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MAGNETIC FLUX LOSSES DURING THE FORMATION OF FIELD-REVERSED CONFIGURATIONS

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A b s t r a c t

Bias-flux trapping during the formation of a field-reversed configuration in theta-pinch has been studied theoretically. It is shown that the rate of bias-flux losses at the reversal phase of the theta-pinch becomes anomalously high due to the plasma motion and formation of a high-density sheath at the tube walls. The conditions for the bias-flux losses to be negligible are defined.

Plasma configurations with reversed magnetic field are considered now as one of the promising directions in controlled thermonuclear research and are studied in many laboratories [1]. Formation of a field-reversed configuration compact toroid with a theta pinch requires the application of an initial reverse-bias-flux. The reverse-flux is lost from the plasma configuration during the dynamic phases of compact toroid formation illustrated in Fig. 1: a) the initial low-β plasma with the bias magnetic field; b) the reversal phase of an external field H_e ; c) the reverse-field plasma configuration isolated from the tube walls, $H_e = -H_i$ (H_i is the internal magnetic field in the plasma). The compact toroid is formed after the field reconnection and axial contraction.

The amount of reverse-flux, retained in the plasma after field reversal, is one of the most important quantities governing the final state. The flux loss during the reversal phase of the theta pinch has been previously studied by Green and Newton [2]. They considered a model of the fast field-reversal, when the plasma and frozen magnetic field expand to the tube walls with Alfven velocity. In the recent experiments, with the typical field-reversal time greatly exceeding the Alfven transit time the bias-flux losses are determined by the resistive magnetic field diffusion to the wall at the phase *b* of Fig. 1. This problem has been studied in papers [3-5], in which the authors come to the conclusion that the observed fast magnetic field diffusion cannot be explained by the classical plasma resistivity and is connected with the anomalous resistivity due to microinstabilities in the current sheath. In [6] attention was paid to an important role of plasma motion. As seen from Fig. 1,b, during the reversal phase the radial plasma motion to the wall arises and pressure bearing plasma sheath with $\beta \sim 1$ is formed. Since the magnetic field is frozen into the internal plasma, this motion leads to a conbective outflow of the magnetic field to the wall. It results in a considerable enhancement of the bias-flux losses. It is convenient to describe this process by the effective magnetic viscosity coefficient $\mathcal{D}_{eff}$, which greatly exceeds the classical value $\mathcal{D}_{cl} = c^2/4\pi\sigma$. The situation here is rather similar to that of the fast cooling of a high- β magnetized plasma [7].

Let us consider now the magnetic field and plasma evolution on the phase $б$, when the external field H_e is less than the internal one H_i, and the plasma expands to the wall. It follows from the pressure balance condition that the plasma sheath with $nT \sim H_i^2/8\pi$ and high density $n \gg n_0$ (n_0 is the density of the homogeneous internal plasma) is formed near the wall. Since its thickness is small compared to the plasma radius R, the geometry of the problem is approximately plane. Let us define the wall as a plane $x = 0$, the plasma occupies the half-space $x > 0$, and the magnetic field is directed along the z-axis. Then the plasma transport equations may be written in the following form [8]:

$$nT + H^2/8\pi = H_i^2/8\pi \tag{1}$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0 \tag{2}$$

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\left(\frac{c^2}{4\pi\sigma}\frac{\partial H}{\partial x} + \frac{c}{e}\beta_\wedge\frac{\partial T}{\partial x} - vH\right) \tag{3}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}nT + \frac{H^2}{8\pi}\right) = \frac{\partial}{\partial x}\left\{\varkappa_\perp\frac{\partial T}{\partial x} + \frac{cT}{4\pi e}\beta_\wedge\frac{\partial H}{\partial x} - \frac{5}{2}nTv + \frac{H}{4\pi}\left(\frac{c^2}{4\pi\sigma}\frac{\partial H}{\partial x} + \frac{c}{e}\beta_\wedge\frac{\partial T}{\partial x} - vH\right)\right\} \tag{4}$$

(the notations of [8] are used).

The boundary conditions at the wall (x = 0) are as follows: constant external magnetic field H_e (its magnitude is of the order of H_i, i.e. $H_i - H_e \sim H_i$), plasma temperature T(0) = 0 and velocity V(0) = 0. We consider the magnetic field H_i and density n_0 of the internal homogeneous plasma as a constant, i.e. the plasma is unbounded along the x-axis, and $H(+\infty) = H_i$, $n(+\infty) = n_0$. In the real problem with the plasma of finite radius it corresponds to the initial phase of the process when the fraction of the lost flux is small. In such a formulation this problem allows the self-similar solution. Introducing the dimensionless variables: $h = H/H_i$, $\rho = n/n_0$,

$\theta = T/T_0$ where $T_0 = H_i^2/8\pi n_0$, in the self-similar solution, we have: $h(x,t) = h(\xi)$, $\rho(x,t) = \rho(\xi)$, $\theta(x,t) = \theta(\xi)$, $v(x,t) = -(\mathcal{D}_0/t)^{1/2}u(\xi)$, where $\xi = x/(\mathcal{D}_0 t)^{1/2}$ and $\mathcal{D}_0 = c^2/4\pi\sigma(T_0)$ is magnetic viscosity of the plasma with temperature T = T_0. In these notations eqs. (1)-(4) take the form:

$$\rho\theta + h^2 = 1 \tag{5}$$

$$-\frac{\xi}{2}\frac{d\rho}{d\xi} = \frac{d}{d\xi}(\rho u) \tag{6}$$

$$-\frac{\xi}{2}\frac{dh}{d\xi} = \frac{d}{d\xi}\left(\theta^{-3/2}\frac{dh}{d\xi} + \alpha\frac{d\theta}{d\xi} + uh\right) \tag{7}$$

$$-\frac{\xi}{4}\frac{d(\rho\theta)}{d\xi} = \frac{d}{d\xi}\left\{\varkappa\frac{d\theta}{d\xi} + \alpha\theta\frac{dh}{d\xi} + \frac{5}{2}\rho\theta u + h\left(\theta^{-3/2}\frac{dh}{d\xi} + \alpha\frac{d\theta}{d\xi} + uh\right)\right\} \tag{8}$$

Here the dimensionless plasma heat conductivity $\varkappa$ and thermoelectric coefficient α have the different dependence on plasma parameters and magnetic field in the magnetized and unmagnetized plasma [8]:

$$\varkappa = \begin{cases} \mu^{-1/2}\rho^2/h^2\theta^{1/2}, & \dfrac{h\theta^{3/2}}{\rho} > \mu^{-1/2}\delta_0^{-1} \quad (\mathrm{I}) \\ \delta_0\rho\theta/h, & \delta_0^{-1} < \dfrac{h\theta^{3/2}}{\rho} < \mu^{-1/2}\delta_0^{-1} \quad (\mathrm{II}) \\ \delta_0^2\theta^{5/2}, & \dfrac{h\theta^{3/2}}{\rho} < \delta_0^{-1} \quad (\mathrm{III}) \end{cases} \qquad \alpha = \begin{cases} \rho/h\theta^{3/2}, & (\mathrm{I}, \mathrm{II}) \\ \delta_0^2\dfrac{h\theta^{3/2}}{\rho}, & (\mathrm{III}) \end{cases} \tag{9}$$

where $\delta_0 \equiv \omega_{He}\tau_e(n_0, T_0, H_i) \gg 1$, $\mu \equiv m_e/m_i \ll 1$.

The enhancement of the magnetic flux losses manifests itself in a large value of the rate of homogeneous plasma expansion: $u(+\infty) = u_0 \gg 1$. It permits to neglect the left-hand parts of eqs. (7) and (8), since

$$\int_0^\infty \frac{\xi}{2}\frac{dh}{d\xi}d\xi \ll u_0, \quad \int_0^\infty \frac{\xi}{4}\frac{d(\rho\theta)}{d\xi}d\xi \ll u_0$$

and the magnetic and energy fluxes are assumed to be constant:

$$q_h = \theta^{-3/2}\frac{dh}{d\xi} + \alpha\frac{d\theta}{d\xi} + uh = u_0 \qquad (10)$$

$$q_W = \varkappa\frac{d\theta}{d\xi} + \alpha\theta\frac{dh}{d\xi} + \frac{5}{2}\rho\theta u + hu_0 = u_0 \qquad (11)$$

From eqs. (10),(11) and (5) it is possible to obtain the profiles of the plasma temperature, density and magnetic field in the near-wall sheath where u_0 enters as a not yet obtained parameter. The value of u_0 should be determined from the plasma continuity equation (6) integrated over ξ

$$u_0 = -\frac{1}{2}\int_0^\infty \xi\frac{d\rho}{d\xi}d\xi \qquad (12)$$

(it means that the plasma, which flows to the wall with the velocity u_0, is accumulated in the near-wall sheath).

Let us start from the unmagnetized plasma near the wall (region III). Since the external magnetic field is $h_e < 1$, the plasma pressure at the wall is $\rho\theta \sim 1$. Here the main contribution to the magnetic (10) and energy (11) fluxes comes from the resistive magnetic diffusion term and heat conductivity:

$$\varkappa\frac{d\theta}{d\xi} = \delta_0^2\theta^{5/2}\frac{d\theta}{d\xi} \sim u_0; \quad \theta^{-3/2}\frac{dh}{d\xi} \sim u_0 \qquad (13)$$

It follows from (13) that $\xi \sim \delta_0^2 u_0^{-1}\theta^{7/2}$ and $dh \sim \delta_0^2\theta^4 d\theta$. This solution is valid up to $\theta \lesssim \theta_1 \sim \delta_0^{-2/5}$, where the electrons become magnetized and the magnetic field increases by the magnitude $\Delta h \sim 1$. At $\theta > \theta_1$ the transition to region II occurs, where the electrons are already magnetized ($\omega_{He}\tau_e > 1$), but the ions are not yet ($\omega_{Hi}\tau_i < 1$). Here eqs. (10) and (11) may be transformed, using (5) and (9), into the form:

$$\varkappa\frac{d\theta}{d\xi} = \delta_0\frac{\rho\theta}{h}\frac{d\theta}{d\xi} \sim u_0(1-h) \sim u_0\rho\theta, \quad \text{or} \quad \frac{d\theta}{d\xi} \sim u_0\delta_0^{-1}$$

$$\theta^{-3/2}\frac{dh}{d\xi} = -\frac{1}{2h\theta^{3/2}}\frac{d(\rho\theta)}{d\xi} \sim -\theta^{-3/2}\frac{d(\rho\theta)}{d\xi} \sim u_0 \qquad (14)$$

Therefore, in region II the plasma pressure decreases from the value $\rho\theta \sim 1$ to $\rho\theta \sim \mu^{1/2} \ll 1$ when the temperature increases by $\Delta\theta \sim \theta_1$ (here the transition from the plasma with $\beta \sim 1$ to the low-β plasma occurs).

As for the magnetized plasma (region I), it follows from (10) and (11) that the plasma pressure decreases rapidly while the temperature remains approximately constant: $\theta \sim \theta_1$ and $d\ln(\rho\theta)/d\ln\theta \sim \mu^{-1/2} \gg 1$. This is valid while the density is high: $\rho \gg 1$. At $\rho \sim 1$ the temperature reaches its maximum $\theta_{max} \sim \theta_1$ and then decreases when moving off the wall: $\theta \sim \mu^{-1}u_0^{-2}\xi^{-2}$ (here $\rho \simeq h \simeq 1$). The qualitative form of the solution is shown in Fig. 2.

With the known structure of the near-wall sheath it is possible to solve eq. (12) and find the value of u_0. The main contribution to this integral comes from the region, when the electron magnetization parameter is $\omega_{He}\tau_e \sim 1$, and plasma pressure is $\rho\theta \sim 1$:

$$\int_0^\infty \xi\frac{d\rho}{d\xi}d\xi \sim \rho_1\xi_1 \sim \theta_1^{-1}\delta_0^2 u_0^{-1}\theta_1^{7/2} \sim \delta_0 u_0^{-1}$$

Therefore, the following estimation for u_0 is valid: $u_0 \sim \delta_0^{1/2} \gg 1$. It means that the magnetic flux losses occur with the effective magnetic viscosity

$$\mathcal{D}_{eff} \sim u_0^2\mathcal{D}_0 \sim (\omega_{He}\tau_e)_0 \cdot c^2/4\pi\sigma_0 \sim cH_i/4\pi n_0 e \qquad (15)$$

It is interesting to note that the value of $\mathcal{D}_{eff}$ doesn't depend on the plasma collision frequency, so this estimation may be valid not only for the case of binary Coulomb electron-ion collisions, but also for the anomalous plasma resistivity. Obviously, the amount of bias-flux losses during the formation of a compact toroid would be small, if the typical field-reversal time T satisfies the following condition:

$$T < \tau_h \sim R^2/\mathcal{D}_{eff} \sim 4\pi n_0 e R^2/cH_i \qquad (16)$$

It is seen from (16) that for a given R and T the fraction of bias-flux losses would increase when the magnetic field H_i become stronger and the plasma density n_0 decreases (the same conclusion has been made in [6] by means of numerical computation of the magnetic field evolution in a plasma).

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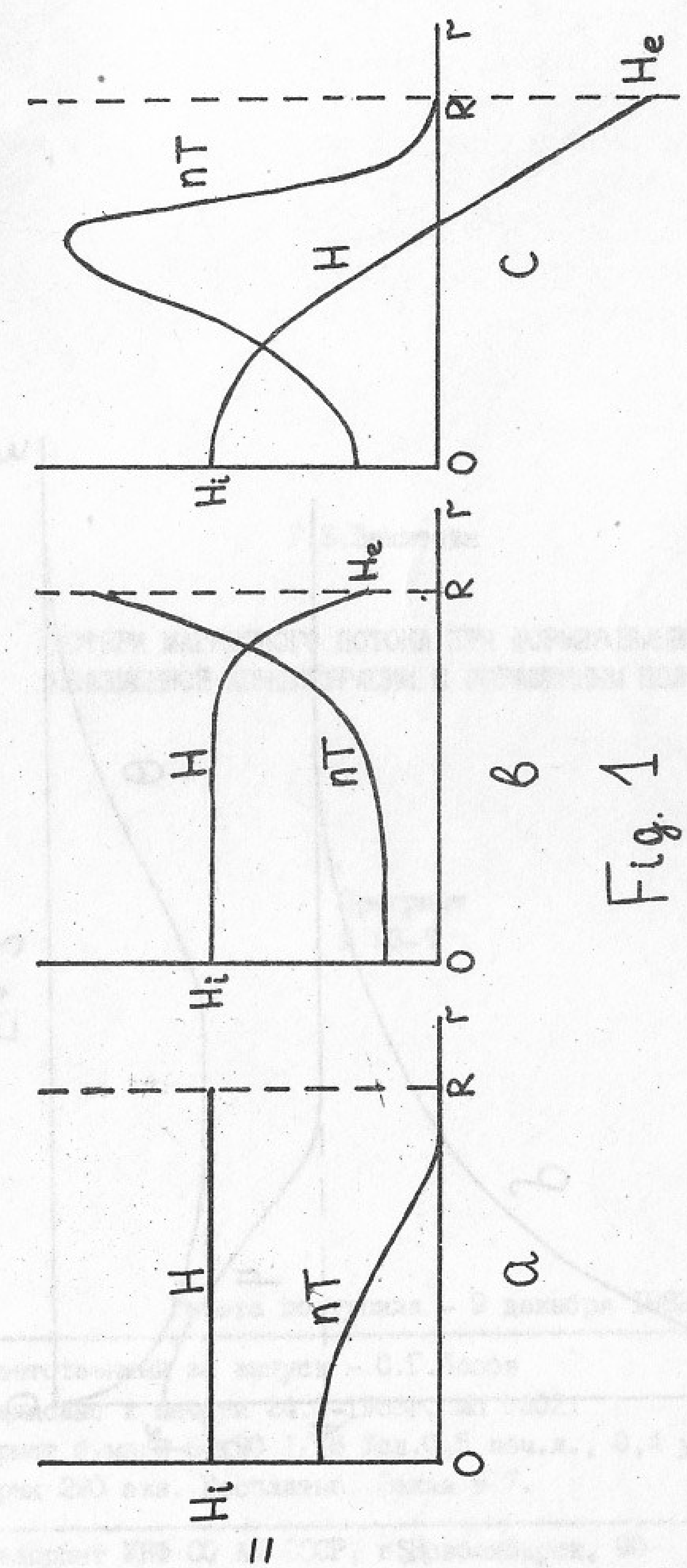
Figure captions

Fig. 1. The different phases of field-reversed configuration formation.

Fig. 2. The profiles of plasma temperature (Θ) density (ρ) and magnetic field (h) in a near-wall sheath during the field-reversal phase δ.



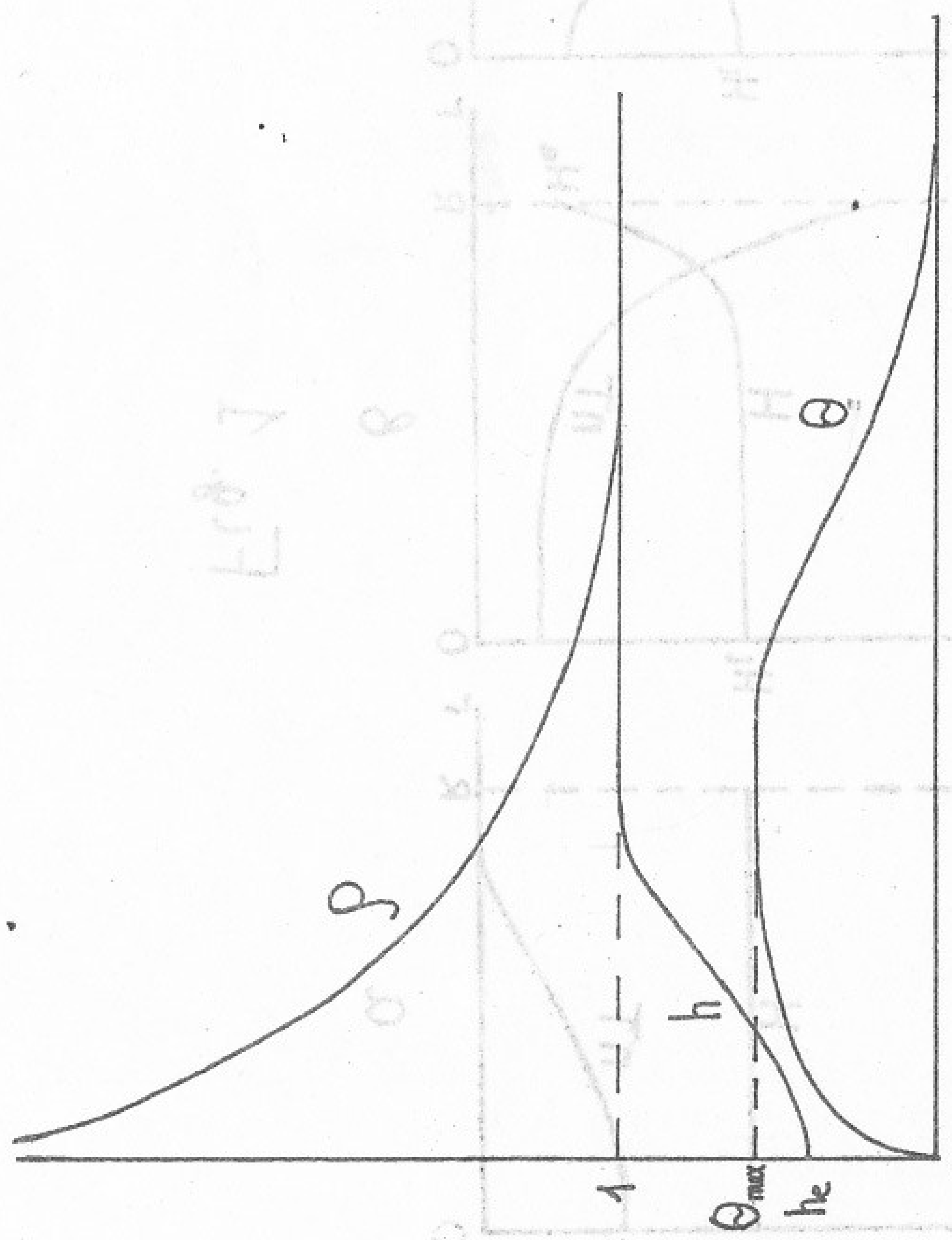


Fig. 2

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ПОТЕРИ МАГНИТНОГО ПОТОКА ПРИ ФОРМИРОВАНИИ ПЛАЗМЕННОЙ КОНФИГУРАЦИИ С ОБРАЩЕННЫМ ПОЛЕМ

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