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# RADIATION OF ULTRARELATIVISTIC PARTICLES MOVING NEAR CRYSTALLINE AXES

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# RADIATION OF ULTRARELATIVISTIC PARTICLES MOVING NEAR CRYSTALLINE AXES

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A b s t r a c t

The total radiation intensity of ultrarelativistic electrons and positrons moving near crystalline axes in thin crystals is considered. The approach avoids combersome analysis of the radiation from concrete trajectories. A general expression for the intensity which is valid for any axial symmetric potential is derived.

The radiation of relativistic particles which move near the crystalline planes and axes in single crystals has widely been discussed in recent years (see [1-3] and the references there). At present a theoretical analysis of the radiation of the particles moving near the crystalline planes (one-dimensional problem) permits one to obtain a quite satisfactory quantitative description of experiment. We have recently obtained such a description [4,5], which is based on the authors' earlier papers [1,6]. The radiation of the particles moving near the crystalline axes (two-dimensional problem) is studied mainly on a qualitative level (see, e.g. [7,8]; in Ref.[8] the radiation of the particles which move near the axes was considered using the Coulomb type potential; it will be shown that this type of the potential is unadequate for this problem). The present paper is devoted to a general approach to the total radiation intensity of electrons and positrons moving near the crystalline axes in thin crystals and its orientation dependence.

If one is interested in such characteristics of the radiation, which only depend on the instant values of coordinates and momenta of particles, then one must know only a distribution of particles in phase space which is transverse to the direction of axes. The problem is substantially simplified for this case, since it is possible to avoid a very cumbersome analysis of the radiation from concrete trajectories. So, in the present paper we will use a statistical description of the motion of particles near the crystalline axes. The particle distribution can be written in the form [9]

$$dN(\varepsilon_\perp, \underline{\varrho}, \ell) = F(\varepsilon_\perp, \underline{\varrho}, \ell)\, d\varepsilon_\perp d^2\underline{\varrho} = g(\varepsilon_\perp, \ell)\, f(\varepsilon_\perp, \underline{\varrho})\, d\varepsilon_\perp d^2\underline{\varrho} \quad (1)$$

where

$$f(\varepsilon_\perp, \underline{\varrho}) = \frac{1}{S(\varepsilon_\perp)}\,\vartheta(\varepsilon_\perp - U(\underline{\varrho})) \quad (2)$$

with normalization conditions

$$\int F(\varepsilon_\perp, \underline{\varrho}, \ell)\, d\varepsilon_\perp d^2\underline{\varrho} = \int g(\varepsilon_\perp, \ell)\, d\varepsilon_\perp = \int f(\varepsilon_\perp, \underline{\varrho})\, d^2\underline{\varrho} = 1 \quad (3)$$

Here $\varepsilon_\perp = \varepsilon\vartheta_\perp^2/2 + U(\underline{\varrho})$ is the transverse particle energy,

$\underline{\rho}(\underline{v}_\perp)$ is the transverse coordinate (velocity) of the particle, $U(\underline{\rho})$ is the potential of the transverse motion, $\vartheta(x)=\begin{cases}1 & x>0\\ 0 & x<0\end{cases}$; $S(\varepsilon_\perp)$ is an available area of the transverse motion at a fixed value of $\varepsilon_\perp$, within one elementary cell; l is a penetration depth of the particle inside the crystall.

By definition, in a thin crystal (see Ref.[I]) a distribution function $g(\varepsilon_\perp)$ is determined by initial conditions for incident particles, when one can neglect the change of distribution function $g(\varepsilon_\perp,l)$ with l due to multiple scattering. There is also a limitation of the crystal thickness from below, which is connected with an establishment of the equilibrium distribution. This means that the crystal thickness L is much larger than the free path length $\lambda_\perp$ [9]. Moreover, one can consider the distribution over transverse coordinates as an uniform one, since the transverse dimensions of an incident beam are much larger than the distances between axes. Then the initial distribution function over transverse energy $\varepsilon_\perp$ at a fixed incident angle $\vartheta_0$ is

$$g(\varepsilon_\perp)d\varepsilon_\perp = n_\perp d^2\rho_0 \qquad (4)$$

where $n_\perp$ is a density of the chains of atoms in the crystal. For the axially symmetric potential one has

$$g(\varepsilon_\perp)d\varepsilon_\perp = 2\pi n_\perp \rho_0 d\rho_0 = 2\pi n_\perp \rho_0(\varepsilon_\perp,\vartheta_0)\left|\frac{dU(\rho_0)}{d\rho_0}\right|^{-1} d\varepsilon_\perp \qquad (5)$$

where $\rho_0(\varepsilon_\perp,\vartheta_0)$ is a solution of equation

$$\varepsilon_\perp(\rho_0) = \frac{\varepsilon}{2}\vartheta_0^2 + U(\rho_0) \equiv \varepsilon_0 + U(\rho_0) \qquad (6)$$

At motion of ultrarelativistic particle in the potential $U(\underline{\rho})$ an instantaneous radiation intensity is determined by the known equation (see, e.g. [10]):

$$I(\underline{\rho}) = \frac{2}{3}e^2\underline{w}^2\gamma^4 = \frac{2}{3}\frac{e^2\gamma^2}{m^2}(\underline{\nabla}U)^2 \equiv A(\underline{\nabla}U)^2 \qquad (7)$$

where $\underline{w}$ is an acceleration, $\gamma=\frac{\varepsilon}{m}$ is the Lorentz factor. Using the distribution function (I), one obtains



$$I(l) = A\int d\varepsilon_\perp g(\varepsilon_\perp,l)\frac{l}{S(\varepsilon_\perp)}\int(\underline{\nabla}U)^2\vartheta(\varepsilon_\perp-U(\rho))d^2\rho \qquad (8)$$

for the radiation intensity of particles moving near the crystalline axes in a single crystal. The integrals over $d^2\rho$ in (8) are taken within one elementary cell. The contribution to the radiation of the particles moving above the barrier (when $\varepsilon_\perp \geq U_0$, $U_0 = \max U, U \geq 0$) is given by

$$I_{nc} = An_\perp\int(\nabla U)^2 d^2\rho\int_{\varepsilon_\perp\geq U_0} g(\varepsilon_\perp,l)d\varepsilon_\perp \qquad (9)$$

Since at $\varepsilon_\perp \geq U_0$ the whole transverse coordinate space is accessible, then $s(\varepsilon_\perp) = I/n_\perp$. In the case $\varepsilon_0 \geq U_0$ all the particles are moving above the barrier, then using the normalization condition for the function $g(\varepsilon_\perp,l)$ one has for the radiation intensity,

$$I_{as} = An_\perp\int(\underline{\nabla}U)^2 d^2\rho\,,\quad \varepsilon_0 \geq U_0 \qquad (10)$$

Thus, the radiation intensity at $\varepsilon_0 \geq U_0$ ($\vartheta_0 \geq \vartheta_c$, $\vartheta_c=\sqrt{\frac{2U_0}{\varepsilon}}$ is the Lindhard angle) does not depend on the incident angle and a charge sign of the particle for any potential (let us remark, that in Fig. 2 of Ref. [8] the curves do not follow such a behaviour). This situation differs from the case of particle motion near the crystalline planes, where the difference between radiation intensities of the particle with negative and positive charge sign and their dependence on $\varepsilon_0$ vanish in the limit $\varepsilon_0 \gg U_0$ only.

For the electrons moving in the axial channel, the above consideration is based on the fact that for initial trajectories which give the main contribution to the radiation, an angular momentum fails to be a good integral of motion. Due to this, an averaging over momenta has been carried out in the distribution function $g(\varepsilon_\perp)$. Generally speaking, this approach is valid for electrons with not very high energies ( < 10 GeV).

Let us discuss some general properties of the radiation

intensity of the particles moving near the crystalline axes. One can restrict oneself to a consideration of one cell in the plane transverse to the axes, which contains a projection of one chain of atoms forming the axis. The main contribution to integrals (8) - (10) is given by the region, where the gradient of the potential $U(\varrho)$ is large. This region has some characteristic scale $a_s$ which is a screening radius. Inside this region the potential is axially symmetric with a good accuracy. Let us represent the elementary cell as a circle with the radius $r_0 = 1/\sqrt{\pi n_\perp}$. For a real crystal, $r_0 \gg a_s$. This means that one can extend to $\infty$ the upper limit of the integration over $\varrho$ in Eq. (10). Then,

$$I_{as} \simeq 2\pi A n_\perp \int_0^\infty \left(\frac{dU}{d\varrho}\right)^2 \varrho\, d\varrho = 2\pi A n_\perp U_0^2 \int_0^\infty f'^2(\varrho)\, \varrho\, d\varrho \qquad (11)$$

where $f(\varrho) = U(\varrho)/U(0)$. If f is a function of $\varrho/a_s$ i.e. $f = f(\varrho/a_s)$, then after the substitution $\varrho \to \varrho a_s$ one can see that $I_{as}$ is independent on a specific value of the screening radius and depends on a shape of the potential only (for real crystals one should consider more complicated case, where actually there are two characteristic scales). This result differs essentially from the case of the particle moving near the crystalline planes where the intensity of the radiation is increasing when $a_s \to 0$.

Let us consider now qualitative features of an orientation dependence of the radiation of the particles moving near the crystalline axes in thin crystals. At $\vartheta_0 = 0$ a portion of the positrons $\Delta N^+$, which approach the axis at a distance $\varrho \lesssim a_s$, where the radiation mainly takes place, $\Delta N^+ \sim a_s^2/r_0^2$, so

$$I^+(\vartheta_0 = 0) \sim \frac{a_s^2}{r_0^2} I_{as} \qquad (12)$$

For electrons there is a different situation. The portion of electrons, which has an impact parameter $\varrho_0$ inside $d^2\varrho$ is given by

$$n_\perp \frac{d^2\varrho\, d^2\varrho_0}{\pi \varrho_0^2}\, \vartheta(\varrho_0 - \varrho)$$



Averaging I (7) with this distribution and taking into account that in the integral over $\varrho$ the main contribution comes from the region $\varrho \lesssim a_s$, we obtain up, to logarithmic accuracy,

$$I^-(\vartheta_0 = 0) \simeq I_{as} \int_{a_s}^{r_0} \frac{d^2\varrho_0}{\pi \varrho_0^2} = I_{as} \ln \frac{r_0^2}{a_s^2} \qquad (13)$$

Thus, at zero incident angle the radiation intensity of electrons exceeds considerably the radiation intensity of positrons.

For the axially symmetric potential, we obtain the radiation intensity substituting (5) into Eq. (8):

$$I(\vartheta_0) = 4\pi^2 A n_\perp \int_0^{r_0} \varrho_0 d\varrho_0 \frac{1}{s(\varepsilon_\perp, \varrho_0)} \int \left(\frac{dU}{d\varrho}\right)^2 \varrho\, d\varrho\, \vartheta(\varepsilon_\perp(\varrho_0) - U(\varrho)) \qquad (14)$$

where $s(\varepsilon_\perp(\varrho_0)) = 2\pi \int \varrho\, d\varrho\, \vartheta(\varepsilon_\perp(\varrho_0) - U(\varrho))$, value $\varepsilon_\perp(\varrho_0)$ is given by Eq. (6). Going over to the variables $\alpha = \varrho^2/a_s^2$, $\gamma = \varrho_0^2/a_s^2$, and after some manipulation, we have for the radiation intensity of positrons

$$I^+(\vartheta_0) = 4\pi A n_\perp \left\{ Q \left[ \vartheta(\varepsilon_0 - U_0) + \frac{y_+}{x_0} \vartheta(U_0 - \varepsilon_0) \right] + \right.$$
$$\left. + \vartheta(U_0 - \varepsilon_0) \int_{y_-}^{x_0} \frac{dy}{x_0 - x_+(y)} \int_{x_+(y)}^{x_0} x\, U'^2(x)\, dx \right\} \qquad (15)$$

where $\varepsilon_0 = \varepsilon \vartheta_0^2/2$, $x_0 = r_0^2/a_s^2$, $U_0 = U(0) > 0$, $U(x_0) = 0$; the quantities $y_+$, $x_+(y)$ are determined by equations

$$U(x_+(y)) = U(y) + \varepsilon_0, \quad U(y_+) = U_0 - \varepsilon_0; \qquad (16)$$
$$Q = \int_0^{x_0} x\, U'^2(x)\, dx$$

and for the radiation intensity of electrons

$$I^-(\vartheta_0) = 4\pi A n_\perp \left\{ Q \left[ 1 - \frac{y_-}{x_0} \vartheta(U_0 - \varepsilon_0) \right] + \right.$$
$$\left. + \vartheta(U_0 - \varepsilon_0) \int_0^{y_-} \frac{dy}{x_-(y)} \int_0^{x_-(y)} x\, U'^2(x)\, dx \right\} \qquad (17)$$

where

$$U(x_-(y)) = U(y) - \varepsilon_0, \quad U(y_-) = \varepsilon_0 \qquad (18)$$

In some papers (see e.g. Ref. [8]) the potential was used

$$U(\varrho) = \frac{\alpha}{\varrho}\vartheta(\varrho-u_\perp) + \frac{\alpha}{u_\perp}\vartheta(u_\perp-\varrho) \qquad (19)$$

In this case, there is one characteristic scale (it is the thermal vibration amplitude $u_I$). So, in accordance with above results, the radiation intensity of the particles moving above the barrier ( $\mathcal{E}_\perp > U_0 = \alpha/u_\perp$) does not depend on the amplitude of thermal vibrations.

Substituting (19) in Eq. (II), we have

$$I_{as} \doteq 2\pi A n_\perp U_0^2 \int_0^\infty f'^2(z) z\,dz = 2\pi A n_\perp U_0^2 \int_\perp^\infty \frac{dz}{z^2} = \frac{2\pi e^2 U_0^2 \gamma^2}{3m^2} n_\perp \qquad (20)$$

The main contribution to the intensity is given by the region $\varrho \sim u_\perp$ (at $\varrho \gtrsim \varrho_1 \gg u_\perp$ this contribution is $\sim u_1^2/\varrho_1^2$). One can readily obtain a simple analytical expression for the radiation intensity in the potential (19) using the developed approach. In particular, at $\vartheta_0 = 0$ we have, from (15) - (18),

$$I^+ = \frac{u_1^2}{z_0^2}\left(\ln\frac{z_0^2}{u_1^2}+1\right) I_{as}\,, \quad I^- = \left(\ln\frac{z_0^2}{u_1^2}-1\right) I_{as} \qquad (21)$$

where $I_{as}$ is given by Eq. (20). This results agree naturally with the performed qualitative analysis (compare with (12)-(13)). However, one should bear in mind that in a realistic potential of the chain of atoms the region of integral convergence is $\varrho \sim a_s \gg u_I$. This means that Eq. (21), in particular, diminishes substantially the radiation intensity of positrons since $a_s^2/u_I^2 \gtrsim 10$. Moreover, one can obtain erroneous notion of a radiation spectrum using the potential (19), because the main contribution in this case is given by the trajectories with $\varrho \sim u_I$, while actually the whole interval from $u_I$ to $a_s$ contributes. For these reasons it seems unadequate an utilization of the potential (19) for the description of the radiation of the particles moving near the crystalline axes.

We have used the Moliere potential for isolated atom, taking into acccunt thermal (zero) vibrations. Then we have summed over lattice to obtain "exact" potential. As it was stressed, the main contribution to the radiation intensity is given by the region where the gradient of the potential is large. This takes place when the distance from the axis is rather small and the potential is axially symmetric with a good accuracy. In this region it is possible to use a simple approximation of exact potential (in some sense this is the so called standard potential [9])

$$U(\varrho) = V_0\left[\ln\left(1+\chi\frac{a_0^2 z^{-1/3}}{\varrho^2+2\beta u_1^2}\right) + \text{const}\right], \quad V_0 = \frac{Ze^2}{d} \qquad (22)$$

where d is an average distance between atoms in the chain, $u_I$ is the thermal vibration amplitude, $a_0$ is the Bohr radius. Parameters $\chi$ and $\beta$ are fitted by comparing Eq. (22) with numerical calculations of the exact potential. The difference between these potentials does not exceed 10% in the main contribution region. Substituting the potential (22) in to Eqs. (15)-(18) one can obtain explicit expressions for the radiation intensities $I^+(\vartheta_0)$ and $I^-(\vartheta_0)$. The results for axis $\langle 100\rangle$ in tungsten are shown in Fig. I ($I^+(\vartheta_0)$ is curve I and $I^-(\vartheta_0)$ is curve 2). Details of the calculations will be published elsewhere.

So, for thin crystals, Eqs. (15)-(18) give an opportunity to obtain easily the radiation intensity for any axially symmetric potential. We would remind that this consideration was carried out in the frames of classical theory. This means that the quasiclassical nature of the particle motion was assumed (in real conditions this is valid starting from energy a few tens MeV) and a recoil at the radiation was neglected (this is possible up to an energy of $\sim$ TeV). The results obtained are valid within this energy interval, which is most important from the experimental point of view.

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## Figure Caption

Fig. 1. The dependence on $\frac{\varepsilon_0}{\varepsilon_{i0}} = \left(\frac{\vartheta_0}{\vartheta_c}\right)^2$ of radiation intensity at axial channeling in W (axis $<100>$) for positrons (curve I) and electrons (curve 2). Quantity $I_0 = \frac{8\pi e^2 N_0^2 \gamma^2 n_L}{3m^2}$ (see Eq.(22)).

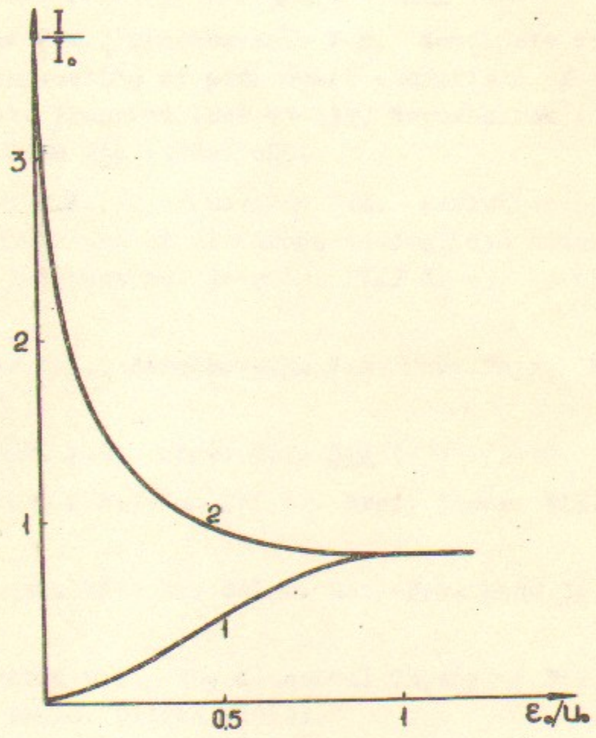


Fig. 1.



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# ИЗЛУЧЕНИЕ УЛЬТРАРЕЛЯТИВИСТСКИХ ЧАСТИЦ, ДВИЖУЩИХСЯ ВБЛИЗИ КРИСТАЛЛИЧЕСКИХ ОСЕЙ

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