6

ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

V.M.Khatsymovsky

# ON MATRIX ELEMENT FOR THE PROTON DESAY

PREPRINT 85-7

НОВОСИБИРСК

ON MATRIX ELEMENT FOR THE PROTON DECAY

V.M.Khatsymovsky

Institute of Nuclear Physics,
630090, Novosibirsk, U S S R

ABSTRACT

The correlator of axial-vector current and of current with the proton quantum numbers in an external fermion field is considered. The QCD sum rules for this correlator are used to find $p \rightarrow e^+\pi^0$ or $p \rightarrow \tilde{\nu}_e \pi^+$ proton decay rates in minimal SU(5) grand unified theory (GUT). The result obtained is closed to that of ref. [15] found either in pole approximation or by extrapolating PCAC formulas from nonphysical region.

To describe properties of the lowest hadron states the QCD sum rules method originally proposed in ref. [1] is now widely used. Meson [1,2] and baryon [3-5] masses, meson formfactors and couplings [6-8] had been calculated using this method. In refs. [9,10] the QCD sum rules for polarization operator of nucleon current in an external electromagnetic field first suggested in ref. [9] were applied to calculation of nucleon and hyperon magnetic moments. Analogous sum rules in an external axial field were used in ref. [11] to calculate vector and axial constants of octet baryons.

In this paper the QCD sum rules for the correlator of axial and proton currents in an external fermion field are applied to finding matrix elements of the following operators

$$\eta_p = (u^a C \gamma_\mu u^b)\gamma^\mu d^c \varepsilon_{abc}\,, \quad C = \gamma_0 \gamma_2 \qquad (1)$$
$$\eta_n = -(d^a C \gamma_\mu d^b)\gamma^\mu u^c \varepsilon_{abc}$$

between proton p and pion $\pi$ with the momentum $k_\mu$ being of order of the proton mass $m$. Here $u$, $d$ are the quark field operators, a,b,c are the colour indeces. These matrix elements arise, for example, at calculation of amplitudes of the decays $p \to e^+ \pi^0$ (the dominant mode) and $p \to \tilde{\nu}_e \pi^+$ in minimal SU(5) GUT. These amplitudes are connected with each other by means of isotopic transformation. For definiteness, we consider the decay $p \to e^+ \pi^0$.

Calculations of the nucleon matrix element of the SU(5) baryon number changing Lagrangian were repeatedly made using different quark models for nucleon (for reviews see ref. [13]). The results were scattered in a rather large interval leading to uncertainty of one and a half-order of magnitude in the proton lifetime. In ref. [15] this matrix element was first calculated without using any nucleon model with the help of the matrix element $\langle 0 | \eta_p | p \rangle$ previously found in refs. [3,4] by the QCD sum rules method. Calculations [15] had been made in pole approximation retaining the lowest baryon state in the lepton channel of a decay $p \to lM$ ($l = e^+, \mu^+, \tilde{\nu}_e, \tilde{\nu}_\mu$, $M = \pi, \rho, \omega, K$). In the case $M = \pi$ the pole approximation was checked by some extrapolation of PCAC formula from $k_\mu = 0$ to physical point $|\vec{k}| \approx \frac{1}{2} m$ (earlier the PCAC method was used in ref. [14]). However, the extrapolation mentioned is not, generally speaking, unambigious procedure. So it is desirable to give an independent ground for the pole model. The following calculation is not restricted by the lowest baryon state in the lepton channel but takes into account contribution of the higher hadron states in this channel as well. The latter turns out to be small, approximately 10%, thus ensuring the validity of the pole approximation.

More specifically, let us introduce the notation

$$\langle \pi^0 | \eta_p | p \rangle = i \frac{\lambda_\pi}{\sqrt{2}} \phi_\pi u_p \qquad (2)$$

where $\phi_\pi$, $u_p$ are the pion and proton wavefunctions, respectively, $\bar{u}u$ being equal to 2m. Then the matrix element of interest takes form

$$M(p \to e^+\pi^0) = \lambda_\pi \frac{A}{\sqrt{2}} G_{GUM} \bar{u}_{e^+} (\gamma_5 - 3) u_p \qquad (3)$$

where $G_{GUM}/\sqrt{2} = g^2_{GUM}/8m_X^2 = g^2_{GUM}/8m_Y^2$ is the effective four-fermion constant of the SU(5) low-energy Lagrangian, A is a factor responsible for the renormalization of operators entering the Lagrangian, $u_{e^+}$ is the lepton spinor. It is found in ref. [15] within the pole model that

$$\lambda_\pi^{pole} = \beta \frac{g_\pi \sqrt{2}}{m} = \frac{\beta}{f_\pi} \cdot 2 g_A \qquad (4)$$

(the second equality implies Goldberger-Treiman relation) where $\langle 0 | \eta_p | p \rangle = \beta \gamma_5 u_p$, $g_\pi$ is the $\pi$NN coupling, $g_\pi^2/4\pi = 14$, $g_A = 1.25$, $f_\pi = 133$ MeV. The PCAC formula extrapolated from nonphysical region results in

$$\lambda_\pi^{PCAC} = \frac{\beta}{f_\pi} (1 + g_A) \qquad (5)$$

with unity in the parentheses relevant to the commutator term. Then the ratio $\lambda_\pi^{PCAC}/\lambda_\pi^{pole}$ is 0.9. Finally, our result reads



$$\lambda_\pi^{QCD} = \frac{\beta}{f_\pi} (3 g_A - 1) \qquad (6)$$

$\lambda_\pi^{QCD}/\lambda_\pi^{pole}$ being equal to 1.1.

Let us note that the experimental lower limit $\tau(p \to e^+\pi^0) > 10^{32}$ years established by the IMB group [16] more than an order of magnitude exceeds the upper limit [15] for this value in minimal SU(5) and thus it seems to rule out this model. This result, however, would not exclude philosophically similar models such as those based on SO(10) (for reviews see ref. [13]). Restrictions on superheavy boson mass $m_X$ in these models are relaxed as compared to those in minimal SU(5). Then $\tau \sim m_X^4$ can be larger.

Let us explain in what way (6) can arise in the framework of QCD sum rules. The latter follow from calculations considered below and take the form

$$f_\pi \tilde{\lambda}_\pi \tilde{\beta} = \exp\left(\frac{m^2}{M^2}\right)\left(M^6 + \frac{8}{3} a^2\right) \qquad (7)$$

where tilde means multiplication by $(2\pi)^2$, M is a parameter (of some Borel transformation specified below), $a = -(2\pi)^2 \langle \bar{q}q \rangle$ is the quark condensate. On the other hand, one should take into account the sum rules for $\tilde{\beta}^2$ [3,4]

$$2\tilde{\beta}^2 = \exp\left(\frac{m^2}{M^2}\right)\left(M^6 + \frac{4}{3} a^2\right) \qquad (8)$$

and for ($g_A$ - 1) [11] as well

$$(g_A - 1)\tilde{\beta}^2 = \exp\left(\frac{m^2}{M^2}\right) \cdot \frac{4}{9} a^2 \qquad (9)$$

Of course, the continuum contribution must be subtracted from the RHS of (7-9). The dependence of (7) on $M$, $a$, $m$ can be eliminated with the help of (8,9) yielding (6).

Let us proceed now to derivation of (7). Suppose that there is an external positron field $\bar{e}^+$ with a momentum q responsible for appearance of the term $\bar{e}^+ \eta_p(x) \exp(iqx) + \bar{\eta}_p(x) e^+ \exp(-iqx)$ in QCD Lagrangian. The momentum q satisfies the condition $q^2=m_e^2\approx 0$ . Consider the correlator of $\bar{\eta}_p$ and axial current $j_\lambda^5=2^{-1/2}(\bar{u}\gamma_\lambda\gamma_5 u-\bar{d}\gamma_\lambda\gamma_5 d)$ placed in this field:

$$K_\lambda = i\int e^{ikx}\langle 0|T\{j_\lambda^5(x),\bar{\eta}_p(0)\}|0\rangle_{\bar{e}^+} d^4x \qquad (10)$$

It is clear that $K_\lambda$ differs from zero only leading off the first order of it's expansion in $\bar{e}^+$, $e^+$ series in which the term proportional to $\bar{e}^+$ is just of interest for us. Really, contribution of the transition $p\to\pi^0$ into $K_\lambda$ is

$$\frac{1}{\sqrt{2}}\cdot f_\pi\lambda_\pi\beta\,\frac{k_\lambda}{k^2}\,\frac{\bar{e}^+(\not{p}+m)\gamma_5}{p^2-m^2} \qquad (11)$$

where $p=q+k$ is the proton momentum, and we neglect the pion mass. On the other hand, consider operator product expansion (OPE) for the T-product of $j_\lambda^5$ and $\bar{\eta}_p$ (in the external field $\bar{e}^+$) valid for $p^2,k^2$ laying in the deep Euclidean region:

$$K_\lambda=\sum_n\Big(C_n\frac{\not{p}+\not{k}}{2}\frac{(p+k)_\lambda}{2}+D_n\frac{\not{p}-\not{k}}{2}\frac{(p+k)_\lambda}{2}+ \qquad (12)$$
$$+\ldots\Big)\cdot\langle 0|O_n|0\rangle$$

where $O_n$ are operators, $C_n, D_n, \ldots$ are the coefficient functions of $p^2$, $k^2$ at the independent $\gamma$-matrix structures chosen in (12) in a way convenient for us. The dependence on $\bar{e}^+$ can develop itself either in the coefficients $C_n, D_n, \ldots$ or in the vacuum expectation values (VEV's) of the operators $O_n$ in the external field. If a parameter M is considered as the natural normalization point of OPE then the first possibility is realized for configurations in which interaction with external field occurs at the distances smaller than $M^{-1}$ (see Figs. 1,2) while interaction at the distances larger than $M^{-1}$ (which is possible due to $|q^2|\ll M^2$ ) is included into definition of the VEV's in the field $\bar{e}^+$ (see Fig. 3).

The Borel transformation in one [1] or two [6] momentum variables is probably the most convenient means to pick out the lowest resonance contribution into correlator and to improve convergence of OPE series. As it follows from subsequent analy-



sis it is convenient to use in parallels the two methods: the one-dimensional Borel transformation in $-p^2=-k^2$ and the two-dimensional one in $-p^2, -k^2$. The sum rules are formulated for the coefficient at the structure $(k+p)_\lambda(\not{k}+\not{p})$ . Let the coefficients $C_n$ in (12) with numbers n = 1,2,3 correspond to contributions of Figs. 1,2,3 respectively. Let us begin with the asymptotic loop of Fig. 1. The corresponding coefficient $C_1(p^2,k^2)$ is known as an analytical function of the two variables (given in the form of a definite Feynman integral). It is then a straightforward matter to calculate it's image at the double Borel transformation $B_pB_\pi$ in $-p^2,-k^2$ with parameters $t_p$ , $t_\pi$ respectively. The Borel transformation of an analytical function $f(S)$ in $S$ with parameter $t$ is defined as

$$f(t)\equiv B_t f(S)=\int_{\varepsilon-i\infty}^{\varepsilon+i\infty}\frac{ds}{2\pi i t}\exp\left(\frac{s}{t}\right)f(s),\ \varepsilon>0 \qquad (13)$$

Explicit calculation of the diagram of Fig. 1 with taking into account (11) gives the following sum rule for $\lambda_\pi$:

$$\frac{f_\pi\tilde{\lambda}_\pi\tilde{\beta}}{t_\pi t_p}\exp\left(-\frac{m^2}{t_p}\right)+\text{higher states}=$$
$$=\frac{t_p^2}{t_p+t_\pi}+\ldots=\frac{t_p^2}{t_\pi}\left(1-\frac{t_p}{t_\pi}+\frac{t_p^2}{t_\pi^2}-\ldots\right)+\ldots \qquad (14)$$

Multiplying (14) by $t_pt_\pi$, putting $t_\pi=\infty$ and introducing the notation $t_p=M^2$ we get

$$f_\pi\tilde{\lambda}_\pi\tilde{\beta}=\exp\left(\frac{m^2}{M^2}\right)\left(M^6+O(M^2)+\ldots\right) \qquad (15)$$

This expression implies the following three circumstances. Firstly, the other operators developing non-zero VEV's have dimension $d\geqslant 4$. Secondly, positive powers of $t_\pi$ turn into the same powers of the duality interval for pion, i.e. they are finite. Thirdly, exponentially (in masses) suppressed contribution of higher states is considered to be subtracted from the RHS of (15) in a model way following the usual rules, i.e.

the replacements like

$$M^6 \to M^6\left[1-\exp\left(-\frac{S_0}{M^2}\right)\left(1+\frac{S_0}{M^2}+\frac{S_0^2}{2M^4}\right)\right] \qquad (16)$$

are implicit. Here $S_0$ is a continuum threshold in the proton channel.

The rest of power corrections relevant to interaction with the field $\bar{e}^+$ at short distances can be calculated analogously. We have computed the correction due to $\Psi\bar{\Psi}\Psi\bar{\Psi}$ (see Fig. 2). It is the nearest correction with an odd number of $\gamma$ -matrices strengthened by the loss of the two small loop factors $(16\pi^2)^{-1}$. As for the large distance superweak interaction (see Fig. 3) we do not possess sufficient information about spectral properties of the function $C_3(p^2, k^2)$ required to perform the double Borel transformation of this function. Let us parametrize corresponding contribution into the sum rules by a constant $C_3^{(0)}$ and write down

$$f_\pi \tilde{\lambda}_\pi \tilde{\beta} = \exp\left(\frac{m^2}{M^2}\right)\left(M^6 + \frac{4}{3}a^2 + C_3^{(0)}\right) \qquad (17)$$

where the three addends in parentheses are determined by the diagrams of Figs. 1,2,3 respectively. The power of $M$ in the last term is determined from dimensional consideration by comparison with the case $p^2 = k^2$ (see below).

The function $C_3(p^2, k^2)$ is known, however, at $p^2 = k^2$ (theoretically, we know even an arbitrary finite number of terms of it's expansion in $(p^2 - k^2)$ series around the point $p^2 = k^2$ ) entering correlator in combination with the VEV $\langle 0|\nabla_\lambda \Psi\bar{\Psi}\bar{\Psi}\Psi|0\rangle_{\bar{e}^+}$ in the external field $\bar{e}^+$. Let us outline here a way to calculate VEV's of such the kind. First, the general Lorentz covariant structure of these VEV's must be writing out, for example:

$$\varepsilon^{abc}\langle 0|\, \overline{i\nabla_\mu d^a}(\bar{u}^b\gamma_\lambda C\bar{u}^c)|0\rangle_{\bar{e}^+} = \\ = \bar{e}^+(\alpha g_{\mu\lambda} + \beta\sigma_{\mu\lambda}) \qquad (18)$$

To determine the constants $\alpha$ , $\beta$ one should multiply (18) by $g^{\mu\lambda}$ and by $\gamma^\mu$ and take into account the equations of motion of quark fields in the external field $\bar{e}^+$:



$$\overline{i\not\nabla u^c} = 2(\bar{e}^+\gamma_\mu d^a)\,u^b C\gamma^\mu \varepsilon^{abc} \\ \overline{i\not\nabla d^a} = (u^b C\gamma^\mu u^c)\,\bar{e}^+\gamma_\mu \varepsilon^{abc} \qquad (19)$$

As a result, $\alpha$ , $\beta$ can be expressed through the purely QCD VEV's of the four-quark operators calculable within factorization hypothesis.

Let us multiply correlator by $k^2$ and apply the Borel transformation in $-p^2 = -k^2$ with a parameter $M^2$. Doing so we cannot suppress contribution from the transitions $p \to A_1$, ... exponentially (in masses). Forgetting this circumstance for a moment we can write down

$$f_\pi \tilde{\lambda}_\pi \tilde{\beta} = \exp\left(\frac{m^2}{M^2}\right)\left[(M^6 + C_1^{(0)}M^4) + \frac{4}{3}a^2 + \frac{4}{3}a^2\right] \qquad (20)$$

Here the three addends in brackets are like (17) the contributions from the diagrams of Figs. 1,2,3, respectively. The unknown constant $C_1^{(0)}$ has appeared because of the presence of UV divergent subdiagram I in the asymptotic loop of Fig. 1 and should be specified by imposing a normalization condition on this subdiagram. The corresponding subtraction term in $C_1(p^2, k^2)$ is proportional to $\ln k^2$ and it is not cancelled by the one-dimensional Borel transformation. However, comparing (17) and (20) allows us to determine the both unknown constants $C_{1,3}^{(0)}$ and to arrive at (7).

Consider now in more detail the exponentially unsuppressed contribution of higher states previously omitted in (20). If only short-distance superweak interaction is taken into account then the double spectral density is known. Therefore the higher states can be accounted for in the model way as it was already done in (16). Thus, it is the contribution from transitions $p \to A_1, \ldots$ induced by superweak interaction at large distances which is potentially dangerous for us. It is possible, however, to estimate this contribution in a modelless way. Such the possibility arises due to existence of $\gamma$ -matrix structures, namely $\gamma_\lambda$ and $[\gamma_\lambda, \not k]$, saturated by transitions of proton into axial-vector states $A$ , not into pion. These transitions can be parametrized by adding the terms

$g\varepsilon^{\mu}\bar{u}_{e^+}\gamma_{\mu}u_p$ ( $\varepsilon^{\mu}$ is the polarization of $A$ ) to the phenomenological Lagrangian for the proton decay. Then the sum rules for $g$ can be derived by investigating coefficients at the structures either $\gamma_{\lambda}$ or $[\gamma_{\lambda}, \hat{k}]$ in the correlator. Explicit calculations with both these methods of the graphs of interest of Fig. 3 yield zero contribution to $g$ . Therefore the transitions into axial-vector states induced by long-distance superweak interaction do not contribute into our sum rules (20) at the level of power corrections accounted for.

Thus, parallel use of both the two- and one-dimensional Borel transformations allows one to extract a sufficient information about superweak interaction in both the short- and large-distance regions. Let us note that taking into account large distances (last term in (20)) enhances $\lambda_{\pi}$ by 40% which leads to the corresponding decay rate two times enlarged.

Finally, some words are in order concerning the structure $k_{\lambda}$(with the even number of $\gamma$ -matrices). We have calculated contribution to this structure of the VEV's $\langle\Psi\bar{\Psi}\rangle$ and $\langle\bar{\Psi}\bar{\Psi}\bar{\Psi}\rangle_{\bar{e}^+}$ , the latter VEV being estimated in the proton dominance model, for example

$$\langle\bar{\eta}_p\rangle_{\bar{e}^+} \equiv +i\int \bar{e}^+ \langle 0|T\{\eta_p(x), \bar{\eta}_p(0)\}|0\rangle d^4x + O((e^+)^2) \simeq -\frac{\beta^2}{m}\bar{e}^+ + O((e^+)^2) \qquad (21)$$

The sum rules take the form

$$f_{\pi}\tilde{\lambda}_{\pi}\tilde{\beta} = exp\left(\frac{m^2}{M^2}\right)\left(aM^4 + \frac{\tilde{\beta}^2}{m}M^2\right) \qquad (22)$$

Now we do not manage to eliminate M-dependence and to get a simple expression of the type of (6) for $\lambda_{\pi}$ in terms of experimentally measured quantities. However, the duality estimate at $M \to \infty$, $S_0 \simeq 2$ GeV can be performed giving $\lambda_{\pi}^{QCD}/\lambda_{\pi}^{pole} \simeq 1$ in this approach and thus demonstrating a consistency of the sum rules.

In conclusion, the author is grateful to V.L.Chernyak who has drawn my attention to this problem, for consideration of the work; to V.M.Belyaev and A.I.Vainshtein for helpful discussions and to I.B.Khriplovich for attention to the work.



References

1. M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Nucl. Phys. B147 (1979) 385, 448.
2. L.J.Reinders, H.R.Rubinshtein and S.Yazaki, Nucl. Phys. B196 (1982) 125.
3. B.L.Ioffe, Nucl. Phys. B188(1981)317; (E)B191(1981)591.
4. V.M.Belyaev and B.L.Ioffe, ZhETF 83(1982)876; ZhETF 84 (1983)1236.
5. Chung et al., Nucl. Phys. B197(1981)55.
6. B.L.Ioffe and A.V.Smilga, Phys. Lett. 114B(1982)353.
7. V.A.Nesterenko and A.V.Radyushkin, Phys. Lett. 115B(1982) 410.
8. V.L.Eletsky, B.L.Ioffe and Ya.I.Kogan, Phys. Lett. 122B (1983)423.
9. B.L.Ioffe and A.V.Smilga, Pis'ma v-ZhETF 37(1983)250; Phys. Lett. 133B(1983)436.
10. I.I.Balitsky and A.V.Yung, Phys. Lett. 129B(1983)388.
11. V.M.Belyaev and Ya.I.Kogan, Pis'ma v ZhETF 37(1983)611; Phys. Lett. 136B(1984)273.
12. H.Georgi and S.L.Glashow, Phys. Rev. Lett. 32(1974)438.
13. P.Langacker, Phys. Rep. 72C(1981).
14. Y.Tomozawa, Phys. Rev. Lett. 46(1981)463.
15. V.S.Berezinsky, B.L.Ioffe and Ya.I.Kogan, Phys. Lett. 105B(1981)33.
16. IMB Collaboration, R.Bionta et al., Phys. Rev. Lett. 51(1983)27.

Figures

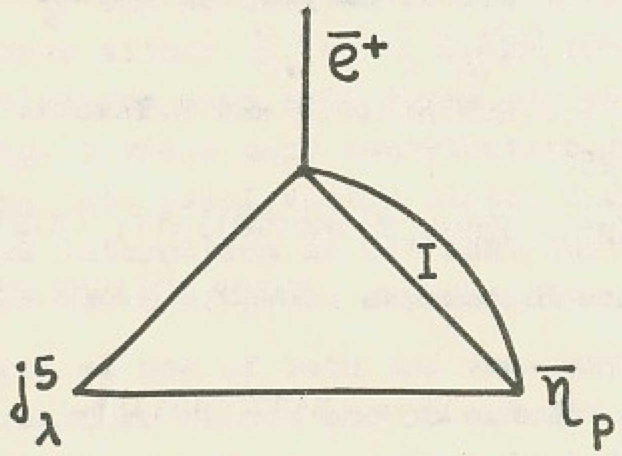


Fig. 1. Asymptotic loop.

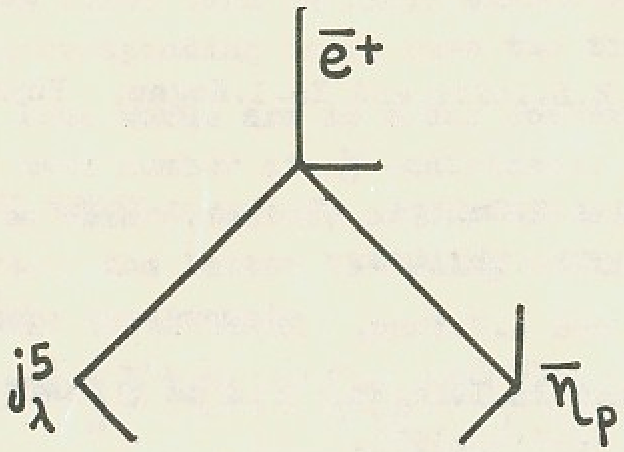


Fig. 2. Graph giving rise to the operator $\Psi\bar{\Psi}\Psi\bar{\Psi}$.

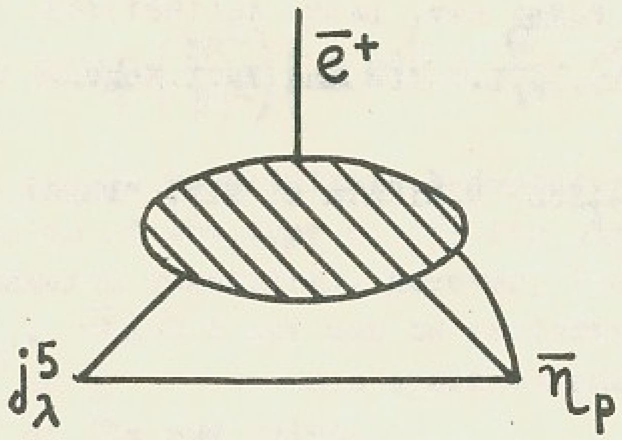


Fig. 3. Graph relevant to the VEV's $\langle\bar{\Psi}\bar{\Psi}\bar{\Psi}\rangle_{\bar{e}^+}, \langle\nabla_\mu\bar{\Psi}\bar{\Psi}\bar{\Psi}\rangle_{\bar{e}^+}, \dots$ in the external field $\bar{e}^+$.



В.М.Хацимовский

О МАТРИЧНОМ ЭЛЕМЕНТЕ ДЛЯ РАСПАДА ПРОТОНА

Препринт
№ 85-7

Работа поступила 24 декабря 1984г.

Ответственный за выпуск - С.Г.Попов
Подписано к печати 30.01.85г. МН 06507
Формат бумаги 60x90 1/16 Усл.0,9 печ.л., 0,8 учетно-изд.л.
Тираж 290 экз. Бесплатно. Заказ № 7

Ротапринт ИЯФ СО АН СССР, г.Новосибирск, 90