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ON THE MESON WAVE FUNCTION PROPERTIES

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A B S T R A C T

The mean value of the quark transverse momentum for the meson leading twist wave function is found:

$$\langle \vec{k}_\perp^{\,2} \rangle_\pi^A \simeq 5\langle i g \bar\psi \sigma_{\mu\nu} G_{\mu\nu}^a \lambda^a \psi\rangle / 72\langle \bar\psi\psi\rangle \simeq 5/72\,(1.5\,\mathrm{GeV}^2) \simeq (320\,\mathrm{MeV})^2$$

Qualitative description of the properties of various leading and nonleading twist hadronic wave functions is presented.

The properties of the wave functions of the vector mesons with the helicity $|\lambda| = 1$ are concidered in detail and used for the calculation of the decay width: $\chi_2(3555) \to \rho\rho$

I. Introduction.

The method for calculation of the asymptotic behaviour of exclusive processes in QCD with the help of the corresponding operator expansions has been proposed ir [1] (farther development see into [2-5]). The main idea of the method is the following. Since in the observed processes the hadrons are always on the mass shell, then even in the exclusive processes with the large momentum transfer the interactions both at small and at large distances are of importance. The interaction at small distances is responsible for the "hard" part of the process which ensures the large momentum transfer. The interaction at large distances is responsible for the formation of the bound states of quarks. In QCD the interactions at small and at large distances are governed by essentially different physics. It is possible to describe the interaction at small distances with a good accuracy by the perturbation theory. At the same time the nonperturbative effects play the main role at the large distance interaction.

It is natural therefore to use the method which allows to separate the contributions into the amplitude which are caused by the small and large distance interactions. The most suitable for this purpose is the method of operator expansions. In this approach the hard kernel of the process is computed in an explicit form with the help of the perturbation theory, while the nonperturbative interaction responsible for the hadron formation is described with the help of the hadron wave functions. For instance, the two-quark meson wave function can be introduced and is described by the matrix element of the bilocal operator: $\langle 0|\bar{\psi}_\alpha(x)\exp\{ig\int_{-x}^{x}d\sigma_\mu B_\mu(\sigma)\}\psi_\beta(-x)|M(q)\rangle$

The hadron wave function is of fundamental importance for the description of any process in which this hadron participate. Therefore, the investigation of the hadron wave function properties is of great importance.

In our previous papers [6,7] the properties of the leading twist wave functions of the mesons with zero helicity have been investigated. In [6] the properties of the π -meson wave function $\varphi_\pi^A(\xi)$ were described:

$$\langle 0|\bar{d}(z)\gamma_\mu\gamma_5 \exp\left\{ig\int_{-z}^{z} d\sigma B(\sigma)\right\} u(-z)|\pi(q)\rangle_{\mu^2} = \qquad (1)$$

$$= i f_\pi q_\mu \varphi_\pi^A(zq), \quad z^2 = 0$$

$$\varphi_\pi^A(zq,\mu^2) = \int_0^1 dx_1 dx_2\, \delta(1-x_1-x_2)\, e^{ix_1(zq)}\, e^{-ix_2(zq)}\, \varphi_\pi^A(\xi = x_1-x_2, \mu^2).$$

The wave function $\varphi_\pi^A(\xi,\mu^2)$ describes the distribution of quarks inside the π -meson (at $q_z \to \infty$) in the longitudinal momentum fractions $0 \le x_{1,2} \le 1$, $\xi = x_1 - x_2$, μ^2 -is the normalization point of the operators in (1). The value of μ^2 for the given process is determined by the characteristic virtuality of the constituents in this process.

The dependence of $\varphi_\pi^A(\xi,\mu^2)$ on μ^2 is caused by the higher order perturbation theory logarithmic corrections. This dependence is determined by the renormalization group and is very weak. The asymptotic form of the wave function $\varphi_\pi^A(\xi,\mu^2)$ at $\mu^2 \to \infty$ is known: $\varphi_\pi^A(\xi,\mu^2\to\infty) = \varphi_{as} = \frac{3}{4}(1-\xi^2)$ However, for the description of the experimentally observable exclusive processes we need to know the wave function with the virtuality $\mu^2 \sim 1\,GeV^2$. The form of this wave function $\varphi(\xi,\mu^2 \sim 1\,GeV^2)$ can differ greatly from those of $\varphi_{as}(\xi)$ In this case at any experimentally possible momentum transfer Q^2 we can't never replace $\varphi(\xi,\mu^2)$ by $\varphi_{as}(\xi)$ with a good accuracy, because the dependence of $\varphi(\xi,\mu^2)$ on μ^2 is very weak.

The form of the wave function $\varphi(\xi,\mu^2 \sim 1\,GeV^2)$ is determined mainly by the nonperturbative interactions. Therefore, for the investigation of its properties the nonperturbative methods are needed. In papers [6,7] the QCD sum rules [8] have been applied for the approximate calculation of the wave function moments: $\langle\xi^n\rangle_{\mu^2} = \int d\xi\, \xi^n \varphi(\xi,\mu^2)$. The knowledge of the values of few first moments $\langle\xi^n\rangle$ supplemented with some general physical considerations allows one to elucidate the main characteristic properties of the wave function $\varphi(\xi,\mu^2)$. In particular, using this method the model wave function of the π -meson was proposed in [6]:

$$\varphi_\pi^A(\xi,\mu \simeq 500\,MeV) \simeq \frac{15}{4}\xi^2(1-\xi^2) \qquad (2)$$

and the K-meson wave function and the wave functions of the vector mesons with the helicity $\lambda = 0$ were obtained in [7].
It has been shown in [6,7] that using these wave functions which satisfy the QCD sum rules one obtains the predictions for a number of exclusive processes in agreement with the experiment.

The goal of this paper is to investigate farther the properties of the pseudoscalar and vector meson wave functions. In sect.II the calculation of the mean quark transvers momentum inside the π -meson, $\langle \vec{k}_\perp^2 \rangle$, is described. This quantity is of the great interest in many respects and, besides, it characterizes the scale of the power corrections in the exclusive processes. The general qualitative consideration of the properties of various hadronic wave functions is presented in sect.III. The properties of the leading twist wave functions of the vector mesons with the helicity $|\lambda| = 1$ are investigated in detail in sect.IV. Our main results are summarized in sect.V.

II. The mean value of the transverse quark momentum.

Let us consider at first the π -meson wave function *

$\varphi_P = \langle 0|\bar d(z)\, i\gamma_5\, u(-z)|\pi(q)\rangle$:

$$\langle 0|\bar d(0)\, i\gamma_5\, i\vec D_\mu\, i\vec D_\nu\, u(0)|\pi\rangle = f_\pi \frac{m_\pi^2}{m_u+m_d}\left[q_\mu q_\nu A + g_{\mu\nu} B\right] \tag{3}$$

the momentum q is directed along the z-axis, the index $\perp$ below denotes the components in the transverse plane. If one takes in (3) $\mu=\nu=z$, $q_z\to\infty$ then it is evident that the coefficient A is $\langle x_u^2\rangle_\pi^P$ -i.e. the mean value of the x_u^2 ,where x_u is the longitudinal momentum fraction carried by the u-quark inside the π -meson. If one introduces the wave function $\psi_\pi^P(x_u, x_d, \vec k_\perp)$ which describes the distribution of quarks inside the π -meson in the longitudinal momentum fractions and in the transverse momentum, then

$$A = \langle x_u^2\rangle = \int_0^1 dx_u\, dx_d\, \delta(1-x_u-x_d)\, x_u^2 \int_0^\infty d^2\vec k_\perp\, \psi_\pi^P(x_u, x_d, \vec k_\perp) \tag{4}$$

$$\int d^2\vec k_\perp \int_0^1 dx_u\, dx_d\, \delta(1-x_u-x_d)\, \psi_\pi^P(x_u, x_d, \vec k_\perp) = 1$$

The natural definition of the quantity $\langle \vec k_\perp^2\rangle$ is:

$$\langle 0|\bar d(0)\, i\gamma_5\, (i\vec D_\perp)^2 u(0)|\pi(q)\rangle \equiv f_\pi \frac{m_\pi^2}{m_u+m_d}\langle \vec k_\perp^2\rangle = f_\pi \frac{m_\pi^2}{m_u+m_d}(-2B) \tag{5}$$

$$\langle \vec k_\perp^2\rangle = \int d^2\vec k_\perp\, \vec k_\perp^2 \int_0^1 dx_u\, dx_d\, \delta(1-x_u-x_d)\, \psi_\pi^P(x_u, x_d, \vec k_\perp) \tag{6}$$

Therefore, we want to find the value of the constant B. Using the equations of motion and PCAC we have from (3) in the chiral limit ($m_u = m_d = m_\pi = 0$):

$$\langle 0|\bar d\, i\gamma_5 (i\vec D_\mu)^2 u|\pi(q)\rangle = f_\pi \frac{m_\pi^2}{m_u+m_d} 4B = \frac{1}{2}\langle 0|\bar d\gamma_5\, g\sigma_{\mu\nu} G^a_{\mu\nu}\frac{\lambda^a}{2} u|\pi\rangle =$$

$$= \frac{1}{f_\pi}\langle 0|\bar u\, ig\sigma_{\mu\nu} G^a_{\mu\nu}\frac{\lambda^a}{2} u|0\rangle;\quad f_\pi \frac{m_\pi^2}{m_u+m_d} \simeq -\frac{\langle \bar u u + \bar d d\rangle}{f_\pi} \tag{7}$$

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* To simplify the notations, here and in what follows we don't write explicitly the gluonic exponents.

Therefore:

$$\langle \vec k_\perp^2\rangle_\pi^P \simeq \frac{1}{8}\,\frac{\langle 0|\bar u\, ig\sigma_{\mu\nu} G^a_{\mu\nu}\lambda^a u|0\rangle}{\langle 0|\bar u u|0\rangle} \tag{8}$$

It is seen from (8) that in the chiral limit $m_\pi\to 0$, $\langle \vec k_\perp^2\rangle \neq 0$ is entirely due to interaction (the presence of $gG_{\mu\nu}$). It is natural, because for the free quarks $k_\perp\to 0$ at $m_\pi\to 0$. The value of the vacuum matrix elements ratio in (8) has been obtained, for instance, in [9] from the investigation of the baryon mass spectrum with the help of QCD sum rules. Using the result [9] one has from (8)* :

$$\langle \vec k_\perp^2\rangle_\pi^P \simeq \frac{1}{8}(1.5\, GeV^2) \simeq (430\, MeV)^2 \tag{9}$$

This value seems very reasonable.

Let us estimate now the value $\langle \vec k_\perp^2\rangle_\pi^A$ for the leading twist wave function: $\varphi_\pi^A = \langle 0|\bar d(z)\gamma_\mu\gamma_5 u(-z)|\pi(q)\rangle$:

$$\langle 0|\bar d(0)\gamma_\mu\gamma_5 \frac{1}{2}\left[i\vec D_\nu i\vec D_\lambda + i\vec D_\lambda i\vec D_\nu\right] u(0)|\pi(q)\rangle = i f_\pi q_\mu q_\nu q_\lambda \langle x_u^2\rangle + \tag{10}$$

$$+ i f_\pi (g_{\mu\nu} q_\lambda + g_{\mu\lambda} q_\nu) A + i f_\pi q_\mu g_{\nu\lambda} B$$

Multiplying (10) by $g^{\mu\nu}$ and using the equations of motion (in the chiral limit), one obtains from (10): 5A+B=0. Multiplying now (10) by $g_{\nu\lambda}$ one has:

$$\langle 0|\bar d\gamma_\mu\gamma_5 (i\vec D_\nu)^2 u(0)|\pi(q)\rangle = i f_\pi q_\mu \frac{18}{5} B = -\frac{1}{2}\langle 0|\bar d\gamma_\mu\gamma_5\, ig\sigma_{\alpha\beta} G^a_{\alpha\beta}\frac{\lambda^a}{2} u|\pi\rangle \tag{11}$$

Let us determine $\langle \vec k_\perp^2\rangle_\pi^A$ as:

$$\langle 0|\bar d\gamma_\mu\gamma_5 (i\vec D_\perp)^2 u|\pi\rangle = i f_\pi q_\mu \langle \vec k_\perp^2\rangle_\pi^A = -2i f_\pi q_\mu B \tag{12}$$

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* Here and in what follows we neglect the effects due to anomalous dimensions.

$$\langle \vec{k}_\perp^2 \rangle_\pi^A = \int d^2\vec{k}_\perp \, \vec{k}_\perp^2 \int_0^1 dx_u dx_d \, \delta(1-x_u-x_d) \, \psi_\pi^A(x_u, x_d, \vec{k}_\perp), \tag{12}$$
$$\int d^2\vec{k}_\perp \int dx_u dx_d \, \delta(1-x_u-x_d) \, \psi_\pi^A(x_u, x_d, \vec{k}_\perp) = 1$$

We have now from (11),(12):

$$i f_\pi q_\mu \langle \vec{k}_\perp^2 \rangle_\pi^A = \frac{5}{18} \langle 0 | \bar{d} \gamma_\mu \gamma_5 ig \sigma_{\alpha\beta} G^a_{\alpha\beta} \frac{\lambda^a}{2} u | \pi(q) \rangle \tag{13}$$

In order to find the value of the matrix element in (13) let us consider the correlator:

$$T_\mu = i \int dx \, e^{iqx} \langle 0 | T \{ \bar{d} \gamma_\mu \gamma_5 g \sigma_{\alpha\beta} G^a_{\alpha\beta} \frac{\lambda^a}{2} u(x), \bar{u}(0) i\gamma_5 d(0) \} | 0 \rangle = q_\mu I(q^2) \tag{14}$$

At $|q^2| \to \infty$ the leading contribution to T_μ (in the chiral limit) gives the fig.1 diagram:

$$T_\mu(q) \to q_\mu / q^2 \langle 0 | \bar{u} ig \sigma_{\alpha\beta} G^a_{\alpha\beta} \frac{\lambda^a}{2} u | 0 \rangle \tag{15}$$

Using the dispersion relation for $I(q^2)$ one has from (15):

$$\frac{1}{\pi} \int ds \, \mathrm{Im}\, I(s) = - \langle 0 | \bar{u} \sigma_{\alpha\beta} ig G^a_{\alpha\beta} \frac{\lambda^a}{2} u | 0 \rangle \tag{16}$$

The spectral density has the form:

$$\frac{1}{\pi} \mathrm{Im}\, I(s) = f_\pi \frac{18}{5} \langle \vec{k}_\perp^2 \rangle_\pi^A \cdot f_\pi \frac{m_\pi^2}{m_u + m_d} \delta(s) + \ldots \simeq \tag{17}$$
$$\simeq -\frac{36}{5} \langle 0 | \bar{u} u | 0 \rangle \langle \vec{k}_\perp^2 \rangle_\pi^A \delta(s) + \ldots$$

The π -meson contribution is shown explicitly in (17). Since the spectral density falls down quickly at large S (the perturbation theory contribution is zero in the chiral limit and we neglect the logarithmic corrections due to anomalous dimensions), it seems reasonable to retain only the π -meson contribution in (17). In this approximation one has from (16),(17):

$$\langle \vec{k}_\perp^2 \rangle_\pi^A \simeq \frac{5}{36} \frac{\langle 0 | \bar{u} ig \sigma_{\mu\nu} G^a_{\mu\nu} \frac{\lambda^a}{2} u | 0 \rangle}{\langle 0 | \bar{u} u | 0 \rangle} \simeq \frac{5}{9} \langle \vec{k}_\perp^2 \rangle_\pi^P \tag{18}$$

Therefore:

$$\langle \vec{k}_\perp^2 \rangle_\pi^A \simeq (320 \, \mathrm{MeV})^2 \tag{19}$$

It is natural to expect that the main value of the quark transverse momentum in the other mesons ($K, \rho, K^* \ldots$) will be approximately the same as in the π -meson.

III. Qualitative description and numerical estimates.

Let us begin with the qualitative comparison of the properties of the π -meson wave function (1) and the $\rho_{|\lambda|=1}$ -meson wave function determined by the bilocal matrix element:

$$\langle 0 | \bar{d}(z) i \sigma_{\mu\nu} u(-z) | \rho_{|\lambda|=1}(q) \rangle = i (e^\perp_\mu q_\nu - e^\perp_\nu q_\mu) f_\rho^T \varphi_\rho^T(zq, \mu^2), \quad \sigma_{\mu\nu} = \frac{1}{2} [\gamma_\mu, \gamma_\nu] \tag{20}$$

Here: $e^\perp_\mu$ -is the polarization vector for $|\lambda|=1$, f_ρ^T is the dimensional constant (analogous to f_π, f_ρ) which determines the value of the wave function at the origin, $\varphi_\rho^T(zq, \mu^2)$ is the dimensionless wave function, μ^2 is the normalization point of the operators in (20). Such dimensionless wave functions

$$\varphi_i(zq, \mu^2) = \int_{-1}^{1} d\xi \, e^{i\xi(zq)} \varphi_i(\xi, \mu^2), \quad \varphi_i(\xi) = \int d^2\vec{k}_\perp \psi_i(x_u, x_d, \vec{k}_\perp), \tag{21}$$
$$\varphi_i(zq=0) = \int_{-1}^{1} \varphi_i(\xi) d\xi = 1, \quad \xi = x_u - x_d, \quad i = \pi, \rho$$

describe the distribution of quarks in the longitudinal momentum fractions (at $q_z \to \infty$). At $\mu^2 \to \infty$ the asymptotic form of $\varphi_\pi^A(\xi, \mu^2)$ and $\varphi_\rho^T(\xi, \mu^2)$ coinside: $\varphi_\pi^A(\xi, \mu^2 \to \infty) = \varphi_\rho^T(\xi, \mu^2 \to \infty) = \frac{3}{4}(1-\xi^2)$.

The function $\frac{3}{4}(1-\xi^2) = \varphi_{as}$ corresponds to the contribution of the free quark loop, fig.2a, in the method of QCD sum rules. The function $\varphi_{as}(\xi)$ describes some characteristic distribution of quarks in the longitudinal momentum: $\langle \xi^2 \rangle_{as} = 0.2$,so that in the state with this wave function the total momentum is more or less equally distributed between the quarks. However, in the nonperturbative interactions of quarks and gluons with the vacuum fluctuations, fig.2b,c, the distribution of the total momentum between the quarks is essentially different. Let us discuss qualitatively the influence of the nonperturbative contributions shown in fig.2c (the role of the fig.2b contributions is, as a rule, small and qualitatively the same). It is seen from fig.2c that these contributions describe such momentum distribution that nearly all the initial momentum is carried by one quark while the other one is "wee". The wave function like $\varphi(\xi)=\frac{1}{2}[\delta(1-\xi)+\delta(1+\xi)]$ corresponds to such momentum distribution. Therefore, the nonperturbative interactions tend to strengthen or weaken the role of such configurations in which the total momentum is very unequally distributed between two quarks. The sign of this effect is determined by the relative sign of the fig.2a and the fig.2c contributions. If the lowest resonance gives large contribution into the spectral density then the characteristic momentum distribution between the quarks in this state and in the total correlator will be the same. If the fig.2a and fig.2c contributions are of the same sign, then the true meson wave function is wider than $\varphi_{as}(\xi)$ i.e. $(\langle\xi^2\rangle) > (\langle\xi^2\rangle_{as})$ Just this variant is realized in the case of the π -meson wave function $\varphi_\pi^A(\xi)$ [6], fig.3. If the fig.2a and fig.2c contributions have opposite signs, the true wave function is more narrow than $\varphi_{as}(\xi)$, i.e. $(\langle\xi^2\rangle) < (\langle\xi^2\rangle_{as})$ As will be shown latter, just this variant is realized in the case of the $\rho_{|\lambda|=1}$ -meson wave function $\varphi_\rho^T(\xi)$, fig.3. The strength of the effect depends on the relative values of the perturbation theory contribution and the nonperturbative contribution and is different for different correlators. (The nonperturbative contributions were very large for the π -meson wave function $\varphi_\pi^A(\xi)$, see [6]).

Let us describe now the simple method which allows one to

estimate the values of various dimensional constants f_i (i.e. the values of the wave functions at the origin) which determine the scales of the wave functions. This constant f_i has the dimensionality $[\mu^1]$ for the two-quark (or two-gluon) leading twist wave functions ($f_\pi \simeq 130\,MeV$, $f_\rho \simeq 200\,MeV$, ...). The lowest resonance contribution into the sum rule has the form (after "borelization"): $f_i^2 \exp\{-M_i^2/M^2\}$ where M_i -is the resonance mass and M -is the scale parameter (see [8]). The perturbation theory contribution, fig,2a, is: $M^2/4\pi^2$ (one loop). The usual scale of nonperturbative contributions is such that at $M^2 \simeq M_\rho^2$ they are $\simeq 20\%$ of the perturbative contribution, fig,2a. Therefore, one can estimate the value of f_i from the relation:

$$f_i^2 \exp\{-M_i^2/M_\rho^2\} = M_\rho^2/4\pi^2$$

This gives for the π -meson: $f_\pi \simeq M_\rho/2\pi \simeq 120\,MeV$ and $f_\rho \simeq \sqrt{e}\, f_\pi \simeq$ 200MeV for the ρ -meson. For the constant f_ρ^T from (20) we have the same situation, so: $f_\rho^T \simeq f_\rho \simeq 200\,MeV$.

For the three-particle component of the $\rho_{|\lambda|=1}$ -meson wave function (twist 3) determined by the matrix element:

$$\langle 0|\,\bar{d}\,\gamma_\lambda\gamma_5\, g\, G_{\mu\nu}\frac{\lambda^a}{2}\, u\,|\rho_{|\lambda|=1}(q)\rangle = q_\lambda\, \epsilon_{\mu\nu\alpha\beta}\, q_\alpha e_\beta\, f_\rho^{3A} \tag{23}$$

the perturbation theory contribution is (fig.4, two loops): $\alpha_s M^4/720\pi^3$ Therefore, the rough estimate gives:

$$(f_\rho^{3A})^2 e^{-1} \sim M_\rho^4\, \bar{\alpha}_s/720\pi^3,\quad f_\rho^{3A} \simeq -0{,}4\cdot 10^{-2}\,GeV^2 \simeq -f_\rho^V(f_\rho^V/10),\ \bar{\alpha}_s=0{,}35$$

More precise treatment of the corresponding sum rule gives [11]:

$$|f_\rho^{3A}| \simeq 0{,}6\cdot 10^{-2}\,GeV^2$$

For the nucleon wave function (twist 3):

$$\langle 0|\,\varepsilon^{abc}\, u^a C \not{z}\, u^b\, d_z^c\,|P\rangle = p_z\,(\gamma_5 N)_z\, f_N$$

(here N -is the nucleon spinor, C is the charge conjugation matrix) the perturbation theory contribution is (fig.6, two loops): $M^4/960\pi^4$ Therefore, the rough estimate gives: $(f_V^N)^2 e^{-m_N^2/M_\rho^2} \simeq M_\rho^4/960\pi^4$, $f_V^N \simeq 0.4\cdot 10^{-2}\,GeV^2$. More precise treatment of the sum rules gives: $|f_V^N| \simeq 0.5\cdot 10^{-2}\,GeV^2$

For the two-particle ρ -meson wave function (twist 3) determined by the matrix element:

$$\langle 0|\bar{d}\, i\overleftrightarrow{D}_\mu\, u|\rho_{\lambda=0}(q)\rangle = e_\mu^{\lambda=0} M_\rho f_\rho^S;\ (e_{\lambda=0})_\mu \cdot M_\rho = q_\mu \tag{24}$$

the perturbation theory contribution is (fig.2a, one loop): $M^4/8\pi^2$ Therefore, the rough estimate gives: $(f_\rho^S)^2 e^{-1} \simeq M_\rho^4/8\pi^2$, $f_\rho^S \simeq -0.11\,GeV^2 \sim 20 f_\rho^{3A}$ The strong inequality $|f_\rho^S| \gg |f_\rho^{3A}|$ is due to smallness of the three-particle phase space (fig.4) as compared with the two-particle one (fig.2a). This is, evidently, the general rule for the analogous constants of the same dimensionality: the value of f_i falls off quickly as the number of constituents encreases.

In connection with the above given estimates it is important to note the following. The meson wave functions with the dimensionality μ^2:

$$\langle 0|\bar{d}(z)\gamma_\mu^\perp u(-z)|\rho_{|\lambda|=1}(q)\rangle = e_\mu^{|\lambda|=1}\cdot[f_\rho^V M_\rho \varphi_\rho^V(zq)] + \ldots$$

$$\langle 0|\bar{d}(z)\, i\overleftrightarrow{D}_\mu\, u(-z)|\rho_{\lambda=0}(q)\rangle = q_\mu f_\rho^S \varphi_\rho^S(zq), \tag{25}$$

$$\langle 0|\bar{d}(z)\, i\gamma_5\, u(-z)|\pi(q)\rangle = f_\pi \frac{m_\pi^2}{m_u+m_d}\varphi_\pi^P(zq),$$

$$\varphi_i(zq=0)=1,\quad |f_\rho^V M_\rho| \simeq 0.15\,GeV^2,\quad |f_\rho^S| \simeq 0.11\,GeV^2,$$

$$\left|f_\pi \frac{m_\pi^2}{m_u+m_d}\right| \simeq 0.21\,GeV^2$$

determine the values of the power corrections to the leading twist contributions in the exclusive processes. It is seen from (25) that the relative value of the power correction is:

$\sim(1\div 2)\cdot(f_\rho M_\rho / f_\rho Q) \sim (1\div 2)\, M_\rho/Q$ and not f_ρ/Q as can be naively expected. Besides, it is seen from (25) that the values of the power corrections will be approximately the same for the π and ρ -mesons. Therefore, it is natural to expect that for other particles the scale of the power corrections will be the same.

Since the values of the three-particle wave functions are $\sim 10^{-1}$ relative to the two-particle ones, one can suppose that the power corrections in the exclusive processes are determined mainly by the two-particle contributions. We want to note that this is not so, in general. Let us consider for definitness the π -meson form factor $F_\pi(Q^2)$ The power corrections Q^{-4} are determined by the two-particle contributions, fig.7, and by the three-particle ones, fig.8 (there are also the four-particle contributions). The two-particle corrections are caused mainly by the wave function $\langle 0|\bar{d}\, i\gamma_5 u|\pi\rangle \sim f_\pi(2M_\rho)$ (correction $\sim$ $\sim(2M_\rho/Q)^2$,for more detail see /12/) and by the quark transverse momentum: $\sim\langle\vec{k}_\perp^2\rangle_\pi^A/\overline{(1-x)}Q^2 \sim (4\div 6)\langle\vec{k}_\perp^2\rangle_\pi^A/Q^2 \sim M_\rho^2/Q^2$

The value of the three-particle wave function is ~10 times smaller than the two-particle one. However, the radiation of the additional gluon at the fig.8 diagram gives the additional propagator and this usually increases the amplitude in 3-4 times. Besides, there are 6 diagrams like that of fig.8 and this can also give the factor 2-3. As a result, in spite of the smallness of the constant $|f^{(3)}| \ll |f^{(2)}|$ the three-particle contributions can compete, in principle, with the two-particle ones.

IV. Wave functions of the helicity one vector mesons.

Decomposing (20) in Z one can express the wave function moments $\langle\xi^n\rangle = \int d\xi\, \xi^n \varphi(\xi)$ via the matrix elements of the local operators:

$$\langle 0|\bar d(0)\, i\sigma_{\mu\nu}(iz\overleftrightarrow{D})^n u(0)|\rho_{\lambda=1}(q)\rangle = i(e^{\perp}_{\mu}q_{\nu}-e^{\perp}_{\nu}q_{\mu})(zq)^n f^T_{\rho}\langle \xi^n\rangle, \qquad (26)$$

$$i\overleftrightarrow{D} = i\overrightarrow{D} - i\overleftarrow{D}, \quad i\overrightarrow{D} = i\overrightarrow{\partial} + g A^a \frac{\lambda^a}{2}, \quad z^2=0, \quad \langle \xi^0\rangle = 1$$

Further we shall widely use the method of QCD sum rules [8] for approximate calculation of these matrix elements.

a) To find the value of the constant f^T_{ρ} in (20) let us use first the same method as in the previous section. Consider the correlator:

$$T_{\mu\nu\lambda} = i\int dx\, e^{iqx}\langle 0|T\{\bar d(x)\gamma_{\mu}u(x)\,\bar u(0)\sigma_{\nu\lambda}d(0)\}|0\rangle = \qquad (27)$$

$$= (g_{\mu\nu}q_{\lambda} - g_{\mu\lambda}q_{\nu})\, I_0(q^2), \quad \sigma_{\nu\lambda} = \frac{i}{2}[\gamma_{\nu},\gamma_{\lambda}]$$

The perturbation theory contribution into $T_{\mu\nu\lambda}$ equals zero in the chiral symmetry limit ($m_u = m_d = 0$) and the whole answer is nonzero only due to spontaneous chiral symmetry breaking effects. At $-q^2\to\infty$ the leading contribution give two diagrams, fig. 9:

$$I_0(q^2) = \frac{1}{\pi}\int_0^{\infty}\frac{ds}{s-q^2}\, Im I_0(s) \to \frac{\langle 0|\bar u u + \bar d d|0\rangle}{q^2} + O\left(\frac{1}{q^4}\right), \qquad (28)$$

$$\frac{1}{\pi}\int_0^{\infty} Im I_0(s)\, ds = -\langle 0|\bar u u + \bar d d|0\rangle$$

As the spectral density falls off quickly at large S, we can confine ourselves in (29) by the ρ -meson contribution only:

$$\frac{1}{\pi} Im I_0(s) = (f^V_{\rho} M_{\rho})(f^T_{\rho})\,\delta(s - M^2_{\rho}) + \ldots \qquad (29)$$

$$\langle 0|\bar d\gamma_{\mu}u|\rho_{\lambda}(q)\rangle = e^{\lambda}_{\mu} f^V_{\rho} M_{\rho}, \quad f^V_{\rho} \simeq 200\, MeV$$

One has in this approximation:

$$f^T_{\rho} \simeq -\frac{\langle 0|\bar u u + \bar d d|0\rangle}{f^V_{\rho} M_{\rho}} \simeq 200\, MeV \simeq f^V_{\rho} \qquad (30)$$

The fact that $f^T_{\rho}\neq 0$ is caused by the spontaneous chiral symmetry breaking is evident beforehand, because in the matrix element which determines f^T_{ρ} : $\langle 0|\bar d\sigma_{\mu\nu}u|\rho\rangle$ the initial and the final states are chiral invariants while the operator $\bar d\sigma_{\mu\nu}u$ is not. Let us point also that using the nondiagonal correlator (27) we can determine not only the modulus of f^T_{ρ} but its sign as well (i.e. the sign relative to f_{ρ} ,see (30)).

b) Let us consider now in more detail the properties of the wave function and use for this purpose the correlator

$$T_n = i\int dx\, e^{iqx}\langle 0|T\{\bar u(x)\sigma_{\mu\nu}z_{\nu}(iz\overleftrightarrow{D})^n d(x),\ \bar d(0)\sigma_{\mu\lambda}z_{\lambda}u(0)\}|0\rangle \qquad (31)$$

$$= (zq)^{n+2} I_n(q^2)$$

At $-q^2\to\infty$ the leading contribution gives the perturbation theory diagram, fig. 2a. This contribution corresponds to the asymptotic wave function: $\varphi_{as}(\xi) = 3/4\,(1-\xi^2)$,which we use as a zero approximation. The nonperturbative corrections give contributions in addition to the asymptotic wave function and change it in the direction which corresponds to the more realistic wave function.*

The sum rules have the form;

$$\frac{1}{\pi}\int Im I_n(s)\, e^{-s/M^2} ds = \frac{3}{4\pi^2}\frac{M^2}{(n+1)(n+3)} + \frac{n-1}{n+1}\frac{1}{12 M^2}\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle \qquad (32)$$

$$-\frac{64}{81}\pi(n-1)\frac{1}{M^4}\langle 0|\sqrt{\alpha_s}\,\bar u u|0\rangle^2 + \ldots, \quad n = 0, 2, 4 \ldots$$

The matrix elements entering (32) are as follows [8]:

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* Strickly speaking, the sum rules give information not about the wave functions itself ,but only about the values of their moments.

$$\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle \simeq 1.2\cdot 10^{-2}\,GeV^4, \quad \langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle^2 = 1.83\cdot 10^{-4}\,GeV^6$$

The spectral density is chosen in the standard form*:

$$\frac{1}{\pi}Im\,I_n(s) = (f_\rho^T)^2\langle\xi^n\rangle\delta(s-m_\rho^2) + \theta(s-s_n)\frac{3}{4\pi^2(n+1)(n+3)} \tag{33}$$

The standard treatment (see [6] and the Appendix) gives:

$$|f_\rho^T(\mu^2 \simeq 0.5\,GeV^2)| \simeq (200 \pm 20)\,MeV$$
$$\langle\xi^2\rangle_{\mu^2 \simeq 0.5\,GeV^2} \simeq (0.14 \pm 0.2), \quad s_2 \simeq 1.5\,GeV^2, \quad \langle\xi^4\rangle \lesssim 0.05 \tag{34}$$

It is seen that the value $f_\rho^T \simeq 200\,MeV$ obtained with the help of the correlator (31) agrees well with the result (30) obtained with the help of the different correlator (27). In the sum rules (32) for the moments $\langle\xi^2\rangle$ and $\langle\xi^4\rangle$ the correction $\langle\sqrt{\alpha_s}\,\bar{u}u\rangle^2$ plays the main role and it has here the opposite sign with respect to the perturbation theory contribution. As was expected beforehand (see sect.III) this diminishes the values of the moments $\langle\xi^2\rangle$ and $\langle\xi^4\rangle$ in comparison with their values for the asymptotic wave function: $\langle\xi^2\rangle_{as} = 0.2$ and $\langle\xi^4\rangle_{as} \simeq 0.086$. As a result, <u>the wave function $\varphi_\rho^T(\xi,\mu^2)$ is narrower than the asymptotic one.</u>

Taking into account that at $|\xi| \to 1$ the wave function $\varphi_\rho^T(\xi)$ has, in general, the same behaviour as $\varphi_{as}(\xi)$, i.e. $\sim(1-\xi^2)$ (see Appendix), let us choose as in previous papers [6,7] the simplest model form as the superposition of two lowest Gegenbauer polinomials:

$$\varphi_\rho^T(\xi,\mu^2) = \frac{3}{4}(1-\xi^2)\left[1 + a(\mu^2)\left(\xi^2 - \frac{1}{5}\right)\right] \qquad \frac{a(\mu_2^2)}{a(\mu_1^2)} = \left(\frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)}\right)^{50/81} \tag{35}$$

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* The account of the B(1235)-meson contribution does not change the results.

The value $\langle\xi^2\rangle \simeq 0.14$ from (34) determines: $a(\mu^2 \sim 0.5\,GeV^2) \simeq -1.35$ and therefore:

$$\varphi_\rho^T(\xi,\mu^2 \sim 0.5\,GeV^2) \simeq \frac{15}{16}(1-\xi^2)^2, \quad \langle\xi^2\rangle \simeq 0.142, \quad \langle\xi^4\rangle \simeq 0.048 \tag{36}$$

c) It seems that one of the most accesible ways to check the properties of the $\varphi_\rho^T(\xi)$ wave function is to measure the branching ratio: $\chi_2(3555) \to \rho\rho$. The χ_2 -meson can decay both into $\rho_{\lambda=0}\rho_{\lambda=0}$ -pair and into $\rho_{\lambda=1}\rho_{\lambda=-1}$ -pair. The leading (at $M_c \to \infty$) contribution to the decay $\chi_2 \to \rho_\perp\rho_\perp$ ($\rho_\perp \equiv \rho_{|\lambda|=1}$) give the wave function $\varphi_\rho^T(\xi)$. The expression for the decay amplitude in terms of $\varphi_\rho^T(\xi)$ can be found, for instance, in [13]. In our notations:

$$W(\chi_2 \to \rho_\perp^+\rho_\perp^-)/W(\chi_2 \to \pi^+\pi^-) \simeq 12\left(\frac{f_\rho^T}{f_\pi}\right)^4\left(I_2^\perp/I_2\right)^2$$

$$I_2^\perp = \int_{-1}^{1}\frac{d\xi_1}{1-\xi_1^2}\varphi_\rho^T(\xi_1)\int_{-1}^{1}\frac{d\xi_2}{1-\xi_2^2}\varphi_\rho^T(\xi_2)\frac{\alpha_s^2}{1-\xi_1\xi_2}, \tag{37}$$

$$I_2 = \int_{-1}^{1}\frac{d\xi_1}{1-\xi_1^2}\varphi_\pi^A(\xi_1)\int_{-1}^{1}\frac{d\xi_2}{1-\xi_2^2}\varphi_\pi^A(\xi_2)\frac{\alpha_s^2}{1-\xi_1\xi_2}\left[1 - \frac{1}{2}\frac{(\xi_1-\xi_2)^2}{1-\xi_1\xi_2}\right],$$

$$\int_{-1}^{1}\varphi_\rho^T(\xi)d\xi = \int_{-1}^{1}\varphi_\pi^A(\xi)d\xi = 1$$

Using in (37) the wave functions (2) and (36) one obtains*:

$$W(\chi_2 \to \rho_\perp\rho_\perp)/W(\chi_2 \to \pi\pi) \simeq 1 \tag{38}$$

We want to stress the following. The coefficient in the ratio (37) is very large: $12(f_\rho^T/f_\pi)^4 \simeq 60$. Therefore, if the wave functi-

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* Since $\varphi_\rho^T(\xi)$ is narrower than $\varphi_\pi^A(\xi)$, $\bar{\alpha}_s$ in $I_2^\perp$ will be somewhat smaller than $\bar{\alpha}_s$ in I_2 and this can diminish somewhat the value (38).

ons $\varphi_\rho^T(\xi)$ and $\varphi^A(\xi)$ were much alike to each other, then one has*: $(\chi_2 \to p\bar{p})/(\chi_2 \to \pi\pi) \simeq 60$ while the experiment gives [14]: $(\chi_2 \to p\bar{p})/(\chi_2 \to \pi\pi) \lesssim 1$ So, the experiment shows unambiguously that the wave function $\varphi_\rho^T(\xi)$ is much more narrow than $\varphi^A(\xi)$ This agrees with our results obtained with the help of the QCD sum rules.**

The ρ_L -meson ($\rho_L \equiv \rho_{\lambda=0}$) wave function has been determined in [7] and, besides, there was obtained: $(\chi_2 \to \rho_L \rho_L)/(\chi_2 \to \pi\pi) \simeq 0.5$
Using this result and (38) one has:

$$W(\chi_2 \to \rho^+\rho^-)/W(\chi_2 \to \pi^+\pi^-) \simeq 1.5 \qquad (39)$$

Let us point out also that if the gluon were not a vector but a scalar particle, then the decay $\chi_2 \to \rho_\perp \rho_\perp$ would be power suppressed in comparison with the decay $\chi_2 \to \rho_L \rho_L$.

As a whole, the experimental observation of the $\chi_2 \to \rho\rho$ decay with the branching ratio: $Br(\chi_2 \to \rho\rho) \simeq Br(\chi_2 \to \pi\pi) \simeq 0.25\%$ can be a good check of the above described ρ -meson wave function properties. Let us point also that the information about the $\rho_\perp$ -meson wave function can, in principle, be obtained by measuring the $\gamma\gamma \to \rho_\perp \rho_\perp$ cross-section at high energies [15,16]. The main contribution into the $\gamma\gamma \to \rho_\perp^+ \rho_\perp^-$ cross-section give the diagramms with the t-channel quark exchange, while for the $\gamma\gamma \to \rho_\perp^0 \rho_\perp^0$ cross-section the main contribution comes from the two-gluon exchange diagrams (using the VDM).

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* The term $\frac{1}{2}(\xi_1 - \xi_2)^2/(1 - \xi_1\xi_2)$ in I_2 gives a small contribution in comparison with the unity.

** For the case $\varphi_\rho^T(\xi) \simeq \varphi_{as}(1-\xi^2)\frac{3}{4}$ one has: $(\chi_2 \to \rho_\perp \rho_\perp)/(\chi_2 \to \pi\pi) \simeq 4$

d) SU(3)-symmetry breaking effects.

Our goal in this paragraph is to investigate the properties of the $K^*_{|\lambda|=1}$ and $\varphi_{|\lambda|=1}$ -meson wave functions. Similar program has been carried out in [7] for the vector mesons with $\lambda=0$ We therefore give below mainly the results.

For the case of the $K^*_{|\lambda|=1}$ -meson the term

$$m_s\left[\frac{\langle 0|\bar{s}s|0\rangle}{M^2} - \frac{n}{6M^4}\left(\langle 0|\bar{s}\, i g \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a s|0\rangle - 4m_s^2\langle \bar{s}s\rangle\right)\right] \qquad (40)$$

sould be added to the r.h.s. of (32). This term leads both to the shift of the mass value and to the change of the wave function. For the $\varphi_{|\lambda|=1}$ -meson the additional term is two times larger than (40). The matrix element $\langle \bar{s}\, i g \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a s\rangle$ in (40) is equal [9]: $\langle \bar{s}\, i g \sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a s\rangle \simeq 1.5\,\mathrm{GeV}^2 \langle \bar{s}s\rangle \simeq -(1.5\,\mathrm{GeV}^2)\cdot 0.8\cdot(0.25\,\mathrm{GeV})^3$
Standard treatment of the sum rules gives:

$$\left(f^T_{K^*}/f^T_\rho\right)^2 \simeq 1.1 \qquad \left(f^T_\varphi/f^T_\rho\right)^2 \simeq 1.3$$

$$\left.\frac{\langle \xi^2\rangle^T_{K^*}}{\langle \xi^2\rangle^T_\rho}\right|_{\mu^2 \simeq 0.9\,\mathrm{GeV}^2} \simeq 0.75 \qquad \left.\frac{\langle \xi^2\rangle^T_\varphi}{\langle \xi^2\rangle^T_\rho}\right|_{\mu^2 \simeq 0.9\,\mathrm{GeV}^2} \simeq 0.6 \qquad (41)$$

In the qualitative respect the results (41) for the $|\lambda|=1$ -vector mesons are analogous to those obtained in /7/ for the $\lambda=0$ vector mesons. Namely, when the light u- or d-quark is replaced by the s-quark the value of the constant f_i (i.e. the wave function at the origin) increases and the distribution of quarks in the longitudinal momentum becomes narrower. The wave functions $\varphi^T_{K^*,\varphi}(\xi)$ are narrower than $\varphi^T_\rho(\xi)$ because the additional term (40) has the opposite sign with respect to the perturbation theory contribution, fig.2a (see sect.III).

V. Summary.

Let us enumerate the main results.

1. Hadronic wave function has many components- two-particle components with different spin structures, three-particles ones, ets. The components with the minimal number of constituents are of main interest for the investigation of the asymptotic behaviour of exclusive processes.

We have calculated the mean value of the quark transverse momentum for the two-particle π -meson wave functions defined by the matrix elements: $\langle 0|\bar{d}(z)\gamma_\mu\gamma_5 u(-z)|\pi(q)\rangle = i f_\pi q_\mu \varphi_\pi^A(zq) + \ldots$

$\langle 0|\bar{d}\, i\gamma_5 u(-z)|\pi(q)\rangle = f_\pi m_\pi^2/(m_u+m_d)\, \varphi_\pi^P(zq)$

It has been obtained:

$$\langle \vec{k}_\perp^{\,2}\rangle_\pi^P \simeq \frac{1}{8}\frac{\langle 0|\bar{u}\, ig\sigma_{\mu\nu}G_{\mu\nu}^a\lambda^a u|0\rangle}{\langle \bar{u}u\rangle} \simeq \frac{1}{8}(1.5\,GeV^2) \simeq (430\,MeV)^2$$

$$\langle \vec{k}_\perp^{\,2}\rangle_\pi^A \simeq 5/9\,\langle \vec{k}_\perp^{\,2}\rangle_\pi^P \simeq (320\,MeV)^2$$

The value of the mean quark momentum in the bound state is of great interest. It is natural to expect that the mean quark momentum in other hadrons is approximately the same: $|\vec{k}_\perp| \simeq (300 \div 400)\,MeV$

2. The numerical values of various hadronic wave functions at the origin are estimated (the dimensional constants f_i). The most characteristic properties are the following.

a) The leading twist components of the $(n+1)$ particle wave function has the dimensionality $[\mu^n]$ and its characteristic value is: $(f_\pi \div f_\rho)^n \sim (150\,MeV)^n$ (or smaller). Using the duality relations it is easy to see that here the smallness of the scale ($\mu \simeq$ 150MeV) is due to the smallness of the n-particle phase space. For instance, the leading twist 3particle components of the ρ -meson and of the nucleon wave functions have the value $\sim 10^{-2}\,GeV^2$

b) The nonleading twist two-particle components of the meson wave functions have the dimensionality $[\mu^2]$ and their characteristic values are: $(f_\rho M_\rho) \simeq 10^{-1}\,GeV^2$ (for baryons: $f_\rho^2 M_\rho$).

c) For the two-particle wave functions $\varphi_i(zq, \mu_0^2 z^2)$ at $z^2 \neq 0$ the characteristic scale is: $\mu_0^2 \simeq \langle \vec{k}_\perp^{\,2}\rangle \simeq (300 \div 400)^2\,MeV^2$ (and analogously for the many-particle wave functions).

3. The properties of the helicity one vector meson wave functions are investigated. It is shown that unlike the case of pseudoscalar and helicity zero vector meson wave functions, the nonperturbative interaction of the quarks with the vacuum fields makes the distribution of quarks in the longitudinal momentum more narrow. In particular, it is obtained for the $\rho_{|\lambda|=1}$ -meson: $\langle (x_d - x_u)^2\rangle \simeq 0.14$ where $\langle x_{d,u}\rangle$ is the mean longitudinal momentum fraction carried by the quark (at $q_z \to \infty$). (For the π -meson: $\langle (x_d - x_u)^2\rangle \simeq 0.4$,for the $\rho_{\lambda=0}$ -meson: $\langle (x_d - x_u)^2\rangle \simeq 0.3$, $\langle (x_d - x_u)^0\rangle = 1$)

4. It is emphasized that if the $\rho_{|\lambda|=1}$ -meson wave function were much alike the π -meson wave function then we would have: $Br(\chi_2 \to \rho\rho)/Br(\chi_2 \to \pi\pi) \simeq 60$ ($\chi_2(3555)$ - is the $^3P_2$ -charmonium level), while the experimental data show that this ratio is less or about of the unity. It is shown that for the realistic π -meson and $\rho_{|\lambda|=1}$ -meson wave functions satisfying the QCD sum rules this ratio is $\simeq 1$

5. For the helicity one vector meson wave functions the same regularity is observed as for the pseudoscalar and helicity zero vector meson wave functions: the replacement of the light u- or d-quark by the s-quark increases slightly the wave function value at the origin and narrows somewhat the distribution of quarks in the longitudinal momentum fractions (at $q_z \to \infty$).

APPENDIX

It is the purpose of this Appendix to describe in more detail(as compared with[6])our method of treatment of the QCD sum rules. Moreover, we presented arguments in favour that the wave function $\varphi_\pi(\xi,\mu^2)$ have at $|\xi|\to 1$ the same behaviour $\sim(1-\xi^2)$ as the asimptotic wave function $\varphi_{as}\equiv\varphi(\xi,\mu^2\to\infty)=\frac{3}{4}(1-\xi^2)$

The sum rules for the moments $\langle\xi^n\rangle\equiv\int_{-1}^{1}d\xi\,\xi^n\varphi_\pi(\xi)$, $n=0,2,4..$ have the form [6]:

$$\frac{4\pi}{M^2}\int_0^\infty ds\,e^{-s/M^2}\,\mathrm{Im}\,I_n(s)=\frac{3}{(n+1)(n+3)}+\frac{\pi^2}{3}\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{M^4}+\frac{64}{81}\frac{\pi^3(11+4n)}{M^6}\langle\sqrt{\alpha_s}\bar{u}u\rangle^2 \tag{A1}$$

$$\frac{1}{\pi}\mathrm{Im}\,I_n(s)=f_\pi^2\langle\xi^n\rangle_\pi\,\delta(s)+f_{A_1}^2\langle\xi^n\rangle_{A_1}\delta(s-m_{A_1}^2)+\theta(s-s^{(n)})\frac{3}{4\pi^2(n+1)(n+3)} \tag{A2}$$

The first term at r.h.s. (A1) represents the contribution of the free quark loop and corresponds to the asimptotic wave function $\varphi_{as}(\xi)$. The sum rule(A1)at $n=0$ has been derived first in [8] and used for the determination of f_π. Neglecting in (A2), $n=0$ the A_1 -meson contribution, we have from the fit in M^2(the scale parametr M^2 is varied in such limits that the power corrections are $(5\div35)\%$ of the perturbation theory contribution): $f_\pi\simeq 130\,MeV$, $S_\pi^0\simeq 0.8\,GeV^2$ It is clear that the value $S_\pi^0\simeq0.8\,GeV^2$ detemines in this case the π -meson duality interval,not the beginning of the "continuum".

In order to determine f_{A_1} and to kill the π -meson contribution, let us differentiate (A1),$n=0$ in $1/M^2$. It is evident,that when the A_1 -meson contribution is taken into account, the "continuum" starts at $S^0\gtrsim m_{A_1}^2$. The fit in M^2 gives for $170\le|f_{A_1}|\le190\,MeV$, $S^0\simeq(1.7\div2)\,GeV^2$(Weinberg's sum rule gives $|f_{A_1}|\simeq|f_\pi|\simeq133\,MeV$,see also[8,17]).

Returning now to (A1),(A2), $n=0$ and substituting into (A2): $|f_{A_1}|\simeq(0.17\div0.19)\,GeV$, $S^{(0)}\simeq(1.7\div2)\,GeV^2$ we have from the fit $f_\pi\simeq(125\div140)\,MeV$ Therefore, the total duality interval at $n=0$ is $S^{(0)}\sim 2\,GeV^2$ while the π -meson duality interval is $S_\pi^{(0)}\simeq0.8\,GeV^2$

Let us take now (A1) with $n=2$ and differentiate it in $1/M^2$ Let the $S^{(2)}$ duality interval to be $S^{(2)}\simeq(1.8\div2)\,GeV^2$ and then we can obtain the upper bound $\langle\xi^2\rangle_{A_1}<0.07\ll\langle\xi^2\rangle_{as}^*$ .Returning now to (A1), (A2), $n=2$ and substituting into (A2): $\langle\xi^2\rangle_{A_1}\simeq0.04\div0.07$, $S^{(2)}\simeq(1.8\div2)\,GeV^2$, we have from the fit $\langle\xi^2\rangle_\pi\simeq0.4\div0.5$. Therefore, we see the essential redistribution of the contributions in the real spectral density $f_\pi\sim f_{A_1}$ while $\langle\xi^2\rangle_{A_1}$ is considerably smaller and $\langle\xi^2\rangle_\pi$ is considerably larger than $\langle\xi^2\rangle_{as}=0.2$. At the same time, the "continuum" starts at the values: $S^{(0)}\simeq S^{(2)}\simeq 2\,GeV^2$

In order to check the selfconsistence of the above described picture, let us consider also the sum rules for the quantities $\langle1-\xi^2\rangle$ and $\langle1-2\xi^2\rangle$. The sum rules should be much more sensitive to the A_1 -meson contribution and less sensitive(especially $\langle1-2\xi^2\rangle$) to the π -meson contribution in comparison with those of $\langle\xi^2\rangle$. Indeed, the investigation of these sum rules shows that it is impossible to obtain the acceptable fit without the A_1 -meson contribution. At the time, these sum rules allow a good fit for $\langle\xi^2\rangle_{A_1}<0.07$ and $0.4\le\langle\xi^2\rangle_\pi\le0.6$, $S^{(0)}\simeq S^{(2)}\simeq 2\,GeV^2$

As a result ,The complex analysis of the sum rules allows one

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* Investigation of this sum rule shows that there is large contibution from the resonance with the mass $M_R^2\simeq(3.5-4)\,GeV^2$ Taking the contribution of this resonance into account , we have from the fit in M^2: $\langle\xi^2\rangle_{A_1}\simeq0.04\div0.05$

to show that $\langle \xi^2 \rangle_\pi \simeq 0.4 \div 0.5$ and this value is much larger than $\langle \xi^2 \rangle_{as}$.*

The analogous analysis of the sum rules for $\langle \xi^4 \rangle_\pi$ leads to $\langle \xi^4 \rangle_\pi \simeq 0.18 \div 0.32$ for $S^{(4)} \simeq (1.8 \div 3)\ GeV^2$

Let us discuss now the sum rules for $n \gg 1$. It is not difficult to see from (A1) that the power corrections can be neglected only at $M^2 > M_n^2 \simeq n m_\rho^2$. This shows that the spectral density coincides with the asymptotic one at $S > n m_\rho^2$ and deviates considerably from it at $S < n m_\rho^2$. Therefore, the total duality interval is $S_{tot}^{(n)} \simeq n m_\rho^2,\ n \gg 1$. It is clear from the physical considerations that such an enormous duality interval can not be filled with a few resonances. Roughly speaking, there will be $\sim n$ resonance-like structures each of which fills its ows finite duality interval. In particular, the π-meson fills its ows finite duality interval at $n \gg 1$: $S_\pi^{(n)} \to S_\pi^{\infty} \simeq 3 \div 4\ GeV^2$. In this case the duality relations lead to the following important result**:

$$f_\pi^2 \langle \xi^n \rangle_\pi \to \frac{3}{4\pi^2} \frac{S_\pi^{\infty}}{(n+1)(n+3)} \simeq \frac{3}{4\pi^2} \frac{S_\pi^{\infty}}{n^2}, \quad n \gg 1 \qquad (A3)$$

Therefore $\langle \xi^n \rangle_\pi \sim 1/n^2$ at $n \gg 1$ and such a behaviour shows that at $|\xi| \to 1$ the π-meson wave function $\varphi_\pi(\xi)$ has the same behaviour as $\varphi_{as}(\xi) = \frac{3}{4}(1-\xi^2)$, i.e. $\varphi_\pi(\xi) \sim (1-\xi^2)$ at $|\xi| \to 1$. For instance, for the model wave function [6] $\varphi_\pi(\xi) = \frac{15}{4}\xi^2(1-\xi^2)$: $\langle \xi^n \rangle \to \frac{15}{n^2}$ at $n \gg 1$ and this corresponds to the π-meson duality interval $S_\pi^{(n)} \to S_\pi^{\infty} = 20\pi^2 f_\pi^2 \simeq 3.5\ GeV^2$

This value of S_π^{∞} seems reasonable.

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*We neglect the small corrections due to the anomalous dimensions because these corrections do not exceed the uncertainty in the value of $\langle \xi^2 \rangle_\pi$

**We neglect here the loga-rithmic effects due to the anomalous dimensions.

Let us represent now the π-meson wave function in the form

$$\varphi_\pi(\xi) = (1-\xi^2)\sum_{n=0} A_n C_n^{3/2} = (1-\xi^2)\left[\frac{3}{4} + \frac{15}{4}\left(\xi^2 - \frac{1}{5}\right)\right] + (1-\xi^2)\sum_{n=4} A_n C_n^{3/2}(\xi) \qquad (A4)$$

where $C_n^{3/2}$ -are Gegenbauer polinomials. Two first polinomials in (A4) are taken with such coefficients which ensure the good agreement with the above found values of the moments $\langle \xi^0 \rangle$, $\langle \xi^2 \rangle$ and $\langle \xi^4 \rangle$. Because $\varphi_\pi(\xi)$ has the behaviour $\sim (1-\xi^2)$ at $|\xi| \to 1$ the series in n in (A4) is convergent(barring the pathological cases). It is natural therefore to expect that all other coefficients are not large : $\alpha_n \equiv A_n C_n^{3/2}(1) \sim O(1)$, $n \geq 4$.

The typical integral has the form: $I = \int_{-1}^{1} (d\xi/1-\xi^2)\varphi(\xi)$

Substituting (A4) we have $I = [3/2 + 1] + B_0$, $B_0 \simeq \sum_{n=4} \frac{7\alpha_n}{n^2}$. The constant B_0 will be small for $\alpha_n \sim O(1)$ and so the approximation $B_0 \simeq 0$ introduces the error which is not larger than the uncertainty with wich the quantity $\left(\frac{3}{2}+1\right)$ is known . As a result ,we can describe with a good enough accuracy the π-meson wave function with the help of two lowest polinomials in (A4).

The sum rules for $\varphi_\rho^T(\xi)$

The analysis presented above can be used for the moments corresponding to wave function $\varphi_\rho^T(\xi)$. This procedure gives us the next results:the wave function of B(1235)-mesons (just the B-mesons gives the essential contribution to the corresponding spectral density) becomes "wider" $(\langle \xi^2 \rangle_B > \langle \xi^2 \rangle_{as} = 0.2)$ and the wave function of ρ-mesons $(\varphi_\rho^T(\xi))$ becomes more narrow $(\langle \xi^2 \rangle_\rho^T < \langle \xi^2 \rangle_{as} = 0.2)$ than the asimptotic one $(\langle \xi^2 \rangle_{as} = 0.2)$ This leads to the fact that the parametr S_{20} in the sum rules for $\langle \xi^2 \rangle_\rho^T$ is less than parametr S_0 in sum rules determining f_ρ^T.

The estimate is as follows :

$$\frac{\langle \xi^2 \rangle^T_\rho (f^T_\rho)^2}{(f^T_\rho)^2} = \frac{\frac{M^2}{20\pi^2}\left(1-e^{-S_{20}/M^2}\right) + \frac{1}{36}\frac{1}{M^2}\langle \frac{\alpha_s}{\pi} G^2 \rangle - \frac{64}{81}\frac{1}{M^4}\pi \langle \sqrt{\alpha_s}\, \bar{u}u \rangle^2}{\frac{M^2}{4\pi^2}\left(1-e^{-S_0/M^2}\right) - \frac{1}{12}\frac{1}{M^2}\langle \frac{\alpha_s}{\pi} G^2 \rangle + \frac{64}{81}\pi \frac{\langle \sqrt{\alpha_s}\, \bar{u}u \rangle^2}{M^4}} \quad (A5)$$

Note that for $M^2 = m_\rho^2$ and $S_{20} \simeq 1.5\,GeV^2$ we have $0.13 \le \langle \xi^2 \rangle^T_\rho \le 0.16$

The best fit is $\langle \xi^2 \rangle^T_\rho = 0.14$

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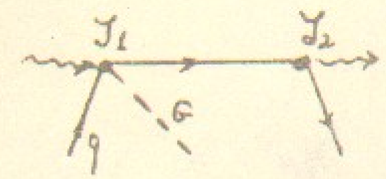


fig.1

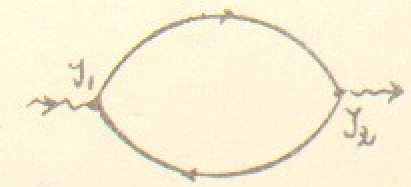


fig. 2 a

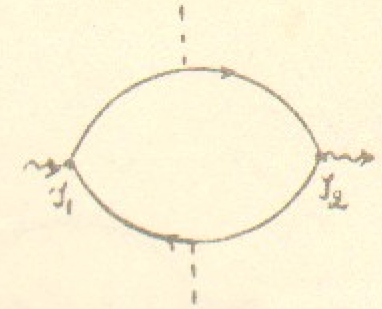
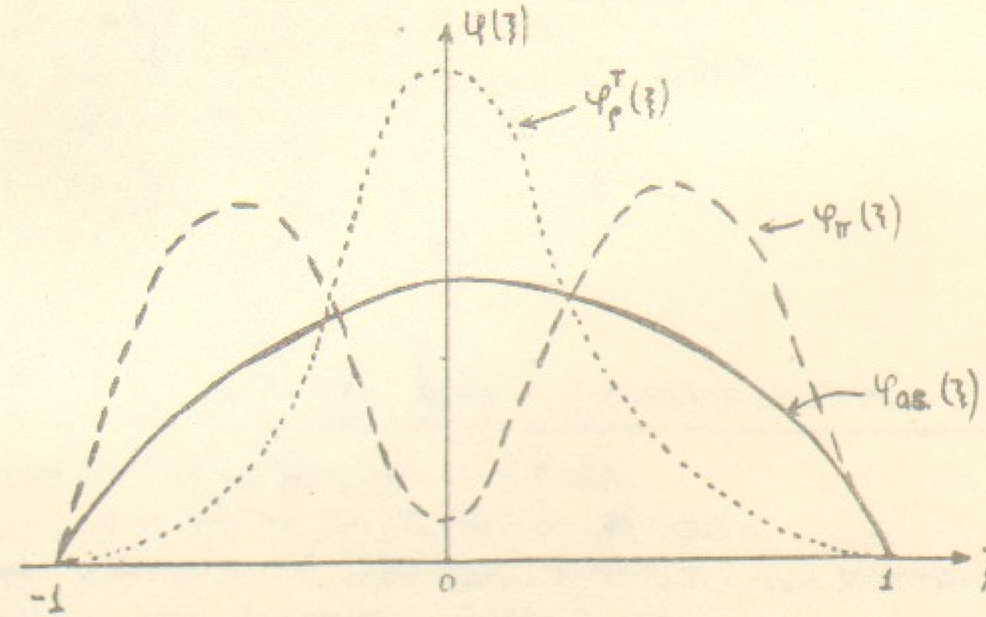


fig.2 b

fig.2 c



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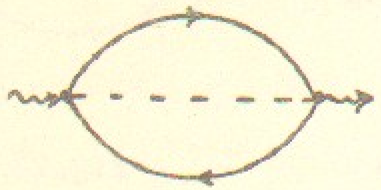

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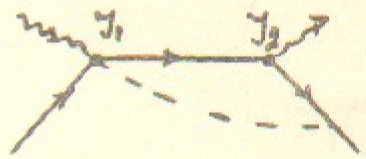

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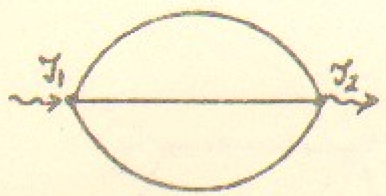

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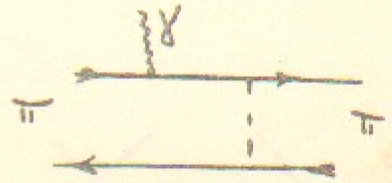

fig.7

fig.8

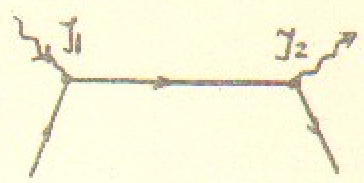

fig.9

А.Р.Житницкий, И.Р.Житницкий, В.Л.Черняк

О СВОЙСТВАХ МЕЗОННЫХ ВОЛНОВЫХ ФУНКЦИЙ

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