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V.N.Baier, V.M.Katkov, V.M.Strakhovenko

MECHANISM OF ELECTRON-POSITRON PAIR PRODUCTION BY HIGH-ENERGY PHOTONS IN A SINGLE CRYSTAL

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НОВОСИБИРСК

MECHANISM OF ELECTRON-POSITRON PAIR PRODUCTION
BY HIGH-ENERGY PHOTONS IN A SINGLE CRYSTAL

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ABSTRACT

A mechanism of pair production by photons in a macroscopic electric field of the axes (planes) of a single crystal has been discussed. Simple expressions for the process probability and spectral distribution at $\varkappa \lesssim 1$ is presented. An analysis of conditions to observe the effect has been made.

A production of particle pairs by high-energy photons in an external macroscopic electromagnetic field is an important mechanism of the pair creation. This mechanism is well known (see , for example, [1]). A character of the process is determined by the basic parameter

$$\varkappa = \frac{e}{m^3}\sqrt{|F_{\mu\nu}k^\nu|^2} = \frac{e}{m^3}\left[(\mathbf{k}\times\mathbf{H}+\omega\mathbf{E})^2-(\mathbf{k}\mathbf{E})^2\right]^{1/2} \xrightarrow[\mathbf{H}=0,\ \mathbf{E}\perp\mathbf{k}]{} \frac{E}{E_0}\frac{\omega}{m} \qquad (1)$$

where $k^\nu(\omega,\mathbf{k})$ is the photon 4 -momentum, m is the electron mass, and $E_0 = m^2/e = 1.32\cdot 10^{16}$ V/ cm is the critical field.

After averaging over the photon polarization the spectrum of the producted electrons (positrons) in a uniform or slightly nonuniform field is given by a formula

$$dW_e = \frac{\alpha m^2}{2\sqrt{3}\pi\,\mathrm{ch}^2 y}\left[4\,\mathrm{ch}^2 y\,K_{2/3}(\xi) - \int_\xi^\infty K_{5/3}(\eta)\,d\eta\right]dy \qquad (2)$$

where K_ν is the McDonald function,

$$\xi = \frac{8\,\mathrm{ch}^2 y}{3\varkappa}, \quad \mathrm{ch}^2 y = \frac{\omega^2}{4\varepsilon_+\varepsilon_-}, \quad \omega = \varepsilon_+ + \varepsilon_- ,$$

where ε_+, ε_- are the energies of the producted particles. Formula (2) is valid at arbitrary $\varkappa$. The total probability of pair production per unit time W_e is obtained after integration of equation (2) over y (see [1] , pp. 173-175).

At $\varkappa \ll 1$ the probability W_e is

$$W_e = \frac{3\sqrt{3}\,\alpha m^2\varkappa}{16\sqrt{2}\,\omega}\, e^{-\frac{8}{3\varkappa}}\left(1-\frac{11}{64}\varkappa+\frac{7985}{73728}\varkappa^2+\dots\right) \qquad (3)$$

Let us remark that the above expression has an accuracy better than 15% up to $\varkappa$ = 2, where the probability is large enough.

The maximum of W_e is attained at $\varkappa \simeq 11$ and at the larger values of $\varkappa$ (at fixed field) $W_e \propto \varkappa^{-1/3}$.

The presented mechanism can manifest itself when the high-energy photon, incident on a single crystal, moves nearly along the crystallographic axes (planes). The reason for this is the following: in this situation the crystal field is described, with a good accuracy, by an averaged potential. The electric fields connected with this potential appear to be very large in magnitude so the effect can be observed for quite accessible photon energies. It consists in a substantial enhancing (starting with a definite energy) of the probability of pair production and in a change of its characteristics, in particular, the dependence on photon polarization and the spectral distribution. One should bear in mind that the standard Bethe-Heitler mechanism of pair production under these conditions can undergo some modifications. Because the pair production mechanism as such is well investigated the problem is to take into account the real configuration of the electric field in a single crystal and to analyse the conditions under which an optimal observation of the effect is possible.

The main contribution to W_e at $\varkappa \lesssim 1$ is given by the crystal region where the electric field is of the maximum value. The maximum averaged fields in single crystals are attained near the crystal axes (planes). To describe these fields we will use the axis potential which appears to be quite adequate in the radiation problem [2]

$$U(x)=V_0\left[\ln\left(1+\frac{1}{x+\eta}\right)-\ln\left(1+\frac{1}{x_0+\eta}\right)\right], \qquad (4)$$

where $x=\varrho^2/a_s^2$, ϱ is a distance from the axis, a_s is a screening radius, $x_0^{-1}=\pi a_s^2 n d$, d is an average distance between the atoms in the chain forming the axis, n is the atom density in the crystal. The parameters of the potential should be fitted by comparing equation (4) with the numerical calculation of the lattice potential within some model, e.g. in the Moliere approximation. We have employed this approach for the axes $\langle 100\rangle$ [3] and $\langle 111\rangle$ (see the Table) for certain single crystals. For estimation, one can use $V_0 \simeq Ze^2/d$, $\eta \simeq 2u_1^2/a_s^2$, where u_1 is the thermal vibration amplitude. The electric field for the potential (4) is

$$eE=-U'(x)=\frac{V_0}{a_s\sqrt{\eta}}\psi(x), \quad \psi(x)=\frac{2\sqrt{\eta x}}{(x+\eta)(1+x+\eta)} \qquad (5)$$

Maximal field eE_m is attained at $x = x_m$, $x_m=\frac{1}{6}\left[\sqrt{1+16\eta(1+\eta)}-1-2\eta\right]$ ($\eta \ll 1$), that is at $\varrho \simeq u_1$, and

$$\varkappa_m=\frac{V_0\omega}{a_s\sqrt{\eta}\,m^3}\psi(x_m). \qquad (6)$$

The equality $\varkappa = 1$ permits one to find out a photon energy at which the discussed effect becomes considerable

$$\omega=\omega_p=\frac{d u_1}{\lambda\!\!\!^{-2}}\frac{1}{Zd}m \qquad (7)$$

where $\lambda\!\!\!^{-}=\frac{1}{m}$ is an electron Compton wavelength. It follows from the estimation (7) that the effect manifests itself first for the crystals with minimal u_1 and maximal Z.

Equation (3) holds in the WKB - approximation. It is easy to estimate that number of state of electrons with energy $\varepsilon_1 < U(\eta)$ is $n \sim \frac{1}{5}\varkappa_m(u_1/\lambda\!\!\!^{-})^3$ and taking into account that at room temperature $u_1/\lambda\!\!\!^{-} \gtrsim 10$ for all the crystals we have $n \gg 1$ for any reasonable $\varkappa_m$. Hence we conclude that the quasiclassical approximation is valid.

In this approximation one can introduce a specific time τ_f of pair formation [1]:

$$\tau_f \sim \lambda \frac{E_0}{E} \simeq \frac{m u_L}{V_0} \quad (8)$$

The constant field limit in the main contribution region can be used if the field difference ΔE, due to the transversal motion with velocity $v_\perp \sim \sqrt{V_0/\omega}$ during the formation time τ_f, satisfies a condition $\Delta E \ll E$. As a result, we obtain an inequality

$$\rho_{ef} \equiv \frac{V_0 \omega}{m^2} \gg 1 \quad (9)$$

At $\varkappa_m \sim 1$ we obtain from (6) and $u_L/\lambda \gg 1$ just the inequality (9). Therefore we conclude that for the conditions considered one can use the results for the constant field. From the inequality (9) it follows also that the transverse motion of the produced particles is a relativistic one. Let us remind that the parameter similar to ρ_{ef} (eq. (9))appears also in the radiation problem [4].

We have considered actually the case when the photon incident angle (the angle between $\underline{k}$ and the axis direction) $\vartheta_0 = 0$. All the estimates remain valid if $\vartheta_0 \lesssim \frac{m}{\omega}$. Under these conditions the angles of the produced pair are $\vartheta_{e^+e^-} \lesssim \frac{m}{\omega}$.

If $\vartheta_0 \gtrsim \frac{m}{\omega}$, then the transverse velocity is $v_\perp \sim \vartheta_0$ and the criterion (9) will be replaced by $\vartheta_0 \frac{m}{V_0} \ll 1$. We conclude that the general criterion for the use of the result for a constant field is

$$\max\left(\frac{m}{\omega}, \vartheta_0\right) \ll \frac{V_0}{m} \quad (10)$$

Equation (3) presents the "instant" probability of the pair production at the distance ρ from the axis. Since the transverse dimension of the photon wave packet are much larger than the distances between the crystal axes (planes), it is necessary to average this probability over the transverse coordinates, i.e. to carry out the integration $nd \int d^2\rho\, W_e(\rho)$ for the axis (for the planar case this will be the integral $d_{pe}^{-1} \int dy\, W_e(y)$). The main contribution to the integral for the potential (4) gives the region $x \sim \eta$, so one can extend the integration to the infinity. Carring out the averaging of the probability (3) and keeping the leading term, we obtain the probability of the pair production in single crystal per unit lenght

$$W_e = \frac{9\alpha\sqrt{\pi}\,\eta}{16\sqrt{2}\,x_0} \frac{V_0}{m a_s} \varphi(\varkappa_m),$$

$$\varphi(\varkappa_m) = \left[\frac{\varkappa_m \psi^3(x_m)}{4\eta^2 \psi''(x_m)}\right]^{1/2} e^{-\frac{8}{3\varkappa_m}} \quad (11)$$

We estimate accuracy of the formulae (11) at $\varkappa_m < 2$ as 25%. It is of evident interest to compare W_e (11) with the standard Bethe-Heitler mechanism of pair production in a screened potential. Taking into account the parameters of the potential (4), we have

$$r(\varkappa_m) \equiv \frac{W_e}{W_{BH}} \simeq \frac{1}{Z\alpha \ln(183 Z^{-1/3})} \frac{u_1}{\lambda} \varphi(\varkappa_m) \quad (12)$$

It is seen from eq. (12) that at a given $\varkappa_m$ the value r is maximum for low Z and large u_1. However, it follows from eqs (6) and (7) that the parameter $\varkappa$ (for fixed ω) is maximum just in the opposite case. We conclude that the effect manifest itself with the growth in photon energy first of all at large Z and small u_1. If one introduces the "threshold" energy $\omega = \omega_t$, by definition $z(\omega_t) = 1$, then for this case the threshold energy is minimum. But a maximum enhancement will be just in the opposite case of small Z and large u_1. The threshold energy has the minimum value for the $\langle 111 \rangle$ axis in tungsten. For this case, the requirements to the incident angle $(\vartheta_0 \ll V_0/m)$ and to the single crystal quality (including mosaic structure) will be also weaker.

Let us consider now the spectral distribution over the energy of one of the produced particles $\varepsilon_\pm$. When $\varkappa \ll 1$ one can expand K_ν in eq. (2). Then, averaging over the transverse coordinates (as it was done in eq. (11)) we obtain, for moderate photon energy $(\varkappa_m \lesssim 1)$, the following distribution

$$\frac{dW_e}{d\varepsilon_\pm} = \frac{\sqrt{3}\,\alpha V_0}{m x_0 \omega a_s}\left[\frac{\psi^3(x_m)}{4\eta\,|\psi''(x_m)|}\right]^{1/2}\left[1-\frac{\varepsilon_\pm}{\omega}\left(1-\frac{\varepsilon_\pm}{\omega}\right)\right]\exp\left[-\frac{2\omega^2}{3\varkappa_m \varepsilon_\pm(\omega-\varepsilon_\pm)}\right] \quad (13)$$

This distribution is symmetrical with respect to $\varepsilon_\pm = \frac{\omega}{2}$ and has in this point sharp maximum (when $\varkappa_m \ll 1$ this distribution tends to a δ-function). The distribution (13) differs substantially from the Bethe-Heitler spectrum, which varies very slowly with the energy.

The results obtained (eq. (11), (13)) have a very simple form, so one can easily obtain a prediction for any single crystal. Nevertheless, we illustrate them in Fugures 1 and 2. Fig. 1 presents the probability W_e (11) for the $\langle 111 \rangle$ axis in tungsten for different temperatures T = 77° (curve 1) and

T = 293° (curve 2). In Fig. 2 the same is presented for iron and diamond (T = 293°). The Table contains the parameters of the potential (4) and the threshold energy ω_t for which the probability of this effect and Bethe-Heitler mechanism become equal. It is seen that for tungsten ω_t = 15 GeV (1) and ω_t = 23 GeV (2). The photon 10÷30 GeV beams are available in several laboratories, so an experimental investigation of the effect is quite possible now.

The criterion, when the radiation mechanism at planar channeling becomes quantum due to a recoil at radiation was earlier discussed in the author's paper [4]. The results of our recent paper [3] permit one to evaluate the values of particle energy when the quantum effects become significant at the radiation by electrons moving near the crystal axes. These values are $\varepsilon \gtrsim$ 15 GeV for W, $\varepsilon \gtrsim$ 80 GeV for Ge etc., in the general case $\varepsilon \sim \omega_t$. A coincidence of ω_t and ε is not an occasional one. Indeed, when the radiation becomes a quantum one, we go out the region of exponential suppression of pair production.

The pair production by a high-energy photon in a single crystal was recently been discussed in Ref. [5]. The authors of [5] did not use the previously obtained results and solved the problem as a whole from the very beginning. The calculations were carried out numerically (for the diamond $\langle 110 \rangle$ axis) so the only result is Figs. 2 and 3 in [5] which displayed $W_e(\omega)$ and $\frac{dW_e}{d\varepsilon_-}$ (at ω = 51,1 GeV), respectively. We have carried out a calculation under the same conditions and compared our results with those in [5]. It appears that W_e in [5] exceeds the ours by 9 times at ω = 50 GeV, by 6 times at ω = 80 GeV, and by

5 times at ω_t = 150 GeV. The spectral distribution (eq. (13)) also appears to be narrower than that in [5]. Let us observe that the calculations in [5] was also made using the WKB approximation. The effect under consideration has been discussed very recently in Ref. [6], using the approach of Ref. [1], but the authors of Ref. [6] have made a mistake in evaluation of the initial matrix element and, because of this, all the following results are wrong.

In this paper we have considered the pair production by a high-energy photon which is aligned along the crystal axes at a single crystal at $\varkappa \lesssim 1$. The general analysis will be published elsewhere.

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Table

Parameters of the potential (4) for axis ⟨III⟩ and values of threshold energies ω_t

Crystal	$u_1(\AA)$ (T = 293°)	V_0(eV)	η	$a_s(\AA)$	x_0	ω_t(GeV)
$C_{(d)}$	0.040	29	0.025	0.326	5,5	99
$Si_{(d)}$	0.075	54	0.150	0.30	15	150
Fe	0.068	180	0.145	0.276	20	50
$Ge_{(d)}$	0.085	91	0.130	0.30	16	111
W(293°)	0.050	417	0.115	0.215	40	23
W(77°)	0.030	348	0.027	0.228	35	15

Figures captions

Fig. 1. The probability (per unit time) of electron-positron pair production $W_e(\omega)$ in tungsten (axis ⟨III⟩) as a function of photon energy at temperature T = 77° (curve 1) and T = 293° (curve 2) and at incident angle $\vartheta_0 \ll V_0/m$. The dashed line is the Bethe--Heitler probability for the screened potential.

Fig. 2. The same as in Fig. 1 at T = 293° and for iron and diamond.

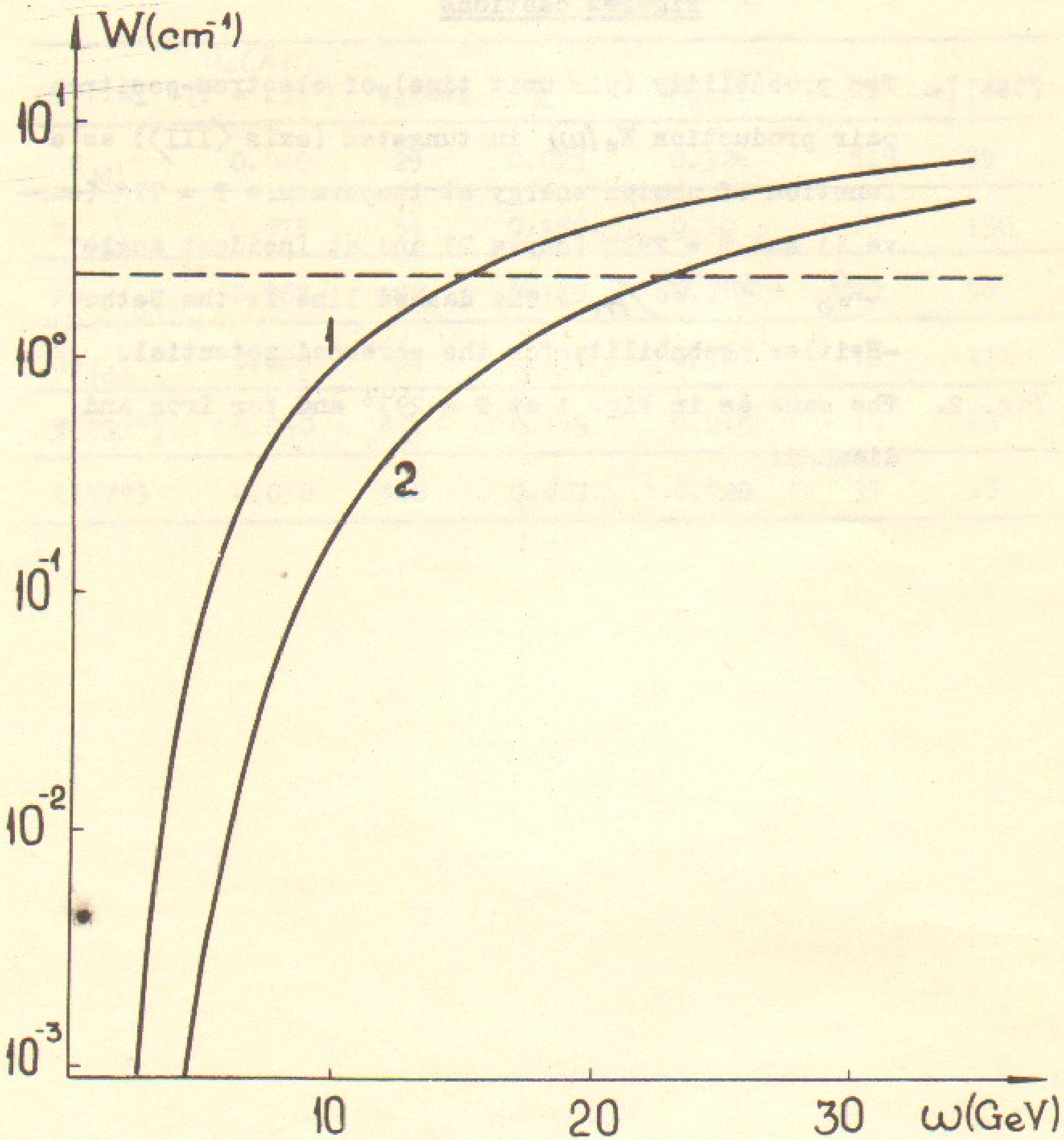
W(cm⁻¹)
10¹
10°
10⁻¹
10⁻²
10⁻³
1
2
10
20
30
ω(GeV)

Fig.1

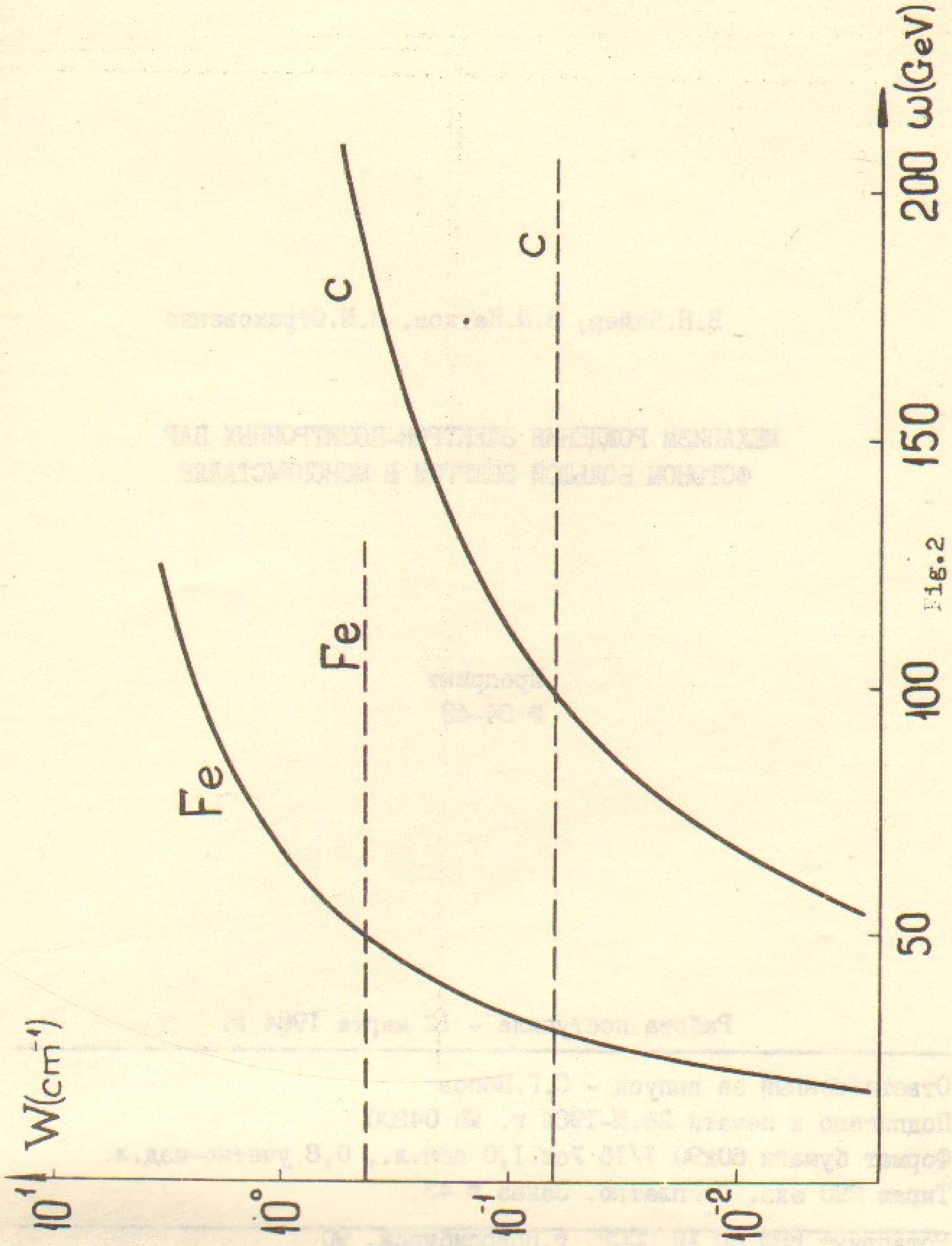
W(cm⁻¹)
10¹
10°
10⁻¹
10⁻²
Fe
C
Fe
C
50
100
150
200
ω(GeV)

Fig.2

В.Н.Байер, В.М.Катков, В.М.Страховенко

МЕХАНИЗМ РОЖДЕНИЯ ЭЛЕКТРОН-ПОЗИТРОННЫХ ПАР ФОТОНОМ БОЛЬШОЙ ЭНЕРГИИ В МОНОКРИСТАЛЛЕ

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