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НОВОСИБИРСК

COMPLETE DESCRIPTION OF DOUBLE BREMSSTRAHLUNG IN THE REGION OF SMALL ANGLES FOR THE POLARIZED PARTICLES

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ABSTRACT

Amplitude of the $e^{\pm}e^{-} \to e^{\pm}e^{-}\gamma\gamma$ process is found in the main region of scattering angles $m/E \lesssim \theta_i \ll 1$. The result has a simple form convenient for the calculations of various cross sections. Energy $d\sigma/d\omega_1 d\omega_2$ and energy-angular $d\sigma/d^3k_1 d\omega_2$, $d\sigma/d^3k_1 d^3k_2$ distributions of the photons are found for the case when initial electrons are polarized and the photon polarizations are measured. It is shown that the measurement of the mean photon helicities allows one to determine polarizations of the initial electrons.

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1. INTRODUCTION

1. The double bremsstrahlung (DB) process is of great interest for experiments on the $e^{\pm}e^{-}$ colliding beams since it is used as the standard calibrating process for the luminosity measurement on a number of the $e^{+}e^{-}$ colliders at Novosibirsk, Frascati and Orsay. It was suggested and the method of it's theoretical description was developed in ref. [1] where the differential cross section and photon spectra was found for the case of the unpolarized particles. As a rule, it is registered by coincidence of both photons emitted in opposite directions at small emission angles $\theta_{1,2} \sim m/E$ where m is electron mass and E is its energy in the c.m.s. of the beams. The scattering angles of electrons are also small $\theta_{3,4} \sim m/E$. In the present paper we consider the $e^{\pm}e^{-} \to e^{\pm}e^{-}\gamma\gamma$ process when the emission and scattering angles may be larger or of order of m/E but much less than unit

$$m/E \lesssim \theta_{1-4} \ll 1. \tag{1}$$

Besides the useful application mentioned above DB is a background for a number of experiments. All that determines the interest to DB both from the experimentalists and theoreticians.

The first experiments on DB were performed at $e^{-}e^{-}$ and $e^{+}e^{-}$ beams in refs. [2,3]. Earlier DB was used in such a set-up when one measured the photon energies ω_1 and ω_2 only. Moreover, the angular dimensions of counters were considerably larger than the characteristic emission angles $\sim m/E$. Such a set-up corresponds to the cross section $d\sigma/d\omega_1 d\omega_2$ integrated over scattering angles of photons and electrons. Recently, in experiment [4] one studied not only energy but angular distribution of photons as well, to be exact, in ref. [4] the cross section $d\sigma/d^3k_1 d\omega_2$ in the region $\theta_1 \lesssim 17 m/E$ was measured (here k_i is momentum of the i -th photon).

2. Calculations of various cross sections for the unpolarized particles were performed in a number of papers [1,5-7] (see reviews [8,9]). The experimental needs demand the calculations of the radiative corrections to DB in ref. [7,10].

On the other hand, in literature there are practically no calculations of the polarization effects (cf. ref. [8]). But such calculations will be very useful when discussing the possibility to measure initial electron polarization by means of DB. Besides, the calculation methods used as a rule are rather cumbersome even for unpolarized particles, therefore, it is quite desirable to develop a simple and convenient method of calculation of DB in region (1). Both these problems have been solved in the present paper.

We obtain analytical expressions for all 64 helicity amplitudes of DB. They have such a simple form that the whole result occupies only a few lines - see formulae (9, 13, 4). This allows one to calculate without difficulties the cross sections $d\sigma/d\omega_1 d\omega_2$ and $d\sigma/d^3k_1 d\omega_2$ for the most interesting case when initial electrons are polarized and the photon polarizations are measured.

Finally, we calculate the inclusive for both photons cross section $d\sigma/d^3k_1 d^3k_2$ for the case when at least one of them is emitted at the angle considerably larger than m/E. This cross section is of practical interest for the accelerators with high luminosity. The point is that in the case of a collider with high luminosity the use of DB presents a problem because of the background of random coincidences due to independent emission of two photons in two single bremsstrahlung (SB) processes. However, the cross section of SB decreases with the growth of emission angle of photon sharper than the cross section of DB (see refs. [5,9]). Therefore, registration of both photons in the region $m/E \ll \theta_{1,2} \ll 1$ can help in suppressing the SB background.

Our method of calculation is close to that used in refs. [6,11], but we made two new steps: a) we present amplitude in such a form in which dependence on E and dependence on transverse momenta and energy fractions ω_i/E are evidently separated; b) additional simplification is achieved taking into account a hidden symmetry of our problem. This symmetry is the reflection of the symmetry for the cross-reaction $\gamma\gamma \to e^+e^-e^+e^-$ under the replacement $e^+ \leftrightarrow e^-$.

3. The basic notations are given in fig. 1. We use the

centre of mass frame of the colliding beams

$$p_1 = (E,0,0,p_1); \; p_2 = (E,0,0,-p_1); \quad s = 4E^2 \gg m^2.$$

Instead of the photon energies ω_i, photon emission angles θ_i, φ_i (i = 1,2) and the scattering angles of electrons it is convenient to use the dimensionless quantities

$$x_i = \frac{\omega_i}{E}, \; \vec{n}_i = \frac{\vec{k}_{i\perp}}{m x_i}, \; \vec{n} = \frac{\vec{k}_{1\perp} + \vec{p}_{3\perp}}{m} = -\frac{\vec{k}_{2\perp} + \vec{p}_{4\perp}}{m}. \qquad (2)$$

Here x_i is the fraction of the initial electron energy which is carried out by the i-th emitted photon (we assume that the quantities x_i are finite at $E \to \infty$). The $\vec{n}_i$ vector modulus is equal to the emission angle of the i-th photon in the units m/E:

$$n_i \equiv |\vec{n}_i| = \theta_i E/m; \quad i = 1,2.$$

Vector $\vec{n}$ is connected with the total transverse momentum of electron and photon in the upper or lower block in fig. 1a; for soft photons its modulus equals $n = \theta_3 E/m$, where θ_3 is the scattering angle of the electron. We show below that the energy of the virtual photon is very small therefore the energies of the scattered electrons E_3 and E_4 are equal

$$E_3 = (1-x_1)E, \quad E_4 = (1-x_2)E. \qquad (3)$$

The DB amplitude depends on vectors $\vec{n}_i$ in the form of the following combinations only

$$\vec{Q} = \frac{\vec{n}_1}{1+\vec{n}_1^2} + \frac{\vec{n}-\vec{n}_1}{1+(\vec{n}-\vec{n}_1)^2}, \quad R = \frac{1}{1+\vec{n}_1^2} - \frac{1}{1+(\vec{n}-\vec{n}_1)^2},$$
$$\tilde{\vec{Q}} = \vec{Q}(\vec{n}_1 \to \vec{n}_2, \; \vec{n} \to -\vec{n}), \; \tilde{R} = R(\vec{n}_1 \to \vec{n}_2, \; \vec{n} \to -\vec{n}). \qquad (4)$$

Among them the quantity $\vec{Q}$ is symmetric and R is antisymmetric under the replacement*)

$$\vec{n}_1 \leftrightarrow \vec{n} - \vec{n}_1 \tag{5}$$

2. AMPLITUDE FOR DOUBLE BREMSSTRAHLUNG

The DB process in region (1) can be considered as two-jets one (see, for example, ref. [13]). The main contribution to its cross section is given by the block diagram of fig. 1a with photon exchange in the t -channel. Let e_i be the polarization vector of the i -th photon and $u_j = u_{\lambda_j}(p_j)$ be spinor corresponding to electron with momentum p_j and helicity λ_j. Omitting the terms of relative order of

$$m^2/s\,, \quad \theta^2_{1-4} \tag{8}$$

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*) In our paper [11] it was shown that the amplitude of the cross-reaction $\gamma\gamma \to e^+e^-e^+e^-$ (see fig. 2) depends on the transverse momenta of the first e^+e^- pair in the form of the following combinations only

$$\vec{Q}_{\gamma\gamma} = \frac{\vec{p}_{1\perp}}{m^2+\vec{p}_{1\perp}^{\,2}} + \frac{\vec{p}_{3\perp}}{m^2+\vec{p}_{3\perp}^{\,2}}\,, \quad R_{\gamma\gamma} = \frac{m}{m^2+\vec{p}_{1\perp}^{\,2}} - \frac{m}{m^2+\vec{p}_{3\perp}^{\,2}}\,. \tag{6}$$

Among them the first one is symmetric and the second one is antisymmetric under the replacement $\vec{p}_{1\perp} \leftrightarrow \vec{p}_{3\perp}$. Using the substitution rules (see ref. [12])

$$\vec{p}_{1\perp} \to \frac{\vec{k}_{1\perp}}{x_1}\,, \quad \vec{p}_{3\perp} \to \vec{p}_{3\perp} - \frac{1-x_1}{x_1}\vec{k}_{1\perp} = \vec{q}_\perp - \frac{\vec{k}_{1\perp}}{x_1} \tag{7}$$

we obtain that the DB amplitude depends on quantities (4) which have a certain symmetry under replacement (5).

one can represent the DB amplitude M_{fi} in a simple factorized form (see appendix A)

$$M_{fi} = -\frac{s}{\vec{q}_\perp^{\,2}}\, J_1 J_2 \tag{9}$$

where the vertex factors J_1 and J_2 correspond to the upper and lower blocks in fig. 1a (in particular, J_1 corresponds to the process of virtual photon Compton scattering, fig. 1b). These factors depend on variables (2) only, but not on s :

$$J_1 = \sqrt{2}\,\frac{4\pi\alpha}{s m x_1}\,\bar{u}_3 \hat{p}_2 \left[2\vec{Q}\vec{e}_1^{\,*} + \vec{\gamma}_\perp \vec{e}_1^{\,*}\,(\vec{Q}\vec{\gamma}_\perp - R)\right] u_1 \tag{10}$$

where $\vec{\gamma}_\perp = (\gamma_x, \gamma_y)$ are the Dirac matrices. The expression for J_2 can be obtained from (10) by the substitutions $\vec{Q} \to \vec{\tilde{Q}}$, $R \to \tilde{R}$, $1 \leftrightarrow 2$, $3 \to 4$.

To prove J_i independence of s, one should replace spinors u_j by two-component spinors φ_{λ_j} which obey the equations

$$\vec{\sigma}\vec{\nu}_j\,\varphi_{\lambda_j} = 2\lambda_j\,\varphi_{\lambda_j}\,, \quad \vec{\nu}_j = \frac{\vec{p}_j}{|\vec{p}_j|} \tag{11}$$

where $\vec{\sigma}$ are the Pauli matrices. Indeed,

$$u_1 = \sqrt{E}\begin{pmatrix} 1 \\ \vec{\sigma}\vec{\nu}_1 \end{pmatrix}\varphi_{\lambda_1}\,, \quad \bar{u}_3 \hat{p}_2 = 2E^{3/2}\sqrt{1-x_1}\,\varphi^+_{\lambda_3}\,(1, \vec{\sigma}\vec{\nu}_1)$$

and therefore

$$J_1 = \sqrt{2}\,\frac{4\pi\alpha}{m x_1}\sqrt{1-x_1}\,\varphi^+_{\lambda_3}\left\{(2-x_1)\vec{Q}\vec{e}_1^{\,*} + i x_1 \vec{\sigma}\left[\vec{Q} + R\vec{\nu}_1, \vec{e}_1^{\,*}\right]\right\}\varphi_{\lambda_1}\,. \tag{12}$$

Formulae (10) and (12) are given for an electron, for a positron J_1 can be obtained from eq. (12) by the substitutions $\vec{Q}\vec{e}_1^{\,*} \to -\vec{Q}\vec{e}_1^{\,*}$, $\varphi_{\lambda_1} \to -2\lambda_3 i \varphi_{-\lambda_3}$, $\varphi^+_{\lambda_3} \to 2\lambda_1 i \varphi^+_{-\lambda_1}$. As a result, we get helicity amplitude

$$\mathcal{J}_1(e^{\pm}_{\lambda_1} \to e^{\pm}_{\lambda_3} \gamma_{\lambda_\gamma}) = 4\pi\alpha \frac{\sqrt{1-x_1}}{m x_1} \Big[\pm(2 - x_1 + 2\lambda_1 \lambda_\gamma x_1)(\lambda_\gamma Q_x - i Q_y)\delta_{\lambda_1 \lambda_3} \mp \tag{13}$$

$$\mp 2 x_1 R\, \delta_{\lambda_1, -\lambda_3}\, \delta_{\lambda_\gamma, 2\lambda_1} \Big]$$

(the quantities $\vec{Q}$ and R are defined in (4)).

Let us discuss briefly the results obtained above (9-13).

a) When the photon energies are small, $x_i \ll 1$, our result

$$\mathcal{J}_1 = \mp 8\sqrt{2}\,\pi\alpha \frac{\vec{Q}\vec{e}_1^{\,*}}{m x_1} \delta_{\lambda_1 \lambda_3} \tag{14}$$

coincides with that obtained in the approximation of the classical currents. Indeed, in this approximation the amplitude of the ee $\to$ ee$\gamma\gamma$ process in the region of small angles has the form

$$M_{fi} = 4\pi\alpha\, A_1 A_2 M_0\,; \quad M_0 = 4\pi\alpha\, \bar{u}_3 \gamma^\mu u_1 \cdot \bar{u}_4 \gamma_\mu u_2 / q^2\,;$$

$$A_i = \left(\frac{p_i}{k_i p_i} - \frac{p_{i+2}}{k_i p_{i+2}} \right) e_i^* \tag{15}$$

where M_0 is the amplitude of the elastic process ee $\to$ ee and A_i is the factor of the accompanying classical emission. The calculations similar to those done in appendix A lead to (at $x_i \ll 1$ and with the accuracy (8))

$$M_0 = -8\pi\alpha \frac{s}{\vec{q}_\perp^{\,2}} \delta_{\lambda_1\lambda_3} \delta_{\lambda_2\lambda_4}\,; \; A_1 = \frac{2}{m x_1} \vec{Q}\vec{e}_1^{\,*}, \; A_2 = \frac{2}{m x_2} \tilde{\vec{Q}} \vec{e}_2^{\,*}. \tag{16}$$

This result is identical to (9), (14). Note that at small energies of photons $M_{fi} \propto 1/\omega_1\omega_2$ and that the electron helicities are not altered.

b) Useful relations take place for the quantities defined by eq. (4)

$$\vec{Q}^2 + R^2 = \frac{\vec{n}^2}{(1+\vec{n}_1^2)[1+(\vec{n}-\vec{n}_1)^2]}\,, \quad \tilde{\vec{Q}}^2 + \tilde{R}^2 = \frac{\vec{n}^2}{(1+\vec{n}_2^2)[1+(\vec{n}+\vec{n}_2)^2]}\,. \tag{4a}$$

From here and from definition (4) one can see that at small n quantities (4) vanish

$$|\vec{Q}|, R \propto n \quad \text{at} \quad n \ll n_1\,; \quad |\tilde{\vec{Q}}|, \tilde{R} \propto n \quad \text{at} \quad n \ll n_2\,. \tag{4b}$$

Therefore, at small $q_\perp$ we have

$$\mathcal{J}_i \propto q_\perp \tag{4c}$$

As a result, the amplitude M_{fi} has the finite limit at $q_\perp \to 0$ and the region of small values $q_\perp \ll m$ gives small contribution to the cross section.

c) The photon helicity in the amplitudes with the change of the electron helicity is strictly connected with the helicity of the initial electron $\lambda_\gamma = 2\lambda_1$:

$$\mathcal{J}_1(e^{\pm}_{\lambda} \to e^{\pm}_{-\lambda} \gamma_{2\lambda}) = \mp 8\pi\alpha \frac{\sqrt{1-x_1}}{m} R\,; \quad \mathcal{J}_1(e_\lambda \to e_{-\lambda} \gamma_{-2\lambda}) = 0\,. \tag{17}$$

d) Relative magnitude of the spin-flip amplitude is given by the relation

$$\frac{|\mathcal{J}_1(\lambda_3 = -\lambda_1)|}{|\mathcal{J}_1(\lambda_3 = +\lambda_1)|} \sim \frac{x_1 |R|}{|\vec{Q}|} \tag{18}$$

which vanishes in the soft photon limit (at $x_1 \to 0$) as well as in the limit of large angles (at $m/k_{1\perp}, m/q_\perp \to 0$).

3. DIFFERENTIAL CROSS SECTION

Results (9-13) are convenient both for analytical and numerical calculations of various cross sections. In particular, the cross section for unpolarized particles can be obtained by summing up the squares of helicity amplitudes (13). Therefore, here there is no need to calculate the traces which earlier was the most cumbersome procedure.

Consider now the case of polarized particles. The polarization state of the j -th electron is determined by the polarization vector $\vec{\xi}_j$, its longitudinal component

$$\vec{\xi}_j \vec{\nu}_j = 2\langle\lambda_j\rangle \, , \quad \vec{\nu}_j = \frac{\vec{p}_j}{|\vec{p}_j|} \tag{19}$$

where $\langle\lambda_j\rangle$ is the mean electron helicity. Introduce the usual density matrices of electrons

$$\varphi_{\lambda_j} \varphi^+_{\lambda_j} \to \frac{1}{2}(1+\vec{\xi}_j \vec{\sigma})$$

(for the positrons the vector $\vec{\xi}_j$ should be replaced by $(-\vec{\xi}_j)$).

The polarization states of the first and the second photons are described by the Stokes parameters $\xi_{1,2,3}$ and $\tilde{\xi}_{1,2,3}$ corresponding, among them ξ_2 and $\tilde{\xi}_2$ is the mean helicity of the first and second photons and the degrees of their linear polarizations ℓ_1 and ℓ_2 are equal to

$$\ell_1 = \sqrt{\xi_1^2+\xi_3^2} \, , \quad \ell_2 = \sqrt{\tilde{\xi}_1^2+\tilde{\xi}_3^2} \tag{20}$$

The density matrices of photons are

$$e_{1\alpha} e^*_{1\beta} \to \frac{1}{2}(1+\vec{\xi}\vec{\sigma});$$

$$e_{2\alpha} e^*_{2\beta} \to \frac{1}{2}(1-\tilde{\xi}_1\sigma_1-\tilde{\xi}_2\sigma_2+\tilde{\xi}_3\sigma_3); \qquad \alpha,\beta = x,y.$$

When writing down the density matrix of the second photon we take into account that the natural reference frame for the second photon coincides with our one after reflecting y and z axes.

This cross section (and all ones obtained below) depend on the initial electron polarizations via the combinations $\xi_2 \vec{\zeta}_1$ and $\tilde{\xi}_2 \vec{\zeta}_2$ only. Therefore, to find the polarizations of the initial electrons, it is necessary to measure the circular polarization (helicity) of the photons. The dependence on the most interesting longitudinal polarization of the beams is connected with the quantities Λ_i while their transverse polarizations are connected with the interference of spin flip and non-flip amplitudes. For example, the corresponding term in U_1 equals

$$-x_1(1-x_1)\,\xi_2\,\vec{\zeta}_1'\vec{Q}\,R. \tag{23a}$$

This term changes its sign under replacement (5), it leads to azimuthal asymmetry of the photon and it vanishes after averaging over azimuthal angle of the photon.

To get the cross section for the unpolarized particles, it is enough to put $\xi_{1,2,3} = \tilde{\xi}_{1,2,3} = 0$ and to multiply cross section (22) by the factor 2·2 = 4 which corresponds to summing over photon polarizations. Thus obtained cross section coincides (after some transformations) with the result of ref. [6].

4. ENERGY DISTRIBUTIONS OF THE PHOTONS

To obtain this distribution it is convenient to integrate U_i over $\vec{n}_i$ and then to perform the integration over $\vec{n}$. Due to rapid convergence of the integrals over $\vec{n}_i$ and $\vec{n}$ the upper limits of integration can be extended up to infinity. Using formulae (A.11) from appendix A one obtains

$$\int U_i \frac{d^2\vec{n}_i}{2\pi} = F_i\,\Phi\left(\frac{n}{2}\right) + \left[G_i + 2(1-x_i)\ell_i \cos 2\gamma_i\right] \varphi\left(\frac{n}{2}\right),$$

$$\Phi(z) = -1 + \frac{2z^2+1}{z\sqrt{z^2+1}}\,\mathrm{Arsh}\,z, \quad \varphi(z) = \frac{1}{2} - \frac{\mathrm{Arsh}\,z}{2z\sqrt{z^2+1}}. \tag{24}$$

Here ℓ_i is the degree of the linear polarization of the i-th photon (20) and γ_i is the azimuthal angle between the vector $\vec{n}$ and the direction of the linear polarization of the i-th photon. The function $\Phi(z)$ is well-known in the soft photon approximation; the function $\varphi(z)$ is everywhere smaller than $\Phi(z)$. The ratio $\varphi(z)/\Phi(z)$ is equal to 0.25 at z = 0 and to 0.1 at z = 1; when $z \gg 1$ it becomes equal to $1/(4\ln z)$.

Having substituted eq. (24) into eq. (22) we perform the integration over $\vec{n}$. After integration over angles of the vector $\vec{n}$ the integrals appear

$$[A, B, C] = \int_0^\infty \frac{dz}{z^3}\left[\Phi^2(z),\ \Phi(z)\varphi(z),\ \varphi^2(z)\right].$$

As a result, we get

$$d\sigma = \frac{2\alpha^4}{\pi m^2}\frac{dx_1}{x_1}\frac{dx_2}{x_2}\left[F_1F_2A + (G_1F_2 + F_1G_2)B + G_1G_2C + 2(1-x_1)(1-x_2)C\,\ell_1\ell_2\cos 2\gamma_{12}\right],$$

$$A = \frac{7}{8}\zeta(3) + \frac{5}{4} = 2.30; \quad B = \frac{7}{16}\zeta(3) - \frac{1}{8} = 0.401; \tag{25}$$

$$C = \frac{7}{32}\zeta(3) - \frac{3}{16} = 0.0754; \quad \zeta(3) = 1.202.$$

Here γ_{12} is the angle between the direction of the linear polarizations of the first and second photons, the quantities F_i, G_i are defined in (21). For unpolarized particles eq. (25) coincides with the result of ref. [1].

Formula (25) can be written to within a sufficiently high accuracy (< 1%) in an "almost multiplicative" form using approximate numerical equality $AC \approx B^2$

$$d\sigma = \frac{2\alpha^4}{\pi m^2}\frac{dx_1}{x_1}\frac{dx_2}{x_2}\left[P_1 P_2 A + 2(1-x_1)(1-x_2)\, C\, \ell_1 \ell_2 \cos 2\gamma_{12}\right], \tag{26}$$

$$P_1 = F_1 + G_1 B/A = 1 - x_1 + 0.67x_1^2 + x_1(1-0.33x_1)\,\xi_2 \vec{\zeta}_1 \vec{\nu}_1 ,$$

$$P_2 = F_2 + G_2 B/A = 1 - x_2 + 0.67x_2^2 + x_2(1-0.33x_2)\,\tilde{\xi}_2 \vec{\zeta}_2 \vec{\nu}_2 .$$

It is seen that in the energy distribution of the photons there are considerable spin effects. Even at $x_i \ll 1$ cross section (25) can change for $4C/A$ = 13% when the angle γ_{12} changes.

5. CROSS SECTION $d\sigma / d^3k_1\, d\omega_2$

To avoid unnecessary complexity, we put here $\xi_{1,3} = \tilde{\xi}_{1,3} = 0$, i.e. we only consider in this section the case when the circular photon polarizations ξ_2 and $\tilde{\xi}_2$ are measured (it should be reminded that the polarizations of the initial electrons are connected with these very Stokes parameters). Cross section can be obtained by integrating eq. (22) over $\vec{n}_2$ and $\vec{n}$. The result of the integration over $\vec{n}_2$ is given in (24). Having integrated over $\vec{n}$, we obtain

$$d\sigma = \frac{2\alpha^4}{\pi m^2}\frac{dx_1}{x_1}dn_1\frac{d\varphi_1}{2\pi}\frac{dx_2}{x_2}\Big[F_1F_2\, a + G_1F_2\, b + F_1G_2\, c + G_1G_2\, d - x_1(1-x_1)\,\xi_2\,\frac{\vec{\zeta}_1\vec{n}_1}{n_1}\left(F_1 f + G_1 g\right)\Big]. \tag{27}$$

Here F_i, G_i are defined in (21) and the quantities a÷g are represented by the following integrals

$$[a,b,c,d,f,g] = \frac{1}{\pi}\int\frac{d^2\vec{n}}{n^4}\Big[2n_1\vec{Q}^2\Phi(\tfrac{n}{2}),\ n_1R^2\Phi(\tfrac{n}{2}),\ 2n_1\vec{Q}^2\varphi(\tfrac{n}{2}),\ n_1R^2\varphi(\tfrac{n}{2}),\ 2\vec{n}_1\vec{Q}R\,\Phi(\tfrac{n}{2}),\ 2\vec{n}_1\vec{Q}R\,\varphi(\tfrac{n}{2})\Big]. \tag{28}$$

All these quantities are functions of $n_1 = E\theta_1/m$; the result of the numerical integration over formulae (28) is given in table. All the quantities a÷g vanish at $n_1 \to 0$ and $n_1 \to \infty$, all of them have their maxima at $n_1 \sim 1$.

At $n_1^2 \gg 1$ the following asymptotic expressions hold (we use integrals (B.5) from our paper [11])

$$a = \frac{3}{n_1^3}(\ell^2 - 2\ell + 2),\quad b = \frac{\ell - 1}{n_1^3},\quad c = \frac{2\ell - 3}{n_1^3},\quad d = \frac{1}{2n_1^3},$$
$$f = \frac{3}{n_1^4}(\ell - 2)^2,\quad g = \frac{2\ell - 5}{n_1^4},\quad \ell = \ln(n_1^2),\quad n_1^2 \gg 1. \tag{29}$$

These expressions agree with the table data at $n_1 \geqslant 4$ for the coefficient a with the accuracy better than 1.5% and with a little worse accuracy for other coefficients. Thus, for the coefficient f the accuracy is better than 3.5% at $n_1 \geqslant 7.5$. The coefficient a has the maximum $(a)_{max}$ = 0,543 at n_1 = 1.2. In the region $n_1^2 \gg 1$ this coefficient is larger than the others due to the fact that ratios $\varphi(z)/\Phi(z)$ and $R^2/\vec{Q}^2$ are small in this region. Even for n_1 = 1.5 the coefficient a is at least 5 times as large as any of the b÷g coefficients. Note that the leading logarithms in a arise from two different regions: $1 \ll (\vec{n}-\vec{n}_1)^2 \ll n_1^2$ and $1 \ll \vec{n}^2 \ll n_1^2$, the first region giving two times as large a contribution as the second region. For the soft photons the first region corresponds to the case when the electron with the momentum p_3 moves close to the direction of the first photon, $\theta_3 \approx \theta_1$, while in the second region $\theta_3 \ll \theta_1$.

Table

$n_1=\theta_1\frac{E}{m}$	$a\cdot \mathrm{I0}^2$	$b\cdot \mathrm{I0}^2$	$c\cdot \mathrm{I0}^2$	$d\cdot \mathrm{I0}^2$	$f\cdot \mathrm{I0}^2$	$g\cdot \mathrm{I0}^2$
0.I5	8.24	8.0I	I.94	I.56	2.23	0.4I5
0.3	I7.8	I3.8	4.08	2.67	7.50	I.38
0.5	3I.8	I6.8	7.04	3.26	I4.3	2.58
0.7	43.8	I6.2	9.42	3.I5	I7.2	3.00
I	53.2	I3.I	II.0	2.60	I5.3	2.50
I.2	54.3	II.2	II.0	2.27	I2.5	I.96
I.5	5I.6	9.I3	I0.2	I.89	8.68	I.29
2	43.3	6.89	8.27	I.45	4.77	0.708
2.5	35.2	5.36	6.54	I.II	2.99	0.483
3	28.7	4.22	5.20	0.846	2.I2	0.382
4	I9.7	2.67	3.4I	0.497	I.32	0.269
5	I4.2	I.75	2.34	0.302	0.906	0.I88
7.5	7.I5	0.730	I.07	0.I07	0.404	0.0784
I0	4.I5	0.368	0.574	0.0478	0.204	0.036
20	0.967	0.0633	0.I09	0.0062	0.03	0.0042

Finally, one points out the relation between the functions a+b and the coefficients A,B,C in eq.(25)

$$A=\int_0^\infty a\,dn_1\,,\quad B=\int_0^\infty b\,dn_1=\int_0^\infty c\,dn_1\,,\quad C=\int_0^\infty d\,dn_1\,.$$

The first four items in cross section (27) only depend on the longitudinal polarizations of the electrons. A azimuthal asymmetry of this cross section is determined by the last term which is proportional to (cf. eq. (23a))

$$x_1(1-x_1)\,\xi_2\,\frac{\vec{\xi}_1\vec{n}_1}{n_1}\,f = x_1(1-x_1)\,\xi_2\,|\vec{\xi}_{1\perp}|\,f\cos\varphi_1 \tag{23b}$$

where φ_1 is the azimuthal angle between $\vec{K}_{1\perp}$ and the direction of the transverse polarization of the first electron $\vec{\xi}_{1\perp}$. Relative magnitude of this asymmetry is $\lesssim f/a$. The quantity f/a has its maximum $(f/a)_{max}=0.45$ at $n_1=0.5$ and it is not small in the region $n_1\lesssim 1$ only. Already at $n_1>2$ this quantity is small, $f/a<0.11$.

For unpolarized particles eq. (27) coincides with the result obtained in ref.[7].

6. DOUBLE INCLUSIVE CROSS SECTION $d\sigma/d^3k_1\,d^3k_2$

As it was mentioned in introduction, the case has a practical interest when both photons are emitted at the angles which are considerably larger than m/E , i.e. when $n_1^2\gg 1$ and $n_2^2\gg 1$. We consider here a more general case when

$$(\vec{n}_1+\vec{n}_2)^2=(\theta_1^2+\theta_2^2+2\theta_1\theta_2\cos\Delta\varphi)\,E^2/m^2 \gg 1 \tag{30}$$

where $\Delta\varphi$ is the difference between azimuthal angles of the first and the second photons. In other words, our calculation is valid when at least one of the photons is emitted at a large angle, say $m/E\ll\theta_1\ll 1$, but $\theta_2\sim m/E$. As for polarization effects, in this section we restrict ourselves only with the consideration of the most interesting case when the mean helicities of the photons and the initial electrons are not equal to zero (cf. section 5 where the effect of the transverse electron polarization has been found to be small).

To get the cross section discussed one needs to integrate cross section (22) over $\vec{n}$ at condition (30). The corresponding integrals are calculated in appendix B. As a result, we obtain

$$d\sigma = \frac{2\alpha^4}{\pi^3 m^2} \frac{1}{(\vec{n}_1+\vec{n}_2)^2} \frac{d^2\vec{n}_1}{1+\vec{n}_1^2} \frac{dx_1}{x_1} \frac{d^2\vec{n}_2}{1+\vec{n}_2^2} \frac{dx_2}{x_2} \left(F_1F_2\tilde{a} + G_1F_2\tilde{b} + F_1G_2\tilde{c} + G_1G_2\tilde{d}\right),$$

$$\tilde{a} = \ln(\vec{n}_1+\vec{n}_2)^2 - 1 + \frac{L+1}{1+\vec{n}_1^2} - \frac{L-1}{1+\vec{n}_2^2} + 4\tilde{d},$$

$$\tilde{b} = \frac{1}{4} - \frac{L+1}{2(1+\vec{n}_1^2)} - 2\tilde{d}, \qquad (31)$$

$$\tilde{c} = \frac{1}{4} + \frac{L-1}{2(1+\vec{n}_2^2)} - 2\tilde{d},$$

$$\tilde{d} = \frac{(\vec{n}_1+\vec{n}_2)^2}{8(1+\vec{n}_1^2)(1+\vec{n}_2^2)}, \qquad L = \ln\frac{1+\vec{n}_1^2}{1+\vec{n}_2^2}.$$

The quantities F_i , G_i were defined in eq. (21). Omitted terms in (31) have the relative value $\sim 1/(\vec{n}_1+\vec{n}_2)^2$.

For the unpolarized particles in the region $\vec{n}_i^2 \gg 1$ and $(\vec{n}_1+\vec{n}_2)^2 \gg 1$ the cross section has a simpler form

$$d\sigma = \frac{8\alpha^4}{\pi^3 m^2} \frac{1}{(\vec{n}_1+\vec{n}_2)^2} \frac{d^2\vec{n}_1}{\vec{n}_1^2} \frac{dx_1}{x_1} \frac{d^2\vec{n}_2}{\vec{n}_2^2} \frac{dx_2}{x_2} \Big\{\left[\ln(\vec{n}_1+\vec{n}_2)^2 - 1\right] F_1F_2 +$$

$$+ \frac{1}{4} x_1^2 F_2 + \frac{1}{4} x_2^2 F_1\Big\}, \quad F_i = 1 - x_i + \frac{1}{2}x_i^2, \qquad (32)$$

$$\vec{n}_i^2 = (\theta_i E/m)^2, \qquad d^2\vec{n}_i = (E/m)^2\, \theta_i\, d\theta_i\, d\varphi_i .$$

The main contribution to this cross section is given by the first term which arises from the amplitudes without changing electron helicities. When integrating over $\vec{n}$, two symmetric regions give the leading logarithm: $1 \ll (\vec{n}-\vec{n}_1)^2 \ll (\vec{n}_1+\vec{n}_2)^2$ and $1 \ll (\vec{n}+\vec{n}_2)^2 \ll (\vec{n}_1+\vec{n}_2)^2$. For the soft photons the first of them corresponds to the case when electron with the momentum p_3 moves close to the direction of the first photon, $\theta_3 \approx \theta_1$, while in the second region $\theta_4 \approx \theta_1$.

For the soft photons $(x_i \ll 1)$ the leading logarithm in our result (32) coincides with formula (3.51) from ref. [9].

7. DISCUSSION

We have found the DB amplitude in region (1). Our result (9-13) has a simple form which is convenient for the analytica as well as for the numerical calculation of various cross sections.

The most interesting feature of the energy distribution of photons (25-26) is that the longitudinal polarizations of the colliding electrons $\vec{\xi}_1\vec{\nu}_1$ and $\vec{\xi}_2\vec{\nu}_2$ are "transferred" to circular photon polarizations. It is seen from eq. (26) that the degree of circular polarization (i.e. the mean helicity) of the i-th photon is equal to

$$\vec{\xi}_i \vec{\nu}_i \; x_i \, \frac{1-0.33\,x_i}{1-x_i+0.67\,x_i^2} . \qquad (33)$$

This quantity becomes of order of 1 at $x_i = \omega_i/E \sim 1$ only and vanishes for the soft photons $x_i \ll 1$; in particular, at x_i = 1/4 it equals 29%. One can estimate from eq. (33) at what level the measurement of the circular photon polarization allows one to determine the longitudinal polarization of the initial electrons.

Photon spectra do not depend on the transverse polarization of the initial electrons $\vec{\zeta}_{i\perp}$. Dependence on $\vec{\zeta}_{1\perp}$ appears in the energy-angular distribution of the photons (see eq. (27)) and it determines the azimuthal asymmetry of this distribution. The magnitude of this asymmetry is of order of (cf. (23a))

$$\sim x_1(1-x_1)\,\xi_2\,|\vec{\zeta}_{1\perp}|\,\frac{f}{a} \qquad (34)$$

i.e. it is not small in the region $n_1 \lesssim 1$ as one can see from table and formulae (29).

Using the substitution rules of type (7) one can obtain some new results for cross-reaction $\gamma\gamma\to e^+e^-e^+e^-$, they are given in appendix C.

Our results can also be applied for the single bremsstrahlung whose amplitude can be written in the following form

$$M_{e^+e^-\to e^+e^-\gamma} = M_a + M_b,$$

$$M_a = \frac{s}{q^2}\,J_1 J_2, \qquad (35a)$$

$$M_b = M_a\,(p_1\leftrightarrow p_2,\ p_3\leftrightarrow p_4)$$

where J_1 is the same as in eqs. (10), (12), (13) and

$$J_2 = \frac{\sqrt{8\pi\alpha}}{s}\,\bar{u}_4\hat{p}_1 u_2 = \sqrt{8\pi\alpha}\,\delta_{\lambda_2\lambda_4}; \quad q^2 = -\frac{\vec{q}_\perp^{\,2}+m^2\alpha_q^2}{1-\alpha_q},$$

$$\alpha_q = \frac{m^2}{s}\,\frac{x_1}{1-x_1}\,[1+(\vec{n}-\vec{n}_1)^2]. \qquad (35b)$$

Finally, it should be noted that the similar method of the amplitude calculation is quite useful for some quantum chromodynamic processes (such as $\gamma\gamma\to\rho^0+X$, $\gamma\gamma\to\rho^0\rho^0$ and so on) in the region $s \gg \vec{q}_\perp^{\,2} \gg (0.3\ \text{GeV})^2$.

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APPENDIX A

It is convenient to introduce the "almost light-like" 4-vectors p and p':

$$p = p_1 - \frac{m^2}{s}p_2,\quad p' = p_2 - \frac{m^2}{s}p_1,\quad s = 2p_1p_2 = 2pp' + \frac{3m^4}{s},\quad p^2 = p'^2 = \frac{m^6}{s^2}$$

and to decompose 4-vectors k_i, p_j, q into the components in the plane of the 4-vectors p and p' and in the plane orthogonal to them

$$k_i = \alpha_i p' + \beta_i p + k_{i\perp};\quad i=1,2;\quad p_j = \alpha_j p' + \beta_j p + p_{j\perp},\quad j=3,4;$$

$$q = \alpha_q p' + \beta_q p + q_\perp.$$

Parameters α and β are the so-called Sudakov variables, in the c.m.f. of the initial particles the four-vectors $k_{i\perp}$, $p_{j\perp}$, $q_\perp$ have x and y components only, e.g.

$$q_\perp = (0, q_x, q_y, 0) = (0, \vec{q}_\perp, 0),\quad q_\perp^2 = -\vec{q}_\perp^{\,2}.$$

In a jet kinematics 4-vectors k_1 and p_3 have large components along p_1 (or p) while k_2 and p_4 - along p_2 (or p'), therefore at $s\to\infty$ the following quantities are finite (all the formulae given below are valid with accuracy (8))

$$\beta_1 = \frac{2k_1p'}{s} = \frac{\omega_1}{E} = x_1,\quad \beta_3 = \frac{E_3}{E},\quad \alpha_2 = \frac{2k_2p}{s} = \frac{\omega_2}{E} = x_2,\quad \alpha_4 = \frac{E_4}{E}. \qquad (A.1)$$

On the other hand, components of k_1 and p_3 along p_2 as well as those of k_2 and p_4 along p_1 are small. Indeed, using mass-shell conditions $k_1^2 = s\alpha_1\beta_1 + k_{1\perp}^2 = 0$, $k_2^2 = 0$, $p_3^2 = p_4^2 = m^2$ one easily obtains

$$\alpha_1 = \frac{\vec{k}_{1\perp}^{\,2}}{s x_1},\quad \alpha_3 = \frac{m^2+\vec{p}_{3\perp}^{\,2}}{s\beta_3},\quad \beta_2 = \frac{\vec{k}_{2\perp}^{\,2}}{s x_2},\quad \beta_4 = \frac{m^2+\vec{p}_{4\perp}^{\,2}}{s\alpha_4}. \qquad (A.2)$$

$$g^{\mu\nu} \to \frac{2s}{s_1 s_2}\, q_\perp^\mu q_\perp^\nu\,, \quad s_1 = s\alpha_q = 2qp\,,\ s_2 = -s\beta_q = -2qp'. \tag{A.7}$$

From eq. (A.7) one can see that the vertex factor in eq. (9) is equal to

$$J_1 = \frac{\sqrt{2}}{s_1}\, M_1^\mu q_{\perp\mu}\,. \tag{A.8}$$

In other words, it coincides (up to factor $\sqrt{-2q_\perp^2}\,/\,s_1$) with the amplitude of virtual photon Compton scattering (see fig. 1b) in which the virtual photon has the "mass" squared $q^2 \approx q_\perp^2$ and the polarization vector $e^\mu = q_\perp^\mu / \sqrt{-q_\perp^2}$.

It is convenient, however, to use (A.6)

$$J_1 = \frac{\sqrt{2}}{s} M_1^\mu p'_\mu\,, \qquad J_2 = \frac{\sqrt{2}}{s} M_2^\nu p_\nu \tag{A.9}$$

and to calculate J_i in the limit $s \to \infty$ (assuming quantities (2) to be finite in this limit). For the case when the upper black of fig. 1a describes the emission by electron we have

$$J_1 = \frac{\sqrt{2}}{s} 4\pi\alpha\, \bar{u}_3 \left[\frac{\hat{p}'(\hat{p}_1 - \hat{k}_1 + m)\,\hat{e}_1^*}{2p_1k_1} - \frac{\hat{e}_1^*(\hat{p}_3 + \hat{k}_1 + m)\,\hat{p}'}{2p_3k_1} \right] u_1\,. \tag{A.10}$$

Denominator of the first term $2p_1k_1 = s\alpha_1 + m^2\beta_1$ can be transformed, using eqs. (A.1) and (A.2) to the form $2p_1k_1 = x_1 m^2 (1 + \vec{n}_1^{\,2})$. In the numerator of this term we substitute

$$N = \bar{u}_3 \hat{p}' (\hat{p}_1 - \hat{k}_1 + m) \hat{e}_1^* u_1 =$$

$$= \bar{u}_3 \hat{p}' [(1-\beta_1)\hat{p} - \hat{k}_{1\perp} + (-\alpha_1 + m^2/s)\hat{p}' + m] \hat{e}_1^* u_1 ,$$

take into account that $\hat{p}'\hat{p}' = 0$ and transpose $\hat{e}_1^*$ to the left

$$N = \bar{u}_3 \hat{p}' \{ \hat{e}_1^* [-(1-x_1)\hat{p} + m] + 2 e_1^* [(1-x_1)p - k_{1\perp}] \} u_1 .$$

Using Dirac equation $p u_1 = m u_1$ and substituting $e_1 p_1$ = $= - e_1 k_{1\perp}/x_1$ (it follows from gauge invariance condition $e_1 k_1$ = $= e_1 (\alpha_1 p' + x_1 p + k_{1\perp}) = 0$ and from estimation $e_1 p' \sim e_1 p$) we obtain

$$N = \bar{u}_3 \hat{p}' [-2(e_1^* k_{1\perp}/x_1) + \hat{e}_1^* (\hat{k}_{1\perp} + m)].$$

Let us make similar transformations for the second term in eq. (A.10), that leads to eq. (9) and (10).

Integrating U_1 over $\vec{n}_1$, the following integrals arise

$$\int Q_\alpha Q_\beta \frac{d^2\vec{n}_1}{2\pi} = \frac{1}{2} \delta_{\alpha\beta}\, \Phi\left(\frac{n}{2}\right) + \left(2 \frac{n_\alpha n_\beta}{\vec{n}^2} - \delta_{\alpha\beta}\right) \varphi\left(\frac{n}{2}\right); \alpha,\beta = x, y;$$

$$\int \frac{1}{2} R^2 \frac{d^2\vec{n}_1}{2\pi} = \varphi\left(\frac{n}{2}\right); \int \vec{Q} R\, d^2\vec{n}_1 = 0 , \qquad \text{(A.11)}$$

with functions $\Phi(z)$ and $\varphi(z)$ defined in eq. (24).

APPENDIX B

The problem of calculation of the integral

$$\frac{1}{\pi} \int \frac{d^2\vec{n}}{n^4} \left(\vec{Q}^2 F_1 + \frac{1}{2} R^2 G_1\right)\left(\tilde{\vec{Q}}^2 F_2 + \frac{1}{2} \tilde{R}^2 G_2\right) \qquad \text{(B.1)}$$

can be reduced to calculation of four simpler integrals (see eq. (4a))

$$[I_1, I_2, I_3, I_4] = \frac{1}{\pi} \int \frac{d^2\vec{n}}{n^4} \left[\frac{n^4}{a_1 b_1 a_2 b_2}, R^2 \frac{n^2}{a_2 b_2}, \frac{n^2}{a_1 b_1} \tilde{R}^2, R^2 \tilde{R}^2\right]$$

where the notations are used:

$$a_i = 1 + \vec{n}_i^2, \; b_1 = 1 + (\vec{n} - \vec{n}_1)^2, \; b_2 = 1 + (\vec{n} + \vec{n}_2)^2, \; \vec{n}_{12} = \vec{n}_1 + \vec{n}_2 .$$

Using Feynman parametrization, the first of them can be calculated exactly

$$I_1 = \frac{4}{a_1 a_2\, n_{12} \sqrt{n_{12}^2 + 4}} \ln\left(\frac{n_{12}}{2} + \sqrt{\frac{n_{12}^2}{4} + 1}\right) . \qquad \text{(B.2)}$$

As for I_2 , we consider first the case $a_1 \gg a_2$. It is easy to see that the main contribution arises from the region $(\vec{n} - \vec{n}_1)^2 \lesssim 1$. Introducing the variable $\vec{\rho} = \vec{n} - \vec{n}_1$, we obtain

$$I_2 = \frac{1}{\pi a_2} \int \frac{d^2\vec{\rho}}{(\vec{\rho} + \vec{n}_1)^2 [1 + (\vec{\rho} + \vec{n}_{12})^2](1 + \vec{\rho}^2)^2} = \frac{1}{a_1 a_2 n_{12}^2} , \quad a_1 \gg a_2 . \qquad \text{(B.3)}$$

In the case $a_2 \gg a_1$ we divide the n^2 integration region into two intervals: $n^2 < \sigma$ and $n^2 > \sigma$, where $a_1 \ll \sigma \ll a_2$. The contribution of the first interval is equal to (after integration over angles of the vector $\vec{n}$)

$$\frac{1}{a_2^2}\int_{\varepsilon}^{\sigma}\frac{dn^2}{n^2}\left(\frac{1}{a_1^2}-\frac{2}{a_1 r_1}+\frac{a_1+n^2}{r_1^3}\right)=\frac{1}{a_1 a_2^2}\left[1-\frac{1}{a_1}\left(\ln\frac{a_1^2}{\sigma}+2\right)\right],$$

where $r_i = \sqrt{(a_i-n^2)^2+4n^2}$ and the quantity $\varepsilon \to 0$ is introduced to avoid the divergences in some items. In the second interval one can put $R^2 \to 1/a_1^2$ and after that the contribution of this interval equals

$$\frac{1}{a_1^2 a_2}\int_{\sigma}^{\infty}\frac{dn^2}{n^2 r_2} = \frac{1}{a_1^2 a_2^2}\ln\frac{a_2^2}{\sigma}.$$

In the sum of two intervals the parameter σ cancels and

$$I_2 = \frac{1}{a_1 a_2^2}\left[1+\frac{2}{a_1}\left(\ln\frac{a_2}{a_1}-1\right)\right], \quad a_2 \gg a_1. \tag{B.4}$$

Both the results (B.3) and (B.4) can be written in the single formula

$$I_2 = \frac{1}{a_1\, a_2\, n_{12}^2}\left[1+\frac{2}{a_1}\left(\ln\frac{a_2}{a_1}-1\right)\right]. \tag{B.5}$$

One can show that this formula is also valid in the case $a_1 \sim a_2 \sim n_{12}^2 \gg 1$ as well as in the case $a_1 \approx a_2 \gg n_{12}^2 \gg 1$. As a result, formula (B.5) is valid by the single condition $n_{12}^2 \gg 1$.

Similar consideration leads to

$$I_3 = \frac{1}{a_1 a_2 n_{12}^2}\left[1+\frac{2}{a_2}\left(\ln\frac{a_1}{a_2}-1\right)\right],$$
$$I_4 = \frac{1}{a_1^2 a_2^2}. \tag{B.6}$$

Substituting I_{1-4} to (B.1), one obtains eq. (31).

APPENDIX C

Notations for the $\gamma\gamma \to e^+e^-e^+e^-$ reaction are given in fig. 2 and in formula (6). The amplitude of this process can be obtained from formulae (9), (12), (13) by means of the substitution rules of type (7)

$$M_{\gamma\gamma} = -\frac{2K_1K_2}{\vec{q}_\perp^{\,2}} J_1^{\gamma\gamma} J_2^{\gamma\gamma}; \quad J_1^{\gamma\gamma}(\gamma_{\lambda_\gamma} \to e^+_{\lambda_1} e^-_{\lambda_3}) = -i\sqrt{2}\, 8\pi\alpha \cdot$$
$$\cdot \lambda_1 \sqrt{y_1(1-y_1)}\, \varphi^+_{\lambda_3} \{ (2y_1-1)(\vec{Q}_{\gamma\gamma}\vec{e}_1)(\vec{\sigma}\vec{v}_1) + i\vec{v}_1[\vec{Q}_{\gamma\gamma}, \vec{e}_1] -$$
$$- R_{\gamma\gamma}\vec{\sigma}\vec{e}_1 \} \varphi_{-\lambda_1} = -i\, 4\pi\alpha \sqrt{y_1(1-y_1)}\, [(2y_1 - 1 + 2\lambda_1\lambda_\gamma) \cdot \tag{C.1}$$

$$\cdot (\lambda_\gamma Q_{\gamma\gamma x} + iQ_{\gamma\gamma y})\, \delta_{\lambda_1, -\lambda_3} + 2R_{\gamma\gamma}\, \delta_{\lambda_1\lambda_3}\, \delta_{\lambda_\gamma, 2\lambda_1}]; \quad \vec{v}_1 = \frac{\vec{k}_1}{|\vec{k}_1|}.$$

Here $y_i = E_i/\omega_i$, $E_1 + E_3 = \omega_1$, $E_2 + E_4 = \omega_2$. The e^+e^- pair, produced by two photons with momenta k_1 and q has charge parity $C = +1$, that is why the vertex factor J_1 changes its sign under the substitutions $\vec{p}_{1\perp} \leftrightarrow \vec{p}_{3\perp}$, $y_1 \leftrightarrow 1 - y_1$, $\lambda_1 \leftrightarrow \lambda_3$.

We also give the formula which is analogous to eq. (27), but for unpolarized particles only

$$d\sigma = \frac{8\alpha^4}{\pi m^2}\, dy_1\, dy_2\, dn_1 \left(\tilde{F}_1 \tilde{F}_2 a + \tilde{F}_2 b + \tilde{F}_1 c + d \right),$$
$$n_1 = \frac{|\vec{p}_{1\perp}|}{m}, \quad \tilde{F}_i = y_i^2 - y_i + \frac{1}{2}, \tag{C.2}$$

where the functions $a \div d$ are the same as in eqs. (28) and (29).

REFERENCES

1. V.N.Baier, V.M.Galitsky. Phys. Lett. 13(1964) 355; Pisma v ZhETF 2(1965) 259; ZhETF 49 (1965) 661.
2. П.И.Голубничий и др. Атомная энергия, 22(I966) I68.
3. I.Augustin et al. Preprint LAL-1249 (1971).
4. В.М.Аульченко, А.П.Онучин, Ю.А.Тихонов, А.Г.Шамов. Препринт ИЯФ 82-I33, Новосибирск, I983.
5. В.Н.Байер, В.С.Фадин, В.А.Хозе. ЖЭТФ 50(I966) I6II.
6. Э.А.Кураев, В.С.Фадин. Препринт ИЯФ 78-56, Новосибирск, I976.
7. Э.А.Кураев, В.Г.Сербо. ЯФ 38(I983) 4II.
8. В.Н.Байер, В.М.Катков, В.С.Фадин. Изучение релятивистских электронов. М. Атомиздат, I973.
9. V.N.Baier, B.S.Fadin, V.A.Khoze, E.A.Kuraev. Phys. Reports 78(1981) 293.
10. В.Н.Байер, В.В.Гейдт. ЯФ I3(I97I) 350.
11. E.A.Kuraev, A.Schiller, V.G.Serbo. Phys. Lett. 134 B(1984) 455. Preprint INP 82-107, Novosibirsk (1982).
12. Э.А.Кураев, Л.Н.Липатов, М.И.Стрикман. ЖЭТФ 66(I974) 838.
13. А.И.Ахиезер, В.Б.Берестецкий. Квантовая электродинамика, § 4.8.4. М.: Наука, I98I.

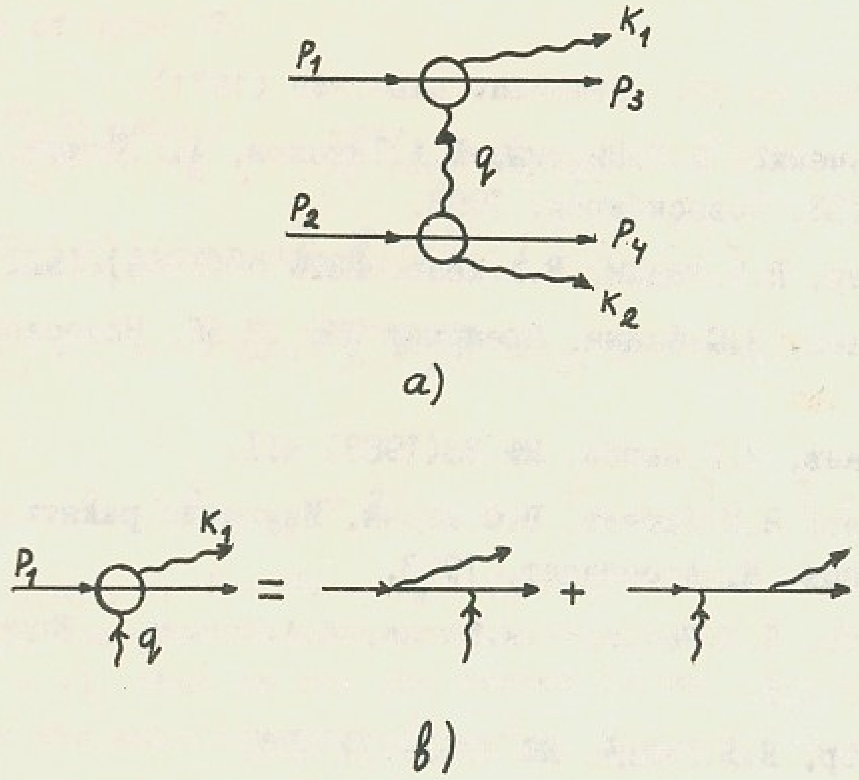

Fig. 1

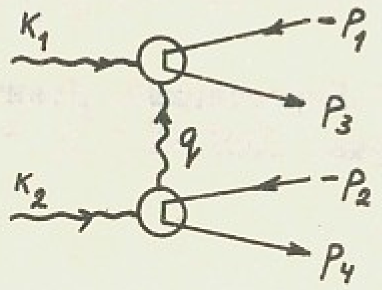

Fig. 2

Э.А.Кураев, В.Г.Сербо, А.Шиллер

ПОЛНОЕ ОПИСАНИЕ ДВОЙНОГО ТОРМОЗНОГО ИЗЛУЧЕНИЯ В ОБЛАСТИ МАЛЫХ УГЛОВ РАССЕЯНИЯ В СЛУЧАЕ ПОЛЯРИЗОВАННЫХ ЧАСТИЦ

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