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THEORY AND PHENOMENOLOGY OF THE QCD VACUUM

5. CORRELATORS AND SUM RULES THE APPLICATIONS

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ABSTRACT

This preprint contains discussion of the various applications of sum rule method, based on the comparison of experimental data with theoretical expressions for the correlators. In section 5.1 we discuss recent works on hadronic spectroscopy on the lattice. While in sections 5.2—5.5 we consider sum rules based on the operator product expansion. They are devoted to mesons containing light quarks, baryons, currents with derivatives and the three-point correlators, respectively. In section 5.6 we discuss sum rules for heavy quarkoniums, in this case it is necessary to evaluate the correlator at somewhat larger distances because their interaction with vacuum fields is suppressed. In sections 5.7 and 5.8 we discuss pseudoscalar quark currents and gluonic currents, for which interaction with vacuum is, on the contrary, strongly enhanced.

5. CORRELATORS AND SUM RULES. THE APPLICATIONS

Violating the historical order we start this chapter with consideration of rather recent spectroscopic calculations on the lattice, which had strong resonance in high energy physics community. In some respects it is justified by the fundamental character of such an approach, attempting to calculate hadronic masses directly from the first principles of QCD.

However, more deep investigations of this approach reveal many open questions, connected with great difficulties met by this ambitious programm. Because of this real accuracy of the results obtained is not so far sufficient for quantitative analysis. Sceptics make jokes, saying that it is hardly necessary to make about 10^{12} arithmetical operations (about 100 hours at best computers) in order to learn that baryons are about 3/2 times heavier than mesons and the proton magnetic moment is about 3 magnetons: all these facts were explained by the quark model 20 years ago. Optimists note that few years ago nobody could even dream that so straightforward approach to QCD is practically possible, proposing to wait a little. The subject of the present paper is the situation at the present moment, so I have tryed at least to explain in section 5.1 the numbers involved, presumably understandable for non-experts.

The situation with the sum rule method (in its more developed OPE-based version) is discussed in sections 5.2—5.5. It is remarkable that it is completely different from that for lattice calculations. It can not be argued on general grounds that the OPE series (valid at small distances) and data about lowest states (prescribing the large distance behaviour of the correlators) can be connected at some intermediate region (the «window», or «fiducial volume») in meaningful way. However, as it was noticed by Shifman, Vainshtein and Zakharov [5.13], it happens to be the case, providing very useful possibility to connect hadronic phenomenology with that of the QCD vacuum. Among particular applications of this method we discuss mesons containing light quarks (section 5.2), the baryons (5.3), currents with derivatives (5.4) and the three-point correlators (5.5).

In some cases this approach also was found to come into troubles. The first such case connected with heavy quarkonia is discussed in section 5.6. It is easy to understand why for heavy enough quarks the OPE method becomes inapplicable: quark and antiquark created by the current at the origin ($x=0$) never goes far from it at limited time t (say, $t<1$ fm) for $x<(t/mass)^{1/2}$. Thus at large mass interaction with vacuum fields becomes too weak to be noticable. At large t they show up, but here the OPE is inapplicable.

Quite opposite complications arise for the correlators of spin-zero currents: in these cases interaction with vacuum fields is too strong, limiting applicability region of the OPE by very small distances. As a result, the «window» for fitting practically disappears. These observations were first made by Novikov et al. [5.56], who have suggested that such anomalously strong effects are caused by instantons. Attempts to connect different phenomena of this type in the framework of some semi-quantitative model were made in my works [5.57], leading to «instanton liquid» model considered in chapter 2. It turns out, that with some simplified assumptions on the instanton parameters one may really connect a lot of facts, in particular explicitly obtain the massless pion or very heavy η'. Even such details as SU(3) violation effects (kaon mass, $\eta - \eta'$ mixing) are reproduced well, presumably indicated that the underlying physics is correct.

Correlators of gluonic currents are so far understood much worse than quark ones. In this case we have only very limited experimental information coming from Ψ, Υ decays, as well as some general low-energy theorems [5.56]. However, it is sufficient to conclude that in this sector we find much stronger effects. Their physical nature is discussed in section 5.8. It is interesting, that similar trend is found in lattice calculations for pure gauge theories.

5.1. Hadronic spectroscopy on the lattice

It was already noted in the introduction to this chapter that lattice methods come across rather severe problems. However they are «only technical» ones, and presumably they will be overcomed with future progress in programming and computer technology.

The first obvious difficulty is connected with rather limited space-time volume of the QCD vacuum, for which the calculations can be made. We have already explained in chapter 3 that the physical correlation length should be much smaller than lattice dimensions and much larger than its spacing. In practical calculations «much larger» means only the factor 2—3, at best.

In order to have more quantitative feeling of the numbers involved let us consider the «finite size effects» at somewhat different angle. In chapter 3 it was explained that periodicity in Euclidean time τ corresponds to calculations for the nozero temperature $T=1/\tau$. In calculations made by Hamber and Parisi [5.1] (to be systematically used as the most detailed studies of the type) the temperature T varies between 110 and 260 MeV, and even in so far record (Crey) calculation [5.6] with 20 points along the time axis $T=110$ MeV. Thus, the superdense matter (being the experimentalist's dream) is in fact the best approximation to vacuum state which is technically possible to investigate on the lattice!

Considering small spatial size of the «box» we may mention that in [5.1] it varies between 0.5 and 1.1 fm. One may indeed imagine that a hadron is contained in such box, but where is the place for the surrounding vacuum? It is clear that we deal here with some densely packed «hadronic crystal» and, as always in problems of such kind, the spectrum is given by some Brilloin zones with widths proportional to tunneling probability between neighbouring sites. Rather instructive calculation of such widths was made by Hasenfratz and Montvay [5.11] by numerical solution of Schredinger equation in similar conditions. The conclusion is rather disappointing: it is rather large for all ordinary hadrons and reasonably small only for upsilons. (Obviously, upsilons can not be studied because the lattice is not only small but also too «coarse grained».) In real calculation hadronic masses are also found to fluctuate in computer time. At Fig.1 we show how such fluctuations depend on the lattice size according to [5.12, 5.6].

Now a few words on how hadronic masses are extracted from the measured correlators. In contrast to the sum rule method where the external currents are assumed to be pointlike, the ones traditionally used in lattice studies are assumed to be homogeneous in space. The signal therefore depends only on (Euclidean) time variable and decreases as $\exp(-m\cdot\tau)$ at large τ. However, periodic boundary condition leads to the signal of the form

$$K_1(\tau)\sim\{\exp(-m_1\tau)+\exp(m_1(\tau-\tau_0))\}\tag{5.1}$$

so that practically useful region of τ is equal to half of the lattice dimension $\tau_0/2$. Obviously, the signal is averaged over discrete «layers». The typical curve for the logarithmic derivative of the baryon current correlator taken from Ref. [5.6] is shown at Fig.2.

(Very similar curves, but following from the analysis of experimental data for c and b quarks production in e^+e^- annihilation will be discussed in sect. 5.6.) Approach to the limit from above is just the consequence of the contribution of heavier states at smaller time values. As noted in section 4.1, there exist simple and well justified parametrization of the spectral density (4.14), containing apart from the resonance parameters the «continuum threshold» w where asymptotic freedom predictions become valid. I think that this important parameter can be (and therefore should be) determined from lattice data. In addition, with parametrization (4.14) one obtains more accurate values for the masses (in usual practice contribution of heavier states is just ignored).

Finally, there are uncertainties connected with putting the results into physical unites. We have already explained in chapter 3 that the coupling g and lattice spacing a correspond (in scaling region) to single parameter Λ, while hopping parameters K_F are connected with quark masses by the relation

$$\exp(m_F a) = 1 + (K_F^{-1} - K_C^{-1})/2 \quad (5.2)$$

where K_C is some critical value related to massless quarks. In reality evaluation of the latter quantity is rather uncertain because one uses as a signal the zero mass of the pseudoscalar meson. Obviously, it is not possible to find exactly massless excitation in finite (and rather restricted) volume, so it is made by some extrapolation. Its quality can be explained by Fig.3, where typical results of Ref. [5.1] are given.

Unfortunately, we do not know the fundamental parameters Λ, m_q good enough, so in real calculations some masses are used as «input». With those of ϱ, π^+, π^0, K, Ψ mesons used in [5.1], it was found that

$$\Lambda_{\overline{MS}} \simeq 70 \text{ MeV}$$

$$m_u \simeq 4.5 \text{ MeV}, \quad m_d \simeq 8 \text{ MeV}, \quad (5.3)$$

$$m_s \simeq 160 \text{ MeV}, \quad m_c \simeq 1300 \text{ MeV}$$

where the normalization point is about .3 fermi or 600 MeV. We remind that renorminvariant combination is equal to

$$\hat{m} = [\alpha_s(\mu)]^{-4/b} m(\mu) \quad (5.4)$$

These numbers are «reasonable» but not quite accurate, especially lambda.

Now we come to the most serious defect of the calculations under consideration, being the so-called «quenching» approximation neglecting virtual quarks. This point is of particular importance because calculations with quarks are extremely time-consuming. Unfortunately, as it was already mentioned in section 3.5 they are indeed quantitatively important. (More convincing facts on this statement will be presented below in chapters 7 and 8.) As an example we show at Fig.4 results of the calculations made in Ref. [5.10] for the pseudoscalar (pion) current correlator including virtual quarks by the pseudofermion method, to be compared to previous calculations [5.1] made in quenching approximation. Apart from completely different behaviour, it turns out that in this case strong splitting between pion and rho meson [5.1] seen at Fig.3 have disappeared!

Note, that introduction of quark loops leads to new observable effects, say to ϱ—ω splitting. It is interesting that in [5.10] it is found to be of correct sign and magnitude, in contrast to the well known wrong sign given by perturbative effects.

Another nontrivial effect connected with quark loops is the η' mass. Although calculations [5.1] were made at zero quark flavours, it was attempted to expand in this parameter and evaluate the first derivative. The results obtained are rather disappointing, for the effect obzerved is too weak and η' turns out to be even lighter than «normal» ϱ meson. We return to this point in section 5.7 where it is argued that large η' mass is the instanton-induced effect. Thus, the result [5.1] mentioned is well correlated with general failure to find instantons in lattice studies discussed in section 3.4.

Concluding this section I may only once more comment that lattice calculations are at the moment at their initial stage and technical limitations are rather severe. Appropriate situation with confinement on the lattice ensure reasonable gross features of hadronic spectroscopy, but important phenomena connected with chiral symmetry breaking are not reproduced so well.

5.2. Meson currents containing light quarks

As it was already mentioned in the introduction to this chapter, nontrivial (and very fortunate) feature of the correlators is the existence of some «window» in which both OPE formulae and available data have reasonable accuracy, so that their comparison makes sense. In this section we discuss examples of analysis made in the pioneer work [5.13].

Let us start with vector current with ϱ^0 quantum numbers

$$j_\mu = (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/2 \tag{5.5}$$

and calculate first OPE corrections to its correlator as explained in section 4.2. Using Borel transformation (see section 4.1) of the polarization operator one may obtain the following sum rule [5.13] connecting the OPE series in l.h.s. with the cross section of e^+e^- annihilation into $I=1$ channel

$$1 + \frac{\alpha_s(m)}{\pi} + \frac{1}{12m^4}\langle(gG)\rangle - \frac{\langle O_4\rangle}{m^6} + \ldots = \frac{2}{3m^2}\int ds \exp\left(-\frac{s}{m^2}\right) R(s)$$

$$R(s) = \sigma(e^+e^- \to hadrons, I=1)/\sigma(e^+e^- \to \mu^+\mu^-) \tag{5.6}$$

We have ignored here operators $m\bar{\Psi}\Psi$ small due to small masses of light quarks, therefore quark-induced effects appear starting from rather complicated four-fermion operator

$$Q_4 = 2\pi^3\alpha_s(\bar{u}\gamma_\alpha\gamma_5 t^a u - \bar{d}\gamma_\alpha\gamma_5 t^a d)^2 +$$

$$+ \frac{4\pi^3}{9}\alpha_s(\bar{u}\gamma_\alpha t^a u + \bar{d}\gamma_\alpha t^a d)\sum_{u,d,s}(\bar{q}\gamma_\alpha t^a q) \tag{5.7}$$

Note that normalization is chosen so that free loop corresponds to unity in l.h.s. In principle, operator average values in (5.6) are some unknown constants which can be found from the sum rules. Such approach will be considered in section 8.1, but in [5.13] the value of the gluon condensate was taken from charmonium sum rules and that of the operator O_4 from factorization hypophesis (see section 8.2):

$$\langle 0|O_4|0\rangle = \frac{7\cdot 2^6}{3^4}\alpha_s\langle\bar{\Psi}\Psi\rangle^2 \tag{5.8}$$

and the «standard» value of the quark condensate $\langle\bar{\Psi}\Psi\rangle = -0.013$ GeV^3. With these numbers the l.h.s. is fixed, the corresponding line is shown at Fig.5. In the r.h.s. $R(s)$ was parametrized as follows:

$$R(s) = \frac{12\pi^2 m_\varrho^2}{g_\varrho^2}\delta(s-m_\varrho^2) + \frac{3}{2}\left(1+\frac{\alpha_s(s)}{\pi}\right)\theta(s-W^2) \tag{5.9}$$

Fitting values of these three parameters in the «window» shown by arrows at Fig.5 (marking places where uncertainty reach 30%) it was found that

$$m_\varrho^2 \simeq 0.6 \text{ GeV}^2, \quad g_\varrho^2/4\pi \simeq 2.3, \quad W^2 \simeq 1.5 \text{ GeV}^2 \tag{5.10}$$

which is indeed very close to experimental data

$$(m_\varrho^2)_{exp} = .592 \text{ GeV}^2, \quad (g_\varrho^2/4\pi)_{exp} = 2.36 \pm 0.18 \tag{5.11}$$

Note that operator O_4 contributes somewhat stronger than gluon condensate, and recollecting (5.7) one may comment that, roughly speaking, the rho meson mass value is fixed from that of the quark condensate. This may appear strange for it is not clear why vector mesons (and not only pseudoscalar ones) have anything to do with chiral symmetry breaking. Similar fact will be shown below for B and D type mesons (containing light and heavy quarks) and in the next section for baryon masses. Thus, it is the general feature of any problem involving light quarks! (Discussion of relevant physics is postponed till section 6.1.)

In order to test the method and demonstrate that good results (5.9) are not due to some coincidence the authors of [5.13] have also considered the correlator of similar axial current

$$j_\mu^A = \frac{1}{2}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d) \tag{5.12}$$

Its spectral density is connected not only with axial A_1 meson, but also with the pion. Experimentally it is therefore completely different from that in vector channel: instead of the resonance at $m^2 = .5$ GeV^2 there are resonances at $m=0$ and $m=1$ GeV. Gluonic corrections are the same, so in the approximation considered only four-fermion operators may lead to such striking variations. Can they really do the job?

Now let us present few formulae. Defining two invariant structures (the axial current is not conserved)

$$i\int dx\, e^{iqx}\langle T\{j_\mu^A(x) j_\nu^A(0)\}\rangle = -\Pi_1 g_{\mu\nu} + \Pi_2 q_\mu q_\nu \tag{5.13}$$

we obtain the following sum rules for Π_2

$$1 + \frac{\alpha_s(m)}{\pi} + \frac{\langle(gG)^2\rangle}{12m^4} + \frac{\langle O_4'\rangle}{m^6} + \ldots = \frac{4\pi}{m^2}\int ds \exp\left(-\frac{s}{m^2}\right) \mathrm{Im}\,\Pi_2(s) \tag{5.14}$$

where

$$\langle O_4' \rangle \equiv \langle 4\pi^3 \alpha_s (\bar{u}\gamma_\alpha \gamma_5 t^a d)(\bar{d}\gamma_\alpha \gamma_5 t^a u) - \frac{4}{9}\pi^3 \alpha_s (\bar{u}\gamma_\alpha t^a u +$$

$$+ \bar{d}\gamma_\alpha t^a d) \sum_{u,d,s} (\Psi \gamma_\alpha t^a \Psi) \rangle \simeq 5 \cdot 10^{-2}\ \mathrm{GeV}^6$$

(the estimate is again made using factorization hypothesis).

The following parametrization of the spectral density is used

$$\mathrm{Im}\,\Pi_2 = \pi f_\pi^2 \delta(s) + \frac{\pi m_{A_1}^2}{g_{A_1}^2} \delta(s - m_{A_1}^2) + \frac{O(s - W^2)}{4\pi}(1 + \alpha_s/\pi + \dots) \qquad (5.15)$$

where standard notations are as follows

$$\langle 0 | \bar{u}\gamma_\mu \gamma_5 d | \pi \rangle = i f_\pi p_\mu$$

$$\langle 0 | \bar{u}\gamma_\mu \gamma_5 d | A_1 \rangle = \varepsilon_\mu m_{A_1}^2 / g_{A_1} \qquad (5.16)$$

and p_μ, ε_μ are momentum and polarization vector of hadrons. Again, parameters in (5.14) can be fitted and they are found to be very reasonable, say the pion decay constant was found to be

$$f_\pi \simeq 130\ \mathrm{MeV} \qquad (5.17)$$

with accuracy of the order of 20%, to be compared to experimental value

$$(f_\pi)_{exp} \simeq 133\ \mathrm{MeV} \qquad (5.18)$$

These results were so unexpectedly accurate, that many new calculations for other channels where initiated, some of them to be discussed below. Now let us make few remarks on applications related to light mesons. Much more detailed analysis than that given above for vector and axial channels (including mixing phenomena etc.) is contained in the original papers [5.13]. Tensor channel was successfully studied in Ref. [5.19]. Some other references are collected in [5.15—5.20], but one should note that for scalar and pseudoscalar channels specific phenomena (to be discussed in section 5.7) are important, not always considered in these papers. There are also first calculations for «exotic» currents of the structure $\Psi G \Psi$ [5.21—5.23, 5.42, 5.52], but it is probably to early to comment on this field of applications for even experimental expectations are here too uncertain.

Our next topic is connected with mesons made of light and heavy quarks (antiquarks). It was used in section 4.2 as the most simple example of the OPE calculations. More important, in this case main corrections are those connected directly with quark condensate, so assumptions like (5.7) are not needed. Now there are several independent analysis for the same quantities, which are not experimentally known. Their variation may serve as a measure of the method accuracy. (Similar situation is with baryon couplings, see the next sections.)

The main unknown parameters evaluated in these works are the decay constants for D and B mesons (containing c and b quarks) defined as

$$\langle 0 | \bar{Q}\gamma_\mu \gamma_5 q | meson(p) \rangle = i f_Q p_\mu \qquad (5.19)$$

Its dependence on large mass is determined by the nonrelativistic limit [5.37]: the combination

$$f_Q^2 m_Q \xrightarrow[m_Q \to \infty]{} \mathrm{const} \qquad (5.20)$$

remains fixed for it is proportional to density of the light quark at the origin. We do not give the sum rules and just make few remarks on the results contained in Table 1.

Reinders et al. [5.36] have obtained values much larger than all others because their «continuum threshold» W was taken to be very large. In my paper [5.37] only the nonrelativistic limit is considered, and numbers come from extrapolation of (5.20). In Refs [5.38] calculations are made in relativistic formalism, with results shifted to still smaller values.

Interest to such parameters was initiated by discussion of the problem of D^0 and D^+ lifetime difference. A number of authors have suggested that such particles are essentially more «compact» that ordinary hadrons, but most results obtained from the sum rules do not confirm it. (Similar conclusion was also drown from first works using MIT bag model, but later works [6.13] have shown that it was essentially due to incorrect account for centre mass motion.) Finally, recent experimental data also point toward more modest effect.

Of course, there are also other predictions not so far tested, in particular relatively strong splitting between states of different parity, say vector and axial ones predicted in [5.36, 5.37]. Note that this effect is directly connected to quark condensate value, thus it is of great interest.

Finally, due to particular simplicity of this problem it was possible to go outside the OPE framework and relatively simply compare it with some other approach, in particular the «instanton liquid model» [5.57]. We remind that leading corrections come from expansion of the following nonlocal object

$$K(x) = \langle \Psi(x) \, \mathrm{Pexp} \left(\frac{ig}{2} \int_0^x A_\mu^a t^a dx_\mu \right) \Psi(0) \rangle \tag{5.21}$$

where the path is just the straight line (it is at this point where large mass of heavy quark is used). With instanton potential for A and zero modes for Ψ one may find the correlator shown at Fig.6, where effects of first terms of the OPE series are also demonstrated. It is seen that as soon as corrections become noticeable such expansion is practically useless, while the instanton model produces much more reasonable curve.

5.3. Baryonic currents

Generalization of the methods discussed in the preceeding section to the baryonic case does not contain any qualitatively new moments. However, the corresponding currents are not so familiar as the mesonic ones, so it is reasonable to comment on their general classification.

It is usuful to start with «diquark» states first, using charge conjugation operator C and standard gamma matrix basis:

$$\Psi_\alpha \Psi_\beta = \Psi^T C \Gamma_i \Psi$$
$$\Gamma_i = 1(-),\ \gamma_5(-),\ \gamma_\mu(+),\ \gamma_\mu\gamma_5(-),\ \sigma_{\mu\nu}(+) \tag{5.22}$$

where signs given in brackets correspond to symmetry or asymmetry of the matrix $(C\Gamma_i)^{kl}$. If the diquark is made out of u, d quarks, this sign determins its isospin: $I=1$ for $(+)$ and $I=0$ for $(-)$.

Now let us add the third quark. The simplest possibility is that three quarks are identical and there is complete symmetry of their (noncolour) wave function: say it happens for isobar $I=3/2$. At first sight, one may start from both symmetric diquarks $u^T C \gamma_\mu u$ and $u^T C \sigma_{\mu\nu} u$, but complete symmetry condition reduces them to identical combination.

$$j_\Delta = (u_i^T C \gamma_\mu u_j) u_k \varepsilon^{ijk} \tag{5.23}$$

However, for nucleons there are two possibilities indeed [5.24], it is convenient to write these two currents as follows

$$j_{1N} = \varepsilon^{ijk} [(u_i^T C d_j) \gamma_\mu u_k - (u_i^T C \gamma_5 d_j) \gamma_\mu \gamma_5 u_k]$$
$$j_{2N} = \varepsilon^{ijk} [(u_i^T C \sigma_{\varrho\lambda} d_j) \sigma_{\varrho\lambda} \gamma_\mu u_k - (u_i^T C \sigma_{\varrho\lambda} u_j) \sigma_{\varrho\lambda} \gamma_\mu d_k] \tag{5.24}$$

Substituting here s instead of d quark we have two currents for Σ hyperon. In the case of Λ particle one should start with $i=0$ diquark, thus there are even three independent currents

$$j_{1\Lambda} = (u^T C \gamma_5 d) s$$
$$j_{2\Lambda} = (u^T C \gamma_\mu \gamma_5 d) \gamma_\mu s \tag{5.25}$$
$$j_{3\Lambda} = (u^T C d) \gamma_5 s$$

As it was noticed in Ref. [5.27], the number of currents corresponds to general decomposition of flavour SU(3) group

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1 \tag{5.26}$$

In more familiar nonrelativistic SU(6) nomenclature there is only one octet and the decuplet, but working at small distances with «current» quarks we have to apply relativistic notations. It may of course be so that certain current have larger coupling to nucleons and other ones to «orbital excitations», but in principle all currents mentioned can equally well be used for generation of the sum rules. Their number is further enhanced by several independent spin structures (2 for spin 1/2 and 4 for 3/2), as well as by the possibility to consider «nondiagonal» correlators with different currents. It is evident that it is not possible to consider many details here, so we just present few examples and comment on the results.

Our first example [5.24] considers diagonal correlator of the first nucleon currents j_{IN}

$$i \int dx \exp(iqx) \langle T\{j_{1N}^\alpha(x) j_{1N}^\beta(0)\} \rangle =$$
$$= q_\gamma (\gamma_{\alpha\beta}^\gamma) F_1(q^2) + \delta_{\alpha\beta} F_2(q^2) \tag{5.27}$$

Making Borel transformion for both invariant structures we obtain the following sum rules (in the form (4.15)):

$$\frac{m^6}{8}\left[1-\exp\left(-\frac{W_1^2}{m^2}\right)\left(\frac{W_1^4}{2m^4}+\frac{W_1^2}{m^2}+1\right)\right]+\frac{bm^2}{32}\left(1-\exp\left(-\frac{W_1^2}{m^2}\right)\right)+$$

$$+\frac{a^2}{6}-\frac{a^2m_0^2}{24m^2}+\ldots=(2\pi)^4\lambda_1^2\exp\left(-\frac{m_N^2}{m^2}\right) \tag{5.28}$$

$$\frac{m^4}{4}\left[1-\exp\left(-\frac{W_2^2}{m^2}\right)\left(\frac{W_2^2}{m^2}+1\right)\right]-\frac{ab}{72}+\frac{17}{81}\frac{\alpha_s}{\pi}\frac{a^3}{m^2}=$$

$$=(2\pi)^4\lambda_1^4 m_N\exp\left(-\frac{m_N^2}{m^2}\right)$$

where the following notations are used

$$a=-(2\pi)^2\langle\bar{\Psi}\Psi\rangle,\qquad b=\langle(gG)^2\rangle$$
$$m_0^2\equiv\langle ig\bar{\Psi}\sigma_{\mu\nu}G^a_{\mu\nu}t^a\Psi\rangle/\langle\bar{\Psi}\Psi\rangle \tag{5.29}$$
$$\lambda_1 N_\alpha=\langle 0|j^\alpha_{IN}|N\rangle$$

Here N_α is the nucleon spinor and all quantities are normalized at μ of the order of 1 GeV, and therefore we do not indicate known anomalous dimensions of all operators.

The careful reader may notice that «continuum thresholds» W_1, W_2 are taken to be different in both cases. This is because only first one has positively-defined spectral density while the second one obtains negative contribution from negative parity states and may well be quite different. We also note that (not very important numerically) corrections due to gluonic condensate were calculated in separate paper [5.26] in the fixed-point gauge formalism.

Now we come to the isobar current, using only two tensor structures $g_{\mu\nu}(\hat{q})_{\alpha\beta}$ and $g_{\mu\nu}\delta_{\alpha\beta}$. The corresponding sum rules are as follows [5.28]:

$$\frac{m^6}{5}\left[1-\exp\left(-\frac{W_1^2}{m^2}\right)\left(\frac{W_1^4}{2m^4}+\frac{W_1^2}{m^2}+1\right)\right]-\frac{5}{72}bm^2\left(1-\exp\left(-\frac{W_1^2}{m^2}\right)\right)+$$

$$+\frac{4}{3}a^2-\frac{7}{9}a^2m_0^2/m^2=(2\pi)^4\lambda_\Delta^2\exp(-m_\Delta^2/m^2)$$

$$\frac{4}{3}am^4\left[1-\exp\left(-\frac{W_2^2}{m^2}\right)\left(\frac{W_2^2}{m^2}+1\right)\right]-\frac{2}{3}am_0^2m^2\left(1-\exp\left(-\frac{W_2^2}{m^2}\right)\right)-$$

$$-\frac{ab}{18}-\frac{4}{27}\frac{\alpha_s}{\pi}\frac{a^3}{m^2}=(2\pi)^4\lambda_\Delta^2 m_\Delta\exp(-m_\Delta^2/m^2) \tag{5.30}$$

where

$$\lambda_\Delta\Delta^\mu_\alpha=\langle 0|j_\Delta^{\mu,\alpha}|\Delta\rangle$$

Before we come to detailed analysis of these sum rules, let us note the striking similarity between (5.28) and (5.30). With only leading $O(a)$ corrections left it is possible to connect them just making simple rescaling of variables. In particular, for first sum rules one find the following simple scaling law [5.27], valid for all mass scales, including masses of the lowest resonances

$$(m_\Delta/m_N)=5^{1/6}=1.308;\qquad (m_\Delta/m_N)_{exp}=1.31 \tag{5.31}$$

Of course, (5.31) is «too precise», for «primed» resonances it is 1.12 and the second pair of sum rules produces even worse ratio $(10/3)^{1/3}$, but existence of such simple relations (with calculatable corrections) is interesting (at least as an argument against the popular opinion that «QCD does not make quantitative predictions»).

In Table 2 results of the analysis of baryonic sum rules are collected. Considering them one should keep in mind that the main problem here is reliable values for average values of certain four-fermion operators, producing the mass scale, but in all works mentioned the same estimates based on «factorization hypothesis» was used. Therefore, difference in results are only due to different fitting prescriptions and other details. Let us note in this connection that in first works [5.24, 5.25] no fitting was used at all, substituted by an assumption that at Borel parameter value equal to the resonance masses we are inside the «window», and sum rules where considered as some equation for crossing of their both sides. Rather arbitrary assumptions were also made about continuum threshold. However, with more advanced treatment we now have more detailed results including convincing cross test by nondiagonal correlators [5.28] etc.

Finally, nucleon coupling constants have some phenomenological significance. λ_1 is connected with proton lifetime problem while λ_2 determines the asymptotics of the formfactors, say [6.54]

$$F^{neutron}(Q^2)=\frac{100\,\lambda_2^2(\mu)}{27\,m^2Q^4}[4\pi\alpha_s(Q^2)]^2\left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu)}\right]^{4/27} \tag{5.32}$$

Calculations with strange baryons are sensitive to SU(3) violating effects in the QCD vacuum, see section 8.3. Strong simplification of the sum rules is obtained in the case when one quark is heavy [5.27]. Interesting feature of these sum rules is that states other than lowest resonance are strongly enhanced and the «window» for fitting is much worse than that for examples considered above. Still for spectral density in the form (4.14) there is well defined region of parameters, producing good fit and giving reasonable parameters for charmed baryon masses etc.

5.4. Currents with derivatives

In this section we discuss some applications of the sum rules for currents containing derivatives. There are two extreme cases, depending on whether we are interested in maximal or minimal Lorentz spin of the intermediate states.

In the former case one may investigate properties of the orbital excitations of hadrons, and the simplest case of the kind is the tensor f meson (one derivative), discussed by Shifman and Aliev [5.19]. However, as it was discussed in details in subsequent paper by Shifman [5.39], with increase in the number of derivatives n the nonperturbative corrections to the correlators increase strongly. On the other hand, dominance of lowest states is also weaker at larger n. Therefore the «window» necessary for sum rule applications soon disappears.

Another interesting applications are connected with lowest spin states. In the correlator of axial current considered in section 5.2 its decay constant was evaluated, and now we are going to study the following set of constants C_n:

$$\langle 0|\bar{u}\gamma_\mu\gamma_5(i\overleftrightarrow{D}_{\alpha_1})\dots(i\overleftrightarrow{D}_{\alpha_n})d|\pi(p)\rangle \underset{p\to\infty}{\Rightarrow} p_\mu p_{\alpha_1}\dots p_{\alpha_n} f_\pi C_n \tag{5.33}$$

Each derivative here is assumed to act as follows

$$f_1\overleftrightarrow{D}f_2 = f_1(Df_2)-(Df_1)f_2$$

and such matrix elements are sensitive to momentum distribution inside the pion. Introduction of such quantities was made by Chernyak and collaborators [6.49] in their formulation of exclusive processes theory, see more in chapter 6. It contains the so-called pion wave function $\varphi_\pi(\xi)$

$$\langle 0|\bar{d}(z)\hat{z}\gamma_5 \exp\left(\frac{ig}{2}\int_{-z}^{z} dx_\nu A^a_\nu t^a\right) u(-z)|\pi^+(p)\rangle =$$

$$=\sum_n \frac{i^n}{n!}\langle 0|\bar{d}z\gamma_5(iz\overleftrightarrow{D})^n u(0)|\pi^+\rangle = i(pz)f_\pi\int_{-1}^{1} d\xi\, e^{i\xi(zp)}\varphi_\pi(\xi) \tag{5.34}$$

which is connected with (5.33) by the relation

$$C_n=\int_{-1}^{1} d\xi\,\xi^n\varphi_\pi(\xi)\equiv\langle\xi^n\rangle \tag{5.35}$$

Here $\xi=x_1-x_2$ and x_1, x_2 are momentum fraction carried by quarks. Evidently, evaluation of such parameters is of great interest. Application of the sum rule method for this aim is considered in Refs [5.40—5.44]. As an example let us consider the nondiagonal correlator of the currents

$$j^A_\mu=\bar{u}\gamma_\mu\gamma_5 d$$

$$j^A_{\mu,\alpha_1,\dots,\alpha_n}=\bar{u}\gamma_\mu\gamma_5(i\overleftrightarrow{D}_{\alpha_1})\dots(i\overleftrightarrow{D}_{\alpha_n})d$$

Sum rules in notations analogous to those in section 5.2 are as follows

$$\frac{3}{4\pi^2(n+1)(n+3)}+\frac{\langle(gG)^2\rangle}{48\pi^2m^4}+\frac{16\pi}{81m^6}(11+4n)\alpha_s\langle\bar{\Psi}\Psi\rangle^2+\dots=$$

$$=\frac{1}{\pi m^2}\int ds e^{-s/m^2}\,\mathrm{Im}\,\Pi(s) \tag{5.36}$$

$$\frac{1}{\pi}\mathrm{Im}\,\Pi(s)=f_\pi^2\langle\xi^n\rangle_\pi\delta(s)+f_{A_1}^2\langle\xi^n\rangle_{A_1}\delta(s-m_{A_1}^2)+\frac{3O(s-W^2)}{4\pi^2(n+1)(n+3)}$$

The first term corresponds to free loop, in terms of the wave function it corresponds to

$$\varphi_\pi^{free\,quarks}(\xi)|_{\mu\to\infty}=\frac{3}{4}(1-\xi^2) \tag{5.37}$$

Nonperturbative corrections modify the l.h.s. in (5.36), which corresponds to modified wave function. Of course, considerations made above concerning the absence of the «window» at sufficiently large n is still valid, therefore only few first moments can in practice be estimated [5.40]. Approximate behaviour consistent with their values is provided by the following function

$$\varphi_\pi^{«real»}(\xi)|_{\mu\sim 1\,\mathrm{GeV}}\simeq\frac{15}{4}\xi^2(1-\xi^2) \tag{5.38}$$

which has typical two-maximum shape, demonstrating surprisingly large fraction of cases when nearly all momentum of the pion is carried by one of the quarks. Obviously, this fact strongly enhance probability of various exclusive reactions with pions, providing an explanation to a number of observations (see more in recent review [5.44]).

Much more combersome analysis is necessary in the baryonic case, although ideas are essentially the same. Defining three wave functions for the proton as a function of three quark momenta $x_i = p_i / p_{tot}$

$$|p^\uparrow\rangle = \iiint_0^1 dx_1\, dx_2\, dx_3\, \delta(1-x_1-x_2-x_3) \left\{ \frac{V(x)-A(x)}{2} \times \right.$$

$$\times |u^\uparrow(x_1) u^\downarrow(x_2) d^\uparrow(x_3)\rangle + \frac{V(x)+A(x)}{2} |u^\downarrow(x_1) u^\uparrow(x_2) d^\uparrow(x_3)\rangle -$$

$$\left. - T(x) |u^\uparrow(x_1) u^\uparrow(x_2) d^\downarrow(x_3)\rangle \right\} \tag{5.39}$$

we may then use symmetry properties following from Fermi statistics and find that only one of them is independent. Analysis of its first moments was recently made in Ref. [5.43], and the proposed «realistic» wave function looks as follows:

$$\varphi_N^{\text{«real»}} \equiv V(x) - A(x) =$$

$$= (40\lambda_2/m) x_1 x_2 x_3 (18x_1^2 + 4.6x_2^2 + 8.8x_3^2 - 1.7x_3 - 3.) \tag{5.40}$$

where constant λ_2 is given in Table 2. It is also very asymmetric wave function: about 70% of momentum is at u quark with spin parallel to that of the proton, see Fig.7. This wave function have passed few nontrivial tests, say it generates correct signs and magnitude for proton and neutron formfactors, reasonably reproduce branchings of $\Psi \to \bar{p}\, p$, $\chi \to \bar{p}\, p$ decays etc.

Evidently, these results are rather recent and they should be tested by other data, but they very clearly demonstrate how many new interesting applications may be generated by the evaluation of the correlators in the QCD vacuum, even in rather restricted region.

5.5. The three-point correlators

In this section we consider correlators of three currents

$$K_{ABC}(x, y, z) = \langle 0 | T\{ j_A(x) j_B(y) j_C(z) \} | 0 \rangle \tag{5.41}$$

which are of course much more complicated objects than the two-current ones considered above. At the moment their investigations are at rather preliminary stage, but in principle they may give a lot of useful information such as hadronic coupling constants. This statement is most spectacular if distances between all points are large, so one may consider the correlation as being

due to propagation of three lightest hadrons from the currents to some intermediate point, where they interact.

Historically such kind of applications was first used for charmonium radiative transitions (see Refs [5.64—5.67]), but for illustration we have selected another example, namely evaluation of the pion formfactor at some intermediate momentum transfer made recently in Refs [5.68, 5.69]. In this case two currents are the axial ones, producing the pions, while the third electromagnetic current interact with them.

In momentum representation our notations are as follows

$$K_{\nu\mu\lambda}(p, p', q) = -\int dx\, dy\, e^{ipx - iqy} \langle 0 | T\{ j_\nu^A(x) j_\mu^{E.M.}(y) j_\lambda^A(0) \} | 0 \rangle \tag{5.42}$$

and there are three independent variables q^2, p^2, $p'^2 = (p+q)^2$. If they are all large and negative all distances involved are small, so the correlator in question may be reasonably evaluated by simple «triangular» diagrams containing free quarks. Nonperturbative corrections are evaluated by standard OPE methods. Complications arise at the phenomenological side: one should use more complicated double dispersion relation

$$K_{\nu\mu\lambda}(s, s', Q^2) = \frac{1}{\pi^2} \int \frac{dp^2\, dp'^2}{(p^2+s)(p'^2+s')} \varrho_{\nu\mu\lambda}(p^2, p'^2, Q^2) \tag{5.43}$$

The contribution of two pion poles into spectral density ϱ is as follows:

$$\varrho_{\nu\mu\lambda}^{\pi\pi} = \pi^2 f_\pi^2 p'_\nu p_\mu (p+p')_\lambda F_\pi(Q^2) \delta(p^2) \delta(p'^2) \tag{5.44}$$

where $F_\pi(Q)$ is the pion formfactor under investigations. Unfortunately, there are also contributions with single pion pole, so it becomes much more difficult to suppress the somewhat weaker singularity with unknown coefficient than to suppress nonresonance contributions in ordinary sum rules. By the analogy to «continuum threshold» parameter, the nonresonant part of ϱ is written as

$$\varrho_{\nu\mu\lambda}^{cont} = \varrho_{\nu\mu\lambda}^{pert\,upb} \cdot \theta(p^2 + p'^2 - W^2) \tag{5.45}$$

Borel transformation in this case is also nontrivial. It is not evident that it is possible to put first $p^2 = p'^2$ and than work with the correlator as with function of only one variable: «subtractive polinomials» may appear. However, it does not take place, as it was proved in recent paper [5.78].

Let us now outline the region of parameters in which the method works. Momentum transfer to electromagnetic current should be sufficiently large in order to ensure small distances, while it can not also be too large because here the formfactor in question is too small, and the pion term (5.44) is not seen. As a result, we are limited to Q values of the order of few GeV, see Fig.8 where results of the works [5.58, 5.59] are compared with data.

There is one more interesting case, in which $Q=0$ and one of the currents is distributed homogeneously in space. In this case kinematics is as simple as for two-point correlators, but average values of all operators should be taken not over pure vacuum state, but over that influenced by the current. In particular, not only scalar operators contribute in this case.

Such type of analysis was recently performed in order to evaluate baryon magnetic moments, made both on the lattice [5.70, 5.71] and by OPE sum rules [5.72]. In the former case external weak magnetic field is applied to the system in rather straightforward way, while for the latter approach it makes problems for new «vacuum polarization» parameters should be introduced, say

$$\langle\Psi\sigma_{\mu\nu}\Psi\rangle_F \simeq \chi e_\Psi F_{\mu\nu} \langle\Psi\Psi\rangle_{F=0} \tag{5.46}$$

containing the quark electric charge e_Ψ and the so-called «susceptibility of the quark condensate» χ. I have mentioned this in order to demonstrate once more that different properties of hadrons are connected with more fundamental but physically analogous properties of the QCD vacuum. Unfortunately, direct evaluation of the value of this parameter from sum rules [5.72] are rather uncertain. (With the application of vector dominance it can be evaluated phenomenologically, see paper by Balitsky et al. [5.72], similar estimates were earlier made by V.L. Cernyak, unpublished.)

$\chi=-(6\div8)\ \mathrm{GeV}^{-2}$ Ioffe and Smilga

$\chi=-(2\div3)\ \mathrm{GeV}^{-2}$ Balitsky et al. (5.47)
Chernyak et al.

All papers report good results for magnetic moments, but comparing the details one can see that Refs [5.70—5.72] have made only first steps and left open a lot of questions.

5.6. Sum rules for heavy quarkoniums

Historically first application of the QCD sum rules by Shifman, Vainshtein and Zakharov [5.13] were connected with charmonium states, in particular the so-called «standard» value of the gluon

condensate was fixed in this analysis. There were some dramatic moments connected with further development of this theory, in particular evaluation of $\Psi-\eta_c$ splitting [5.45] have produced the result much smaller than suggested by experimental candidate for η_c at mass around 2.8 GeV. In contrast to potential models, it was found impossible for sum rules to obtain so low mass, which was clearly stated. Later Crystall Ball experiment have found correct η_c mass, being in agreement with sum rules expectations.

However, application of OPE methods to heavy quarkonia is much more difficult, for in this case interaction with vacuum fields is essentially suppressed by small dimensions of the system. Charmonium case is intermediate between light and heavy quark physics in many respects, so accuracy of the original analysis [5.13] remains rather uncertain and was a subject for recent polemics in literature.

In particular, much attention was payed to details of the fitting procedure and some examples of potential motion was used in order to test them. We are not going into such details and refer to recent review on this point [5.52] for critical discussion.

More important development was made in works [5.49], addressing the question of further nonperturbative corrections in the OPE series. With fixed-point gauge method considered in section 4.2 and analytical computer calculations all coefficients for $d=6$ and $d=8$ operators where calculated for vector and pseudoscalar currents. It is probably not necessary to reproduce here all these lengthy expressions and we only present some typical example. For current $\bar{c}\gamma_\mu c$ polarization operator can be shown to have the following OPE corrections

$$\Pi(Q^2)=-\frac{Q^2}{4\pi^2}\int_0^1\frac{v\,dv\,(3-v^2)}{[4m^2+Q^2(1-v^2)]}\left\{1+\frac{4}{3}\alpha_s\left[\frac{\pi}{2v}-\frac{v+3}{4}\left(\frac{\pi}{2}-\frac{3}{4\pi}\right)\right]\right\}+$$
$$+\frac{\langle(gG)^2\rangle}{48\pi^2Q^4}(-1+3J_2-2J_3)+\ldots+\frac{\langle O_4^{1+2}\rangle}{432\pi^2Q^8}\left(\frac{85}{12}+15J_1-\frac{57}{2}J_2-\right.$$
$$\left.-\frac{301}{3}J_3+\frac{871}{4}J_4-141J_5+30J_6-\frac{3}{2}J_7+\frac{Q^2}{m^2}-\frac{3}{4}\frac{Q^4}{m^4}\right)+\ldots \tag{5.48}$$
$$J_N\left(\frac{Q^2}{m^2}\right)\equiv\int_0^1\frac{dx}{[1+x(1-x)Q^2/m^2]^N}=\frac{(2N-3)!!}{(N-1)!}\left[\left(\frac{v^2-1}{2v}\right)^N v\ln\left(\frac{v+1}{v-1}\right)+\right.$$
$$\left.+\sum_{k=1}^{N-1}\frac{(k-1)!}{(2k-1)!!}\left(\frac{v-1}{2v}\right)^{N-k}\right],\qquad v^2\equiv1-4m^2/Q^2$$

where perturbative correction is given by Schwinger interpolation formula and $O(G^2)$ correction according to [5.45]. We have not included relatively small contribution of $d=6$ operators found in [5.49] and show only the coefficients for one typical operator of $d=8$. Complete set of such operators was defined as follows:

$$O_3^1 = g^3 f^{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu}; \qquad O_3^2 = g^4 \Big(\sum_{u,d,s} \Psi \gamma_\mu t^a \Psi\Big)^2$$

$$O_4^{1+2} = g^4 (d^{abc} G^a_{\mu\nu} G^b_{\alpha\beta})^2 + \frac{2}{3} g^4 (G^a_{\mu\nu} G^a_{\alpha\beta})^2$$

$$O_4^{1-2} = g^4 (f^{abc} G^a_{\mu\nu} G^b_{\alpha\beta})^2; \qquad O_4^{3-4} = g^4 (f^{abc} G^a_{\mu\alpha} G^b_{\alpha\nu})^2$$

$$O_4^{3+4} = g^4 (d^{abc} G^a_{\mu\alpha} G^b_{\alpha\nu})^2 + \frac{2}{3} g^4 (G^a_{\mu\alpha} G^a_{\alpha\nu})^2 \qquad (5.49)$$

$$O_4^5 = g^5 f^{abc} G^a_{\mu\nu} \Big(\sum_{u,d,s} \Psi \gamma_\mu t^b \Psi\Big)\Big(\sum_{u,d,s} \Psi \gamma_\nu t^c \Psi\Big)$$

$$O_4^6 = g^3 f^{abc} G_{\mu\nu} G_{\nu\lambda} G_{\lambda\mu;\alpha\alpha}; \qquad O_4^7 = g^4 \Big(\sum_{u,d,s} \Psi \gamma_\mu t^a \Psi\Big)\Big(\sum_{u,d,s} \Psi \gamma_\mu D^2 t^a \Psi\Big)$$

Evidently, it is not reasonable to consider their average value as some independent parameters, for it is hopeless to fit them, therefore some constructive model of the QCD vacuum is needed. One extreme was suggested by Shifman et al. [5.13], the so-called factorization hypophesis, providing in some sense minimal corrections. Another alternative discussed in [5.49] is the instanton model with parameters similar to those considered in chapter 2, which suggests much larger corrections. Fixing all averages of operators (5.49) and using their OPE coefficients evaluated explicitely it is possible to confront theory with data. At Fig.9 they are given as moment ratios (4.8), together with the SVZ curve [5.13] and those including further corrections in both models. This comparison have lead the authors of [5.49] to rather disappointing conclusion that the original analysis was not justified, at least for $n > 4$—6.

Let us look at this problem at somewhat different angle. It is demonstrated that OPE series are not convergent fast enough for certain vacuum models in the region of interest. It does not mean that such models contradict to data, but simply demands that the correlators be evaluated by some more reliable method. It is useful to recollect here the correlator shown at Fig.6, where also OPE

series become useless at some point, while no defects in the exact correlator for the instanton model are seen and it nicely goes on in agreement with data. In fact, similar behaviour is typical in all cases when strongly decreasing function is expanded in power series: the sum is much smaller than each term of the sign-changing series.

As far as the question is quantitative it is desirable to make accurate numerical calculations without any assumptions involved, other than vacuum models under consideration. Results of my calculations [5.53] are demonstrated at Fig.10, given as the ratio of the complete correlator to its «perturbative» version, corresponding to free propagation of charmed quarks with mass 1.3 GeV times the $O(\alpha_s)$ correction. Unit value of this ratio at small Euclidean time τ corresponds to asymptotic freedom, while deviations at larger τ are due to vacuum fields. In short, calculations where made by explicite averaging of Wilson loops in external field

$$W = \langle \mathrm{Tr}\, \mathrm{Pexp}\left(\frac{ig}{2} \int A^a_\mu t^a dx_\mu\right)\rangle \qquad (5.50)$$

over ensemble of free paths. Two models considered where the «homogeneous vacuum» model and the «instanton liquid» one, approximately corresponding to two models considered in Ref. [5.49] and at Fig.9. Both models where found able to fit data well enough, although with different condensate values (being 1.7 and 1.0 in SVZ unites). However, $O(G^4)$ terms are different by one order and OPE terms considered above are completely different. In conclusion, the condensate value found by Shifman et al. is evidently correct (up to probably factor of two) for quite different vacuum models, including those for which the OPE series completely break down.

Unfortunately, calculations reported above are not also free from simplifying approximations. In particular, relativistic effects were ignored. It is also desirable that perturbative effects should be taken into account not only in the first order and such work is now in progress. However, they can not affect qualitative conclusions made above, but only somewhat modify numbers involved.

Coming to theory of bottonium sum rules pioneered by Voloshin [5.56], we start with general comments on very heavy quark case. Obviously this problem can be considered in the nonrelativistic framework. Suppose a quark makes free propagation during the time t: in average its distance from the initial point becomes proportional to $(t/mass)^{1/2}$ and at large mass it is negligible. Therefore, quark-antiquark pair under such conditions have very small dipole moment and respectively their interaction with vacuum fields is suppressed.

This consideration was oversimplified, for close nonrelativistic quarks interact by Coulomb forces, and the Coulomb factor α_s/v (where v stands for velocity) is not small. Therefore, this interaction produces large effect and it should be nonperturbatively taken into account. In particular, it makes the pair dipole moment even smaller than considered above.

In order to demonstrate how important are Coulomb effects let me mention Ref. [5.50] where they were ignored, with the conclusion that the gluon condensate value following from charmonium and bottonium sum rules are different by nearly two orders of magnitude! In Ref. [5.54] Coulomb forces where included in very crude approximation (the running coupling constant was substituted by the constant one at the average distance), with the conclusion that the condensate should be increased by one order. So strong deviations are not striking because, say, at distance $\tau_0=2$ fermi Coulomb effects change the correlator by the factor 20—50, while the nonperturbative correction is only about 20—30%. Obviously, bottonium sum rules are not the best place to fix the condensate!

Of course, great sensitivity to perturbative effects of bottonium correlators has also its nice features, for it can be used in order to evaluate the value of the fundamental lambda parameter. Such attempt (although in very simplified approach, containing an assumption about «freezing» of coupling constant at α_s about 0.3) was made in Ref. [5.46].

In my paper [5.53] Coulobm effects were taken into account by the averaging over ensemble of paths of the following time-ordered exponent

$$W^{Coulomb}=\langle \mathrm{Tr\,Texp}\left\{ \frac{ig}{2}\int A^a_\mu t^a_{(q)}\, dx^{(q)}_\mu - \frac{ig}{2}\int A^a_\mu t^a_{(\bar q)}\, dx^{(\bar q)}_\mu + \right.$$
$$\left. +\int d\tau \left[\frac{\alpha_s(R_{q\bar q})}{4R_{q\bar q}}\right]\left[t^a_{(q)}\, t^a_{(\bar q)}\right]\right\}\rangle \qquad (5.51)$$

where R is the distance between quark and antiquark at the given moment and $t^a_{(q)}t^a_{(\bar q)}$ are the product of «colour vectors» for quark and antiquark. Note that three terms do not commute, for both external field and mutial interaction can affect the quark colour state.

Apart from the nonrelativistic approximation used for quark motion and the instantaneous interaction, we have ignored here direct interaction between the external and Coulobm fields, which is possible if quarks are sufficiently close.

Results of these calculations are shown at Fig.11, together with experimental data for the logarithmic derivative of the correlator:

$$F(\tau)=-\frac{d}{d\tau}\ln\langle 0|T\{j(\tau)\, j(0)\}|0\rangle$$

$$F(\tau)\xrightarrow[\tau\to\infty]{} m_\Upsilon \qquad (5.52)$$

The two models are the same as in charmonium case, and they both are consistent with data well enough. Among the conclusions obtained in the course of this work are the following. First, Coulomb effects mainly contribute around the points of current application, were quarks are most close. Second, main effect of external field is «colour rotation», and even small admixture of colour octet state of the pair strongly affects the Coulomb effects. No details of the field other than its general intensity are important.

Concluding this section I may comment, that the QCD sum rules for heavy quarkonia are still very actively developing field. Impressive and still growing amount of phenomenological information involved in it is potentially useful for the understanding of vacuum parameters, in particular Λ and $\langle (gG)^2\rangle$ but a lot of work is still needed in order to extract their reliable values. There is not much hope to obtain more detailed information on the vacuum structure though, for which correlators considered in two next sections are suited much better.

5.7. The pseudoscalar currents and instantons

In the previous section we have considered the complications arising if the interaction with vacuum fields is suppressed. In this and the next sections we are going to discuss what happens if it is too strong.

Before we come to correlators and sum rules let us recall some phenomenological facts connected with pseudoscalar mesons. In many respects they differ from other hadronic multiplets, say from vector or tensor ones. First of all, in this case strange quarks are not so much separated from the nonstrange ones as in latter cases.

On the contrary, η meson is almost SU(3) octet particle, while η' is close to SU(3) singlet. The second strange feature of η', its large mass, is the famous U(1) problem already mentioned in the Introduction.

Somewhat less widely known fact of similar kind is connected with pion. We remind that it is the lightest hadron because of its Goldstone nature and for massless u, d quarks it would be massless too. Thus, in order to discuss whether pion is light or heavy it is needed to single out quark masses as some parameters external to QCD. Introducing the coefficient

$$K=\frac{m_\pi^2}{m_u+m_d}\simeq 1.5\div 2\ \text{GeV} \tag{5.53}$$

one may indeed say that pion is surprisingly heavy particle.

Related fact noticed by Novikov et al. [5.56] is that for pseudoscalar currents with pion quantum numbers the «continuum threshold» parameter is relatively large as well. We remind that in this case this parameter is defined by the following parametrization of the spectral density:

$$\text{Im}\ \Pi(s)=\pi f_\pi^2 K^2\delta(s)+\theta(s-W^2)\ \text{Im}\ \Pi^{pert}(s) \tag{5.54}$$

and the so-called «duality» relation between the resonance and Π^{pert}: the integrated cross section should correspond to free quark behaviour, otherwize asymptotic freedom at small distances becomes violated. Writing this condition as follows

$$\frac{3}{16\pi^2}W^2\left(\ln\frac{W}{\Lambda}\right)^{-8/9}=f_\pi^2K^2 \tag{5.55}$$

one obtains relatively large value $W^2=2$—$3\ \text{GeV}^2$, about twice larger than that found in vector and axial channels.

Now we come to OPE evaluation of the relevant correlator [5.13]

$$\Pi(Q)^2=\left\{\frac{3}{8\pi^2}Q^2\ln\frac{Q^2}{\mu^2}-\frac{1}{2Q^2}(m_u\bar{u}u+m_d\bar{d}d+\right.$$
$$\left.+\frac{\alpha_s}{8\pi Q^2}\langle(G^a_{\mu\nu})^2+\langle O_1+O_2\rangle/Q^4+\ldots\right\}\left(\ln\frac{\mu}{\Lambda}\right)^{-8/9} \tag{5.56}$$

where two four-fermion operators are defined as follows

$$O_1=-\pi\alpha_s(\bar{u}\sigma_{\mu\nu}t^a u)(\bar{d}\sigma_{\mu\nu}t^a d)$$

$$O_2=\frac{\pi\alpha_s}{2}[(\bar{u}\sigma_{\mu\nu}t^a u)^2+(\bar{d}\sigma_{\mu\nu}t^a d)^2]+$$
$$+\frac{\pi\alpha_s}{3}[(\bar{u}\gamma_\mu t^a u)+(\bar{d}\gamma_\mu t^a d)][\sum_{u,d,s}(\bar{\Psi}\gamma_\mu t^a\Psi)]$$

Using the same estimates as in section 5.2 for their average values it is easy to see that these effects are too weak to produce the necessary value of parameter W. Respectively, pion coupling to pseudoscalar current if fitted turns to be too small as well.

With such impressive list of puzzles in mind let us consider their possible resolution. Nearly all ideas suggested for it are connected with instantons. In chapter 2 we have already mentioned t'Hooft's comment that the instanton-induced interaction of light quarks explicitely violates the U(1) symmetry. Possible role of this interaction in mixing of strange and nonstrange quark sectors was first discussed by Geshkenbein and Ioffe [5.55], and they even made attempt to estimate η—η' mixing angle. More systematic discussion of such anomalies was made by Novikov et al. [5.56], who have presented convincing list of arguments that all zero-spin channels are characterized by unexpectedly strong nonperturbative effects.

Attempts to connect all these phenomena by some simplified model were made in my works [5.57]. It was already discussed in chapter 2 and contains two parameters being the instanton density n and their typical dimension

$$\frac{du^+}{d^4x\,d\varrho}=n_+\,\delta(\varrho-\varrho_c) \tag{5.57}$$

Fitting their values in one way it is interesting whether other effects can really be reproduced. But before we come to their discussion let me emphasize another important aspect of the problem: we have also to understand why the instanton-induced effects do not spoil good results obtained in many other cases, say in vector and axial channels discussed in section 5.2. To avoid misunderstanding we may say here that power correction included in this analysis may also be the instanton effect. Moreover, the instanton density was in [5.57] fixed from the condensate value

$$n_+\simeq\langle(gG)^2\rangle/64\pi^2 \tag{5.58}$$

By «instanton effects» we mean here corrections which are not given by standard power terms of the OPE series. Examples of such nonpower terms were discussed in section 4.3, they correspond to part of the correlator regular at $x \to 0$ and they where not considered in the preceeding sections.

In Refs [5.55, 5.56] it was emphasized that such contributions are suppressed in vector and axial channels due to chiral properties of t'Hooft zero modes. However, it is true only if quarks are sufficiently light. In the presence of spontaneous chiral symmetry breaking it is not at all evident that its remnants (such as considered selection rule) are left. In order this to be the case one needs that [5.57]

$$[M_{eff}(\varrho_c)\varrho_c]^2 \ll 1 \tag{5.59}$$

Important, that evaluation of quark effective mass made in chapter 2 indeed leads to its value about 0.1.

Let us now return to sum rules for the pseudoscalar current. In section 4.3 it was explained that in the leading order in (5.59) the instanton contribution is simply given by the substitution of zero modes instead of quark fields in the currents. This simple calculation leads to somewhat lengthy formula (in order to make contact with traditionally used sum rules one need to perform Fourier and Borel transformations):

$$\frac{1}{\pi}\int_0^{W^2} \operatorname{Im}\Pi^{pert}(s)\,e^{-s\tau^2}\,ds + \frac{\langle (gG)^2\rangle}{32\pi^2} + \tau^2\langle O_1 + O_2\rangle + \Pi^{inst}(\tau) + \ldots =$$

$$= f_\pi^2 K^2 \exp(-m_\pi^2\tau^2) \tag{5.60}$$

where

$$\Pi^{inst}(\tau) = \left\{ \frac{\sqrt{2}\,n_+}{M_{eff}^2\tau^2}\iint_0^\infty d\alpha\, d\beta \operatorname{ch}\alpha \operatorname{ch}\beta \times \right.$$

$$\left. \times\left(\frac{\varrho_c^5}{\tau^5}t^3 - \frac{3\varrho_c^3}{2\tau^3}t\right)\exp\left(-\frac{\varrho_c^2 t^2}{\tau^2}\right)\right\}\left(\ln\frac{1}{\tau\Lambda}\right)^{-8/9} \tag{5.61}$$

$$t \equiv (\operatorname{ch}\alpha + \operatorname{ch}\beta)/2$$

One should also note that instanton model leads also to some amplification of power four-fermion operators of the OPE series. Note that in contrast to operator O_2 (and those met in section 5.2), the operator O_1 obtains the nonzero average already at one-instanton level:

$$\langle O_1\rangle = \frac{24\alpha_s n_+}{5\pi M_{eff}^2 \varrho_c^4} \tag{5.62}$$

and therefore it is several times larger than $\langle O_2\rangle$, estimated by factorization hypothesis.

The correlator following from such calculation is shown at Fig.12, it should be compared to the pion contribution. The normalization is such that resonances contribution is proportional to $\exp[-(mass\cdot\tau)^2]$, and in the approximation considered pion mass should be equal to zero. And indeed, up to about $\tau < 1.5$ (in 1/GeV) the correlator is nearly independent on τ. It is evident from that Figure that without large instanton effect it is not possible to obtain the massless pion.

Let us now continue this game and consider the correlator for kaons. The instanton contribution in this case is suppressed by the following factor:

$$\xi_k = \frac{M_{eff}}{M_{eff}+m_s} \simeq 0.6 \tag{5.63}$$

(which is also true for instanton-induced power correction).

Comparizon of the correlator with $\exp[-(M_k\cdot\tau)^2]$ made at Fig.13 really produces reasonable mass for the kaon. Note the important difference with standard SU(3) breaking caused entirely by additive effect of the strange quark mass. Now effect is not additive and rather nontrivial: kaon is heavier because the instanton-induced effects which make pion massless are suppressed!

Analogous results are obtained for the η, suppression factor is now

$$\xi_\eta = \frac{M_{eff} - m_s/3}{M_{eff}+m_s} \tag{5.64}$$

However, the most interesting channel is the famous η' one, in this case the instanton contribution differs with that for the pion one by the factor

$$\xi_{\eta'} = -\frac{2}{3}\,\frac{m_s + 3M_{eff}}{m_s + M_{eff}} \simeq -1.4 \tag{5.65}$$

which is negative and therefore correlator fall down with τ even faster than the perturbative one, see Fig.14. In some sense, this effect is too strong for fit give mass value around 1.5 GeV. However, it may well be the contribution of the meson $i(1420)$ discovered recently by Crystal Ball, its production rate in $\Psi \to i + \gamma$ is comparable to that for $\Psi \to \eta' + \gamma$.

Finally, in [5.57] was considered the nondiagonal correlator of $\eta - \eta'$ type, which is also the SU(3) violating effect caused by strange quark mass. It also corresponds to phenomenology and produce the mixing angle of correct magnitude.

Note the general tendency of these calculations: the resonance couplings and continuum thresholds are about the same in all cases (some phenomenological argument for it is that «primed» resonances all are around 1400 MeV), but masses of lowest resonances are completely different. This is because the latter parameters are connected with «small distance physics», being nearly completely perturbative, while the masses of lowest resonances are completely determined by nonperturbative effects. Interesting that the latter are completely SU(3)-asymmetric, for the strange quark mass is not small compared to «effective mass» entering the factors above.

Of course, the model used is oversimplified, also it is not quite clear whether all numbers included in these calculations are sufficiently accurate. However, great success of these simple calculations presumably indicate that we are on the right way.

5.8. Correlators of gluonic currents

In literature this topic is considered in two different frameworks, sometimes without their clear separation. The first one is connected with pure gauge theories while the latter with gluonic correlators in real world. Although these questions may indeed be related, it is far from being clear what is the qualitative effect of the virtual quarks.

We start with the former question, which was studied mainly on the lattice. We have already discussed in chapter 3 some facts connected with glueball spectroscopy and remind here that all states are relatively heavy: the scalar one has the mass of about 1.3 GeV and all the rest at least two times larger (the numbers come from the assumption that the string tension in this case is the same as in real world, but if one would consider instead lambda parameters to be fixed they become even larger). Now we emphasize another aspect of these calculations: the coupling constants to currents are very large as well, signalizing via the duality arguments that asymptotic freedom is violated at very small distances, at Q^2 about 10 GeV^2 or so.

It is clear that if one «switch on» the quark degrees of freedom, the glueballs become unstable, may be even very wide so that their experimental observation becomes difficult. However, qualitative features of the correlators such as strong violation of asymptotic freedom at small distances may in this case persist.

A set of arguments that it is indeed the case in QCD was suggested by Novikov et al. [5.58]. We start their discussion with the pseudoscalar gluonic current defined as follows

$$j_P = i\alpha_s G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} \tag{5.66}$$

$$P(Q^2) = i\int dx\, e^{iqx}\langle 0|T[j_P(x)j_P(0)]|0\rangle \tag{5.67}$$

In this case argumentation is essentially the same as used for the pion in the preceeding section. Coupling constants for resonances η' and i to this current can be estimated from decay rate of $\Psi\to\eta'+\gamma$, $i+\gamma$. (The main assumption here is that they proceed through $\Psi\to gg\gamma$ at sufficiently small distances.) The next step is based on duality argument: the resonance contribution should be equal to the «eaten» part of the continuum. As a result, one really finds surprisingly large value for continuum threshold

$$W^2 \sim 10\ \mathrm{GeV}^2 \tag{5.68}$$

For scalar current in [5.58] it was proven the following low energy theorem:

$$S(0) = \frac{24}{11N-2N_f}\langle (gG)^2\rangle \tag{5.69}$$

where

$$S(Q^2) = i\int dx\, e^{iqx}\langle T[j_S(x)j_S(0)]\rangle$$

$$j_S = \alpha_S (G^a_{\mu\nu})^2$$

Its proof is rather simple and similar to that of relation (1.24). With such important information at hand as (5.69) it is possible to write down «improved» sum rules with one subtraction:

$$S(Q^2) = S(0) - \frac{Q^2}{\pi}\int \frac{\mathrm{Im}\, S(s)\, ds}{s(s+Q^2)} \tag{5.70}$$

which results after Borel transform in the following sum rule

$$\frac{2m^4}{\pi^2}\alpha_s^2 + S(0) + \frac{\alpha_s}{\pi}\langle (gG)^2\rangle + \ldots = \frac{1}{\pi}\int \mathrm{Im}\, S(s) e^{-s/m^2}\, ds \tag{5.71}$$

Note that $S(0)$ is much larger than ordinary power correction (the third term in (5.71)), so it is this parameter which breakes down the asymptotical freedom (the first term in (5.71)). Comparing two first terms one again finds that they are comparable at $m^2 \sim 20$

GeV^2. This conclusion makes a problem if one is interested in further corrections: it is apparent that further OPE terms [5.58]

$$S^{power}(Q^2)=\frac{\alpha_s}{\pi}\langle(gG)^2\rangle+\frac{\alpha_s}{\pi Q^2}\langle g^3 f^{abc}G^a_{\alpha\beta}G^b_{\beta\gamma}G^c_{\gamma\alpha}\rangle-$$
$$-\frac{\alpha_s}{2\pi Q^4}\langle(g^2 f^{abc}G^a_{\mu\nu}G^b_{\alpha\beta})^2\rangle+\frac{7\alpha_s}{\pi Q^4}\langle(g^2 f^{abc}G^a_{\mu\alpha}G^b_{\alpha\nu})^2\rangle+\dots \qquad (5.72)$$

are not large enough in order to continue this trend if, one estimates the operator average values by factorization hypophesis.

Again, the explanation proposed is connected with sufficiently small instantons. Their effect are seen at two levels (as for pion correlator considered above). First, the gluonic matrix elements entering (5.72) are enhanced, see [5.57]. Second, there appear «nonpower effects», related to regular terms in the correlators. The latter effect is evaluated by very simple calculation: gluonic field in the current should be substituted by the instanton field. In the correlator (5.72) the contribution is than equal to

$$\beta S^{inst}(Q^2)=\int dn(\varrho)\cdot 32\pi^{3/2}(m\varrho)^5\exp(-m^2\varrho^2); \qquad (m\varrho\gg 1) \qquad (5.73)$$

and using parameters of the instantons from the «instanton liquid» model we indeed obtain effect of needed sign and magnitude, supporting the trend started by $S(0)$ term. Moreover, fitting of the mass value from the correlator behaviour with m produces 1.4 GeV [5.63], similar to that obtained from quite different considerations by Shifman [5.59] and to lattice results for gauge systems. Note also that experimentally there exists the suitable resonance $\varepsilon(1300)$. It is not the lowest scalar one, being a close doublet S^*, δ with different isospins, but the latter are (according to common wisdom) attributed to four-quark states.

Now we turn to comparizon between scalar and pseudoscalar gluonic correlators, which is extremely instructive. Novikov et al. [5.58] have made very radical assumption that vacuum fields are «locally selfdual» $G^a_{\mu\nu}=\pm i\tilde{G}^a_{\mu\nu}$ with plus or minus sign depending on the place. (This assumption is of course motivated by the picture of separated instantons). If so, the correlators considered are identical

$$S(Q^2)=-P(Q^2) \qquad (5.74)$$

(Up to the general sign: to avoid misunderstanding we remind here that all correlators and sum rules in this chapter are written

in Minkowsky notations, therefore the imaginary unit in selfduality condition. Note also that both $S(Q)$ and $P(Q)$ have positive spectral density.)

If (5.74) is assumed, it is naturally to find the low energy theorem for $P(0)$ similar to (5.69). Unfortunately, it can not take place because due to «anomaly relation» the pseudoscalar gluonic current is complete divergence of the quark axial current

$$\partial_\mu\Big(\sum_{u,d,s}\Psi\gamma_\mu\gamma_5\Psi\Big)=\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} \qquad (5.75)$$

and therefore in chiral limit $P(0)$ should be zero!

In order to reconcile selfduality condition (5.74) with this fact Novikov et al. have suggested the following interesting assumption: by «switching on» the quarks one modifies only the low-energy behaviour of the correlator $P(Q)$, in particular large constant $P(0)$ turns to large coupling to low-lying η' meson. Following this idea one may apply data for η' coupling mentioned above and evaluate $P(0)$ in the absence of quarks:

$$-P^{(no\,quarks)}(0)\simeq\frac{1}{\pi}\int\frac{dS}{S}(\operatorname{Im}P(S))_{\eta'}=\Big(\frac{4\pi}{3}f_{\eta'}m_{\eta'}\Big)^2\simeq .4\ \text{GeV}^4 \qquad (5.76)$$

We remind that $P(0)$ is proportional to «topological susceptibility» discussed in chapter 3, and that its value was earlier estimated in other ways in Refs [5.60—5.62] with the results of the same order as (5.76). In any case, there are good reasons to believe that «local selfduality» condition is fulfilled in the QCD vacuum, and absolute numbers like (5.76) suggest about one instanton or antiinstanton per (fermi)^4. As we have already mentioned, this feature is not observed so far on the lattice, producing much smaller $P(0)$.

Concluding this section we may say that there are many intriguing questions concerning gluonic correlators which are left open, partly because phenomenological information is so far very limited. More efforts are needed, including those by experimentalists, in order to obtain it.

REFERENCES

5. CORRELATORS AND SUM RULES. THE APPLICATIONS

Hadronic spectroscopy on the lattice

The quenched approximation

5. 1. *Hamber H. and G. Parisi.* Phys. Rev. Lett. 47 (1981) 1792.

5. 2. *Marinari E., G. Parisi and C. Rebbi.* Phys. Rev. Lett. 47 (1981) 1795.
F. Fucito, G. Martinelli, C. Omero, G. Parisi, R. Petronzio and F. Rapuano. Nucl. Phys. B210 (1982) 407.
Hamber H. and G. Parisi. Phys. Rev. D27 (1983) 208.
Hamber H., E. Marinari, G. Parisi and C. Rebbi. Phys. Lett. 108B (1982) 314.

5. 3. [illegible] Preprint DESY 83-094. Hamburg, 1983.

5. 4. *Hamber H.* Phys. Rev. D27 (1983) 2239.

5. 5. *Weingarten D.* Nucl. Phys. 215B [FS7] (1983) 1.

5. 6. *Lipps H., G. Martinelli, R. Petronzio and F. Rapuano.* Phys. Lett., B26 (1983) 250.

5. 7. *A. Ali, I. Montvay.* A calculation of the electromagnetic vacuum polarization in lattice QCD.—Preprint DESY 82-081. Hamburg, 1982.

5. 8. *Gupta R. and A. Patel.* Nucl. Phys. 126B (1983) 152.

Fermions included

5. 9. *Hasenfratz A., Z. Kunszt, P. Hazenfratz and C.B. Lang.* Phys. Lett. 110B (1982) 289.
Montvay I. Phys. Lett. 132B (1983) 393.

5.10. *V. Azscoiti, A. Nakamura.* Phys. Rev. D27 (1983) 2559.

Comments on lattice size effects

5.11. *Hasenfratz P. and I. Montvay.* Phys. Rev. Lett. 50 (1983) 309.

5.12. *Martinelli G. and G. Parisi.* Phys. Lett. 122B (1983) 283.

Sum rules based on OPE

Mesons made of light quarks

5.13. *M.A. Shifman, A.I. Vainstein and V.I. Zakharov.* Nucl. Phys. B147 (1979) 385, 448, 519.

5.14. *Eidelman S.I., Kurdadze L.M. and Vainstein A.I.* Phys. Lett. 82B (1979) 278.

5.15. *Launer G., S. Narison and R. Tarrach.* Nonperturbative QCD vacuum from data on e^+e^- into $(I=1)$ hadrons. TH 3712-CERN. Geneva, 1983.

5.16. *Reinders L.J., H.R. Rubinstein and S. Yazaki.* Nucl. Phys. B196 (1982) 125.
Reinders L.J. QCD sum rules. An introduction and some applications. TH 3701 CERN. Geneva, 1983.

5.17. *Narisson S. and E.de Rafael.* Phys. Lett. 103B (1981) 57.
Narisson S., N. Paver, E.de Rafael and D. Treleani. Nucl. Phys. B212 (1983) 365.

5.18. *Narison S.* Phys. Lett. 104B (1981) 485.

5.19. *Aliev T.M. and M.A. Shifman.* Phys. Lett. B112 (1982) 401.

5.20. *Chakrabarti J., S. Kalara and L.P.S. Singh.* Phys. Lett. 109B (1982) 493.

Exotic quark-antiquark-gluon currents

5.21. *Balitsky I.I., D.I. Dyakonov and A.V. Yung.* Phys. Lett. 112B (1982) 71.

5.22. *Zhitnitsky A.R. and I.R. Zhitnitsky.* Yad. Fiz. 37 (1983) N 6.

5.23. *Covaets J., F.de Viron, D. Gusbin and J. Weyers.* Phys. Lett. 28B (1983) 262.

Baryons

5.24. *Ioffe B.L.* Nucl. Phys. B188 (1981) 317, (E) B191 (1981) 591.

5.25. *Chung Y., H.G. Dosch, M. Kremer and D. Schale.* Nucl. Phys. B197 (1982) 55.

5.26. *Smilga A.V.* Yad. Fiz. 35 (1982) 473.

5.27. *Shuryak E.V.* Nucl. Phys. B198 (1982) 83.

5.28. *Belyaev V.M. and B.L. Ioffe.* ZHETF (Soviet Phys.—JETP) 83 (1982) 876.

5.29. *Mallik S.* Nucl. Phys. B206 (1982) 90.

5.30. *Reinders L.J., H.R. Rubinstein and S. Yazaki.* CERN preprint TH-3405 1982; Phys. Lett. 120B (1983) 209.

5.31. *Esprin D., P. Pascual and R. Tarrach.* Nucl. Phys. B214 (1983) 285.

5.32. *P. Pascual and R. Tarrach.* Phys. Lett 116B (1982) 443.

5.33. *Zhitnitsky I.R.* Nucleon wave function and nucleon formfactor in QCD. Preprint of INP 82-155. Novosibirsk, 1982.

5.34. *V.M. Belyaev.* Phys. Lett. 127B (1983) 254.

Hadrons made of light and heavy quarks

5.35. *Novikov V.A., M.A. Shifman, A.I. Vainstein and V.I. Zakharov.* Leptonic decays of charmed mesons. Proceedings of International Conference «Neutrino-78». La Fayette, 1978.

5.36. *Reinders L.J., H.R. Rubinstein and S. Yazaki.* Phys. Lett. 104B (1981) 305.

5.37. *Shuryak E.V.* Nucl. Phys. B198 (1982) 83.

5.38. *Chernyak V.L., A.R. Zhitnitsky and I.R. Zhitnitsky.* On the wave functions of the mesons containing *s, c, b* quarks. Preprint INP 82-163. Novosibirsk, 1982.
Aliev T.M. and V.L. Eletsky. On the leptonic decays of pseudoscalar *d* and *b* mesons. Preprint ITEP-47, 1983; Sov. J. of Nucl. Phys., in press.

Currents with derivatives, orbital excitations

5.39. *Shifman M.A.* On quark-hadron duality for orbital excitations. Preprint ITEP-49. Moscow, 1982.

Hadrons with derivatives, hadronic wave functions

5.40. *Chernyak V.L. and A.R. Zhitnitsky.* Nucl. Phys. B201 (1982) 492.

5.41. *Chernyak V.L., A.R. Zhitnitsky and I.R. Zhitnitsky.* Nucl. Phys. B204 (1982) 477.

5.42. *Chernyak V.L., A.R. Zhitnitsky and I.R. Zhitnitsky.* Yad. Fiz. 38 (1983) 1074, 1277.

5.43. *Zhitnitsky I.R.* Nucleon wave function and nucleon formfactor in QCD. Preprint INP 82-155. Novosibirsk, 1982.

5.44. *Chernyak V.L. and A.R. Zhitnitsky.* Asymptotic behaviour of exclusive processes in QCD.—Phys. Reports, in press.

Sum rules for charmonium and bottonium channels

5.45. *M.A. Shifman, A.I. Vainstein and V.I. Zakharov*. Phys. Lett. 77B (1978) 80; Nucl. Phys. B147 (1979) 385, 448.
5.46. *Voloshin M.B.* Yad. Fiz. 29 (1979) 1368; Nucl. Phys. 154B (1979) 365; Yad. Fiz. 34 (1981) 310.
5.47. *Reinders L.I., H.R. Rubinstein and S. Yazaki*. Nucl. Phys. B186 (1981) 109.
5.48. *Bell J.S. and R.A. Bertelmann*. Nucl. Phys. B177 (1981) 218.
5.49. *Nikolaev S.N. and A.V. Radyushkin*. Nucl. Phys. B213 (1983) 285. QCD sum rules with $O(G^4)$ contributions ... Preprints P2-82-720, P2-82-914. Dubna, 1983; Yad. Fis. 39 (1984) 147.
5.50. *Zalewska A. and K. Zalewski*. Phys. Lett. 133B (1983) 441.
5.51. *Baier V.N. and Pinelis Yu.F.* Small size vacuum fluctuations and QCD sum rules. Preprint INP 82-115. Novosibirsk, 1982.
5.52. *Novikov V.A., M.A. Shifman, A.I. Vainstein and V.I. Zakharov*. Use and misuse of QCD sum rules. Preprint ITEP-71. Moscow, 1983; Nucl. Phys., in press.
5.53. *Shuryak E.V.* What can be learned from heavy quarkonium sum rules about QCD vacuum structure. Preprint INP 83-113. Novosibirsk, 1983.

Instanton-induced effects and pseudoscalar currents

Instanton-induced interaction violates U(1) symmetry

5.54. *G. t'Hooft*. Phys. Rev. D14 (1976) 3132.

Instantons and meson mixing

5.55. *Geshkenbein B.V. and Ioffe B.L.* Nucl. Phys. B166 (1980) 340.

Large deviations from OPE in spin-zero channels

5.56. *Novikov V.A., M.A. Shifman, A.I. Vainstein and V.I. Zakharov* Nucl. Phys. B191 (1981) 301.

Sum rules for pseudoscalar currents and instanton density

5.57. *Shuryak E.V.* Nucl. Phys. B203 (1982) 93; 214B (1983) 237.

Gluonic currents

Correlators of gluonic currents, low energy theorems and new large scale

5.58. *Novikov V.A., M.A. Shifman, A.I. Vainstein and V.I. Zakharov* Nucl. Phys. B165 (1980) 67; B191 (1981) 301.

Mass estimates for scalar gluoniums

5.59. *Shifman M.A.* Z. Phys. 9C (1981) 347.

Other phenomenological estimates for topological susceptibility

5.60. *Veneziano G.* Nucl. Phys. B159 (1979) 213.
5.61. *Witten E.* Nucl. Phys. B156 (1979) 269.
5.62. *Dyakonov D.I. and M.I. Eides*. Preprint LINP 639. Leningrad 1981.

Sum rules for scalar gluonic current in the instanton density

5.63. *Shuryak E.V.* Nucl. Phys. B203 (1983) 116.

The three point correlators

Psion radiative transitions

5.64. *Shifman M.A.* Pisma v ZHETF (JETP Letters) 30 (1979) 546.
5.65. *Novikov V.A. et al.* Phys. Reports 41C (1978) 1.
5.66. *Khodjamirian A.Yu.* Phys. Lett. 90B (1980) 460.
5.67. *Reinders L.I., H.R. Rubinstein and S. Yazaki*. Decay of heavy quark systems: effects of gluon condensate. Preprint RL-82-017.

The pion formfactor

5.68. *Ioffe B.L. and A.V. Smilga*. Preprint ITEP-27, 1982.
5.69. *Nesterenko V.A. and A.V. Radyushkin*. Preprint JINR E2-82-204.

Baryon magnetic moments

5.70. *Bernard C., T. Draper, K. Olynyk and M. Rushon*. Phys. Rev. Lett. 49 (1982) 1076.
5.71. *Martinelli G., G. Parisi, R. Petronzio and F. Rapuano*. Preprint CERN-TH 3334. Geneva, 1982.
5.72. *Ioffe B.L. and A.V. Smilga*. Proton and neutron magnetic moments in QCD. Preprints ITEP-24, ITEP-60. Moscow, 1983; Pisma v ZHETF 37 (1983) 250. *Balitsky I.I. and Yung A.V.* Phys. Lett. 129B (1983) 328.

Other hadronic constants

5.73. *Ioffe B.L. and A.V. Smilga*. Meson widths and formfactors at intermediate momentum transfers. ITEP-100, 1982.
5.74. *Reinders L.I., H.R. Rubinstein and S. Yazaki*. Hadron couplings to Goldstone bosons in QCD. Preprint of Weizmann institute WIS-82/24.
5.75. *Eletsky V.L., B.L. Ioffe and Ya.I. Kogan*. Omega-rho-pion coupling... Preprint ITEP-98, 1982.
5.76. *Eletsky V.L. and Ya.I. Kogan*. Rho-pion coupling... Preprint ITEP-170, 1982.
5.77. *Belyaev V.M. and Ya.I. Kogan*. Pisma v ZHETF (JETP Letters) 37 (1983) 611.
5.78. *Smilga A.V. and Shifman M.A.* Yad.Fiz. 37 (1983) 1613.
5.79. *Khatzimovsky V.M.* The omega-rho-pion coupling... Preprint INP 83-54. Novosibirsk, 1983.

T a b l e 1.

Decay constants for D and B mesons (in MeV)

f_D	f_B	Reference	
<250	—	Novikov et al.	[5.35]
220	140	Shuryak	[5.37]
—	276	Reinders et al.	[5.36]
160—170	90—100	Chernyak and Zhitnitsky	[5.38]
170	130	Aliev and Eletsky	[5.38]
$\sim f_\pi$	—	Hamber	[5.4]

T a b l e 2.

Coupling constants Λ_1, Λ_2 of the two nucleon currents and the continuum threshold parameter W_1

λ_1 GeV^3	W_1 GeV	λ_2 GeV^6	Reference	
.5±.07	2.	—	Ioffe	[5.24]
.35±.07	1.4±.15	—	Shuryak	[5.27]
.66±.11	—	1.±.3	Belyaev and Ioffe	[5.28]
—	2.3	—	Reinders et al.	[5.30]
—	—	.66±.06	Zhitnitsky	[5.33]
.71±.27	no cont.	—	Hamber and Parisi	[5.1]

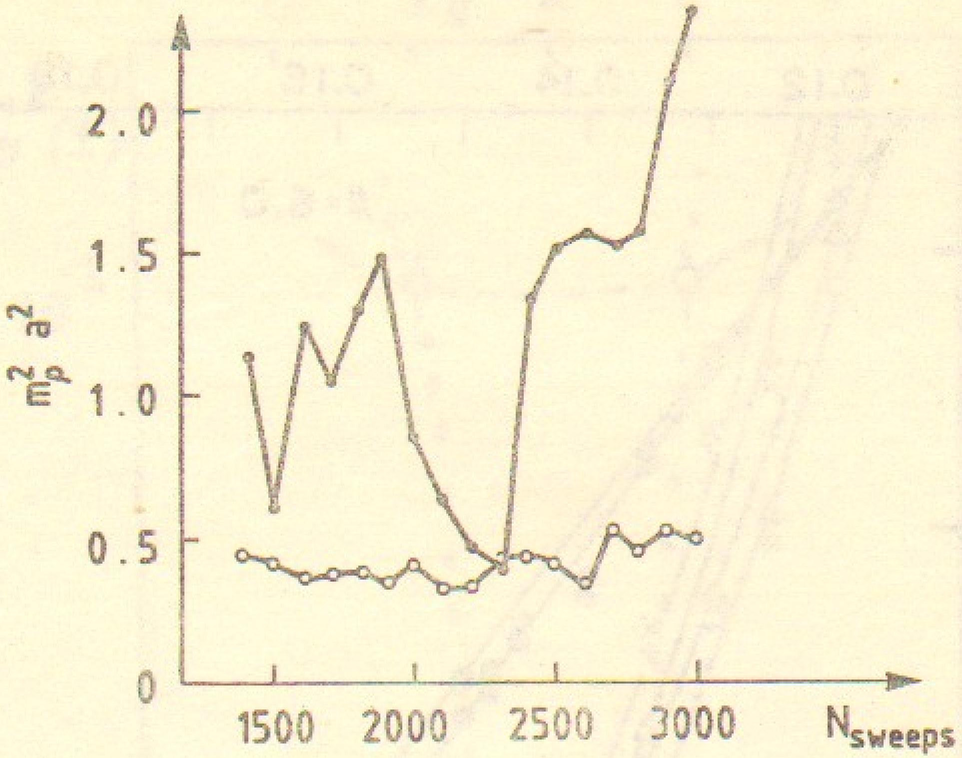

Fig.1. Fluctuations of the hadronic mass values as a function of iteration number. Closed and open points are given acccording to Refs [5.1, 5.6], suppression of the fluctuations with larger number of the lattice sites is clearly seen.

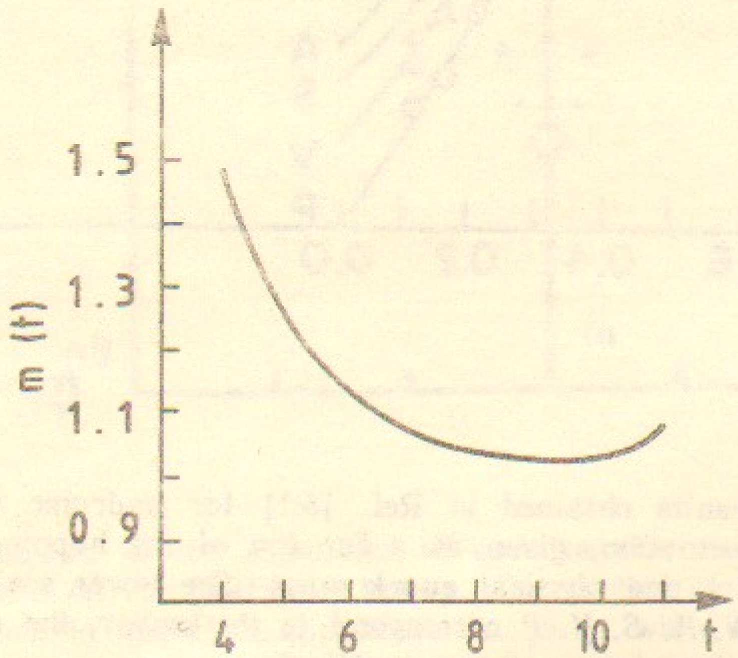

Fig.2. Logarithmic derivative of the correlator of nucleon current versus Euclidean time according to Ref. [5.6]. Approach to asymptotic value at large time (the nucleon mass) is made from above due to the contribution of higher states.

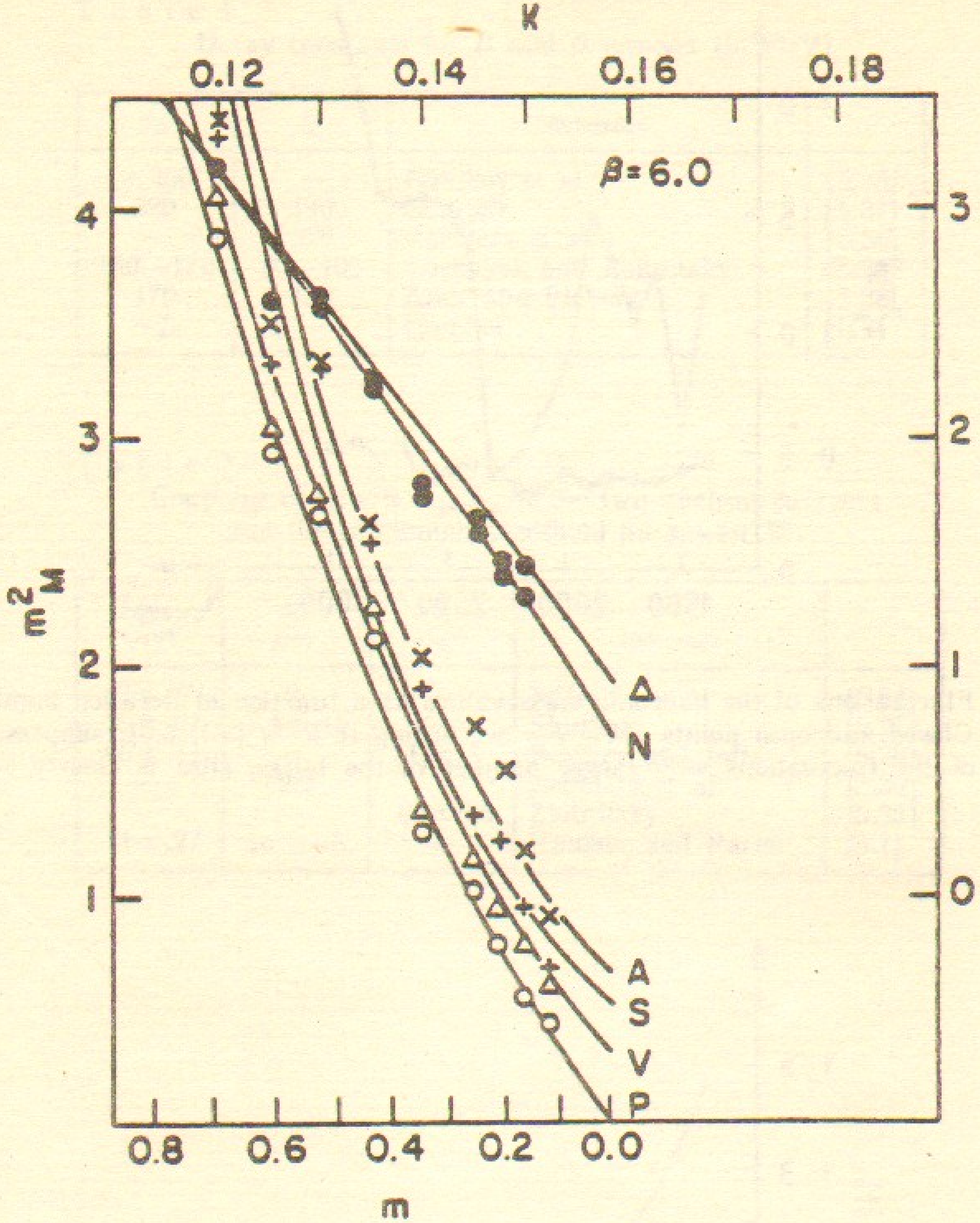

Fig.3. Some set of results obtained in Ref. [5.1] for hadronic spectroscopy in quenched approximation, given as a functon of the hopping parameter K (the upper scale) and physical quark mass (the lower scale). The curves marked by Δ, N, A, S, V, P correspond to the isobar, the nucleon, scalar, vector and pseudoscalar mesons, respectively.

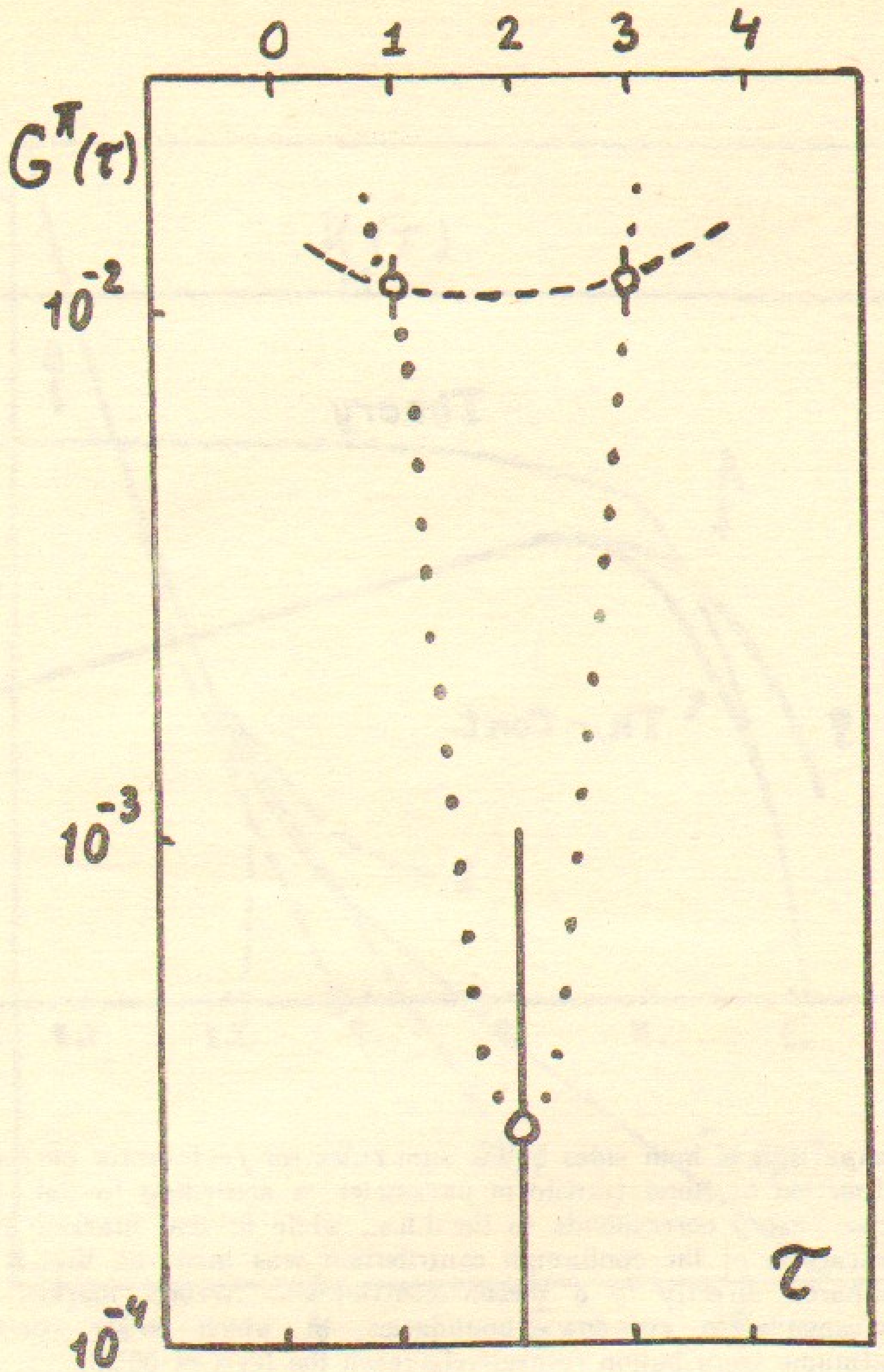

Fig.4. Dependence of the correlator of pseudoscalar current with pion quantum numbers on the Euclidean time τ. The dashed curve corresponds to results obtained in quenched approximation [5.1], while the points and the dotted line (drown just to guide the eye) correspond to the calculation [5.10] accounting for virtual quarks by the pseudofermion method.

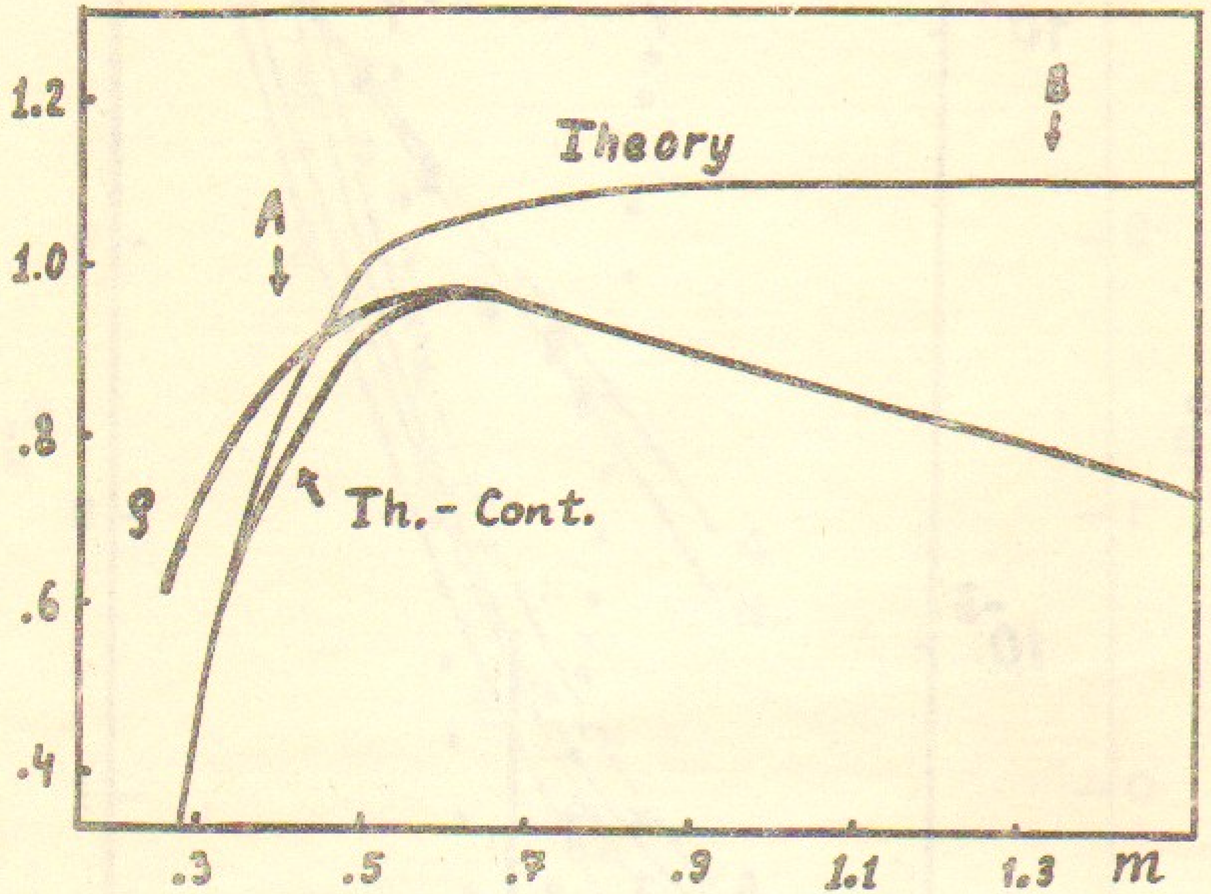

Fig.5. Comparison of both sides of the sum rules for $I=1$ vector current (5.6) as a function of Borel transform parameter m according to Ref. [5.13]. The curve *Theory* corresponds to the l.h.s., while in that marked by *Th.-cont.* subtraction of the continuum contribution was made so that it should be compared directly to ϱ meson contribution. Arrows marked A and B corresponds to «window» boundaries, at which power correction and continuum contribution respectively reach the level of 30%.

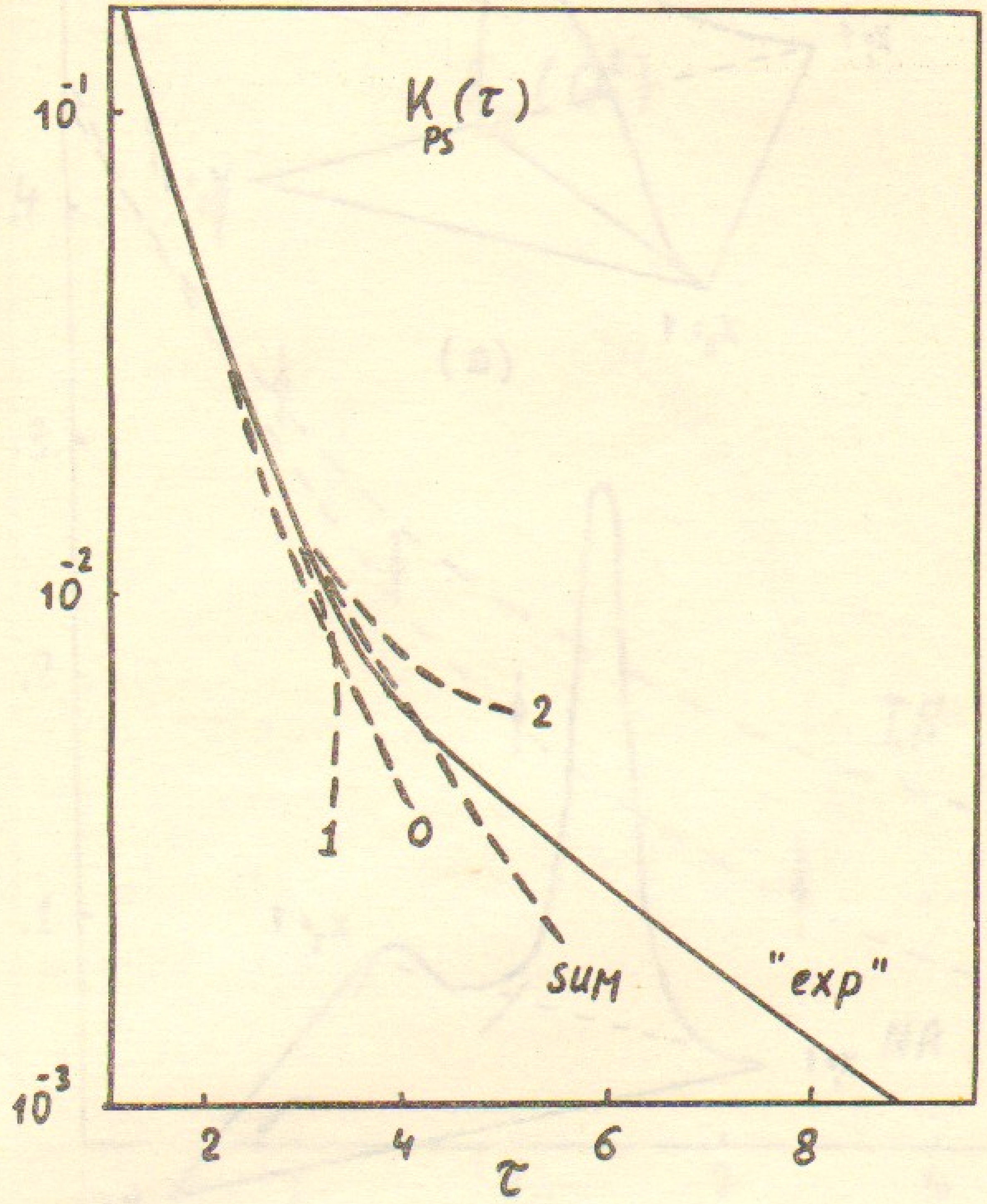

Fig.6. Correlator of pseudoscalar current consisting of light and heavy quark fields versus Euclidean time τ. The solid line marked «*exp*» substitute for experimental data, it corresponds to spectral density with parameters defined by fitting of the sum rules in [5.37]. The dashed curves marked *0*, *1*, *2* include respectively no power corrections to the free correlator, the leading $O(\langle\bar{\Psi}\Psi\rangle)$ one, and two corrections. The curve marked *SUM* refers to direct evaluation of the correlator in the «instanton liquid» model [5.57].

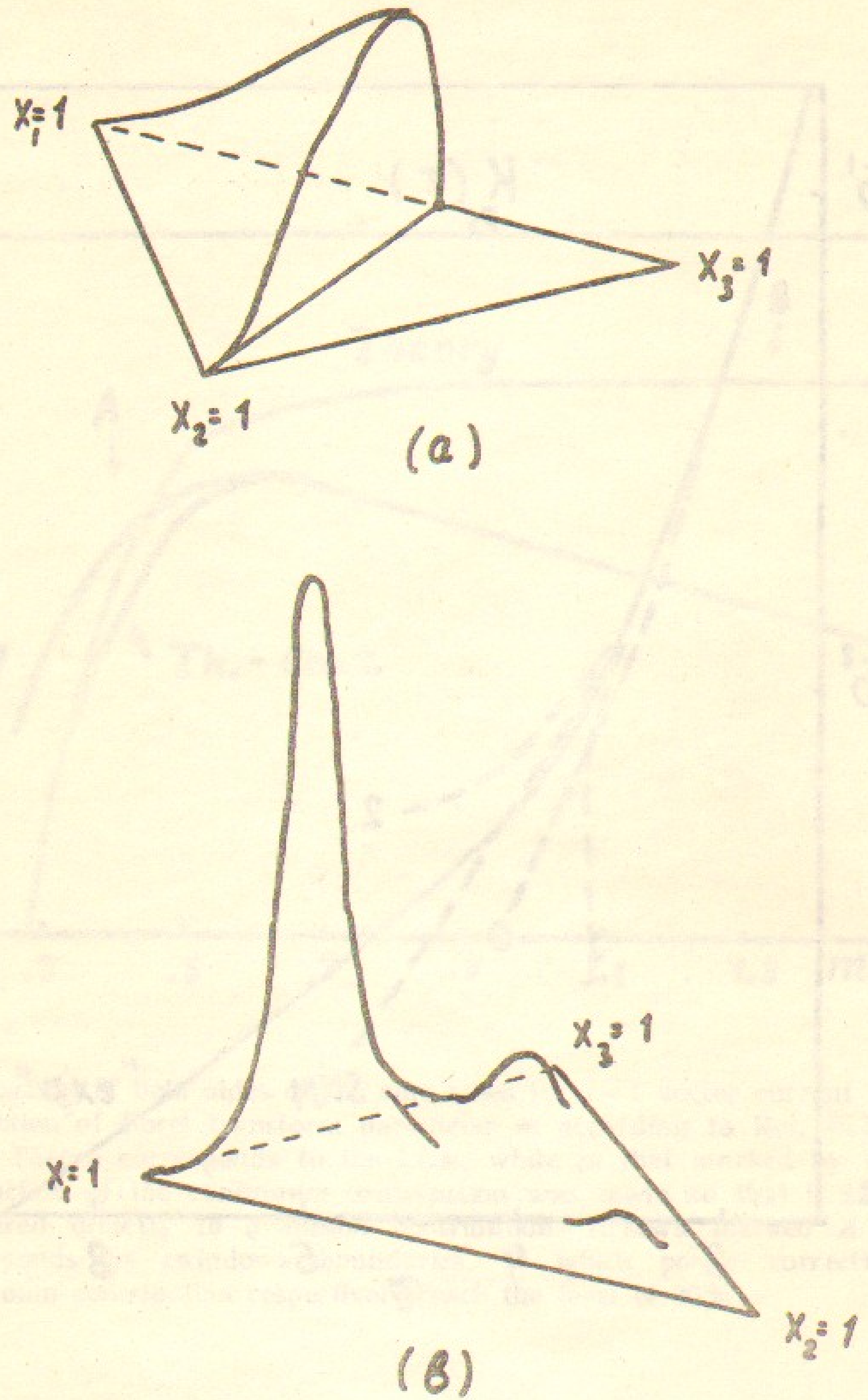

Fig.7. Comparison of the asymptotic nucleon wave function (at log (μ/Λ) going to infinity) (*a*) with the «realistic» one, (*b*) at μ about 1 GeV with parameters defined in Ref. [5.43].

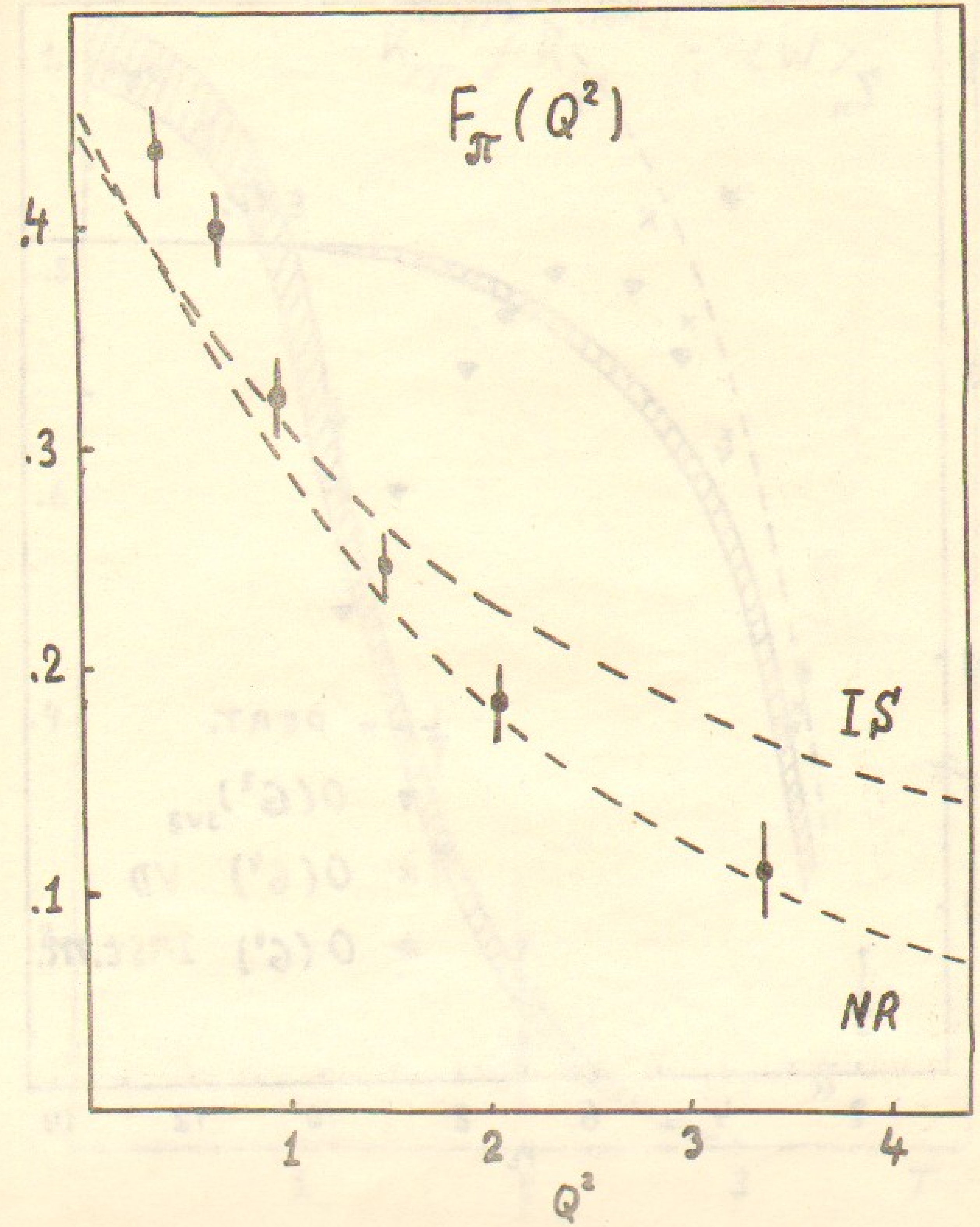

Fig.8. The pion formfactor versus momentum transfer *Q*. Points are (somewhat averaged) experimental data, two curves marked *IS* and *NR* refers to results of Refs [5.68] and [5.69], respectively, based on OPE sum rules for the three-point correlators.

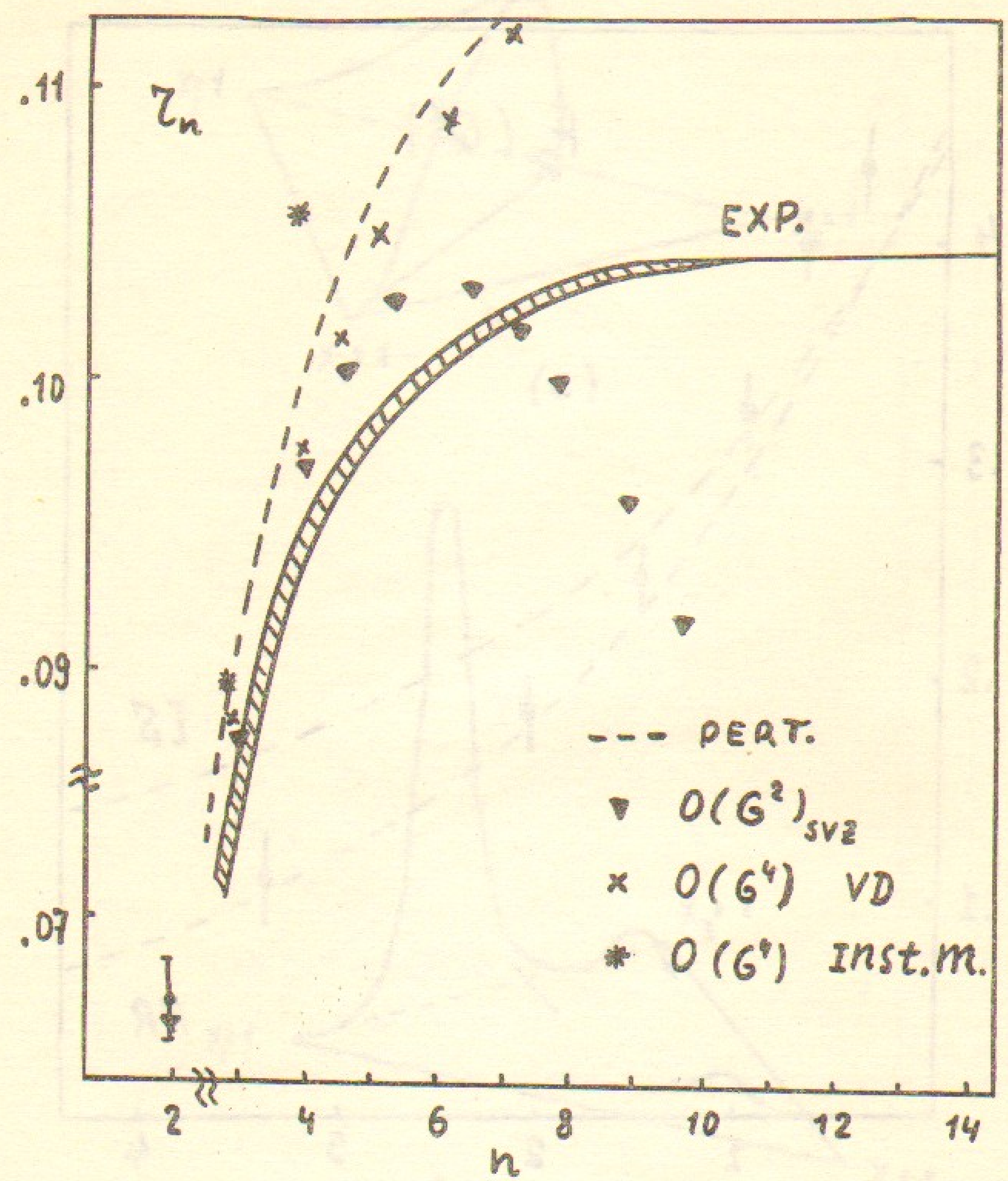

Fig.9. Moments ratios $r_n \equiv M_{n+1}/M_n$ (M_n defined in (4.8)) versus n for vector current of charmed quarks. The dashed region corresponds to experimental uncertainties, at large n it disappears since the ratios limit is connected only with psion mass. The points $\triangledown$ refer to account for $O(G^2)$ corrections in the pioneer work [5.13], and $\times$, $*$ to account for further corrections $O(G^3)$ and $O(G^4)$ [5.49] for «homogeneous» and «instanton liquid» models of the QCD vacuum, respectively.

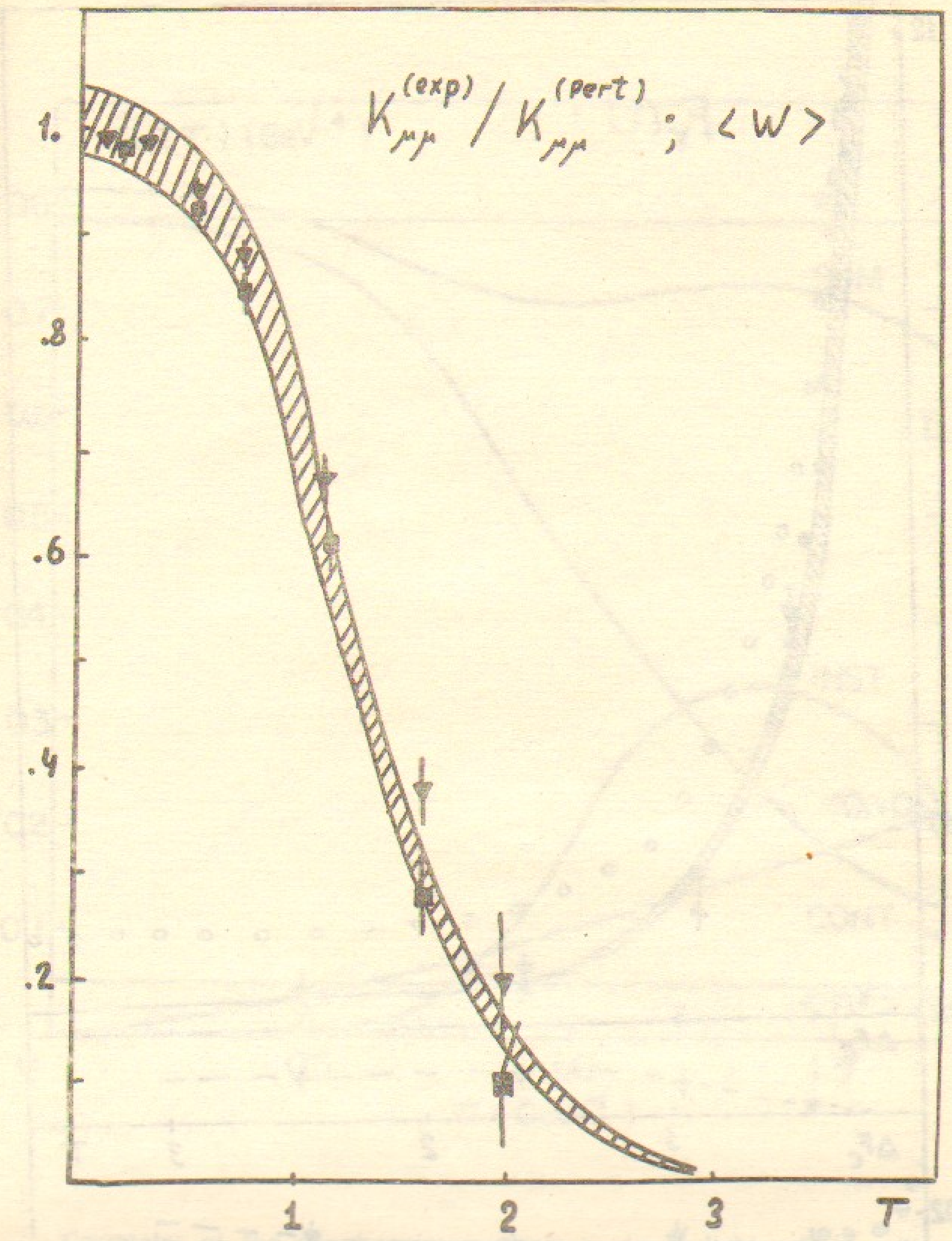

Fig.10. Ratio of the correlator for the vector currents of charmed quarks to its perturbative analog (propagation of free quarks with mass 1.3 GeV times the $O(\alpha_s)$ correction) as a function of Euclidean time τ. The shaded region correspond to experimental data with their uncertainty, while points $\triangledown$, $\square$ refer to «homogeneous» and «instanton liquid» models [5.53].

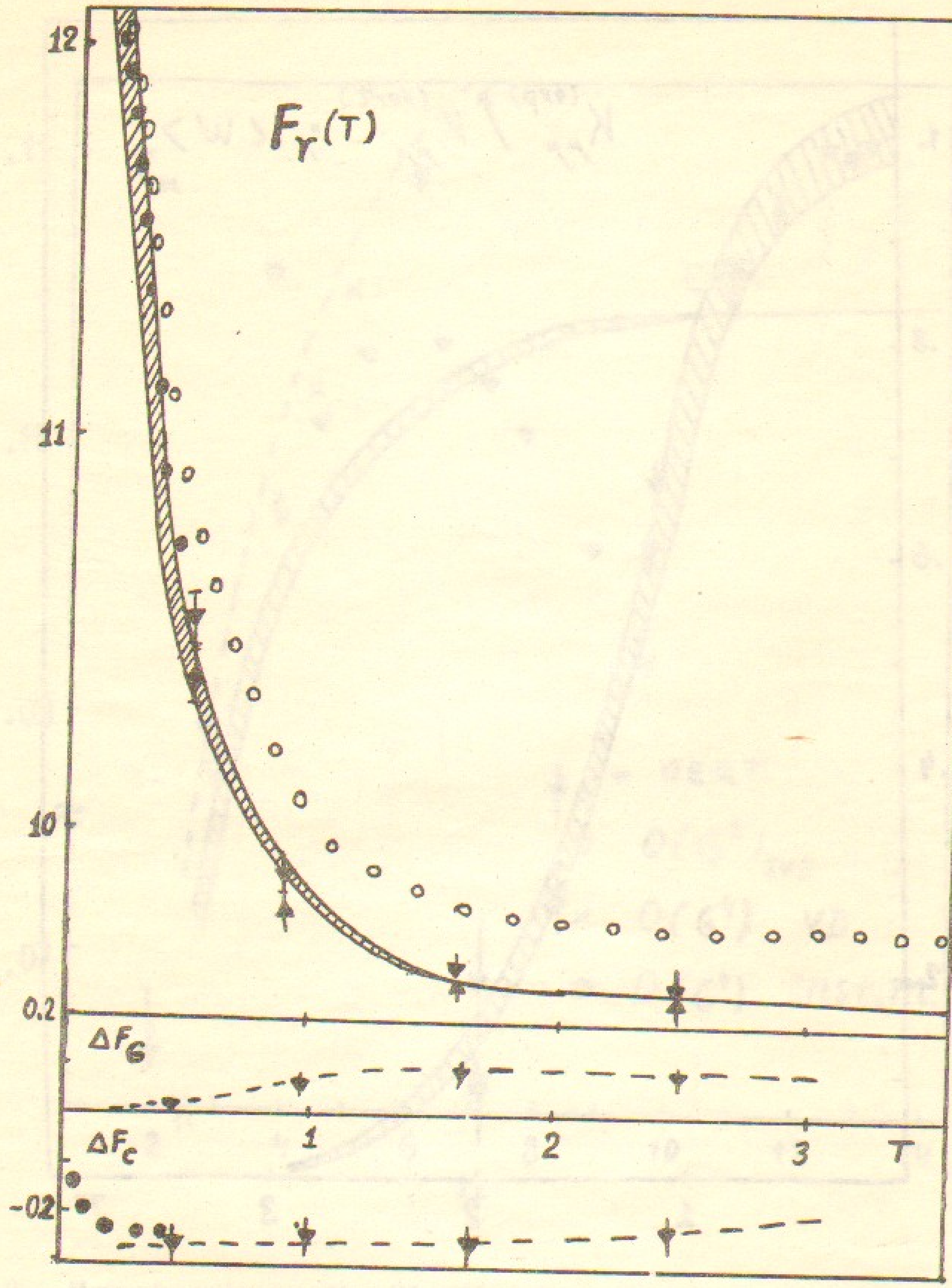

Fig.11. The logarithmic derivative of the correlator of vector current made of b quarks depending on Euclidean time τ (in fermi). The closed and open points correspond to free quark propagation with mass 4.8 GeV and that corrected for $O(\alpha_s)$ effects. Other notations as at Fig.10. In the lower part of the figure we show separately contributions of Coulomb and nonperturbative effects.

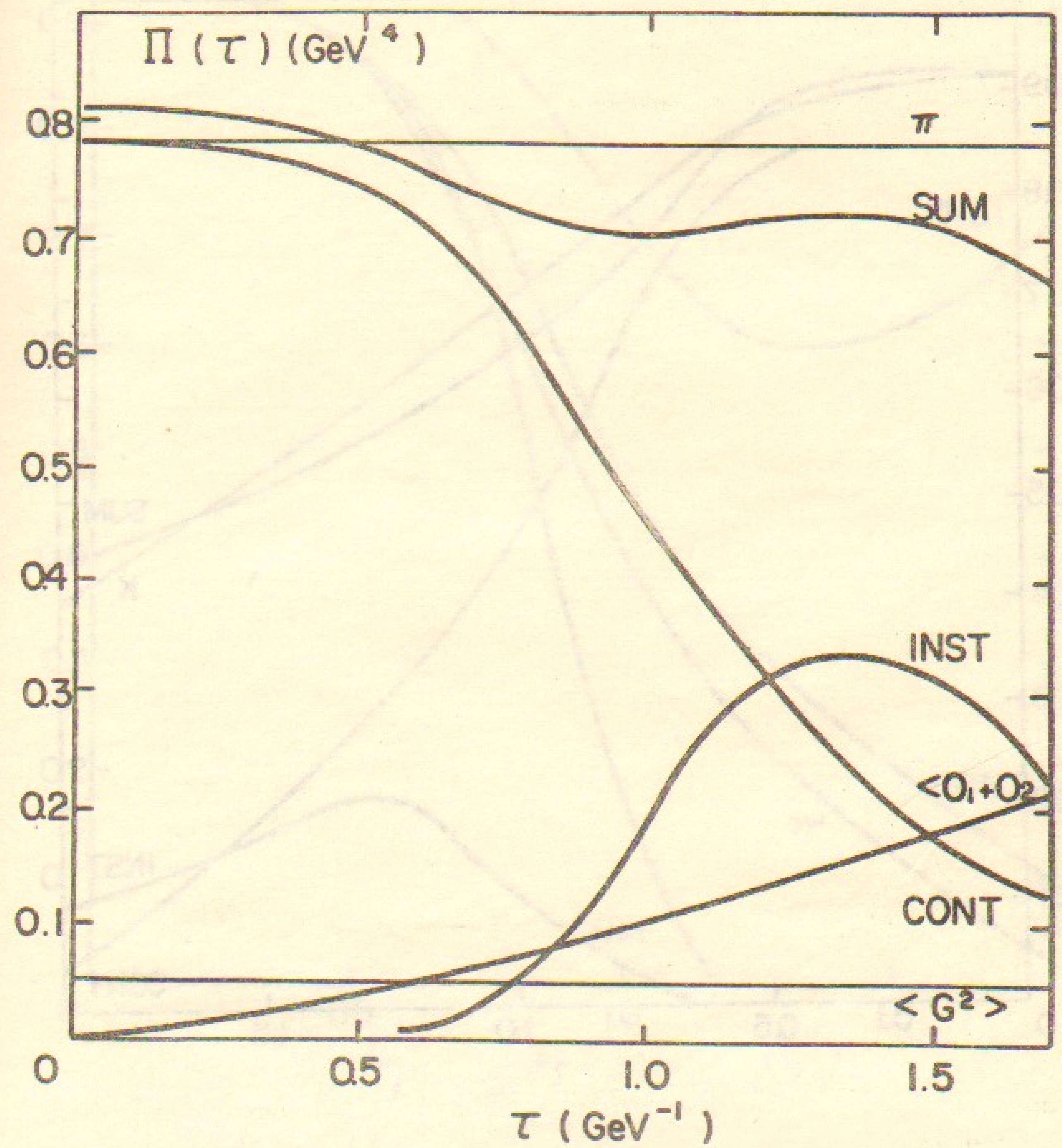

Fig.12. Correlator of the pseudoscalar currents made of light quarks with pion quantum numbers versus the Borel parameter $\tau = 1/m$. The curves marked $\langle G^2 \rangle$, $\langle O_1 + O_2 \rangle$, *CONT* and *INST* correspond to main power corrections, perturbative contribution with subtracted continuum and the nonpower effect due to instantons. Their sum shown by the curve *SUM* weakly depends on τ, in agreement with zero pion mass in the approximation considered.

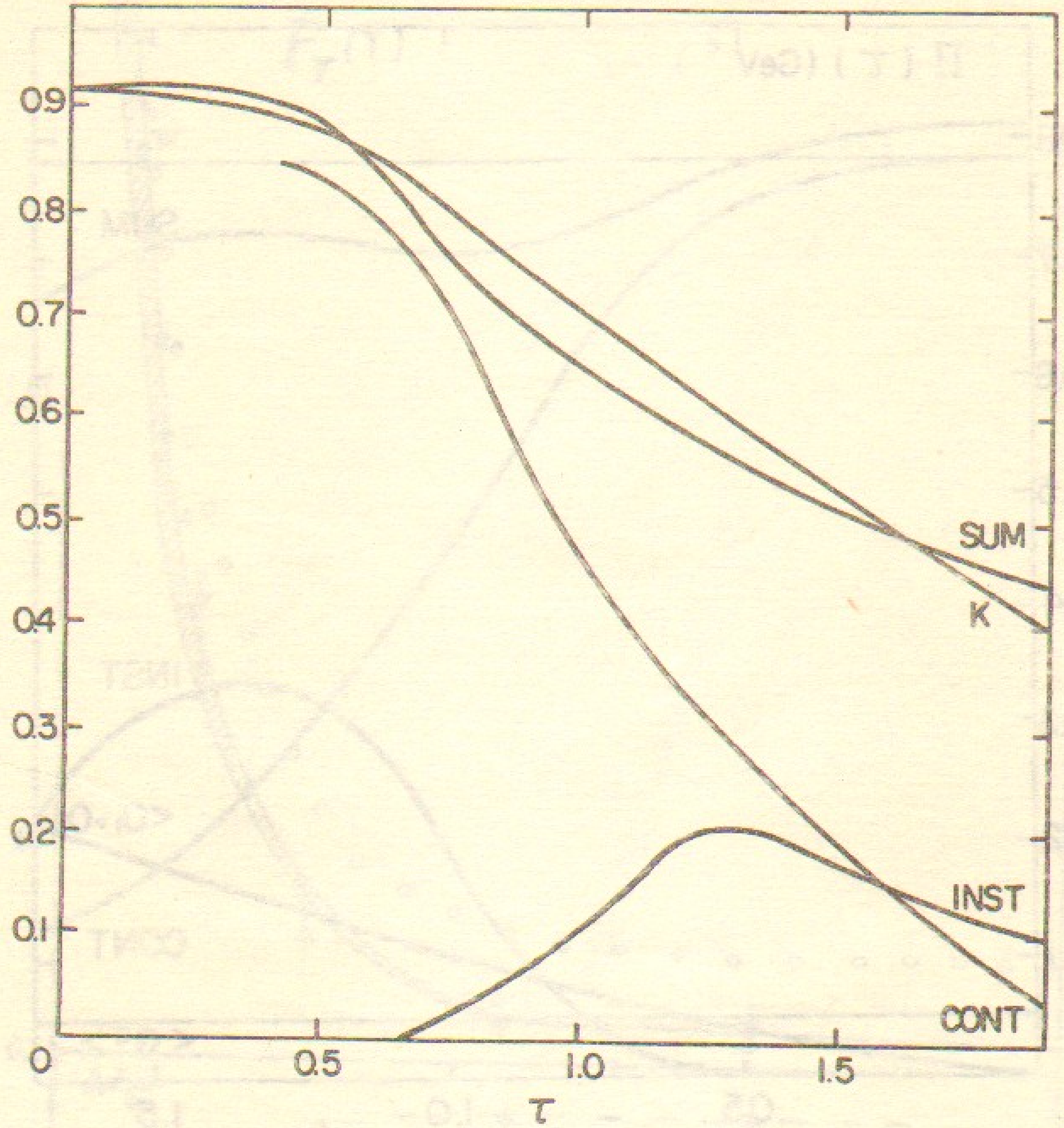

Fig.13. The same as at Fig.12 but for K meson. The curve marked by K is proportional to exp $[-(M_K \cdot \tau)^2]$, it agrees reasonably with the theoretical curve marked *SUM*.

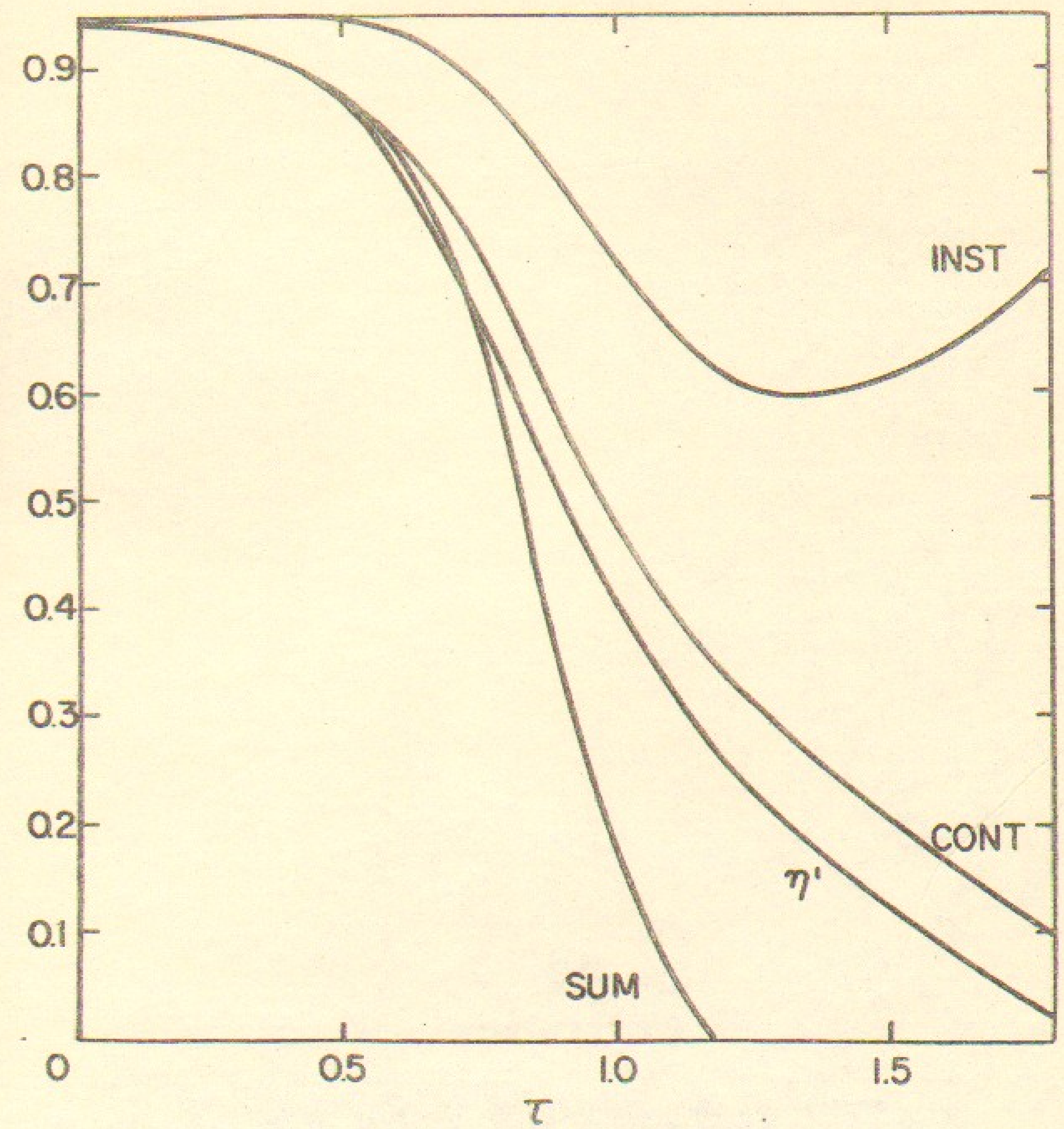

Fig.14. The same as at Fig.12 but for η' channel. The instanton contribution is in this case negative and it is shown at the upper part of the figure.

E.V. Shuryak

Theory and phenomenology of the QCD vacuum
5. Correlators and sum rules. The applications

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