate $\langle\bar{\psi}\psi\rangle$ in this case must vanish, according to the theorem by Coleman [11]).

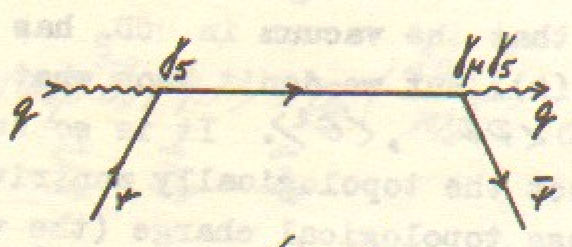


fig. 1

fig. 2

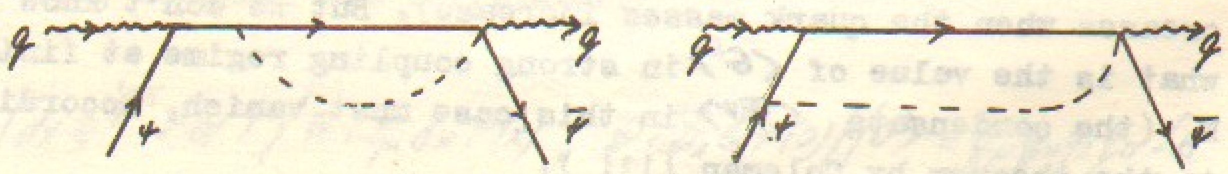
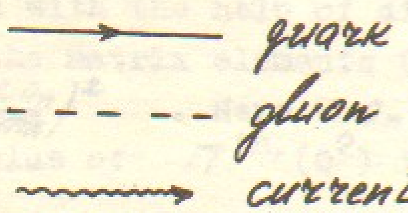


fig. 3



References

1. G't Hooft, Nucl. Phys. B75(1974),461.
2. W.R.Gutierrez, Nucl. Phys. B176(1980),185.
3. Buchmüller et al. Phys. Lett. 108B(1982), 426.
4. C.J.Hamer Nucl. Phys. B195(1982),503.
   K.Barad e.a. Phys. Lett. 143B(1984),222.
5. V.Baluni, Phys. Lett. 90B(1980), 407.
6. G.Bhattacharya, Nucl. Phys. B205(FS'5),(1982),461.
7. P.J.Steinhardt, Nucl. Phys. B176(1980),100.
8. R.E.Gamboa Saravi e.a. Nucl. Phys. B185(1981),239.
9. D.Amati, E.Rabinovici, Phys. Lett. 101B(1981),407.
10. S.Elitzur, Y.Frishman, E.Rabinovici, Phys. Lett. 106B(1981), 403.
11. S.Coleman, Commun. Math. Phys. 31(1973), 259.
12. D.J.Gross, A.Neveu, Phys. Rev. D10(1974),3235.
13. E.Witten, Nucl. Phys. B145(1978),110.
14. Proc. of the 1975 Int. School of Subnuclear. Phys., Erice, Italy, Ed. A.Zichichi, N.Y. Plenum, 1977.
    G't Hooft, page 294,
    S.Coleman, page 420.
15. V.L.Berezinski, JETP (Sov. Phys.) 32(1970), 493.
    J.M.Kosterlitz and D.J.Thouless, J. Phys. C6(1973), 1181.
16. M.Durgut, N.K.Pak, Phys. Lett. 117B(1982),453.
17. G't Hooft, in Recent Developments in Gauge Theories, Plenum Press, N.Y. 1980.
18. Y.Frishman, Nucl. Phys. B148(1979),74.
19. J.Kogut, L.Susskind, Phys. Rev. D11(1975), 3594.
20. C.G.Callan Jr., N.Coote, and D.J.Gross, Phys. Rev. D13 (1976),1649.
21. C.Vafa and E.Witten, Nucl. Phys. B234(1984),173.

А.Р.Житницкий

О НАРУШЕНИИ КИРАЛЬНОЙ СИММЕТРИИ В КХД$_2$ ($N_c \to \infty$)

Препринт
№ 85-78

Работа поступила - 27 мая 1985 г.

Ответственный за выпуск - С.Г.Попов
Подписано к печати 19.06.1985 г. МН 06683
Формат бумаги 60x90 1/16 Усл.0,9 печ.л., 0,8 учетно-изд.л.
Тираж 290 экз. Бесплатно. Заказ № 78.

Ротапринт ИЯФ СО АН СССР. г.Новосибирск, 90