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WHAT QUARKONIUM SUM RULES CAN TELL US ABOUT QCD VACUUM STRUCTURE?

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НОВОСИБИРСК

What quarkonium sum rules can tell us about QCD vacuum structure ?

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Abstract

OPE methods used previously for evaluation of vacuum field effects on two-current correlators are inadequate for heavy quark case. Numerical analysis confirms that with $\langle G^2 \rangle$ value around SVZ one it is really possible to describe data, even in wider region. However, these sum rules are insensitive to details of the vacuum structure, e.g. to $\langle G^4 \rangle$.

Sum rules suggested by Shifman, Vainshtein and Zakharov [1] connect properties of the QCD vacuum to experimental data, and their successful applications to different channels are so far known. In particular, lowest charmonium levels were described, with the following fundamental result as a byproduct

$$\langle (g\, G^{a}_{\mu\nu})^2 \rangle_{SVZ} \simeq 0.5 \text{ GeV}^4 \qquad (1)$$

Recently accuracy of this analysis was questioned by several authors [2], suggesting to increase (1) by the factor 2, 10 or even 50! Most relevant questions are already explained in [3,4].

In this Letter we show that some points in the dispute are created by the method used, the operator product expansion, which is not adequate for heavy quark case. Using more powerful methods we have found no trubbles with (1), and even demonstrate that it ensures agreement with data over wider region.

Quite different situation takes place with further corrections, in particular with $O(G^4)$ ones, for which OPE coefficients were recently calculated in [5]. As it was emphasized in [6], this point is very important for qualitative understanding of the vacuum structure, say instanton models [6,7] suggest much larger values than the factorisation hypothesis suggested in [1]. In contrast to conclusions of Ref. [3], we have found that heavy quarkonium sum rules are insensitive to details of the vacuum structure. The general reason is that heavy quarkonium interact weakly with the vacuum fields, so their effect show up only at larger distances between the currents. Evidently, the "probe" with length of about 1 fm can only measure some integral effect.

It is convenient to present these statements starting from some model example in which the problem can be solved analytically, then we proceed to semirealistic discussion for charmed quarks

and finish with fully realistic discussion of b quark sum rules. Calculations containing relativistic effects and quark mutial interaction for c quarks will be presented elsewhere.

We start with some reformulation of the sum rules. As usual the basic expression is the dispersion relation

$$\Pi(Q) = \frac{1}{16\pi^3\alpha^2 e_q^2}\int ds\, \frac{s}{Q^2+s}\, \sigma(e^+e^-\to \bar q q, s) \qquad (2)$$

$$\Pi(Q^2)(q_\mu q_\nu - g_{\mu\nu} q^2) = i\int dx\, e^{iqx} K_{\mu\nu}(x), \quad Q^2 = -q^2 > 0$$

$$K_{\mu\nu}(x) = \langle 0|T[\bar q(x)\gamma_\mu q(x), \bar q(0)\gamma_\nu q(0)]|0\rangle$$

where $\sigma(e^+e^-\to\bar q q)$ is the cross section of the production of flavour q quarks and e_q is their electric charge. But, instead of tricks suggested in [1] such as differenciation or Borel transformation, we just return into coordinate representation [1)]

$$K_{\mu\mu}(x) = \frac{3}{16\pi^3\alpha^2 e_q^2}\int ds\, s^2 \sigma(e^+e^-\to\bar q q, s)\, \mathcal{D}(s, x^2)$$
$$\mathcal{D}(s, T^2 = -x^2) = \frac{\sqrt{s}}{4\pi^2 T} K_1(\sqrt{s}\, T) \qquad (3)$$

Note that $\mathcal{D}(s, x^2)$ is the ordinary propagator, so the physical meaning of (3) is selfevident. As in sum rules used before, at large enough T only lowest states contribute, but this form is much better suited for numerical methods. However, the correlator is not the best object for the comparizon with data because it is changed by several orders of magnitude and contains rather large uncertainties of resonance widths etc. It is more convenient to consider its logarithmic derivative

$$F(T) = -d\ln K_{\mu\mu}(T)/dT \qquad (4)$$

which at large T is $M + 3/2 * T^{-1} + \ldots$ where M is (well known) mass of the lowest resonance, say ψ or Υ meson.

Such approach is essentially equivalent to moment ratios used in [1]. Experimental data for c and b quarks in vector channel translated to F(T) are shown at Fig.1 and2. At the former one we have also plotted OPE results [1,5], so one can see that $O(G^4)$ effects really look disturbing.

However, OPE is valid only for T small compared to variation scale of the vacuum fields, while in [1] and [8] the region around 1 and 1-2 fermi was used, for c and b quarks respectively. Smaller T it is difficult to use because here effect is too small, which is related to large quark mass. The typical dipole moment of $\bar{q}q$ system is $(T/m)^{1/2}$, so with large enough m effect becomes smaller than data uncertainties in the OPE validity region.

But do we really need OPE ? Of course, it is valid for <u>any</u> field, while in numerical studies one should use some particular model for it. However, are the correlators in question sensitive to details ?

Even simplest examples show that it is not the case. Consider nonrelativistic system with abelian field $E(\tau)$ and the action

$$S = \int d\tau \left[\frac{m\dot{x}_q^2}{2} + \frac{m\dot{x}_{\bar{q}}^2}{2} + \frac{g}{2} E(\tau)(x_q - x_{\bar{q}})^2 \right] \quad (5)$$

which is Gaussian, so that modification of F(T) due to $E(\tau)$ are just expressed in terms of the action on the extreme path

$$\Delta F(T) = \frac{d}{dT}\left\{ S[x_0(E,\tau)] - S[x_0(0,\tau)] \right\}$$

$$x_0(E,\tau) = \frac{g}{2m}\left\{ \int_0^\tau d\tau' \int_0^{\tau'} d\tau'' E(\tau'') - \frac{\tau}{T} \int_0^T d\tau' \int_0^{\tau'} d\tau'' E(\tau'') \right\} \quad (6)$$

E.g. in constant field $\Delta F(T) = -g^2E^2T^2/16m$, note that in QCD vacuum $\langle E^2 \rangle = -\frac{1}{4}\langle G_{\mu\nu} \rangle^2 < 0$, so $\Delta F > 0$.

We do not have space enough to demonstrate particular examples, but with (5,6) the reader can easily find effect of any $E(\tau)$

and see that three subsequent integration wash away information on inhomoginuities of $E(\tau)$ and rather different fields can produce very similar F(T), provided their strength is properly adjusted. Connections to OPE series can also be easily traced, with the conclusion that it is very sensitive to inessential inhomoginuites (if one considers its terms separately, not in sum).

Of course, these conclusions depend on particular parameters of the problem, so it is desirable to supply them by some more realistic examples. For this aim the program was written for the evaluation of propagator in arbitrary nonabelian field. In the nonrelativistic approximation and ignoring mutial interaction of $\bar{q}q$ by gluon exchanges one can express it via Wilson loop

$$W(T) = \left\langle \frac{1}{3} \mathrm{Tr} \left\{ T\exp\left(\frac{ig}{2} \oint A_\mu^a dx_\mu t^a \right) \right\} \right\rangle \quad (7)$$

where averaging is made over ensemble of $\bar{q}$ and q paths, generating numerically by Methropolis algorithm.

Let us present results only for two extreme models of QCD vacuum, essentially the same as discussed in [5]. "Homogineous" vacuum has constant $G_{\mu\nu}^a$ randomly oriented in ordinary and colour spaces. "Instanton vacuum" [6,7] is highly inhomogeneous, only few percent of space-time is occupied by the field. At Fig.3 we show the ratio of experimental correlator for c quarks to "perturbative" one. Both models fit the data equally well, their $\langle G^2 \rangle$ are 1.7 and 1.0 in unites (1), while $\langle G^4 \rangle$ differ by one order. Unfortunately, relativistic and Coulomb effect are not in fact negligible for charmed quark, so the example is not completely realistic.

Finally, let us discuss sum rules for b quarks, for which relativistic effects are small but Coulomb one shold be accurately taken into account. For this aim one should average the following T-exponential

$$W_K = \left\langle \frac{1}{3} Tr \left\{ T exp\left[\frac{ig}{2}\int A^a_\mu t^a_{(1)} dx^{(1)}_\mu - \frac{ig}{2}\int A^a_\mu t^a_{(2)} dx^{(2)}_\mu + \int \frac{\alpha_s(R)}{4R} t_{(2)} t_{(1)} d\tau \right] \right\} \right\rangle$$

where R is the interquark distance. The nontrivial points are that α_s depends on R due to asymptotic freedom relation[2], and the "relative colour charge" ($t_1 t_2$) is time dependent due to external field and deviates from 16/3 for pure singlet state. These two points were not treated properly in [2,8].

Our results are shown at Fig.2 for the same models of the vacuum as those at Fig.3, we show for comparizon the free quark line with m_b=4.8 GeV and first order perturbative corrections. One can see that there is no ground for radical change of the gluon condensate value, but one can hardly extract more information on details of the vacuum structure from such sum rules.

In conclusion, OPE methods are progressively inadequate for heavy quarkonium sum rules. With the help of more powerful numerical methods we have confirmed standard value of $\langle g G^2 \rangle$ and explicitely demonstrated that further local averages is not possible to extract, unless the accuracy of both theory and data is essentially increased. In order to understand vacuum structure one should better use sum rules with light quarks and gluons, especially with zero spin, which much stronger interact with vacuum fields so that analysis can be made at really small distances. Examples of such approach can be found in [6].

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Footnotes

1. In the nonrelativistic limit these sum rules coinside with those used by Voloshin [8].

2. Starting with Voloshin's work [8], formulae with <u>fixed</u> α_s where used for Coulomb factor. Taking $\alpha_s(\bar{R})$, $\bar{R}=(T/m)^{1/2}$ being the typical distance, one finds too strong effect. So, Voloshin has suggested to "freeze" $\alpha_s(R)$ and Baier and Pinelis [2] to increase $\langle G^2 \rangle$ by one order. However, smaller ($R \ll \bar{R}$) dominate in Coulomb effects.

Figure captions

1. Dependence of the logarithmic derivative F(T) (4) (GeV) on T (fm), Euclidean interval between two currents. The dashed region corresponds to experimental uncertainties of the cross section, open and closed points - to free quarks with m=1.35 GeV and first perturbative correction. The dashed curves show OPE results up to $O(G^2)$ and $O(G^4)$ terms (the former with (1), the latter for the instanton model).

2. The same as at Fig.1 but for b quarks, points ▼ and ■ refer to "homogeneous" and instanton models with parameters given in the text. Lower part of the figure show effect of vacuum fields and Coulomb forces separately.

3. The dashed region is the ratio of experimental correlator for c quarks to "perturbative"one, free quarks with $O(\alpha_s)$ correction. The points ▼ and ■ refer to "homogeneous" and instanton vacuum models.

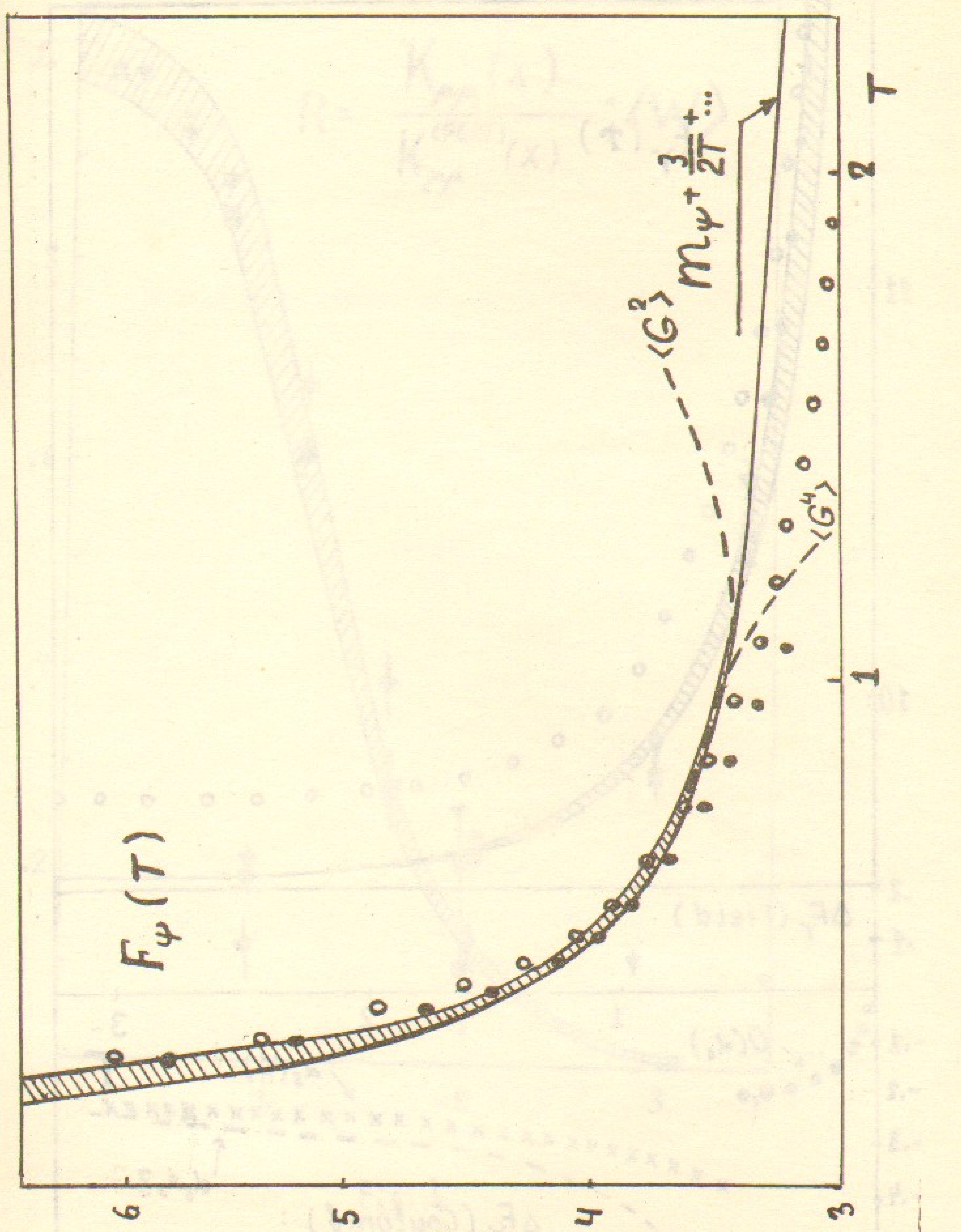
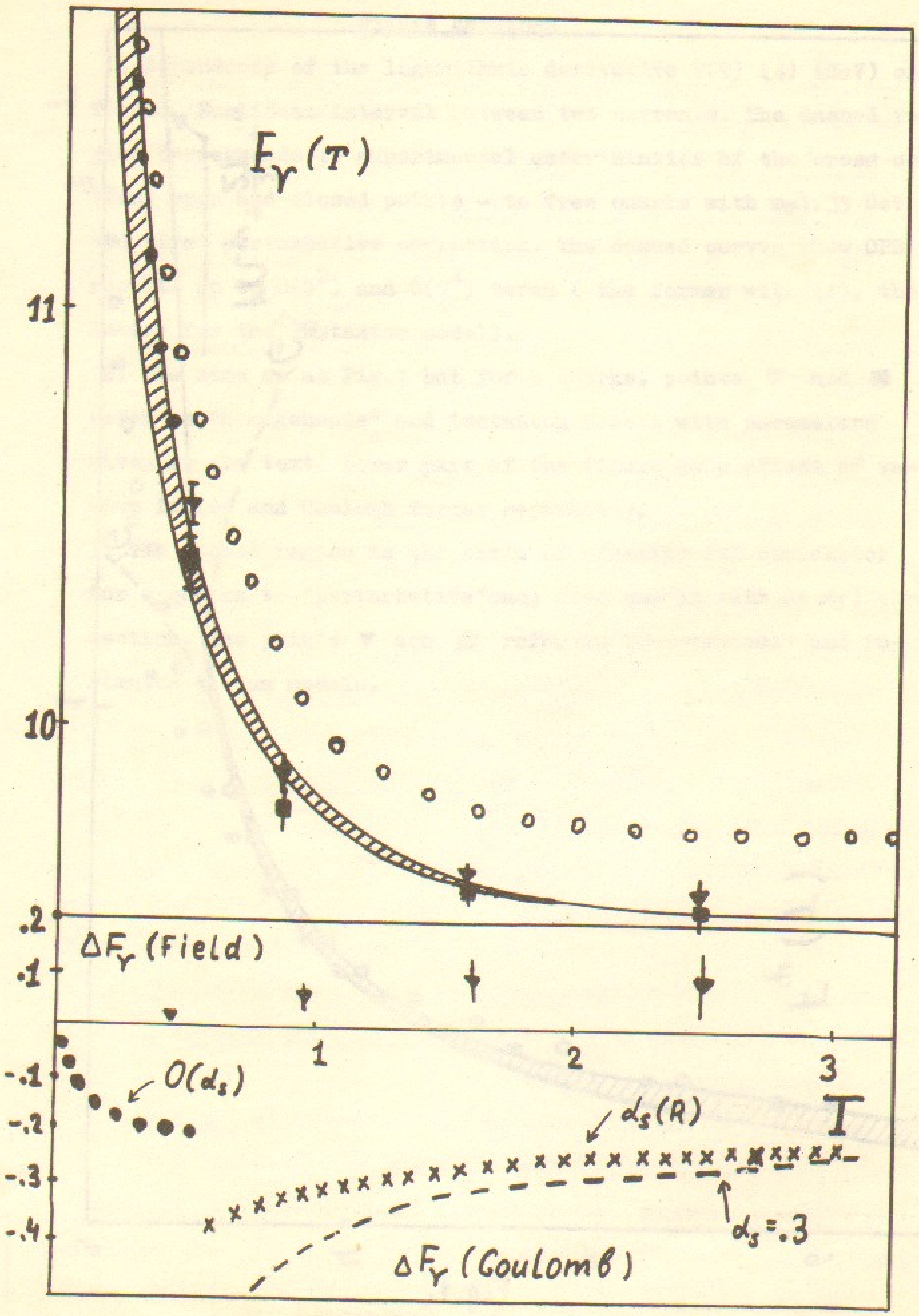
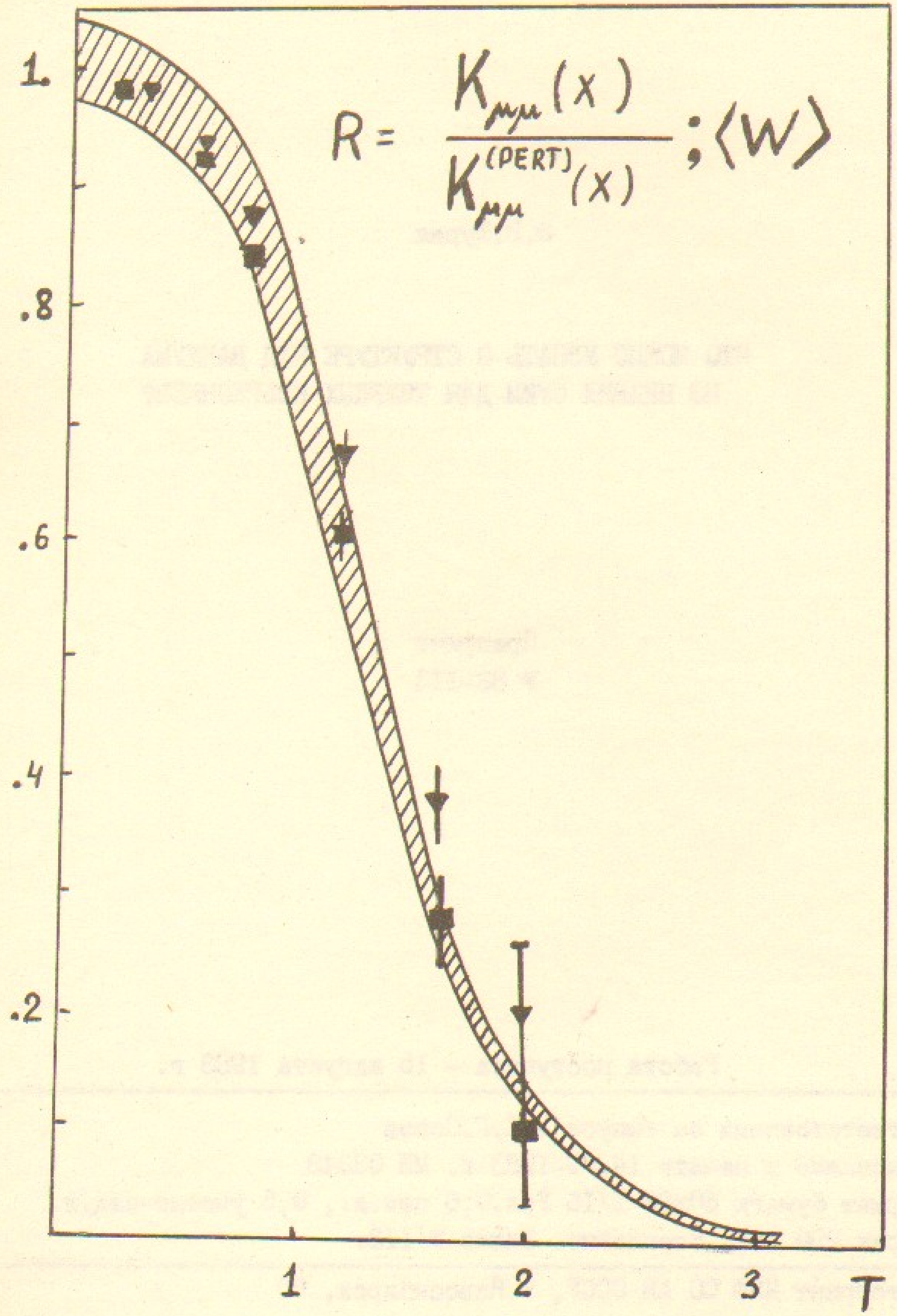


Fig.1.



$F_\gamma(T)$
11
10
.2
.1
-.1
-.2
-.3
-.4
ΔF_γ (Field)
1
2
3
T
$O(\alpha_s)$
$\alpha_s(R)$
$\alpha_s=.3$
ΔF_γ (Coulomb)

Fig.2.



1.
.8
.6
.4
.2
1
2
3
T
$R = \dfrac{K_{\mu\mu}(x)}{K_{\mu\mu}^{(PERT)}(x)}$; $\langle W \rangle$

Fig. 3

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ЧТО МОЖНО УЗНАТЬ О СТРУКТУРЕ КХД ВАКУУМА
ИЗ ПРАВИЛ СУММ ДЛЯ ТЯЖЕЛЫХ КВАРКОНИЕВ?

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