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WAVE FUNCTIONS OF THE MESONS WHICH CONTAIN S,C,B QUARKS

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A B S T R A C T

The properties of the pseudoscalar and vector meson wave functions which are antisymmetric under the interchange of the quark momenta are investigated. We obtain: $\langle x_s - x_u \rangle \simeq 0.1$ for the K-meson and $\langle x_s - x_u \rangle \simeq 0.15 \div 0.2$ for the $\overset{*}{K}$-meson, where $\langle x_s \rangle$ ($\langle x_u \rangle$) is the mean longitudinal momentum fraction carried by the s (u)-quark. The results are applied to the calculation of the asymptotic behaviour of the K- and $\overset{*}{K}$-meson form factors and to the $\eta_c \to K^* K$ decay.

As byproduct we have also estimated the following vacuum averages: $\langle 0|\bar{u}u - \bar{s}s|0\rangle / \langle 0|\bar{u}u|0\rangle \simeq 0.2 \div 0.25$; $\langle 0|\bar{u}u - \bar{d}d|0\rangle / \langle 0|\bar{u}u|0\rangle \simeq 8 \cdot 10^{-3}$

The properties of the D(1865) and B(5200)-meson wave functions are investigated. The main results are: $f_B \simeq 90$ MeV, $f_D \simeq 160$ MeV and $\langle x_d \rangle \simeq 0.10$ for the B-meson. We argue that the processes of D-meson production are not, in general, enhanced in comparison with those of K-mesons.

It has been shown in [1,6] that the method of QCD sum rules allows one to find the most characteristic properties of the hadronic wave functions. In this paper we investigate the properties of those wave functions which are antisymmetric under interchange of the quark momenta. For the pseudoscalar and vector mesons these wave functions are nonzero due to the SU(3) or isotopic spin symmetry breaking effects.

In sect.II the most characteristic properties of D(1865)- and B(5200)-meson wave functions are described.

I. Antisymmetric components of the pseudoscalar and vector meson wave functions.

For the K, $K^*_{\lambda=0}$ and $K^*_{\lambda=1}$ -meson wave functions of the leading twist:

$$\langle 0|\bar{s}(z)\gamma_\mu\gamma_5 u(-z)|K(q)\rangle = i q_\mu f_K \varphi^A_K(zq) + \dots$$

$$\langle 0|\bar{s}(z)\gamma_\mu u(-z)|K^*_{\lambda=0}(q)\rangle = q_\mu f^V_{K^*}\varphi^V_{K^*}(zq) + \dots \quad , \quad e^{\lambda=0}_\mu m_{K^*} \simeq q_\mu$$

$$\langle 0|\bar{s}(z)i\sigma_{\mu\nu} u(-z)|K^*_{\lambda=1}(q)\rangle = i(e^\perp_\mu q_\nu - e_\nu q_\mu) f^T_{K^*}\varphi^T_{K^*}(zq) + \dots \qquad (1)$$

$$\varphi_i(zq) = \int_{-1}^{1} d\xi\, e^{i\xi(zq)}\varphi_i(\xi) \quad , \quad \varphi_i(\xi) = \varphi_i^{(+)}(\xi) + \varphi_i^{(-)}(\xi)$$

$$\varphi_i^{(\pm)}(\xi) = \pm\varphi_i^{(\pm)}(-\xi) \quad , \quad \int_{-1}^{1} d\xi\,\varphi_i(\xi) = 1$$

the SU(3)-symmetry breaking effects lead to a number of effects.

1) $f_K \neq f_\pi$, $f^{V,T}_{K^*} \neq f^{V,T}_\rho$ i.e. the values of the wave functions at the origin are unequal.

2) $\varphi^{A(+)}_K(\xi) \neq \varphi^A_\pi(\xi)$, $\varphi^{V,T(+)}_{K^*}(\xi) \neq \varphi^{V,T}_\rho(\xi)$ i.e. the symmetric components of the wave functions also differ.

3) $\varphi^{A(-)}_K(\xi) \neq 0$, $\varphi^{V,T(-)}_{K^*}(\xi) \neq 0$, i.e. there arise nonzero antisymmetric

components. In the exact symmetry limit

The properties of the N°1 and N°2-effects have been investigated by us in the previous papers [1]. It is the goal of this section to investigate the properties of the wave functions $\varphi_i(\xi)$

These wave functions determine the asymptotic behaviour of the exclusive processes which are caused by the SU(3)-symmetry breaking effects. Below we calculate the values of few lowest moments: $\langle \xi^n \rangle = \int_{-1}^{1} d\xi\, \xi^n \varphi_i(\xi)$, $n=1,3$ for these wave functions. The lowest moment $\langle \xi^1 \rangle$ determines the normalization of the wave function and therefore-the characteristic values of the symmetry breaking effects (for instance, the quantity $f_K \langle \xi \rangle_K$ is analogous to $f_K \sqrt{\langle \xi^2 \rangle_K}$). The ratio $\left(\langle \xi^3 \rangle / \langle \xi \rangle \right)$ determines the characteristic width of the wave function $\varphi_i(\xi)$.

As one can see from (1), the values of the moments $\langle \xi^{2K+1} \rangle$ are determined by the matrix elements of the local operators $\langle 0 | \bar{s}\, \Gamma\, (iz\overleftrightarrow{D})^{2K+1} u | M \rangle$ with odd number of derivatives.

a) Qualitative considerations and estimates.

Consider the correlator:

$$T_A = i \int dx\, e^{iqx} \langle 0 | T\{ \bar{u}(x)\, \hat{z} \gamma_5 (iz\overleftrightarrow{D})^n s(x),\ \bar{s}(0) \gamma_5 u(0) \} | 0 \rangle = \tag{2}$$

$$= (zq)^{n+1} I_A(q^2), \quad z^2 = 0, \quad n = 1, 3, 5 \ldots$$

The spectral density has the form:

$$\frac{1}{\pi} \operatorname{Im} I_A(s) = \left[i f_K \langle (x_s - x_u)^n \rangle_K \right] \left[-i f_K \frac{m_K^2}{m_s + m_u} \right] \delta(s - m_K^2) + \ldots \tag{3}$$

where $\langle x_s \rangle$ is the mean fraction of the K-meson longitudinal momentum carried by the s-quark (at $q_z \to \infty$), $\langle \xi \rangle \equiv \langle x_s - x_u \rangle$

To obtain the estimate let us confine ourselves here by the fig.1 and fig.2 contributions and the K-meson contribution into (3). Then we have at $n=1$

$$e^{-\frac{m_K^2}{M_0^2}} f_K^2 \frac{m_K^2}{m_s + m_u} \langle x_s - x_u \rangle_K \simeq \langle 0 | \bar{s}s - \bar{u}u | 0 \rangle + \frac{m_s - m_u}{8\pi^2} M_0^2 + \ldots \qquad M_0^2 = M_{K^*}^2 \simeq 0.8\,GeV^2 \tag{4}$$

Consider also the correlator ($n = 1, 3, 5 \ldots$):

$$T = i \int dx\, e^{iqx} \langle 0 | T\{ \bar{s}(x)\, \hat{z} \gamma_5 u(x)\ \ \bar{u}(0)\, \hat{z} \gamma_5 (iz\overleftrightarrow{D})^n s(0) \} | 0 \rangle = (zq)^{n+2} I(q^2); \quad z^2 = 0 \tag{5}$$

In the same approximation:

$$e^{-\frac{m_K^2}{M_{K^*}^2}} f_K^2 \langle x_s - x_u \rangle_K = \frac{m_s^2 - m_u^2}{4\pi^2} + \frac{m_u \langle \bar{u}u \rangle - m_s \langle \bar{s}s \rangle}{M_{K^*}^2} \tag{6}$$

where $M_{K^*}^2 \simeq 0.8\,GeV^2$ is the characteristic scale at which the following power corrections at r.h.s. of (4),(6) are 20%. We have from (4),(6) ($m_s \simeq 150\,MeV$, $m_u = 4.5\,MeV$, $m_d = 7.5\,MeV$, $f_K \simeq 160\,MeV$, $\langle \bar{u}u \rangle \simeq -(0.25\,GeV)^3$)

$$\langle x_s - x_u \rangle_K \simeq 0.12 \quad ; \quad \frac{\langle \bar{u}u \rangle - \langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} \simeq 0.22 \tag{7}$$

For the case of the π -meson we have the analogous relations*:

$$f_\pi^2 \frac{m_\pi^2}{m_u + m_d} \langle x_d - x_u \rangle_\pi^{strong} \simeq \frac{1}{8\pi^2} (m_d - m_u) M_\rho^2 + \langle 0 | \bar{d}d - \bar{u}u | 0 \rangle,$$

$$f_\pi^2 \langle x_d - x_u \rangle_\pi^{strong} \simeq \frac{1}{4\pi^2} (m_d^2 - m_u^2) + \frac{m_u \langle \bar{u}u \rangle - m_d \langle \bar{d}d \rangle}{M_\rho^2} \tag{8}$$

$$\langle x_d - x_u \rangle_\pi^{strong} \simeq 0.45 \cdot 10^{-2} \qquad \frac{\langle \bar{u}u - \bar{d}d \rangle}{\langle 0 | \bar{u}u | 0 \rangle} \simeq 0.8 \cdot 10^{-2}$$

It follows from (7),(8) that the s-quark in the K-meson and the d-quark in the π -meson carry larger longitudinal momentum fractions as compared with the u-quark. This result were trivial if the s-quark mass be much larger than the inverse confinement radius, but for the light s- and d-quarks it is highly nontrivial.

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* There are also the electromagnetic contribution into $\langle x_d - x_u \rangle_\pi$.

The experience with the sum rules shows that the properties of the second resonance in the spectral density are, as a rule, opposite to that of the lowest one. For instance, while the π-meson wave function is wider than the asymptotic wave function $\varphi_{as}(\xi)=\frac{3}{4}(1-\xi^2)$ [6], the A_1-meson wave function is narrower than $\varphi_{as}(\xi)$, etc. Such behaviour seems natural taking into account the duality relations. While the properties of the true spectral density are, on the average, the same as in the perturbation theory (i.e. $\tilde{\varphi} \simeq \varphi_{as}$) some redistribution of the properties takes place really so that the contributions of the separate resonances lie above or below the average. Therefore, while $\langle x_d - x_u \rangle > \frac{M_\rho^2 (m_d^2 - m_u^2)}{8\pi^2 f_\pi^2 m_\pi^2}$ for the π-meson and $\langle x_s - x_u \rangle > \frac{(m_s^2 - m_u^2) M_{K^*}^2}{8\pi^2 f_K^2 m_K^2}$ for the K-meson, one can expect that these quantities will be opposite sign for the next resonances.

It is seen from (7),(8) that the heavier is the quark, the smaller is the absolute value of its vacuum condensate. For the light d- and s-quarks this is also highly nontrivial.*

Let us discuss now in short the situation with the higher moments $\langle \xi^3 \rangle$, $\langle \xi^5 \rangle$ which characterize the width of the wave function $\varphi(\xi)$. Since at the fig.1 diagram the whole momentum is carried by one quark and the other one is "wee", this contribution corresponds to the wave function of the type: $[\delta(1-\xi) - \delta(1+\xi)]$. It is evident beforehand that the true wave function is more narrow, so that the true values of the moments fall off as "n" increases. This decrease of the moment values is ensured by the next nonperturbative corrections in the sum rules. Indeed, the next correction to the fig.1 contribution $m_s \langle \bar{s}s \rangle$ in the correlator (5) is given by the fig.3 diagram contribution

* See section III for more detail.

on and is proportional to: $-n\, m_s \langle \bar{s}\, ig\, \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a s \rangle / M^2$

This contribution has the sigh opposite to the fig.1 diagram contribution and its absolute value grows with n. Therefore, it ensures the decrease of the moment values $\langle \xi^n \rangle$ with increase of n.

The next power corrections in the correlator (2) sould also provide the decrease of the moment values with the growth of n. In this case the next correction to the main contribution from the fig.1 diagram: $\langle \bar{s}s - \bar{u}u \rangle$ is proportional to: $-n[\langle \bar{s}\, ig\, \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a s \rangle - s \to u]$ Therefore, this last term should have the sign opposite to $[\langle \bar{s}s - \bar{u}u \rangle] > 0$, i.e.:

$$[\langle \bar{s}\, ig\, \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a s \rangle - \langle \bar{u}\, ig\, \sigma_{\mu\nu} \lambda^a G^a_{\mu\nu} u \rangle] > 0 \qquad (9)$$

Since $\langle \bar{u}\, i\sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a g u \rangle < 0$, the absolute value of this vacuum condensate also decreases as the quark mass increases (at least in the region $m_q \leq m_s$).* It seems natural to assume that at $m_q \leq m_s$ the presence of $G_{\mu\nu}$ in $\langle ig\, \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a \psi \rangle$ does not influence the dependence of this matrix element on m_q**.

In this case:

$$\frac{\langle \bar{s}\, ig\, \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a s \rangle}{\langle \bar{u}\, ig\, \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a u \rangle} \simeq \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} \simeq 0.75 \div 0.8 \qquad (10)$$

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* It is known /2/ that at large values of the quark mass the matrix element $\langle \bar{q}\, ig\, \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a q \rangle$ grows with m_q!

$$\langle \bar{q}\, ig\, \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a q \rangle_Q = -m_q \left(\ln \frac{\bar{Q}^2}{m_q^2} - 1\right) \langle \frac{\alpha_s}{\pi} G^2 \rangle - \frac{1}{m_q} \frac{1}{12\pi^2} \langle g^3 f^{abc} G^a G^b G^c \rangle$$

This formula is, in general, unapplicable at $m_q \simeq m_s \simeq 150\,MeV$, but iven in this case using: $\langle \frac{\alpha_s}{\pi} G^2 \rangle \simeq 1.2 \cdot 10^{-2}\,GeV^4$, $\langle g^3 G^3 \rangle \simeq 0.4\,GeV^6$ it is seen that the second term is larger than the first one and therefore at $m_q \simeq m_s$ this matrix element can still decrease with m_q

** If at $m_q < m_s$ this matrix element is dominated by the instanton contribution, this is the case.

b) Quantitative analysis of the sum rules.

For the determination of the values $\langle \xi^n \rangle_K^A$, $n=1,3$ we use the correlator (2) and for the $\langle \xi^n \rangle_{K^*}^V$ –the correlator

$$T_V = i\int dx e^{iqx} \langle 0 | T\{ \bar{s}(x)\sigma_{\mu\nu} z_\nu u(x)\ \bar{u}(0)\hat{z}(iz\overleftrightarrow{D})^n s(0)\} | 0 \rangle = \tag{11}$$

$$= z_\mu (zq)^{n+1} I_V(q^2), \quad z^2=0, \quad n=1,3 \ldots$$

Finally, for the determination of $\langle \xi^n \rangle_{K^*}^T$ we use the correlator:

$$T_T = i\int dx e^{iqx} \langle 0 | T\{ \bar{s}(x)\sigma_{\mu\nu} z_\nu (iz\overleftrightarrow{D})^n u(x)\ \bar{u}(0)\hat{z} s(0)\} | 0 \rangle = \tag{12}$$

$$= z_\mu (zq)^{n+1} I_T(q^2), \quad z^2=0, \quad n=1,3,\ldots$$

The sum rules have the form ((13),(14),(15) correspond to (2),(11),(12)):

$$\frac{f_K^2 m_K^2}{m_s+m_u} \langle (x_s-x_u)^n \rangle_K^A e^{-m_K^2/M^2} = \frac{3}{8\pi^2}\frac{m_s-m_u}{n+2} M^2 \left[1 - e^{-S_A^n/M^2} \right] +$$

$$+ \langle \bar{s}s - \bar{u}u \rangle - \frac{1}{3} n \frac{1}{M^2} \left[\langle 0 | \bar{s} ig\sigma_{\mu\nu} G_{\mu\nu}^a \lambda^a s | 0 \rangle - (s \to u) \right], \tag{13}$$

$$f_{K^*}^T f_{K^*}^V m_{K^*} \langle (x_s-x_u)^n \rangle_{K^*}^V e^{-m_{K^*}^2/M^2} = \frac{3}{8\pi^2}\frac{m_s-m_u}{n+2} M^2 \left[1 - e^{-S_V^n/M^2} \right] +$$

$$+ \langle \bar{s}s - \bar{u}u \rangle - \frac{2n+1}{6} \frac{1}{M^2} \left[\langle 0 | \bar{s} ig\sigma_{\mu\nu} G_{\mu\nu}^a \lambda^a s | 0 \rangle - (s \to u) \right], \tag{14}$$

$$f_{K^*}^T f_{K^*}^V m_{K^*} \langle (x_s-x_u)^n \rangle_{K^*}^T e^{-m_{K^*}^2/M^2} = \frac{3}{8\pi^2}\frac{m_s-m_u}{n+2} M^2 \left[1 - e^{-S_T^n/M^2} \right] +$$

$$+ \langle \bar{s}s - \bar{u}u \rangle - \frac{n+1}{6} \frac{1}{M^2} \left[\langle 0 | \bar{s} ig\sigma_{\mu\nu} G_{\mu\nu}^a \lambda^a s | 0 \rangle - (s \to u) \right], \tag{15}$$

We use at the numerical treatment of (13)-(15):*

$$\langle \bar{s}s - \bar{u}u \rangle \simeq -0.2 \langle 0 | \bar{u}u | 0 \rangle \simeq 3 \cdot 10^{-3}\, GeV^3$$

$$\langle \bar{s}\sigma_{\mu\nu} G_{\mu\nu}^a \lambda^a ig s \rangle - \langle \bar{u} ig\sigma_{\mu\nu} G_{\mu\nu}^a \lambda^a u \rangle \simeq -0.2 \langle \bar{u} ig\sigma_{\mu\nu} G_{\mu\nu}^a \lambda^a u \rangle \simeq \tag{16}$$

$$\simeq -0.2 (1.5\, GeV^2) \langle 0 | \bar{u}u | 0 \rangle \simeq 4.7 \cdot 10^{-3}\, GeV^5$$

As a result:

$$\langle x_s - x_u \rangle_K^A \simeq 0.10, \quad \langle x_s - x_u \rangle_{K^*}^V \simeq 0.15 \div 0.20, \quad \langle x_s - x_u \rangle_{K^*}^T \simeq 0.15 \div 0.2$$

$$\left[\frac{\langle (x_s-x_u)^3 \rangle}{\langle x_s - x_u \rangle} \right]_K^A \simeq 0.55 \div 0.60, \quad \left[\frac{\langle (x_s-x_u)^3 \rangle}{\langle x_s - x_u \rangle} \right]_{K^*}^{V,T} \simeq 0.5 \div 0.55 \tag{17}$$

(the normalization point in (17) is: $\mu^2 \simeq \bar{M}^2 \simeq m_{K^*}^2 \simeq 0.8\, GeV^2$).

The asymptotic form of the leading twist wave function $\varphi_i^-(\xi,\mu^2)$ is: $\varphi^-(\xi, \mu^2 \to \infty) \sim \xi(1-\xi^2)$. It has been argued in [1][6] that $\varphi_i^{(\pm)}(\xi, \mu^2 \sim 1\, GeV^2)$ have, in general, the same behaviour at $\xi \to \pm 1$ i.e. $\sim (1-\xi^2)$ Confining ourselves, as usual, by two lowest Gegenbauer polinomials, let us choose the model wave functions in the form:

$$\varphi_i^-(\xi, \mu^2 \simeq 0.5 \div 1\, GeV^2) = (1-\xi^2)\xi(A\xi^2 + B) \tag{18}$$

One has now from (17),(18):

$$\varphi_i^-(\xi, \mu^2 \simeq 0.5 \div 1\, GeV^2) \simeq \langle \xi \rangle_i \frac{35}{4} \xi^3 (1-\xi^2), \quad \xi \equiv x_s - x_u \tag{19}$$

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* The values (16) agree with our estimate (6) and with the results [3,4]. It has been obtained in [5]: $\frac{\langle \bar{u}u - \bar{s}s \rangle}{\langle \bar{u}u \rangle} \simeq 0.5$ We want to emphasize that this value is highly overestimated, from our point of view, because it seems impossible to obtain the selfconsistent results at the sumiltaneous treatment of the various correlators in this case.

For these wave functions: $\langle\xi^3\rangle/\langle\xi\rangle \simeq 0.56$ that agrees with (17) and exceeds considerably the corresponding ratio for the asymptotic wave function (=0.33). In other words, the realistic wave functions $\varphi_i^-(\xi,\mu^2\sim 0.5\,GeV^2)$ are much wider than the asymptotic wave function $\varphi^-(\xi,\mu^2\to\infty)$.

At fig.4 the K-meson wave functions $\varphi_K^{A(+)}(\xi)$ [1], $\varphi_K^-(\xi)$ and $\varphi_K^A(\xi)=\varphi_K^{A(+)}(\xi)+\varphi_K^{A(-)}(\xi)$ are shown. At the characteristic values of $|\varphi_K^{(-)}(\xi)/\varphi_K^{(+)}(\xi)| \simeq 0.2\div 0.3$, $0.6\leq|\xi|\leq 0.8$, and this value seems reasonable enough.

c) Applications.

The leading term in the asymptotic behaviour of the K°-meson electromagnetic form factor has the form:

$$\langle K^o(q)|J_\mu|K^o(P_2)\rangle = (P_1+P_2)_\mu F_{K^o}(q^2)\ ,\ F_{K^o}=-F_{\bar K^o}\ ,\ q=P_2-P_1$$

$$F_{K^o}(q^2)\to\frac{32\pi\alpha_s}{9q^2}\cdot\frac{4}{3}f_K^2\, I_K^{(+)}I_K^{(-)} \tag{20}$$

$$I_K^{(+)}=\int_{-1}^{1}\frac{d\xi}{1-\xi^2}\varphi_K^{(+)}(\xi)\ ,\qquad I_K^{(-)}=\int_{-1}^{1}\frac{d\xi}{1-\xi^2}\xi\,\varphi_K^{(-)}(\xi)$$

Using $\varphi_K^{(+)}(\xi)$ from [1] and (17),(19) one has in the region

$|q^2|\simeq 10\div 15\,GeV^2$

$$F_{K^o}(q^2)\simeq\frac{0.1\,GeV^2}{q^2}\simeq -0.2\,F_{K^+}(q^2) \tag{21}$$

(in this region: $F_{K^+}(q^2)\simeq -\frac{(0.5\div 0.6)GeV^2}{q^2}$). For the cross-section $e^+e^-\to\bar K^o K^o$ one has:

$$\sigma(e^+e^-\to K^o\bar K^o)=\frac{1}{4}|F_{K^o}(q^2)|^2\sigma(e^+e^-\to\mu^+\mu^-)\simeq 2\cdot 10^{-37}cm^2,\ q^2\simeq 10\,GeV^2$$

Analogously, using for the $\overset{*}{K}$°-meson the wave function from [1] and (17),(19) one has:

$$F_{K^{o*}}(q^2)=F_{K^o}(q^2)\,\frac{f_{K^*}^2\ \langle\xi\rangle_{K^*}\ I_{K^*}^+\ I_{K^*}^-}{f_K^2\ \ \langle\xi\rangle_K\ \ I_K^+\ I_K^-}\simeq\frac{(0.2\div 0.28)GeV^2}{q^2}$$

The wave function $\varphi_K^-(\xi)$ can be used also for the calculation of the fig.5 diagram contribution into the decay $\psi\to K^+K^-$. This contribution is nonzero due to the SU(3)-symmetry breaking effects and the estimate shows [6] that it can compete with the photon contribution, fig.6. If this is indeed the case, then the ratio $(\psi\to K^+K^-)/(\psi\to\pi^+\pi^-)$ deviates noticeably from the unity. The contributions of the diagrams like those shown at fig.5 into the $\psi\to K^+K^-$ decay amplitude is calculated in [7] in terms of the K-meson wave function $\varphi_K^A(\xi)$, however it is difficult to use this result.*

As an another application let us consider the decay of the charmonium ground state: $\eta_c(2980)\to K^*K$ [8,9]:

$$Br(\eta_c\to K^*K)=(4\pi\alpha_s)^2\frac{4}{9}\left(f_K^A f_{K^*}^V/M_{\eta_c}^2\right)^2 I^2 \tag{22}$$

$$I=\int_{-1}^{1}\frac{d\xi_1}{1-\xi_1^2}\varphi_K^A(\xi_1)\int_{-1}^{1}\frac{d\xi_2}{1-\xi_2^2}\varphi_{K^*}^V(\xi_2)\,\frac{(\xi_1-\xi_2)}{1-\xi_1\xi_2}$$

Using the wave functions $\varphi_K^{A(+)}(\xi)$ and $\varphi_{K^*}^{V(+)}(\xi)$ from [1] and (17),(19) we have:

$$Br(\eta_c\to K^*K)\simeq 2\cdot 10^{-2}\%\qquad I\simeq 1.25 \tag{23}$$

The characteristic relative value of the SU(3)-symmetry breaking effects in the wave functions is $\simeq$ 0.2. The characteristic branching ratio for the two-particle charmonium decays is: $(0.1\div 1)\%$ Therefore, the characteristic branching ratio for

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* Let us note also that the contributions due to $\varphi_K^{(-)}(\xi)\neq 0$ into the decays $\chi_0(3415),\chi_2(3555)\to K^+K^-$ are negligible.

the charmonium decays which are caused by the SU(3)-symmetry breaking effects is: $\sim (0.2)^2 \cdot (0.1 \div 1)\% \simeq 4\cdot 10^{-2}(0.1 \div 1)\%$

II. <u>Wave functions of the mesons containing one heavy quark.</u>

We consider in this section the properties of the wave functions of those mesons which contain one light and one heavy quark (i.e. $\bar{q}Q$). These wave functions are of interest for the following reasons. The amplitudes contain usually the integrals of the form: $\int dx\, \varphi(x)/(1-x)$,where $\varphi(x)$ -is the meson wave function. The largest part of the $\bar{q}Q$-meson momentum is carried, of course, by the heavy quark Q. Therefore, the wave function $\varphi(x)$ has the strong extremum at $(1-x) \ll 1$ and this enhances the amplitudes. It is the goal of this section to investigate the properties of the $\bar{q}Q$-meson wave functions in more detail and to elucidate the characteristic properties of the processes which contain such mesons.

Let us denote the mean longitudinal momentum fractions carried by the light and by the heavy quarks (at $P_z \to \infty$) by $\langle x_q \rangle$ and $\langle x_Q \rangle$ correspondingly. We have for the nonrelativistic bound state: $\langle x_q \rangle / \langle x_Q \rangle \simeq m_q/m_Q \ll 1, \langle x_Q \rangle \approx 1$, i.e. the mean momentum fractions are determined mainly by the mass values while the interaction effects can be neglected. The corresponding estimate for the bound state of one light relativistic quark and one heavy nonrelativistic quark has the form: $\langle x_q \rangle / \langle x_Q \rangle \simeq \mu/m_Q \ll 1$,where $\mu \sim k_\perp \simeq 350 \div 400 MeV$ is the characteristic QCD scale. This gives for the B-meson($m_B \simeq 4.7 GeV$): $\langle x_q \rangle \simeq 0.07 \div 0.08$, $\langle x_B \rangle \simeq 0.92 \div 0.93$ and for the D-meson ($m_c \simeq 1.5 GeV$): $\langle x_q \rangle \simeq 0.20$, $\langle x_c \rangle \simeq 0.80$

Let us compare this with the realistic K-meson wave function (see [1] and (17),(19)):

$$\varphi_K^A(\xi) = \frac{15}{4}(1-\xi^2)\left[0.6\xi^2 + 0.25\xi^3 + 0.08\right] \quad ; \quad \xi = x_s - x_u$$

As an illustration, the expected characteristic form of the K,D and B-meson wave functions is presented at fig. 7 The D-meson wave function has one strong extremum at $x_c \simeq 0.8, x_u \simeq 0.2$ while the K-meson wave function has two extrema (each about two times smaller than for D) at $x_s \simeq 0.2, x_u \simeq 0.8$ and $x_s \simeq 0.8, x_u \simeq 0.2$. We conclude from this that <u>the processes with the D-meson are not, in general, enhanced as compared to that of K or π -mesons.</u> For instance, we expect(at $f_K \simeq f_D \simeq 160 MeV$):

$$\frac{Br\left(\Upsilon(^3P_{0,2}) \to D^+D^-\right)}{Br\left(\Upsilon(^3P_{0,2}) \to K^+K^-\right)} \simeq O(1) \qquad (24)$$

Therefore, our viewpoint here is opposite to those expressed in the paper [7].

At the same time the ratio D/K can be large if the K-meson amplitude is suppressed for some reason. This is just the case for the ratio:

$$\frac{Br\left(\Upsilon(^3S_1) \to D^+D^-\right)}{Br\left(\Upsilon(^3S_1) \to K^+K^-\right)} \simeq \left(\frac{1}{0.2}\right)^2 \gg 1 \qquad (25)$$

This ratio is large not because the D^+D^--decay is enhanced, but because the K^+K^--decay is suppressed. The reason is as follows. The diagram at fig.5 gives the main contribution into the decays $^3S_1 \to D^+D^-, D^0\bar{D}^0$ These decays are zero in the SU(4)-symmetry limit, but the SU(4)-symmetry is badly broken ($\simeq$ 100%) and so there are no really any suppression. At the same time the contribution of this diagram into the $^3S_1 \to K^+K^-, K^0\bar{K}^0$ decays is indeed suppressed (by the factor ~ 1/5), because it is zero in the SU(3)-symmetry limit and the SU(3)-symmetry breaking effects are small ($\simeq$ 20%).

The estimate presented in [6] shows that the photon exchange diagram, fig.6, and the diagram at fig.5 give roughly the same contributions into the $^3S_1 \to K^+K^-$ decay.* The contribution of the fig.6 diagram into $^3S_1 \to K^0\bar{K}^0$ decay amplitude is about 5 times smaller than to the $^3S_1 \to K^+K^-$ decay amplitude (see sect. I) and so the diagram at fig.5 gives here the main contribution.

Since the SU(2)-symmetry breaking effects are very small ($\sim 1\%$) the main contribution in to the $^3S_1 \to \pi^+\pi^-$ decay gives the diagram at fig.5.

2. The wave function of the heavy meson $\bar{q}Q$ is: $f_Q \varphi_Q(x_Q - x_q)$ $\int_{-1}^{1} d\xi \varphi_Q(\xi) = 1,\ \xi = x_Q - x_q$ and is defined analogously to (1). For the determination of f_Q and $\langle x_q^n \rangle$ let us consider the correlator:

$$T_{\mu\nu}^{(n)} = i\int dx\, e^{iqx} \langle 0 T\{\bar{q}(x)\gamma_\mu\gamma_5 Q(x)\ \bar{Q}(0)\gamma_\nu\gamma_5 (iz\overleftrightarrow{D})^n q(0)\}|0\rangle =$$
$$= (zq)^n \{ q_\mu q_\nu T_1^{(n)}(q^2) + (q_\mu q_\nu - g_{\mu\nu} q^2) T_2^{(n)}(q^2) \},\quad z^2 = 0 \tag{26}$$

The mesons with the quantum numbers 0^{-+} we are interested in contribute into the spectral density $Im\, T_1^{(n)}$. We use below the technique suggested in [10]: the energy E is used instead of q^2: $q^2 = (M_Q + E)^2$, $E \ll M_Q$. But in contrast with [10] we put $(q^2)^n \simeq (M_Q^2 + 2M_Q E)^n$ and keep all the terms $\sim (2M_Q E/M_Q^2)^k$ when calculating the perturbation theory contribution, fig.2. At the same time one can neglect the corrections and confine himself by the leading (at $M_Q \to \infty$) terms only when calculating the nonperturbative contributions, figs.1,3...

As a result, we have (after "borelization"):

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* Therefore, one can neglect the photon exchange contribution into the $^3S_1 \to D^+D^-, D^0\bar{D}^0$ decays.

$$(f_Q^2 M_Q) e^{-E_t/M} + \Omega_0(\beta)\frac{6}{\pi^2} M^3 e^{-E_0/M}\left(1 + \frac{E_0}{M} + \frac{1}{2!}\left(\frac{E_0}{M}\right)^2\right) = \Omega_0(\beta)\frac{6}{\pi^2}M^3 -$$
$$- \langle 0|\bar{u}u|0\rangle_M + \frac{1}{32}\frac{1}{M^2}\langle 0|\bar{u}\sigma_{\mu\nu} ig G^a_{\mu\nu}\lambda^a u|0\rangle + \frac{\pi}{162}\frac{1}{M^3}\langle 0|\sqrt{\alpha_s}\bar{u}u|0\rangle^2 + \ldots \tag{27}$$

$$(f_Q^2 M_Q)(\langle x_q\rangle M_Q) e^{-E_t/M} + \Omega_1(\beta)\frac{24}{\pi^2} M^4 e^{-E_1/M^2}\left(1 + \frac{E_1}{M} + \frac{1}{2!}\left(\frac{E_1}{M}\right)^2 + \frac{1}{3!}\left(\frac{E_1}{M}\right)^3\right) =$$
$$= \Omega_1(\beta)\frac{24}{\pi^2}M^4 + \frac{1}{12}\frac{1}{M}\langle 0|\bar{u} ig\sigma_{\mu\nu} G^a_{\mu\nu}\lambda^a u|0\rangle + \frac{\pi}{144}\frac{1}{M^2}\langle 0|\sqrt{\alpha_s}\bar{u}u|0\rangle^2 + \ldots \tag{28}$$

$$\Omega_k(\beta) = \frac{1}{\Gamma(k+3)}\int_0^\infty dx\, e^{-x}\frac{x^{k+2}}{(1+\beta x)^{k+3}},\quad \Omega_k(\beta=0) = 1,\quad \beta = \frac{2M}{M_Q}$$

Here: $E_{0,1}$ are the corresponding duality intervals, $E_t \simeq 0.46\,GeV$ is the energy of the lowest resonance, $f_Q^2 M_Q \sim const$, $\langle x_q^n\rangle M_Q^n \sim const$ at $M_Q \to \infty$, $\langle \sqrt{\alpha_s}\bar{u}u\rangle = -1.35\cdot 10^{-2} GeV^3$, $\langle \bar{u} ig\sigma_{\mu\nu} G^a_{\mu\nu}\lambda^a u\rangle \simeq -2.4\cdot 10^{-2} GeV^5$

Let us point out here some characteristic features of (27),(28).

a) Each derivative $\vec{D}$ in the correlator (26) introduces the factor $\sim M/M_Q$. Therefore, the mean momentum fraction of the light quark is: $\langle x_q\rangle \sim \bar{M}/M_Q$. This agrees, of course, with our estimate presented above, but taking now into account the nonperturbative corrections we shall determine more precisely the characteristic scale M and so- the value of $\langle x_q\rangle$

b) The power corrections in the correlator $T_{\mu\nu}^{(n)}$ begin with the operator of the dimensionality (n+3). The pure gluonic corrections like $\langle \frac{\alpha_s}{\pi}G^2\rangle$,etc., give, however, small contributions and can be neglected. Therefore, the main power corrections are: $\langle\bar{u}u\rangle$ -for f_Q, $\langle \bar{u} ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a u\rangle$ -for $f_Q\langle x_q\rangle$ $\langle \bar{u}g^2 GGu\rangle$ -for $f_Q\langle x_q^2\rangle$, etc.

c) The main power correction $\langle \bar{u} ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a u\rangle$ in the sum rule (28) for $\langle x_q \rangle$ have the sign opposite to those of the perturbation theory contribution, fig.2 and increases $\langle x_q \rangle$. Indeed, in the diagram at fig.3 which gives this power correction, the light quark is "wee". Therefore, this correction tends to diminish the role of such configurations where the light quark is "wee", i.e. it increases $\langle x_q \rangle$.

For the case of the b-quark ($M_B \simeq 4.7\,GeV$) the sum rules (27), (28) have been treated in the standard way [6, 10]. (The scale parameter M has been varied within the limits: $0.4 \le M \le 0.8\,GeV$ for (27) and $0.4 \le M \le 0.7\,GeV$ for (28)). The results of the best fits are:

$$f_B \simeq 90\,MeV, \quad E_0 \simeq 0.8\,GeV; \quad \langle x_q \rangle \simeq 0.1, \quad E_1 \simeq 1.6\,GeV \qquad (29)$$

The quantities (29) present one of the main results of this section. The value $f_B \simeq 110\,MeV$ has been obtained in [10] and this does not differ greatly from (29). This small difference seems surprising at first sight as $\Omega_0(\beta) \simeq 0.2$ in (27) while $\Omega_0(\beta) \simeq 1$ has been used in [10]. The reason is as follows. The spectral density in (27) has the dimensionality $[\mu^3]$ and so the main power correction $\langle \bar{u}u \rangle_\mu$ which determines the scale enters with the coefficient ~ 1 The sum rule (27) is fitted in the region of such values of M that: $|\langle \bar{u}u \rangle| \simeq \frac{1}{5} \cdot \frac{6}{\pi^2} \Omega_0(\beta) M^3$ and therefore the change of the value $\Omega_0(\beta)$ change mainly the scale $\bar{M}$ *. As was expected beforehand, the nonperturbative corrections enhance somewhat the value of $\langle x_q \rangle$ as compared with our estimate above.

We do not write here the sum rules for $\langle x_q^2 \rangle$ as the main power correction in this sum rule ($\sim \langle \bar{u}\, G G\, u \rangle$) is poorly known. The estimate is: $\langle x_q^2 \rangle \simeq (1 \div 2) 10^{-2}$.

* The result obtained in [11] for   is overestimated from our point of view.

The numbers are such that the sum rules (27), (28) can not, strictly speaking, be used for D-meson. We can obtain, however, the reasonable estimate for f_D from (27). It is not difficult to see that for the case of b-quark at $0.4 \le M \le 0.7\,GeV$ the perturbation theory contribution into the sum rule (27) can be neglected. There are every reason to believe that for the c-quark it can be neglected all the more. Therefore, we can rewrite (27) with the good accuracy in the form*:

$$f_Q^2 M_Q e^{-E_1/M} \simeq -\langle 0|\bar{u}u|0\rangle_M + \frac{1}{32 M^2} \langle 0|\bar{u}\, ig\sigma_{\mu\nu} G_{\mu\nu}^a \lambda^a u|0\rangle \simeq \qquad (30)$$

$$\simeq -\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle \left\{ \frac{1}{\sqrt{\alpha_s}} - \left(\frac{0.23\,GeV}{M}\right)^2 \right\}; \qquad E_1 \simeq 400\,MeV$$

Using: $M_B \simeq 4.75\,GeV$, $M_C \simeq 1.5\,GeV$, $\langle \sqrt{\alpha_s}\,\bar{u}u \rangle \simeq 1.35 \cdot 10^{-2}\,GeV^3$

$\langle \bar{u}\, ig\sigma_{\mu\nu} G_{\mu\nu}^a \lambda^a u \rangle \simeq -2.35 \cdot 10^{-2}\,GeV^5$

we have from (30):

$$f_Q^2 M_Q \simeq -2.85 \langle \sqrt{\alpha_s}\,\bar{u}u \rangle, \quad f_B \simeq 90\,MeV, \quad f_D \simeq 160\,MeV \qquad (31)$$

The value of f_B in (31) agrees well with (29) and so we expect that the value $f_D \simeq 160\,MeV$ is close to the true value.

In conclusion of this section let us note the following. The estimates obtained above for the B- and D-mesons look like: $\langle x_q \rangle_B \simeq 0.08$, $\langle x_q \rangle_D \simeq 0.2$. More precize treatment of the sume rules shows that the nonperturbative corrections tend to increase $\langle x_q \rangle$ (see (29)). Therefore, there are every reason to expect that $(\langle x_q \rangle) > 0.2$ for D-meson. This confirm our quantitative conclusion made above that the processes with the D-mesons are not enhanced as compared with those with the K, π-mesons. This can be checked in the following way.

* The anomalous dimension of the operator $\langle \bar{\psi}\, ig\sigma_{\mu\nu} G_{\mu\nu}^a \lambda^a \psi \rangle$ is very small.

Consider the inclusive reaction: $e^+e^- \to M(P) + X$ where M is the meson with the momentum P. At large P: $P = z\frac{Q}{2}$, $(1-z) \ll 1$ the missing mass is: $M_X^2 \simeq (1-z)Q^2 \ll Q^2$ and so the process is quasiexclusive. Based on the above considerations we expect that at large Q and $z \to 1$ The D- and π,K-meson production cross sections are roughly the same:

$$\frac{d\sigma(e^+e^- \to D^+ + X)/dz}{d\sigma(e^+e^- \to K^+ + X)/dz} \simeq 1 \;, \quad z \to 1$$

III. Summary.

Let us enumerate the main results.

1. Antisymmetric wave functions of the pseudoscalar and vector mesons are nonzero due to the SU(2) or SU(3)-symmetry breaking effects. We obtained: $\langle x_s - x_u \rangle \simeq 0.1$ for the K-meson, $\langle x_d - x_u \rangle \simeq 0.5 \cdot 10^{-2}$ - for the π^+-meson, $\langle x_s - x_u \rangle_{K^*} \simeq 0.15 \div 0.2$, $\langle x_d - x_u \rangle_\rho \simeq (0.7 \div 1)10^{-2}$ for the vector mesons. Therefore, for these mesons: the more heavy is the quark, the larger fraction of the longitudinal momentum it carries. This result is highly nontrivial for the light d- and s-quarks.

2. The selfconsistency conditions for the various sum rules require:

$$\frac{\langle \bar{u}u - \bar{s}s \rangle}{\langle \bar{u}u \rangle} \simeq 0.2 \div 0.25 \,, \qquad \frac{\langle \bar{u}u - \bar{d}d \rangle}{\langle \bar{u}u \rangle} \simeq 0.8 \cdot 10^{-2} \tag{32}$$

Therefore, the absolute value of the vacuum condensate decreases as the quark mass increases. This result is also nontrivial for the light d- and s-quarks. Let us remind that the sum rule for the K-meson constant f_K also prefers the value $\langle \bar{s}s \rangle / \langle \bar{u}u \rangle \simeq 0.8$ [1]. Our result for $\langle \bar{s}s \rangle / \langle \bar{u}u \rangle$ agrees with those obtained by different methods in [3,4] and that for $\langle \bar{d}d \rangle / \langle \bar{u}u \rangle$ with [5].

3. In connection with results (32) let us note the following. In the paper [2] there was investigated the dependence of the vacuum condensate $\sigma(m) = \langle 0|\bar{u}u + \bar{d}d|0\rangle$ (at $m = m_u = m_d$) on the mean quark mass: $m = \frac{1}{2}(m_u + m_d)$. At large m: $\sigma(m) \simeq -\frac{1}{12}\frac{1}{m}\langle \frac{\alpha_s}{\pi} G^2 \rangle$, at m=0: $\sigma(m) < 0$ and at small m the derivative $d\sigma(m)/dm$ is determined mainly by the two-pion contribution [2]:

$$\frac{d\sigma(m)}{dm} \simeq -\frac{3}{8\pi^2}\left(\frac{\sigma(0)}{f_\pi^2}\right)^2 \left[\ln\frac{M_0^2}{m_\pi^2} - a\right], \quad M_0^2 = 1\,GeV^2, \; a \simeq 2.5 \tag{33}$$

As a result, as m increases the absolute value of $|\sigma(m)|$ increases at first and then decreases. We want to emphasize that the results [2] do not contradict to the unequalities: $|\langle \bar{u}u \rangle| > |\langle \bar{d}d \rangle| > |\langle \bar{s}s \rangle|$. As for the unequality $|\langle \bar{u}u \rangle| > |\langle \bar{d}d \rangle|$, the reason is that the two-pion contribution is zero in the channel with the isotopic spin I=1 and so it is absent for ($\bar{u}u - \bar{d}d$), The value of $\langle \bar{u}u \rangle$ depends in fact both on $(m_u + m_d)$ and $(m_d - m_u)$

$$\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{u}u|0\rangle_0 - (m_d + m_u)A - (m_d - m_u)B$$

$$\langle 0|\bar{d}d|0\rangle = \langle 0|\bar{d}d|0\rangle_0 - (m_d + m_u)A + (m_d - m_u)B$$

$$A = \frac{3}{8\pi^2}\left(\frac{\langle 0|\bar{u}u|0\rangle}{f_\pi^2}\right)^2 \left[\ln\frac{M_0^2}{m_\pi^2} - a\right] \simeq 0.04\,GeV^2 \tag{34}$$

$$B = \frac{m_\pi^2}{m_u + m_d}\,\frac{\langle 0|\bar{u}u|0\rangle}{2M_\rho^2} \simeq \frac{1}{4\pi^2}\left(\frac{\langle 0|\bar{u}u|0\rangle}{f_\pi^2}\right)^2 \simeq 0.02\,GeV^2 \quad (\text{see } (8))$$

Here the constant A is taken from (33) and B-from our estimate (8) ($M_\rho^2 \simeq 4\pi^2 f_\pi^2$, $m_\pi^2/m_u + m_d \simeq -2\frac{\langle \bar{u}u \rangle}{f_\pi^2}$), the quantity $\langle \bar{u}u \rangle_0$ is the value of the condensate in the SU(2)×SU(2) chiral symmetry limit. The coefficient A in (34) determines the correction due to the chiral symmetry breaking but in the limit of the exact SU(2)-isotopic spin symmetry. The coefficient B determines the correction due to the isotopic symmetry breaking.

As for the sign of the difference $\left(\langle \frac{\bar{u}u+\bar{d}d}{2} - \bar{s}s\rangle\right)/\langle\bar{u}u\rangle$ the numbers are such that it is impossible to draw the definite conclusion about this sign from the results [2], We conclude that the results [2] do not cantradict to the unequalities (32) and so the interpretation of the signs (32) in the literature [3,5] is misleading.

4. It is shown from the analysis of the sum rules that

$$|\langle\bar{u}ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a u\rangle| > |\langle\bar{d}ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a d\rangle| > |\langle\bar{s}ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a s\rangle| ,$$

i.e. the absolute value of this vacuum average also decreases as the quark mass m_q increases (at $m_q \lesssim m_s$). Supposing that for the light u,d and s-quarks the factor $\sigma_{\mu\nu}$ is of no importance for the dependence on the quark masses,we have:

$$\frac{\langle 0|\bar{s}ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a s|0\rangle}{\langle 0|\bar{u}ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a u|0\rangle} \simeq \frac{\langle 0|\bar{s}s|0\rangle}{\langle 0|\bar{u}u|0\rangle} \simeq 0.75 \div 0.8$$

$$\frac{\langle 0|\bar{d}ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a d|0\rangle}{\langle 0|\bar{u}ig\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a d|0\rangle} \simeq \frac{\langle 0|\bar{d}d|0\rangle}{\langle 0|\bar{u}u|0\rangle} \simeq (1 - 0.8\cdot 10^{-2})$$

5. Using the antisymmetric wave functions we have found the asymptotic behaviour of the K^0 and K^{0*} -meson electromagnetic form factors

$$\langle K^0(p_2)|J_\mu|K^0(p_1)\rangle = (p_1+p_2)_\mu F_{K^0}(q^2) \quad , \quad q = p_2 - p_1$$

$$F_{K^0}(q^2) \simeq 0.1\,GeV^2/q^2 \quad , \quad F_{K^0}(q^2)/F_{K^+}(q^2) \simeq -0.2$$

$$F_{K^{0*}}(q^2) \simeq \frac{(0.2 \div 0.28)\,GeV^2}{q^2}$$

and the $\eta_c \to K^*K$ decay width

$$Br(\eta_c \to K^*K) \simeq 2\cdot 10^{-2}\%$$

(These form factors are equal zero in the SU(3)-symmetry limit and this decay equals zero in the SU(3) and SU(6)-symmetry limits).

6. For the heavy D(1865) and B(5200)-mesons we calculated the values of their wave functions at the origin (the constants f_D and f_B) and $\langle x_q\rangle_B$ -the mean fraction of the longitudinal momentum carried by the light quark in the B-meson (at $P_z \to \infty$):

$$f_B \simeq 90\,MeV \, , \quad f_D \simeq 160\,MeV, \quad \langle x_q\rangle_B \simeq 0.1$$

Therefore,about 90% of the B-meson momentum is carried by the b-quark. We have argued that for the D-meson $\langle x_q\rangle_D \gtrsim 0.2$,i.e. C-quark carries no more than 75-80% of the whole momentum. Therefore,the D-meson wave function $\varphi_D(x_Q)$ has one strong extremum at $x_Q \simeq 0.8, x_q \simeq 0.2$. Let us remind for comparison that the realistic K-(π) -meson wave function has two extrema (each two times smaller) at $x_s \simeq 0.8, x_u \simeq 0.2$ and at $x_s \simeq 0.2, x_u \simeq 0.8$ [1]. The extremum of the wave function at $(1-x) \ll 1$ enhances the amplitudes with this meson. Therefore,<u>we do not expect that the processes with the D-mesons are enhanced as compared with those with the K- or π -mesons.</u> In particular,we expect:

$$\frac{\Upsilon(^3P_{0,2}) \to D^+D^-}{\Upsilon(^3P_{0,2}) \to K^+K^-} \simeq O(1)$$

$$\frac{\frac{d\sigma}{dz}(e^+e^- \to D^+ + X)}{\frac{d\sigma}{dz}(e^+e^- \to K^+ + X)} \simeq O(1) \qquad P_{D,K} = \frac{zQ}{2} \, , \; z \to 1$$

where $P_{D,K}$ -is the D(K)-meson momentum, Q-is the photon energy.

APPENDIX

We describe here the simple method for finding the form of various asymptotic wave functions.

Consider, for instance, the correlator:

$$T_{\mu\nu} = i\int dx\, e^{iqx} \langle 0 | T\, V_\mu(x)\, V_\nu(0) | 0 \rangle, \quad V_\mu = \bar{d}\gamma_\mu u \tag{A.1}$$

$$T_{\mu\nu} = (q_\mu q_\nu - g_{\mu\nu} q^2)\, I(q^2)$$

The free-quark loop contribution is:

$$I_0(q^2) = -\frac{1}{4\pi^2} \ln q^2 \int_0^1 dx_1 dx_2\, \delta(1-x_1-x_2) \left[6 x_1 x_2 \right] \tag{A.2}$$

It is not difficult to see that the integrand in (A.2) is the asymptotic wave function for the system of the operators $\bar{d}\hat{z}(iz\overleftrightarrow{\partial})^n u$, $z^2=0$ (in the leading logarithm approximation, LLA). Indeed, let O_1 and O_2 to be the operators which belong to the different representation of the conformal group. Then at large q^2 (when the nonperturbative power corrections which violate the conformal symmetry died off) and neglecting temporarily the logarithmic corrections) one has:

$$T_{12} = i\int dx\, e^{iqx} \langle 0 | T(O_1(x)\, O_2(0)) | 0 \rangle \to 0 \tag{A.3}$$

due to the conformal invariance. It has been pointed out in [12] that the conformal spin is still conserved in LLA. Therefore, the conformal operators O_1 and O_2 still have the definite dimensionality (i.e. they are multiplicatively renormalizable) in LLA and (A.3) remains true. Let us take $O_2 = V_\mu$ and now we have from (A.2), (A.3) that the system of the multiplicatively renormalizable operators is $\{\bar{d}\hat{z}\, C_n^{3/2}(iz\overleftrightarrow{\partial}/\partial)u\}$ as just this system of Gegenbauer polinomials is orthogonal with the measure (A.2).

For the two-particle operators the asymptotic wave function determines completely the system of corresponding orthogonal polinomials (i.e. the orthogonality conditions (A,3) are sufficient). Below we give some examples.

a) $\bar{\psi}\psi\ (\bar{\psi}\gamma_5\psi)$, $\varphi_{as}(x_1,x_2) \sim 1 \to C_n^{1/2}(x_1-x_2)$ [13]

b) $\bar{\psi}\sigma_{\mu\nu}\psi$ $\quad \varphi_{as} \sim x_1 x_2 \to C_n^{3/2}(x_1-x_2)$ [14,]

c) $G^a_{\mu\nu} G^a_{\nu\lambda}$ $\quad \varphi_{as} \sim x_1^2 x_2^2 \to C_n^{5/2}(x_1-x_2)$ [14] (A.4)

d) $\bar{\psi}\hat{z}[(iD_\perp)^2]^k\psi$, $\varphi_{as} \sim (x_1 x_2)^{2k+1} \to C_n^{2k+3/2}(x_1-x_2)$

e) $\bar{\psi}[(iD_\perp)^2]^k\psi$, $\varphi_{as} \sim (x_1 x_2)^{2k} \to C_n^{2k+1/2}(x_1-x_2)$

The asymptotic wave functions $\varphi_{as}(x_1 x_2 x_3)$ for the three-particle operators can be found analogously, however the orthogonality conditions (A.3) are unsufficient in this case to determine the explicit form of the polinomials. The two-loop diagrams with the free quarks and gluons give:

f) $\psi\psi\psi$, $\quad \varphi_{as} \sim x_1 x_2 x_3$ [15], [16]

g) $\bar{\psi} G \psi$ $\quad \varphi_{as} \sim x_1 x_2^2 x_3$ (A.5)

h) GGG $\quad \varphi_{as} \sim x_1^2 x_2^2 x_3^2$

(The results (A.4), (A.5) refer to the leading twist for each given operator).

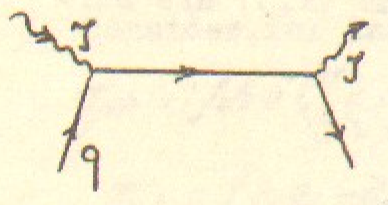

fig.1

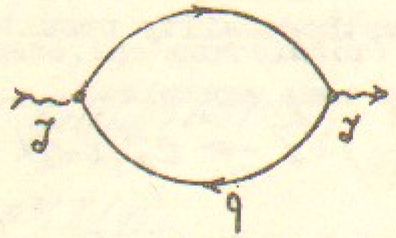

fig.2

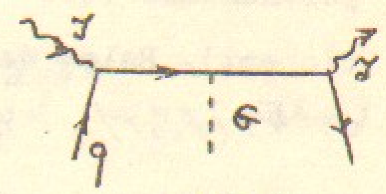

fig.3

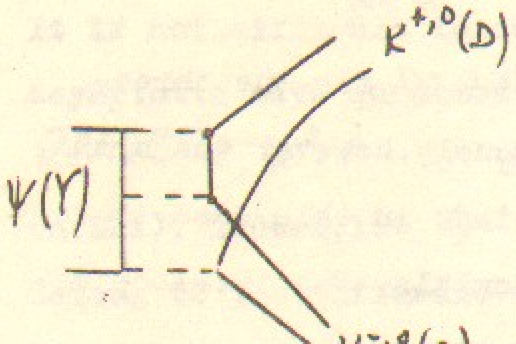

fig.5

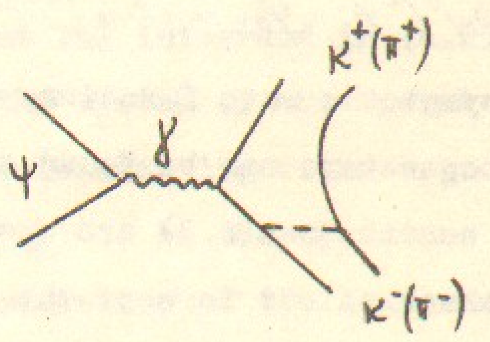

fig.6

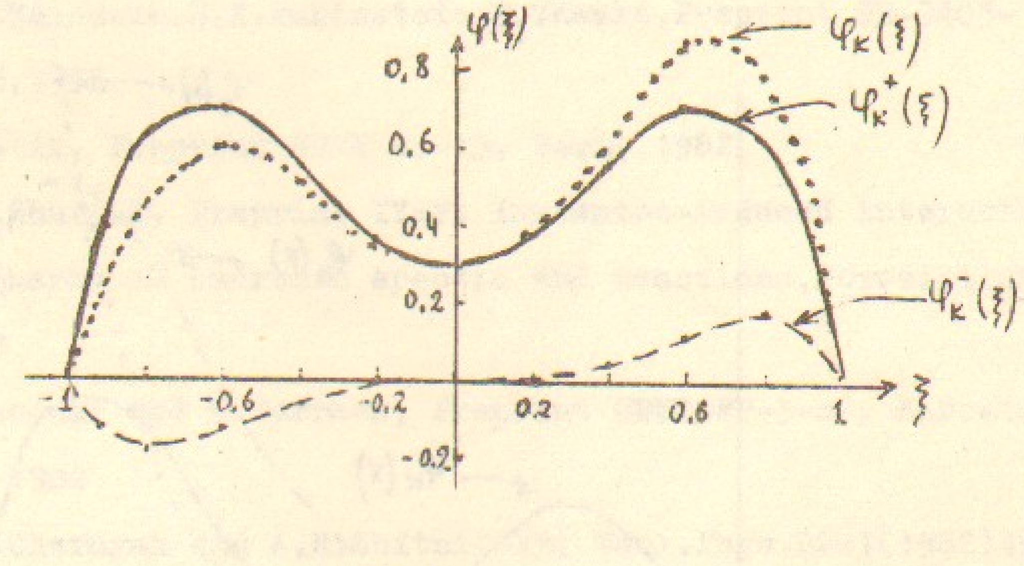

fig.4

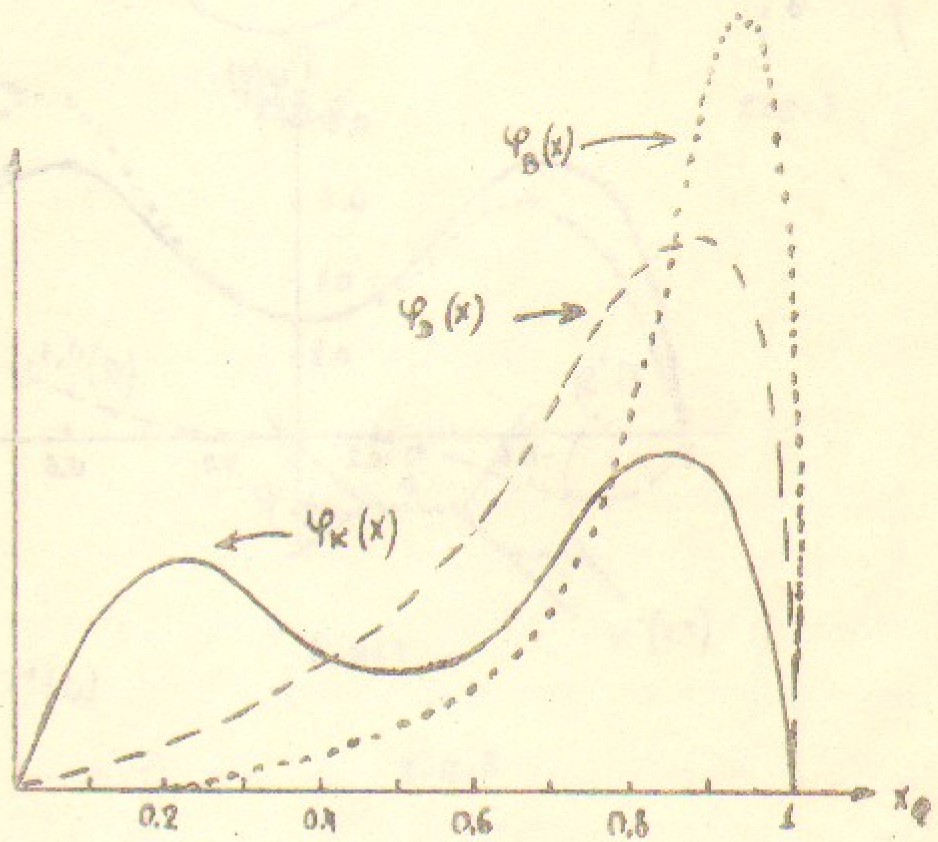

Fig.7

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