26

ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

V.N.Baier, V.M.Katkov, and V.M.Strakhovenko

TO THE THEORY OF PAIR CREATION IN AN ALIGNED SINGLE CRYSTAL

PREPRINT 85-43

НОВОСИБИРСК

TO THE THEORY OF PAIR CREATION IN AN ALIGNED SINGLE CRYSTAL

V.N.Baier, V.M.Katkov and V.M.Strakhovenko

Institute of Nuclear Physics, 630090 Novosibirsk, USSR

A b s t r a c t

A general theory of pair creation in an aligned single crystal is developed. In the limiting cases, it describes coherent pair creation and pair creation in a constant field.

During the interaction of a high-energy photon with a single crystal the e^+e^- pair creation is considerably modified in comparison with the amorphous medium. The coherent creation mechanism due to an interference of the contributions from various centres is well known (see Ref. [1] and the references cited there). Quite recently, it has been revealed that at superhigh energies (tens of GeV), when a photon is incident on a single crystal nearly along the crystallographic axes (planes), the pair creation mechanism in the field of a particular axis (plane) starts acting (Refs. [2-4]). This new mechanism manifests itself when the parameter $\varkappa \gtrsim 1$ $\left(\varkappa(a)=\frac{V_0\omega}{m^3 a}\right.$, where V_0 is the characteristic value of the axis (plane) potential, m is the mass of the electron, ω is the photon energy, a is the range of potential action depending upon the conditions $u_1 \lesssim a \lesssim a_s$, u_1 is the amplitude of thermal vibrations, and a_s is the radius of screening). Strictly speaking, the standard theory of coherent pair creation fails at $\varkappa \gtrsim 1$.

In the present paper short version of a general theory of pair creation in a single crystal is presented which is valid at any energies and describes coherent pair creation and pair creation in a constant field in the limiting cases.

We are mainly interested in the $\varkappa \gtrsim 1$ region when the parameter $\rho_c \equiv \frac{V_0\omega}{m^2} = am\varkappa \gg 1$. In Ref. [5], the general formula was derived in the quasiclassic approximation for the process probability w_e dependent on the trajectory of a particle of the created pair. If the incident angle ϑ_0 (the angle between $\vec{n} = \vec{k}/\omega$, $\vec{k}$ is the photon momentum, and an axis direction) satisfies $\vartheta_0 \gg \vartheta_c \simeq \sqrt{\frac{V_0}{\omega}}$, then the increment in the velocity v is possible to take into account in the straight-path approximation, and $\Delta v \sim V_0/\omega\vartheta_0$. In the limit when $\left(\Delta v\frac{\omega}{m}\right)^2 \simeq \left(\frac{V_0}{m\vartheta_0}\right)^2 \gg 1$, pair creation occurs locally and is described by the formulae for a constant field (Ref. [6]). By virtue of the condition $\varrho_c \gg 1$, there exists the region of overlap of these approximations: $V_0/m \gg \vartheta_0 \gg \vartheta_c$. Since at $\vartheta_0 \lesssim \vartheta_c$ the probability w_e stops to be dependent on the incident angle (accurate up to the terms $\sim \frac{1}{\varrho_c}$), the straight-path approximation can be used up to $\vartheta_0 = 0$. At large entrance angles, $V_0/m\vartheta_0 \ll 1$, the Born approximation holds [5]. In the quasiclassic approximation, we obtain the following expression for the pair creation probability per unit time:

$$W_e = \frac{i\alpha m^2}{2\pi\omega^2}\int\frac{d^3 r_0}{V}\int d\varepsilon\int_{-\infty}^{+\infty}\frac{d\tau}{\tau+i0}\left[1+\frac{\varepsilon^2+\varepsilon'^2}{\varepsilon\varepsilon'}\left(\sum_q \frac{G(\vec{q})e^{-i\vec{q}\vec{r}}}{m q_\parallel}\vec{q}_\perp \sin q_\parallel\tau\right)^2\right]\cdot$$

$$\cdot\exp\left\{\frac{im^2\omega\tau}{\varepsilon\varepsilon'}\left[1+\sum_{\vec{q},\vec{q}'} e^{-i(\vec{q}+\vec{q}')\vec{r}_0}\frac{G(\vec{q})G(\vec{q}')}{m^2 q_\parallel q'_\parallel}(\vec{q}_\perp\vec{q}'_\perp)\left(\frac{\sin(q_\parallel+q'_\parallel)\tau}{(q_\parallel+q'_\parallel)\tau}-\frac{\sin q_\parallel\tau}{q_\parallel\tau}\frac{\sin q'_\parallel\tau}{q'_\parallel\tau}\right)\right]\right\} \qquad (1)$$

where $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$, $\varepsilon' = \omega - \varepsilon$ (ε is the energy of a particle of the pair), $q_\parallel = \vec{n}\vec{q}$, $\vec{q}_\perp = \vec{q} - \vec{n}q_\parallel$; V is the crystal volume, and the crystal potential is chosen as follows: $U(\vec{r}) = \sum_q G(\vec{q})e^{-i\vec{q}\vec{r}}$. Formula (1) contains all the information on the tilt-angle dependence of the process.

We choose the z-axis along the string of atoms forming the axis. In the case when $\vartheta_0 \ll \frac{V_0}{m}$, $q_\parallel\tau \sim \frac{\vartheta_0 m}{V_0} \ll 1$ for the $\vec{q}$ vectors lying in the (x,y) plane, and $q_\parallel\tau \sim \frac{m}{V_0} \gg 1$ for the other $\vec{q}$ (with $q_z \neq 0$). In view of this, one should retain only the terms with $q_z = 0$ in the double sum of eq. (1) (the contribution for $q_z \neq 0$ is suppressed in powers of V_0/m). Expanding in terms of $q_\parallel\tau$ in eq. (1), we find:

$$W_e^{CF} = \frac{\alpha m^2}{\sqrt{3}\pi\omega^2}\int^{\omega} d\varepsilon\int\frac{d^2\varrho_0}{S}\left\{\varphi(\varepsilon)K_{2/3}(\lambda)+\int_\lambda^\infty K_{1/3}(y)dy-\frac{(\vec{b}\cdot(\vec{n}\vec{\nabla})^2\vec{b})}{|\vec{b}|^4}\frac{\varphi(\varepsilon)}{3}\cdot\right.$$

$$\cdot\left(K_{2/3}(\lambda)-\frac{2}{3\lambda}K_{1/3}(\lambda)\right)-\frac{\left[((\vec{n}\vec{\nabla})\vec{b})^2+3(\vec{b}\cdot(\vec{n}\vec{\nabla})^2\vec{b})\right]}{30|\vec{b}|^4}\left[\lambda K_{1/3}(\lambda)-\right. \qquad (2)$$

$$\left.\left.-\frac{4}{3}K_{2/3}(\lambda)-\varphi(\varepsilon)\left(4K_{2/3}(\lambda)-\lambda K_{1/3}(\lambda)-\frac{16}{9\lambda}K_{1/3}(\lambda)\right)\right]\right\}$$

where $\varphi(\varepsilon) = \frac{\varepsilon}{\varepsilon'}+\frac{\varepsilon'}{\varepsilon}$; $\lambda = \frac{2m^2\omega}{3\varepsilon\varepsilon'|\vec{b}|}$, $K_\nu(\lambda)$ is the Macdonald function, and $\vec{b} = \vec{\nabla}\frac{U}{m}$; here the potential $\sum_{q_z=0} G(\vec{q})e^{-i\vec{q}\vec{r}_0}$ is two-dimensional (in the x, y plane), i.e. $\vec{\nabla} \to \frac{\partial}{\partial\vec{\varrho}_0}$. The vector $\vec{b}$ depends only on the local characteristics of the potential (acceleration at the point $\vec{\varrho}_0$). The first two terms in eq. (2), independent on $\vec{n}$ (ϑ_0), give a known probability of pair creation in a constant field (cf. Refs. [2,5]), while the remaining are the correction $\backsim\left(\frac{m\vartheta_0}{V_0}\right)^2$. Calculations show that the coefficient at $(m\vartheta_0/V_0)^2$ in eq. (2) proves to be relatively small, and this makes it possible to make use of formula (2) up to $\vartheta_0 \sim \frac{V_0}{m}$, at not too large values of $\varkappa$. At the threshold of manifestation of the constant field effects ($\varkappa \lesssim 1$) the correction is positive and W_e is minimum at $\vartheta_0 = 0$. With an increase of $\varkappa$, the correction alters its sign and at $\vartheta_0 = 0$ the probability W_e becomes maximum. For sufficiently large values of the parameter $\varkappa$, the theory of coherent pair production is substantially modified. In this case, in the expression for W_e (in eq. (1)) it is necessary to keep, in the exponent, the terms with $q_\parallel + q'_\parallel = 0$ in the first term of the double sum, which contribute the terms

$\sim \left(V_0/m\vartheta_0\right)^2$. The remaining terms are of the order $\frac{1}{\varkappa}\left(\frac{V_0}{m\vartheta_0}\right)^3$ and $\frac{1}{\varkappa^2}\left(V_0/m\vartheta_0\right)^4$. Considering them to be small and expanding W_e, we find

$$W_e = \frac{mc\alpha}{\omega}\sum_q \frac{|G(\vec{q})|^2}{q_{\parallel}^2}\vec{q}_{\perp}^{\,2}\,\vartheta\left(q_{\parallel}-\frac{2m_*^2}{\omega}\right)\left[\left(1+\frac{x-\frac{x^2}{2}}{1+\varrho/2}\right)\ln\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} - \left(1+\frac{x}{1+\varrho/2}\right)\sqrt{1-x}\right], \qquad (3)$$

where

$$m_*^2 = m^2\left(1+\frac{\varrho}{2}\right),\ x = \frac{2m_*^2}{\omega q_{\parallel}},\ \frac{\varrho}{2} = \frac{1}{m^2}\sum_{q_{\parallel}\neq 0}|G(\vec{q})|^2\frac{\vec{q}_{\perp}^{\,2}}{q_{\parallel}^2}.$$

The quantity $\varrho \sim \left(\frac{V_0}{m\vartheta_0}\right)^2$ is the parameter of non-dipoleness for above-barrier trajectories. When $\varrho \to 0$ $\left(\vartheta_0 \gg \frac{V_0}{m}\right)$, formula (3) transforms into the probability of the standard theory of coherent pair creation. The latter is valid for any values of the parameter $\varkappa$ if $\vartheta_0 \gg \frac{V_0}{m}$. At $\varkappa > 1$ the region of applicability of W_e^{mc} (3) turns out, however, to be noticeably wider: it is sufficient only that $\left(\frac{V_0}{m\vartheta_0}\right)^3 \ll \varkappa$.

We proceed now to the discussion of the behaviour of the tilt-angle dependence when varying $\omega(\varkappa)$. At $\varkappa \ll 1$ the constant fields effects are exponentially small and the tilt-angle dependence is completely given by formula (3), the ϑ-function in eq. (3) provides the satisfaction of the condition $\vartheta_0 \gg \frac{V_0}{m}$, i.e. the standard theory of coherent pair creation is applicable (this situation is reflected by curve 1 in Fig.). With increasing $\varkappa$, when $\varkappa \gtrsim 1$, the external field pair creation mechanism "switches on", the minimum at $\vartheta_0 = 0$ (see curve 2 in Fig.) goes gradually to maximum (see curve 3 in Fig.) and up to energies $\omega \sim 25\frac{a_s m^3}{V_0}$ the probability $W_e(\vartheta_0=0)$ grows. The maximum value of W_e ($\vartheta_0 = 0$) can be estimated as follows: $W_{e_{max}} \sim \frac{\alpha n d a_s V_0}{m} \sim \frac{Z\alpha n a_s^2}{m}$, where d is the average

distance between the atoms in the string and n is the density of atoms in the crystal (Ref. [5]). For large $\varkappa$, the angular width of the peak is of the order $\frac{V_0}{m\varkappa^{1/3}}$, in practice, the regions of applicability of formulae (2) and (3) are overlapped, and this enables the entire orientation dependence to be described using simple expressions (2) and (3).

In the recent experiment (Ref. [7]), the enhancement of pair creation by photons with the energy ω = 50-110 GeV in a single Ge crystal was observed, but the observed probability is approximately 3 times lower in comparison with a theoretican one. To clarify the situation, further experiments need to be performed and of particular concern is a comprehensive study of the orientation dependence.

References

1. M.L. Ter-Mikaelian. High-Energy Electromagnetic Processes in Condensed Media (Wiley-Interscience, New York,1972).
2. V.N.Baier, V.M.Katkov and V.M.Strakhovenko. Phys. Lett. 104A, (1984) 231.
3. J.C.Kimball, N.Cue, L.M.Roth and B.B.Marsh. Phys. Rev. Lett. 50 (1983) 950.
4. V.G.Baryshevsky and V.V.Tikhomirov. Zh. Eksp. Theor. Fiz. 85 (1983) 232.
5. V.N.Baier, V.M.Katkov and V.M.Strakhovenko. Particle Pair Production by a Photon Moving along the Axes or Planes in a Single Crystal. Preprint IYaF 84-104 Novosibirsk (1984).
6. V.N.Baier, V.M.Katkov and V.S.Fadin. Radiation from Relativistic Electrons (Atomizdat, Moscow, 1973).
7. A.Belcacem et al. Observation of enhanced pair creation of 50-110 GeV in an aligned Ge crystal. Preprint CERN-EP/ /84-107 Rev.

Caption of the Figure:

Orientation dependence (under experimental conditions of the type described in Ref. [7]) of the pair creation probability for photons incident nearly along the $\langle 110 \rangle$ axis of a single Ge crystal cooled down to T = 100 K. Curve (1) is attached to ω = 30 GeV, curve (2) to ω = 100 GeV, and curve (3) to ω = = 1000 GeV. The dashed sections of the curves are obtained by interpolation.

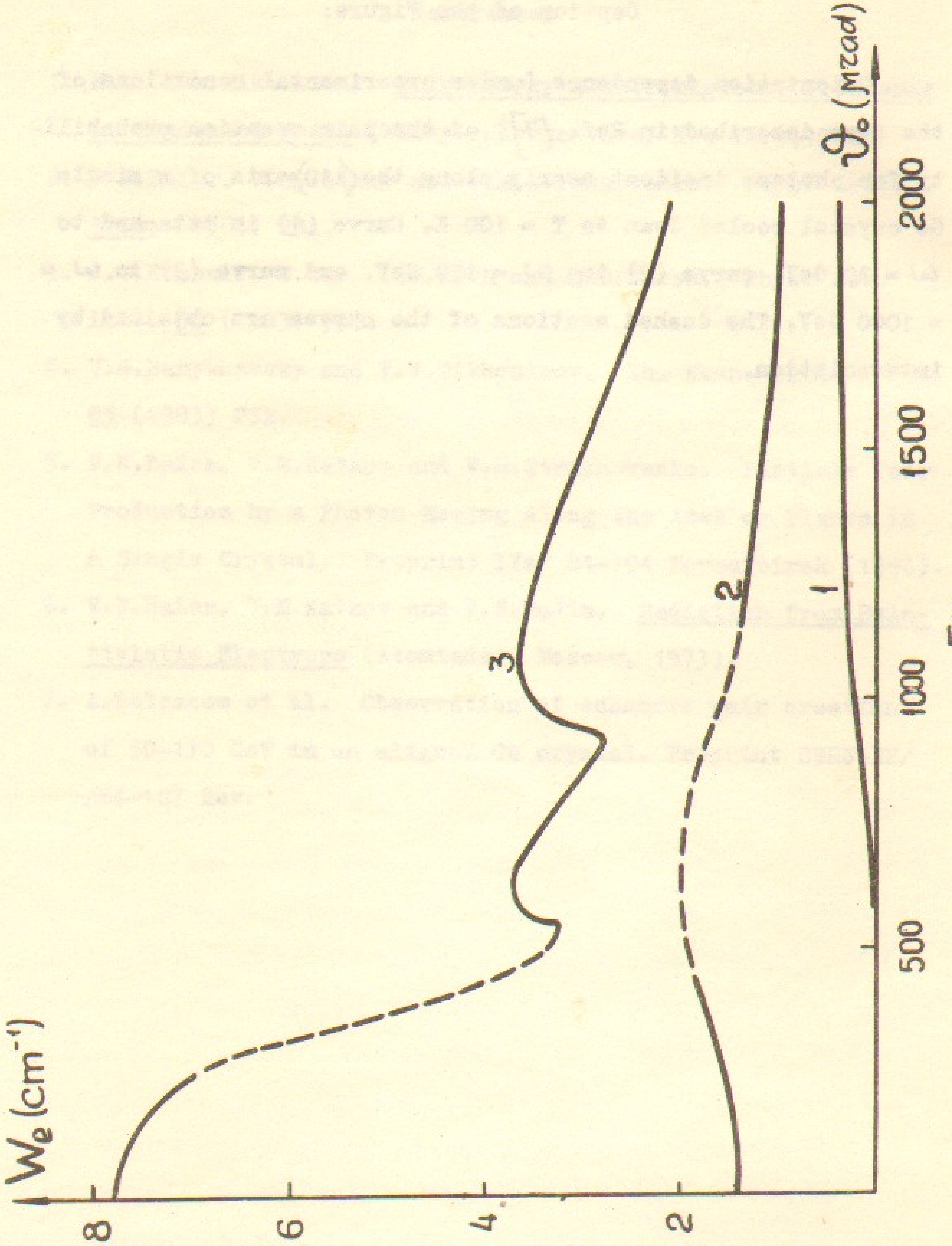

Fig.

В.Н.Байер, В.М.Катков, В.М.Страховенко

К ТЕОРИИ РОЖДЕНИЯ ПАР В ОРИЕНТИРОВАННЫХ МОНОКРИСТАЛЛАХ

Препринт
№ 85-43

Работа поступила - 18 марта 1985 г.

Ответственный за выпуск - С.Г.Попов
Подписано к печати 28.03-1985 г. МН 06627
Формат бумаги 60х90 1/16 Усл.0,7 печ.л., 0,6 учетно-изд.л.
Тираж 290 экз. Бесплатно. Заказ № 43.

Ротапринт ИЯФ СО АН СССР, г.Новосибирск, 90