56

ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
СО АН СССР

I.R.Zhitnitsky

NUCLEON WAVE FUNCTION AND NUCLEON FORM FACTOR IN QCD

PREPRINT 82-155

Новосибирск

NUCLEON WAVE FUNCTION

AND NUCLEON FORM FACTOR IN QCD

I.R. ZHITNITSKY

Institute of Nuclear Physics

630090, Novosibirsk, USSR

A B S T R A C T

The values of few first moments of the nucleon wave function are found by using the method of QCD sum rules. The model for the nucleon wave function is proposed based on the knowledge of these moments. It is shown that with the help of this wave function one obtains the sign and the value of the proton magnetic form factor in agreement with the experiment. The ratio of the proton to neutron magnetic form factors is predicted: $G_M^n/G_M^p \simeq -0.5$.

Predictions for the inclusive nucleon production cross-sections are presented.

I. Introduction.

The asymptotic behaviour of the nucleon electromagnetic form factors in QCD has been investigated in papers /1-4/. It was shown in /1/ that in the formal limit $|q^2|\to\infty$:

$$q^4 G_M^n(q^2) \to |f_n|^2 \left(\alpha_s(q)\right)^{2+4/3b} > 0, \quad G_M^p(q^2)/G_M^n(q^2) \sim \left[\alpha_s(q^2)\right]^{20/9b}, \tag{1}$$

$$b = 11 - \frac{2}{3} n_f ,$$

where $G_M^{p(n)}(q^2)$ - is the proton (neutron) magnetic form factor. Therefore, the neutron magnetic form factor changes the sign at least once, because $G_M^n(q^2=0) \simeq -1.9\mu_N < 0$. The results obtained in /2/ (taking the "erratum" into account) differ from /1/ in overal sign. It was shown in /3/ that if the nucleon wave function is such that each quark carries 1/3 of the total nucleon momentum, then (neglecting the logarithmic corrections): $q^4 G_M^p(q^2) < 0,\ q^4 G_M^n(q^2) > 0$ at $|q^2| \gg 1\,GeV^2$. The general formulae obtained in /1/ coincide with the result /3/ in this limit, while those from /2/ differ in overal sign.

For the reader convenience the contributions of various diagrams into the nucleon form factor asymptotic behaviour are presented in Table I (see /1/). Here $V_\mu(x_i)$, $A_\mu(x_i)$ and $T_\mu(x_i)$ are the leading twist nucleon wave functions which determine the distribution of quarks (with the virtuality up to μ^2) inside the nucleon in the longitudinal momentum: in the $P_z\to\infty$ frame i-th quark carries the fraction $0\le x_i\le 1$ of the total momentum, $\sum_1^3 x_i = 1$.

One can see from Table I that different diagrams contribute with different signs and so the total answer is very sensitive to the precise form of the nucleon wave functions V, A, T. Using the asymptotic wave function: $V(x_i,\mu^2\to\infty) = T(x_i,\mu^2\to\infty) = const\cdot x_1 x_2 x_3$, $A\to 0$, one obtains that the proton magnetic form factor is zero while the neutron one is positive/1,4/. Therefore, if the nucleon wave function with the virtuality $\mu^2 \sim 1\ \mathrm{GeV}^2$ is much alike the asymptotic one, one obtains the same results: $|G_M^p/G_M^n| \ll 1$, $G_M^n > 0$ which are in conflict with the experiment. From our point of view, this shows clearly that the wave functions with the virtuality $\mu^2 \sim 1\ \mathrm{GeV}^2$ differ greatly from the asymptotic one.

It is the main goal of this paper to study the properties of the nucleon wave function with the virtuality $\mu^2 \sim 1\ \mathrm{GeV}^2$. It is just this wave function which determine the asymptotic behaviour of the nucleon form factor (and other amplitudes) at $|q^2| \sim 10^1 \div 10^2\ \mathrm{GeV}^2$. Besides, the dependence of the wave function on the virtuality μ^2 is determined by the renormalization group and is very weak /1,2/. Therefore, <u>if the wave function $\varphi(x_i, \mu^2 \sim 1\ \mathrm{GeV}^2)$ differs greatly from the asymptotic one $\varphi_{as}(x_i, \mu^2 \to \infty)$, then this difference will persist up to enormously large values of μ^2.</u>

To study the properties of the nucleon wave function $\varphi(x_i, \mu^2 \sim 1\ \mathrm{GeV}^2)$ we use the method of the QCD sum rules developed in /5/. This method gives the possibility to calculate the approximate values of the wave function moments:

$$\langle x_1^{n_1} x_2^{n_2} x_3^{n_3} \rangle_{\mu^2} \equiv \int_0^1 d\mu_3\, \varphi(x_i, \mu^2)\, x_1^{n_1} x_2^{n_2} x_3^{n_3}, \quad d\mu_3 = dx_1\, dx_2\, dx_3\, \delta(1 - \sum x_i).$$

Using the found values of moments we propose then the model wave function satisfying sum rules. Such approach has been used in /6,7/ for investigation of the properties of various meson wave functions. It is shown in these papers that wave functions obtained in this way can, in general, differ greatly from the asymptotic ones and lead to the predictions for the two-particle charmonium decay widths which agree with the experiment.

The organization of the paper is as follows. In sect. II main definitions and notations are presented. The sum rules for the nucleon wave function moments are obtained and treated in sect. III and the model wave function satisfying sum rules is proposed here. It is shown in sect. IV that this model wave function leads to the predictions for the nucleon form factors which are in reasonable agreement with the experiment (both in sign and magnitude). The conclusions and predictions for other experiments are presented in sect. V.

II. <u>Main definitions and notations.</u>

Following /1/ let us define the matrix element of the three-local operator (at $p_z \to \infty$):

$$\langle 0 | u_\alpha^i(z_1) u_\beta^j(z_2) d_\gamma^k(z_3) | p \rangle_{\mu^2} \varepsilon^{ijk} = -\frac{1}{4} \Big\{ (\hat{p} C)_{\alpha\beta} (\gamma_5 N)_\gamma V_{\mu^2}(z_i p) + \tag{3}$$
$$+ (\hat{p}\gamma_5 C)_{\alpha\beta} N_\gamma A_{\mu^2}(z_i p) - (\sigma_{\mu\nu} p_\nu C)_{\alpha\beta} (\gamma_\mu \gamma_5 N)_\gamma T_{\mu^2}(z_i p) \Big\}, \quad \sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu].$$

Here: $|p\rangle$ is the proton state with the momentum p, N_γ is the proton spinor, C-is the charge conjugation matrix, u, d - quark fields, i,j,k-color indices. Let us present the functions $V(z_i p)$, $A(z_i p)$ and $T(z_i p)$ in the form:

$$V(z_i p, \mu^2) = \int_0^1 d\mu_3 \exp\{-i \sum x_i (z_i p)\} V(x_i, \mu^2). \tag{4}$$

The wave functions $V(x_i, \mu^2)$, $A(x_i, \mu^2)$ and $T(x_i, \mu^2)$ describe the distribution of quarks inside the proton in the longitudinal momentum fractions: $0 \le x_i \le 1$, $\sum x_i = 1$. The identity of two u-quarks leads to the relations:

$$V(1,2,3) = V(2,1,3), \quad A(1,2,3) = -A(2,1,3), \quad T(1,2,3) = T(2,1,3). \tag{5}$$

Requiring the total isotopic spin of three quarks to be equal 1/2, one obtains the relation:

$$2T(1,2,3)=V(1,3,2)-A(1,3,2)+V(3,2,1)+A(3,2,1). \qquad (6)$$

Therefore, there is only one independent wave function, say $V(x_i)-A(x_i)$.

If the logarithmic corrections due to higher order perturbation theory diagrams are taken into account, the wave function depends weakly on the normalization point μ^2 (the value of μ^2 is equal to the characteristic virtuality of the constituents in the given process). The dependence of the wave function on the μ^2 is determined by the renormalization group /1,2/:

$$V(x_i,\mu^2)=\varphi_{as}(x_i)\cdot\sum f_n P_n(x_i)\,e^{-\varepsilon_n\tau}, \quad \varphi_{as}(x_i)=120x_1x_2x_3,$$

$$f_n=\int_0^1 d\mu_3\,P_n(x_i)\,V(x_i,\mu_0^2), \quad \tau=\frac{1}{6}\ln\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)},$$

where $\{P_n(x_i)\}$ -is the orthogonalized system of the Appel polinomials, ε_n -are the corresponding anomalous dimensions.

The direct investigation of the wave function properties can be replased by the investigation of the values of its moments. It follows from (2)-(4) that the wave function moments* are determined by the following matrix elements of local operators:

$$\langle 0|(i\vec{D}_\mu z_\mu)^{n_1}u^i(0)C\hat{z}(i\vec{D}_\nu z_\nu)^{n_2}u^j(0)(i\vec{D}_\lambda z_\lambda)^{n_3}(\gamma_5 d^k(0))_\tau\,\varepsilon^{ijk}|P\rangle=$$
$$=-(zp)^{n_1+n_2+n_3+1}N_\tau\cdot V^{(n_1n_2n_3)}, \quad z^2=0, \qquad (7)$$

$$\langle 0|(i\vec{D}_\mu z_\mu)^{n_1}u^i(0)C\hat{z}\gamma_5(i\vec{D}_\nu z_\nu)^{n_2}u^j(0)(i\vec{D}_\lambda z_\lambda)^{n_3}(d_\tau^k(0))\varepsilon^{ijk}|P\rangle=$$
$$=-(zp)^{n_1+n_2+n_3+1}N_\tau\cdot A^{(n_1n_2n_3)}. \qquad (8)$$

We use also the following current:

$$\langle 0|J_\lambda^{(n_1)}|P\rangle\equiv\langle 0|(i\vec{D}_\nu z_\nu)^{n_1}u^i(0)C\hat{z}\,u^j(0)(\gamma_5 d)_\lambda^k\,\varepsilon^{ijk}-$$
$$-(i\vec{D}_\nu z_\nu)^{n_1}u^i(0)C\hat{z}\,d^j(0)(\gamma_5 u^k)_\lambda\,\varepsilon^{ijk}|P\rangle= \qquad (10)$$
$$=-(zp)^{n_1+1}N_\lambda\cdot\left[\frac{1}{2}(V-A)^{(n_100)}+T^{(n_100)}\right].$$

The current J_λ has the isotopic spin equal to 1/2 and so we expect that the proton gives large contributions into the correlators containing this current.

Let us consider now the correlator

$$I^{(n_1n_2n_3)}=i\int dx\,e^{iqx}\langle 0|T\,V_\gamma^{(n_1n_2n_3)}(x)\cdot J_{\gamma'}^{(100)+}(0)|0\rangle\,\hat{z}_{\gamma'\gamma} \qquad (11)$$

(and analogous correlator with the replacement $V\to A$) where the factor $\hat{z}_{\gamma'\gamma}$ is introduced to separate out the contribution of the function $V^{(n_1n_2n_3)}$.

The proton contribution into the spectral density (11) is:

$$\frac{1}{\pi}\,\mathrm{Im}\,I^{(n_1n_2n_3)}(s)=V^{(n_1n_2n_3)}\cdot C\cdot\delta(s-M_N^2)+\ldots\equiv r^{(n_1n_2n_3)}\cdot\delta(s-M_N^2)+\ldots$$
$$C=\frac{1}{2}\left[V^{(100)}-A^{(100)}\right]+T^{(100)}, \qquad (12)$$

(the constant $r^{(n_1n_2n_3)}$ in (12) will be called "the residue" in what follows).

In addition to the perturbation theory contribution the nonperturbative corrections $\langle 0|\frac{\alpha_s}{\pi}G_{\mu\nu}^2|0\rangle$ and $\langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle^2$ are taken into account below in the sum rules. We use /5/:

$$\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle\simeq 1.2\cdot 10^{-2}\,\mathrm{GeV}^2, \quad \langle 0|\sqrt{\alpha_s}\,\bar{u}u|0\rangle\simeq 1.8\cdot 10^{-4}\,\mathrm{GeV}^6.$$

It is convenient to calculate the nonperturbative contributions into the sum rules using the gauge $x_\mu A_\mu=0$ /8,9/.

III. <u>Sum rules and nucleon wave function.</u>

The sum rules have the following form (the corresponding diagrams are shown at figs.1,2):

$$3a\, V^{(000)} = \Pi^{(000)} + 0{,}1\, \mathrm{Gev}^4 + 0{,}4\, \frac{\mathrm{Gev}^6}{M^2} \qquad 13$$

$$\frac{42}{5}\, a\, V^{(100)} = \Pi^{(100)} + 0{,}15\, \mathrm{Gev}^4 + 0{,}49\, \frac{\mathrm{Gev}^6}{M^2} \qquad 14$$

$$\frac{21}{2}\, a\, V^{(001)} = \Pi^{(001)} - 0{,}03\, \mathrm{Gev}^4 + 0{,}18\, \frac{\mathrm{Gev}^6}{M^2} \qquad 15$$

$$\frac{56}{3}\, a\, V^{(200)} = \Pi^{(200)} + 0{,}37\, \mathrm{Gev}^4 + 0{,}84\, \frac{\mathrm{Gev}^6}{M^2} \qquad 16$$

$$28\, a\, V^{(002)} = \Pi^{(002)} + 0{,}1\, \mathrm{Gev}^4 + 0{,}25\, \frac{\mathrm{Gev}^6}{M^2} \qquad 17$$

$$28\, a\, V^{(110)} = \Pi^{(110)} + 0{,}06\, \mathrm{Gev}^4 + 0{,}23\, \frac{\mathrm{Gev}^6}{M^2} \qquad 18$$

$$\frac{168}{5}\, a\, V^{(101)} = \Pi^{(101)} - 0{,}03\, \mathrm{Gev}^4 + 0{,}15\, \frac{\mathrm{Gev}^6}{M^2} \qquad 19$$

$$42\, a\, A^{(010)} = \Pi^{(010)} + 1{,}17\, \mathrm{Gev}^4 + 2{,}18\, \frac{\mathrm{Gev}^6}{M^2} \qquad 20$$

$$56\, a\, A^{(020)} = \Pi^{(020)} + 1{,}1\, \mathrm{Gev}^4 + 2{,}43\, \frac{\mathrm{Gev}^6}{M^2} \qquad 21$$

$$168\, a\, A^{(011)} = \Pi^{(011)} + 1{,}38\, \mathrm{Gev}^4 + 1{,}44\, \frac{\mathrm{Gev}^6}{M^2} \qquad 22$$

$$7\, a\, [V-A]^{(100)} = \Pi^{(100)} + 0{,}32\, \mathrm{Gev}^4 + 0{,}77\, \frac{\mathrm{Gev}^6}{M^2} \qquad 23$$

$$\frac{21}{2}\, a\, [V-A]^{(010)} = \Pi^{(010)} - 0{,}1\, \mathrm{Gev}^4 + 0{,}06\, \frac{\mathrm{Gev}^6}{M^2} \qquad 24$$

$$28\, a\, [V-A]^{(020)} = \Pi^{(020)} + 0{,}07\, \frac{\mathrm{Gev}^6}{M^2} \qquad 25$$

$$14\, a\, [V-A]^{(200)} = \Pi^{(200)} + 0{,}45\, \mathrm{Gev}^4 + 1{,}2\, \frac{\mathrm{Gev}^6}{M^2} \qquad 26$$

$$28\, a\, [V-A]^{(101)} = \Pi^{(101)} + 0{,}1\, \mathrm{Gev}^4 + 0{,}35\, \frac{\mathrm{Gev}^6}{M^2} \qquad 27$$

Here: $a = (4\pi^4)\, 160C \cdot \exp\{-M_N^2/M^2\}$, $\Pi^{n_1 n_2 n_3} = M^4 \left\{ 1 - \left(1 + \frac{S_0^{n_1 n_2 n_3}}{M^2}\right) \exp\left(-\frac{S_0^{n_1 n_2 n_3}}{M^2}\right) \right\}$.

Analogous sum rules have been considered previously in /10,11/. The estimated in /10,11/ accurasy of the results obtained from such sum rules is $\simeq$ 10-15%. Our mean accurasy will be the same.

The sum rules (13)-(27) has been treated in the following way.

1. At the first stage the best fit has been performed in such interval of values of M^2 that the nonperturbative power corrections were $\simeq$ 15-40% of perturbation theory contributions. The residues r^i and the duality intervals S_0^i were found, while the proton mass has been considered as known.

2. At the second stage the "effective resonance" contribution has been added to the spectral density: $\mathrm{Im}\, I(s) = (\text{proton}) +$ "eff. res."+continuum (resonance mass: $M_R \simeq 1.5\, \mathrm{GeV}$, that corresponds to the experimental spectrum in the 1/2-isotopic spin channel). Now the best fit was perfofmed with two resonances (p+"eff.res.") in the same interval of M^2-values. (Nearly for all the sum rules this is the interval: $1\, \mathrm{GeV}^2 \lesssim M^2 \lesssim 2\, \mathrm{GeV}^2$). The parameter S_0^i has been varied within $\pm$ 15% around its value found at the first stage. The variation of the residue value as compared with its value determined at the first stage show the uncertainty in the residue which arises when the form of the "continuum" is varied.

The results are presented in Table II.

Using the sum rules (13),(15),(22),(23) and the relation we have for the $V^{(000)}$:

$$f_0 \equiv V^{(000)} = (5.3 \pm 0.2)\, 10^{-3}\, \mathrm{GeV}^2. \qquad (24)$$

The value of $V^{(000)}$ has been determined also in /10,11/, where it was obtained: $V^{(000)}=(8\pm3)10^{-3}\,GeV^2$. This value does not contradict to our result (24). The result /10,11/ has lower accuracy because the sum rules used in /10,11/ are much more sensitive to the precise form of the "continuum".

Let us note here the following.

a) Because in the sum rules we have not introduced explicitly the logarithmic corrections due to the anomalous dimensions, the values of the moments presented in Table II correspond to the normalizations at suitable intermediate points. The mean normalization point for all the sum rules can be taken as: $\mu^2 \simeq 1\,GeV^2$ with a good enough accuracy. (Let us note also that the values of the anomalous dimensions are not large, see /1,2/).

b) It is seen from Table II that the ratios $V^{(100)}/V^{(000)}$ and $V^{(200)}/V^{(000)}$ are much larger and the ratios $V^{(001)}/V^{(000)}$, $V^{(002)}/V^{(000)}$ and $V^{(101)}/V^{(000)}$ are much smaller than the corresponding ratios for the asymptotic wave function: $\varphi_\infty(x_i)=120x_1x_2x_3$. Let us emphasize also that the following ratios

$$(V-A)^{(100)}/(V-A)^{(001)} \simeq (V-A)^{(100)}/(V-A)^{(010)} \simeq 4\div5 \tag{25}$$

are very large in comparison with the corresponding ratios for the wave function $\varphi_\infty(x_i)$.

These results show unambiguously that the largest part of the proton longitudinal momentum is carried by one u-quark with the spin directed along the proton spin. This interpretation becomes especially clear when one rewrites the formula (3) for the proton wave function in the form:

$$|p^\uparrow\rangle = const.\int_0^1 d\mu_3 \Big\{ \Big[\frac{V-A}{2}\Big](x_i)\,|u^\uparrow(x_1)u^\downarrow(x_2)d^\uparrow(x_3)\rangle + \tag{26}$$

$$+\Big[\frac{V+A}{2}\Big](x_i)\,|u^\downarrow(x_1)u^\uparrow(x_2)d^\uparrow(x_3)\rangle - T(x_i)\,|u^\uparrow(x_1)u^\uparrow(x_2)d^\downarrow(x_3)\rangle\Big\}. \tag{26}$$

c) The sum rules for the moments $(V-A)^{(010)}$, $(V-A)^{(020)}$ and $(V-A)^{(011)}$ are sensitive to the contribution of the "effective resonance". This leads to the uncertainty about 30% in the moment values in spite of that the values of nonperturbative corrections are not large here. Newertheless, the values of these moments allowed by the sum rules are essentially smaller than the corresponding values for $\varphi_\infty(x_i)$. Let us emphasize also that using the identities: $\sum_i\langle x_i\rangle=1$, $\langle x_i\rangle=\sum\langle x_i x_j\rangle$ it is possible to diminish essentially the uncertainties at the complex analysis of all the sum rules (17)-(23).

In order to clarify the properties of the nucleon wave function let us introduce the Mandelstam plane for the variables x_1, x_2, x_3, $\sum_i x_i=1$. The maximum of the function is denoted as $\oplus$ and the minimum as $\ominus$. The asymptotic wave function $\varphi_\infty=120x_1x_2x_3$ is shown at fig.3a. The values of the moments presented in Table II imply that the function $V(x_i)$ has its maxima in the vicinity of $x_1\simeq1$ and $x_2\simeq1$, fig.3b, while the function $(V-A)$ has its maximum at $x_1\simeq1$, fig.3c.

Let us discuss now in short the behaviour of the higher moments. As $n=\sum_i n_i$ grows, the relative values of the nonperturbative corrections (with respect to the perturbation theory contributions) increase. This means that the corresponding total duality intervals increase also, so that at large $n\gg1$ they will be very large. However, it is clear from the physical considerations that at large $n\gg1$ the nucleon contribution into the correlators fills not the total duality interval S^n_{tot}, but has its own interval (S_N^n) which is much smaller than S_{tot}^n and tends to some finite value: $S_N^n \to S_N^\infty = \mathrm{const}$ at $n \gg 1$. In this case the duality relations imply that at $n \gg 1$:

$$V^{(n_1 n_2 n_3)} \sim S_N^\infty \cdot \frac{\Gamma(n_1+2)\Gamma(n_2+2)\Gamma(n_3+2)}{\Gamma(n_1+n_2+n_3+6)} . \tag{27}$$

This relation shows that the moments of the true nucleon wave function $V(x_i)$ have the same dependence on n_i at $n_i \gg 1$ as the moments of the asymptotic wave function $\varphi_\infty = 120 x_1 x_2 x_3$. We conclude from this that <u>the true wave functions $V(x_i)$, $A(x_i)$ and $T(x_i)$ have at $x_i \to 1$ the same behaviour as $\varphi_\infty(x_i)$</u>.

Based on these considerations, let us choose the model for the nucleon wave function in the form:

$$\varphi(x_i,\mu^2) = \varphi_\infty(x_i) \cdot \sum_0^{n=2} C_n P_n(x_i) , \quad n = n_1+n_2+n_3 , \tag{28}$$

where $\{P_n(x_i)\}$ are Appel polinomials /1,2/ and C_n -are some constants. In other words, we confine ourselves by two lowest polinomials. The experience with the meson wave functions /15/ shows that such a form can reproduce correctly the main characteristic properties of the true wave function.

We propose the following model wave functions:

$$V(x_i,\mu^2 = 1\,GeV^2) \simeq \varphi_\infty(x_i)\left[11.35(x_1^2+x_2^2) + 8.82 x_3^2 - 1.68 x_3 - 2.94\right] \cdot f_0 ,$$

$$A(x_i,\mu^2 = 1\,GeV^2) \simeq \varphi_\infty(x_i)\left[6.72(x_1^2 - x_2^2)\right] \cdot f_0 , \quad f_0 \simeq 5.3 \cdot 10^{-3}\,GeV^2 , \tag{29}$$

$$(V-A)(x_i,\mu^2 = 1\,GeV^2) \simeq \varphi_\infty(x_i)\left[18.07 x_1^2 + 4.63 x_2^2 + 8.82 x_3^2 - 1.68 x_3 - 2.94\right] \cdot f_0$$

The values of the moments of these functions are **given in Table II** and agree with the **sum rule results. These functions are** shown at figs. 6a,b The asymptotic wave function $\varphi_\infty(x_i) = 120 x_1 x_2 x_3$ is shown for comparison at fig. 6,c Using the relation (6) one has

$$T(x_i,\mu^2 = 1\,GeV^2) \simeq \varphi_\infty(x_i)\left[13.44(x_1^2+x_2^2) + 4.62 x_3^2 + 0.84 x_3 - 3.78\right] \cdot f_0 . \tag{30}$$

To check the consistency of the whole method it is useful to calculate the moments of the wave function $T(x_i)$ from independent sum rules and then to compare the results with the values obtained from (30).

With that end in view let us consider the correlator:

$$I^{(n_1 n_2 n_3)} = i\int dx\, e^{iqx} \langle 0 | T\{ T_\lambda^{(n_1 n_2 n_3)}(x) \cdot \mathcal{J}_{\lambda'}^{(000)}(0) \} | 0 \rangle (\hat{z})_{\lambda'\lambda} , \tag{31}$$

$$T_\lambda^{(n_1 n_2 n_3)} = \varepsilon^{ijk} \{ (iz_\alpha \vec{D}_\alpha)^{n_1} u^i C \sigma_{\mu\nu} z_\nu (iz_\beta \vec{D}_\beta)^{n_2} u^j (iz_\gamma \vec{D}_\gamma)^{n_3} (\gamma_\mu \gamma_5 d)_\lambda \} ,$$

$$\langle 0 | T_\lambda^{(n_1 n_2 n_3)}(0) | P \rangle = (zp)^{n_1+n_2+n_3+2} N_\lambda \cdot 2 T^{(n_1 n_2 n_3)} ; \quad T^{(n_1 n_2 n_3)} = \int_0^1 d\mu_3\, x_1^{n_1} x_2^{n_2} x_3^{n_3} T(x_i) . \tag{32}$$

The corresponding sum rules have the form:

$$a \cdot (T^{(000)})^2 = \Pi^{(000)} + 0{,}066\,GeV^4 + 0{,}13\,GeV^6/M^2 \tag{33}$$

$$a \cdot 3 T^{(100)} T^{(000)} = \Pi^{(100)} + 0{,}1\,GeV^4 + 0{,}25\,GeV^6/M^2 \tag{34}$$

$$a \cdot 3 T^{(001)} T^{(000)} = \Pi^{(001)} \qquad - 0{,}1\,GeV^6/M^2 \tag{35}$$

$$a \cdot 7 T^{(002)} T^{(000)} = \Pi^{(002)} + 0{,}14\,GeV^4 - 0{,}13\,GeV^6/M^2 \tag{36}$$

$$a \cdot 7 T^{(200)} T^{(000)} = \Pi^{(200)} + 0{,}32\,GeV^4 + 0{,}48\,GeV^6/M^2 \tag{37}$$

$$a \cdot 21/2\, T^{(101)} T^{(000)} = \Pi^{(101)} + 0{,}1\,GeV^4 - 0{,}03\,GeV^6/M^2 \tag{38}$$

$$a \cdot 21/2\, T^{(110)} T^{(000)} = \Pi^{(110)} + 0{,}03\,GeV^4 + 0{,}18\,GeV^6/M^2 \tag{39}$$

where:

$$a = (4\pi^4)\, 240\, e^{-M_N^2/M^2} , \quad \Pi^{(n_1 n_2 n_3)} = M^4 \left\{ 1 - \left(1 + \frac{S_0^{n_1 n_2 n_3}}{M^2}\right) e^{-\frac{S_0^{n_1 n_2 n_3}}{M^2}} \right\} .$$

The results obtained from these sum rules are presented in Table II and agree well with (30). It is worth noting that the sum rule (33) allow us to obtain independently the value of $V^{(000)} = T^{(000)}$:

$$f_0 = V^{(000)} = (5.2 \pm 0.3)\, 10^{-3}\, GeV^2 . \qquad (39)$$

This value agrees well with (24).

Let us note also the following.

The treatment of the sum rules for $T^{(001)}$ and $T^{(002)}$ shows that the large contribution to the spectral density gives the resonance with the following values of the mass and the residue:

$$M_R \simeq (1.5 \pm 0.1)\, GeV,$$
$$T_R^{(001)} \cdot \left[T_R^{(000)} + \tfrac{1}{2} V_R^{(000)} \right] = (1 \pm 0.1)\, 10^{-4}\, GeV^4 . \qquad (40)$$

One can see from the sum rules (24) (28) (35) that <u>the largest part of the longitudinal momentum of this resonance is carried by one quark with the spin directed oppositely to the resonance spin</u>.

Let us discuss in short why the currents we use are preferable as compared with those used in /10,11/.

1. We separate the leading twist contribution (i.e. the structure $q_\mu q_\nu q_\lambda$ in the correlator) and so our spectral density has the behaviour $\sim s$ at large $s \gg \mu^2$, unlike the behaviour $\sim s^2$ in /10,11/. Therefore, our spectral density is less sensitive to the precise form of the "continuum".

2. Choosing $J^{(100)}$ as the second current in the correlator one reduces essentially "the effective resonance" contribution into the spectral density and so the continuum begins at larger values of s. Therefore, such sum rules are more sensitive to the nucleon contribution and allow one to obtain the nucleon residue with the better accuracy. Moreover, the relative sign of the residues $V^{(100)}$ and $A^{(100)}$ unambiguously determined here by the perturbation theory contributions.

IV. The nucleon magnetic form factor.

We use below for the calculation of the nucleon magnetic form factor the results obtained in /1/. The contributions of various Born diagrams are shown in Table I and the total answer is /1/:

$$\langle P_2 | J_\mu^{el}(0) | P_1 \rangle \to \frac{\bar N_2 \gamma_\mu N_1}{q^4} \frac{(4\pi \alpha_s)^2}{54} \times \int_0^1 d\mu_3(x)\, d\mu_3(y) \left\{ \sum_{i=1}^{7} q_i \left[T_B^i(x,y) + \tilde T_B^i(x,y) \right] + \sum_{i=8}^{14} q_i T_B^i(x,y) \right\}, \qquad (41)$$

where q_i –is the charge of those quark which interact with the photon in the given diagram (for the case of the proton the lowest line in each diagram is the d-quark line, for the neutron one should replace: $u \leftrightarrow d$).

It has been pointed out in /10,11/ that using the value $V^{(000)} \simeq 0.8 \cdot 10^{-2} GeV^2$ and the asymptotic wave function $\varphi_\infty(x) = 120 x_1 x_2 x_3$ one obtains for the neutron magnetic form factor the value which is two orders of magnitude smaller than the experimental value of the proton magnetic form factor at $|q^2| \simeq 20 GeV^2$. Moreover, the sign of the neutron form factor is positive in this case /1,3,4/.

The main result of this section reads as follows. Using the nucleon wave functions obtained in the previous section and Born diagram contributions from Table I and (41) one obtains (for $10^1 GeV^2 \lesssim |q^2| \lesssim 10^3 GeV^2$):

a) the proton magnetic form factor is positive while the neutron one is negative,

b) the value of $q^4 G_M^p(q^2)$ at $|q^2| \simeq 20 GeV^2$ is:

$$q^4 G_M^p(q^2) \simeq 0{,}6 \ GeV^4 \tag{42}$$

c) the ratio of the neutron to the proton form factor at $|q^2| \simeq 20 GeV^2$ is:

$$G_M^n(q^2) / G_M^p(q^2) \simeq -0.5 \tag{43}$$

These results agree with the experimental data.

Let us describe now some detail. The distribution of the nucleon longitudinal momentum between the quarks is more or less homogenious for the case of the asymptotic wave function $\varphi_\infty(x) = 120 x_1 x_2 x_3$ ($\langle x_i \rangle = 1/3$, $i=1,2,3$). The diagram N°3 in Table I gives large negative contribution into the proton form factor in this case.

The largest part of the proton momentum is carried by first u-quark for the case of the realistic wave function (29),(30) (i.e. $\langle x_1 \rangle \gg \langle x_2 \rangle \simeq \langle x_3 \rangle$). In this case the negative contribution of the diagram N°3 in Table I is diminished while the positive contributions of the diagrams N°1,9,10 in Table I are enhanced. As a result, the proton form factor becomes positive and the neutron one-negative.

Let us discuss in short what is the value of $\bar{\alpha}_s$ in (41) (see also /7/). The typical virtuality of the lightest gluon (see Table I) is: $\overline{q_1^2} \simeq \overline{x_3 y_3 q^2}$,while the rest gluon have the virtuality: $\overline{q_2^2} \simeq \overline{(1-x_1)(1-y_1) q^2}$. The characteristic values of $\bar{x}_i$ for the wave function (29) are: $\bar{x}_1 \simeq 2/3$, $\bar{x}_2 \simeq \bar{x}_3 \simeq 1/6$. Therefore: $\bar{\alpha}_s^2(Q^2) \simeq \alpha_s(Q^2/36)\,\alpha_s(Q^2/9)$, $\bar{\alpha}_s(Q^2 \simeq 20 GeV^2) \simeq 0.3$. The characteristic normalization point " μ^2 " of the nucleon wave function is determined mainly by the smallest virtuality and is about 0.5-1GeV^2 at $Q^2 \simeq 20 GeV^2$. It is worth noting that

the mean normalization point (i.e. the mean virtuality) of the moment values obtained in section III from the sum rules is $\simeq$ 1Gev^2.*

Using the wave functions (29),(30) and Table I and (41) one obtains the behaviour of the proton magnetic form factor shown at fig. 4

We have calculated also the values of the nucleon form factors when the wave function was varied in the limits allowed by the sum rules. The signs of the proton and neutron magnetic form factors can't be changed in any case, the absolute values change within a factor 2-3 and the ratio changes slightly: $G_M^n / G_M^p \simeq -(0.4 \div 0.5)$.

V. Summary.

The values of few first moments of the nucleon wave function with the virtuality $\mu^2 \simeq 1 GeV^2$ has been found above using the method of QCD sum rules. Using these moment values the model wave function of the nucleon is proposed which satisfies the sum rules. The most characteristic property of this wave function is the following: about 60-70% of the proton longitudinal momentum (at $P_z \to \infty$) is carried by one u-quark with the spin parallel to the proton spin, while each of two other quarks carries about 20% of the total momentum.

Using the model wave function of the nucleon one can:

1) to obtain the right sign and the right value of the proton magnetic form factor $G_M^p(q^2)$ at $|q^2| \simeq (15 \div 30) GeV^2$** ; moreover, varying the form of the wave function in the limits allowed by

---

* Let us remind that the dependence of the wave function on the normalization point is extremely weak.

** Our results show that the overall sign obtained in /2/ is wrong.

the sum rules it is impossible to change the form factor sign while its absolute value changes within a factor ~ 2.

2) to predict the ratio: $G_M^n(q^2)/G_M^p(q^2) \simeq -0.5$ at $|q^2| \simeq 10 \div 100\,\mathrm{GeV}^2$, the value of this ratio remains nearly constant when the form of the model wave function is varied in the limits allowed by the sum rules;

3) to obtain the ratio of the proton to neutron deep-inelastic structure functions in the threshold region:

$$F_2^n(x,Q^2)/F_2^p(x,Q^2) \simeq 1/4\,, \quad x \to 1\,,$$

(the SU(6)-symmetric nucleon wave function gives: $3/7$ /12 /);

4) to predict that the number of the leading protons is about four times larger than the number of the leading neutrons in the e^+e^- -annihilation;

5) to expect that in the e^+e^- -annihilation into two jets the presence of the leading proton in one jet will be strongly correlated with the leading $\bar{u}$ (but not $\bar{d}$) -quark into the opposite jet:

$$\frac{1}{4}\,\frac{d\sigma(e^+e^- \to p_{lead} + \bar{u}_{lead} + X)}{d\sigma(e^+e^- \to p_{lead} + \bar{d}_{lead} + X)} \gg 1\,.$$

It has been shown in /13,14/ that at the two-loop level there arise logarithmic corrections to the Born diagram contributions into the nucleon form factor which are not described by the renormalization group. Since they are not caused by the small distance interactions, there are no color neutralization and Sudakov form factor will arise (due to the higher order corrections) multiplying these two-loop contributions. This form factor will suppress these corrections at sufficiently large Q^2. Moreover, because these corrections arise first at

the two-loop level, one can expect they to be numerically small (additional suupression $\sim \frac{\alpha_s^2(Q^2)}{\pi^2} \ln Q^2/\mu^2 \sim 10^{-2}$). Indeed, using the results for these contributions presented in /14/ and our model wave function (29), (30) it is not difficult to obtain that these contributions can be neglected with accuracy about few percents.

I am deeply grateful to V.L.Chernyak for the guidance and numerous fruitful discussions. I am grateful also to A.I.Vainshtein for the useful discussion.

REFERENCES

/1/ V.A.Avdeenko, V.L.Chernyak and S.A.Korenblit, Asymptotic behaviour of nucleon form factors in QCD, Preprint 23-79, Irkutsk, 1979; Yad.Fiz. 33(1981)481

/2/ G.P.Lepage and S.J.Brodsky, Phys.Rev.Lett. 43(1979)545, Phys.Rev.Lett. 43(1979)1625 (E)

/3/ I.G.Aznauryan, S.V.Esaybegyan, K.Z.Hatsagortsyan and N.L. Ter-Isaakyan, Preprint EFI-342(67)-78, Erevan, 1979

/4/ I.G.Aznauryan, S.V.Esaybegyan and N.L.Ter-Isaakyan, Phys.Lett. B90(1980)151

/5/ M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Nucl.Phys. B147(1979)385,448

/6/ V.L.Chernyak and A.R.Zhitnitsky, Nucl.Phys. B201(1982)492

/7/ V.L.Chernyak, A.R.Zhitnitsky, and I.R.Zhitnitsky, Nucl. Phys. B204(1982)477

/8/ E.V.Shuryak and A.I.Vainshtein, Nucl.Phys. B201(1982)141

/9/ A.V.Smilga, Yad.Fiz. 35(1982)473

/10/ B.L.Ioffe, Nucl.Phys. B188(1981)317; B191(1981)591 (E)

/11/ V.M.Belyaev and B.L.Ioffe, Preprint ITEP-59, Moskow, 1982

/12/ G.R.Farrar and D.R.Jackson, Phys.Rev.Lett. 35(1975)1416

/13/ A.Duncan and A.H.Mueller, Phys.Rev. D21(1980)1636

/14/ A.I.Mil'shtein and V.S.Fadin, Yad.Fiz. 33(1981)1391

/15/ V.L.Chernyak, A.R.Zhitnitsky and I.R.Zhitnitsky, Nucl. Phys. B204(1982), Preprint IYaP: On the mezon wave function properties, Novosibirsk, 1983

Table 1

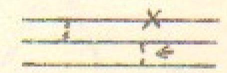
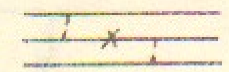
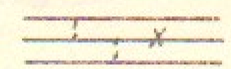
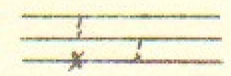
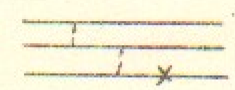
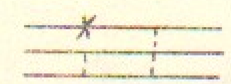
i	diagrams	T_B^i
1		$\frac{[V-A](x_i)\cdot[V^*-A^*](y_i) + 4T(x_i)T^*(y_i)}{(1-x_1)^2 x_3 (1-y_1)^2 y_3}$
2		0
3		$\frac{-4T(x_i)T^*(y_i)}{x_1 x_3 (1-x_2) y_1 y_3 (1-y_1)}$
4		$\frac{[V-A](x_i)[V-A]^*(y_i)}{x_1 x_3 (1-x_3) y_1 y_3 (1-y_1)}$
5		$\frac{-[V+A](x_i)[V+A]^*(y_i)}{x_1 x_3 (1-x_3) y_1 y_2 (1-y_2)}$
6		0
7		$\frac{[V-A](x_i)\cdot[V-A]^*(y_i) + [V+A](x_i)\cdot[V+A]^*(y_i)}{x_1 (1-x_3)^2 y_1 (1-y_3)^2}$
8		0
9		$\frac{[V-A](x_i)[V-A]^*(y_i) + 4T(x_i)T^*(y_i)}{x_2 (1-x_1)^2 y_2 (1-y_1)^2}$
10		$\frac{[V+A](x_i)[V+A]^*(y_i) + 4T(x_i)T^*(y_i)}{x_1 (1-x_2)^2 y_1 (1-y_2)^2}$
11		0
12		$\frac{-[V+A](x)[V+A]^*(y)}{x_1 x_2 (1-x_3) y_1 y_2 (1-y_2)}$
13		$\frac{4T(x)T(y)}{x_1 x_2 (1-x_1) y_1 y_2 (1-y_2)}$
14		$\frac{-[V-A](x_i)[V-A]^*(y_i)}{x_1 x_2 (1-x_1) y_1 y_2 (1-y_3)}$

Table 2.

$n_1 n_2 n_3$	$V^{(n_1 n_2 n_3)}/V^{(000)}$ sum. rul.	$V^{n_1 n_2 n_3}/V^{000}$ Model	$A^{(n_1 n_2 n_3)}/A^{(000)}$ S. R.	$A^{(n_1 n_2 n_3)}/V^{000}$ M.	$[V-A]^{n_1 n_2 n_3}/V^{(000)}$ S. R.	$[V-A]^{n_1 n_2 n_3}/V^{000}$ M	$T^{(n_1 n_2 n_3)}/T^{(000)}$ S. R.	$T^{(n_1 n_2 n_3)}/T^{(000)}$ M.	$\varphi_\infty^{(n_1 n_2 n_3)}/\varphi_\infty^{(000)}$
I00	0,38÷0,42	0,39	-0,I7÷-0,25	-0,24	0,6÷0,75	0,63	0,4÷0,5	0,425	I/3≈0,333
0I0	"	"	0,I7÷ 0,25	0,24	0,09÷0,I6	0,I5	0,40÷0,50	0,425	I/3≈0,333
00I	0,I8÷0,24	0,22	0	0	0,I8÷0,24	0,22	0,05÷0,I6	0,I5	I/3≈0,333
200	0,I8÷0,25	0,2I	-0,II÷-0,I9	-0,I86	0,25÷0,4	0,4	0,22÷0,28	0,26	I/7≈0,I4
020	"	"	0,II÷ 0,I9	0,I86	0,03÷0,08	0,025	"	"	I/7≈0,I4
002	0,08÷0,I2	0,08	0	0	0,08÷0,I2	0,08	0,02÷0,09	0,02	0,I4
II0	0,07÷0,I2	0,II	0	0	0,07÷0,I2	0,II	0,09÷0,I3	0,I	2/2I≈0,I
I0I	0,04÷0,08	0,07	-0,03÷-0,07	-0,053	0,09÷0,I4	0,I23	0,04÷0,08	0,063	2/2I≈0,I
0II	0,04÷0,08	0,07	-0,03÷ 0,07	0,053	?	0,027	0,04÷0,08	0,063	2/2I≈0,I

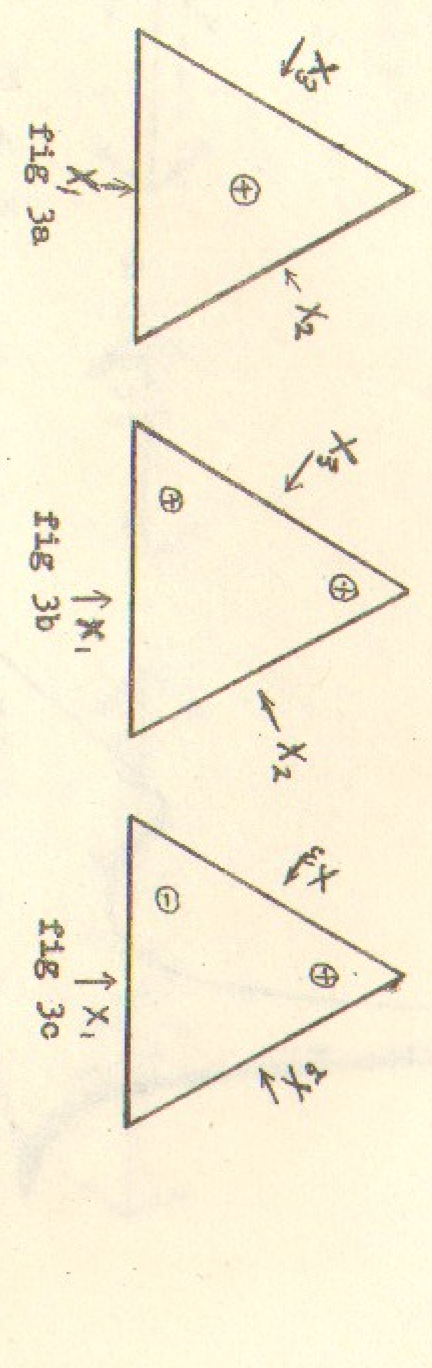


fig 3a fig 3b fig 3c

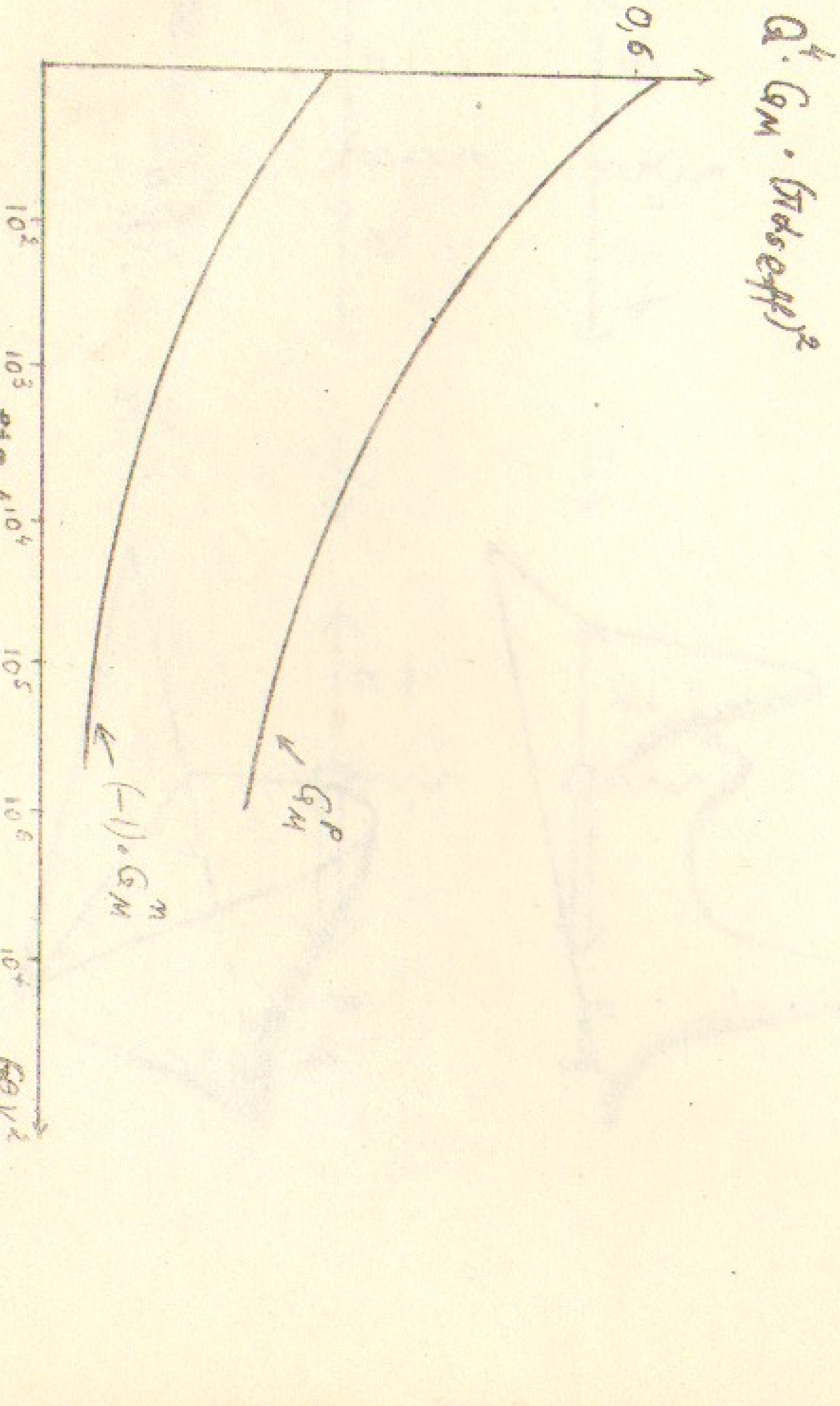


fig 4

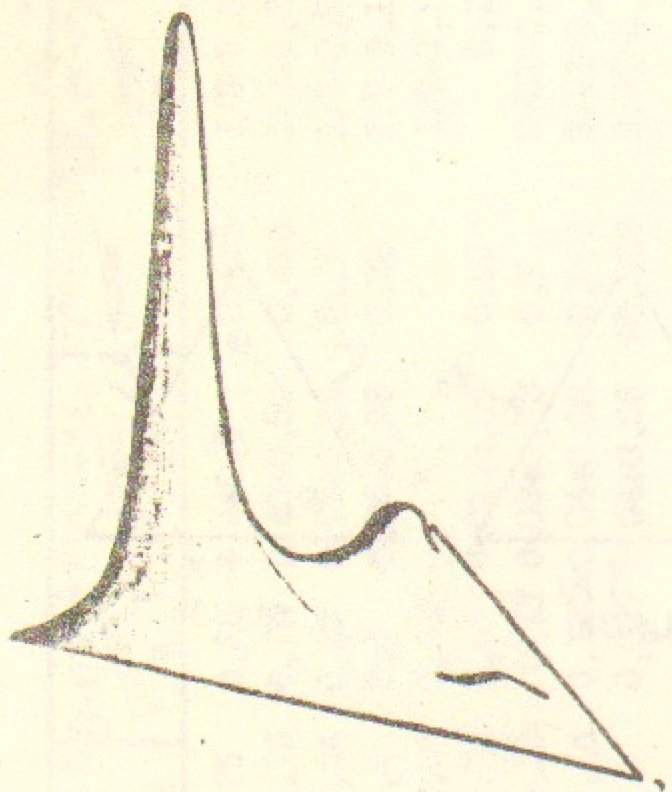


fig 6a

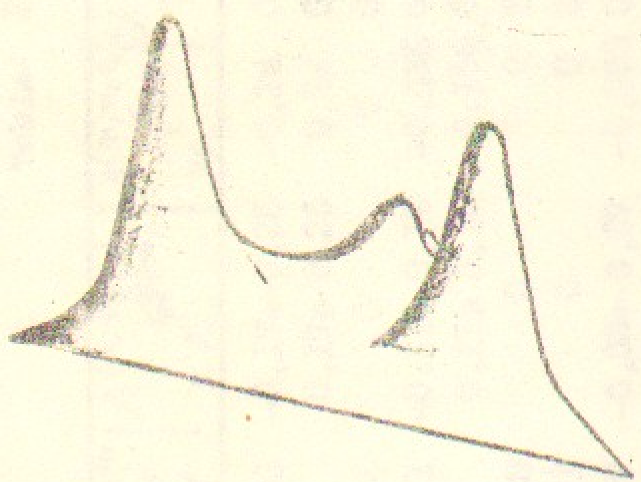


fig. 6в

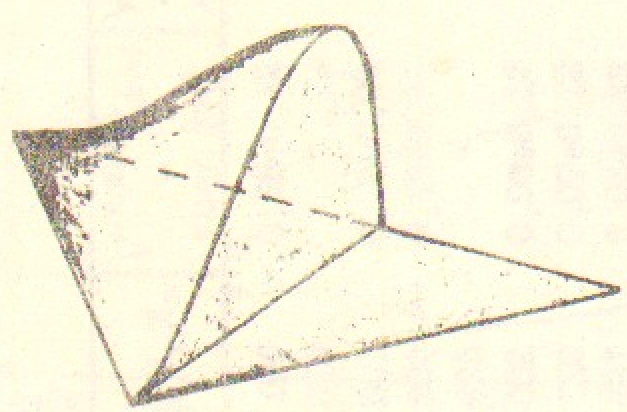


fig. 6c

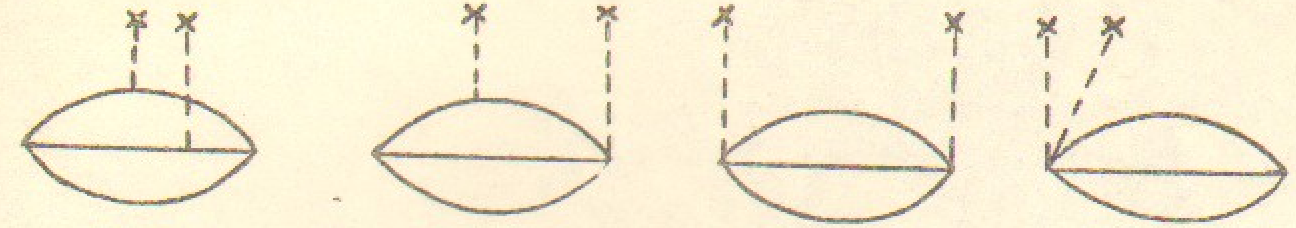


Fig 1

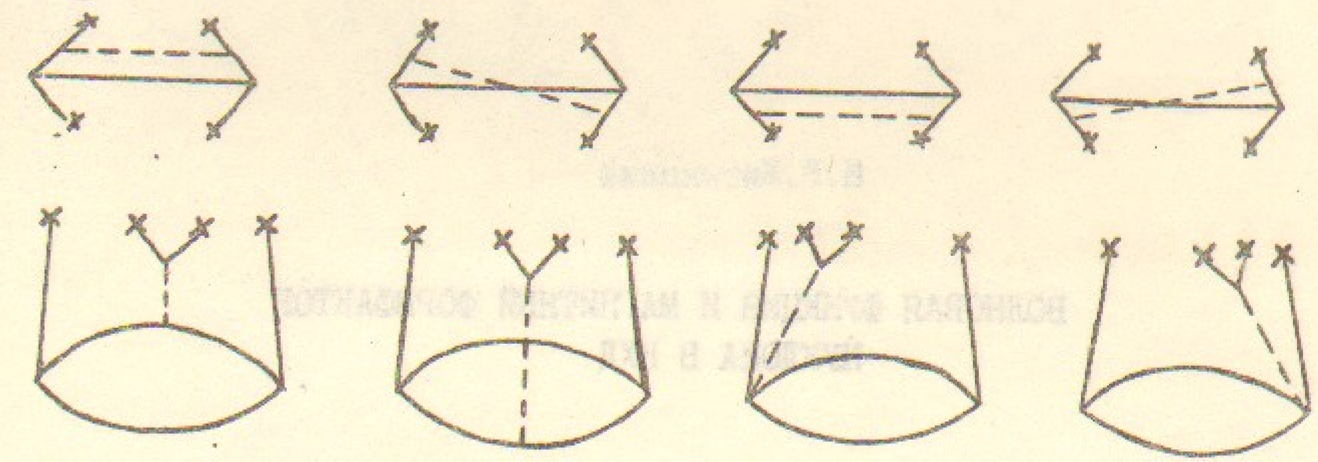


Fig 2

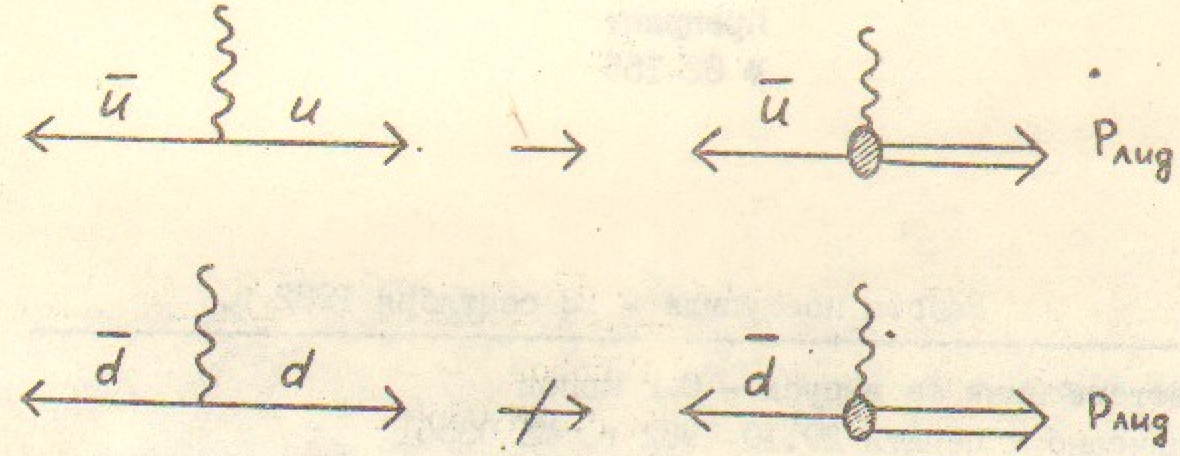


Fig. 5