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ABOUT SUMMING THE GLUONIC CORRECTIONS TO THE WEAK NONLEPTONIC AMPLITUDES

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ABOUT SUMMING THE GLUONIC CORRECTIONS TO
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A b s t r a c t

At a two-loop level leading logarithmic corrections to the coefficients of the operators entering the effective $|\Delta S| = 1$, $|\Delta T| = {}^1/2$ weak lagrangian are computed, taking into account the G I M - mechanism. The results confirm the originally proposed in [1-3] method of the renormalization group improvement of the operator expansion coefficients in theories with more than one mass scales.

1. Introduction

By recently the method of accounting strong interactions at short distances in the analysis of weak nonleptonic decays based on the Wilson operator expansion of a weak current product have been developed in [1-3]. It is equivalent to the introduction of an effective weak lagrangian expressed in terms of some local operators composed of quark and gluon fields. The coefficients of this operator expansion are being renormalized, the W-boson mass M_W being commonly taken as an initial normalization point and a typical hadronic mass m as a final one. This leads to arising terms of the form $(g^2/16\pi^2 \cdot \ln m_W^2/m^2)^n$ in the perturbation expansion in powers of the strong interaction constant g. They can be summed up by the renormalization group technique. The result is

$$C(m) = \left(\frac{g^2(m)}{g^2(m_W)}\right)^{\gamma_1/b_1} \cdot C(m_W) \tag{1}$$

Here γ_1 - the matrix of anomalous dimensions of the operators O_i, $C(\mu)$ - the column of coefficients of these operators, the latter ones being normalized at a point μ. The effective charge ratio is

$$\frac{g^2(m)}{g^2(m_W)} = 1 + b_1 \frac{g^2(m)}{16\pi^2} \cdot \ln\frac{m_W^2}{m^2} \tag{2}$$

When nondegeneracy of the masses of u - and c -quarks is taken into account (because of large mass c - quark occures in decays of ordinary hadrons only in loops at short distances), the terms of the form $\left(\frac{g^2}{16\pi^2}\right)^{k+n} \cdot \ln^k\frac{m_c^2}{m^2} \cdot \ln^n\frac{m_W^2}{m^2}$ arise in the leading log approximation. They can be summed up in two stages. First, one investigates the evolution of the column $C(\mu)$ from the point $\mu = m_W$ to $\mu = m_c$ with anomalous dimension matrix γ_1 and coefficient b_1. Than he moves from $\mu = m_c$ to $\mu = m$ with some another matrix γ_2 and coefficient b_2. As a result, we have*

*) This formula needs some more accurate definition; see consideration following formula (6).

$$C(m) = \left(\frac{g^2(m)}{g^2(m_c)}\right)^{\gamma_2/b_2} \cdot \left(\frac{g^2(m_c)}{g^2(m_W)}\right)^{\gamma_1/b_1} \cdot C(m_W) \qquad (3)$$

However, the question arises which is connected with the introduction of the intermediate normalization point $\mu = m_c$ and the method of passing it. For example, in [4-5] the arguments are given in favour of the necessity to sum up only terms like $(g^2/16\pi^2 \cdot \ln m_c^2/m^2)^n$ but not $(g^2/16\pi^2)^{k+n} \ln^k \frac{m_c^2}{m^2} \ln^n \frac{m_W^2}{m^2}$ for some of the coefficients C_i .

In a recent paper the validity of the formula (3) is verified by means of a direct computation of g^4 -order diagrams in the framework of a specific model. First, in section 2 briefly listed are some notations and definitions and the results of interest of [1-3]. Then in section 3 insufficiency of the argumentation of [4-5] is revealed by the calculation of a concrete two-loop diagram with the leading log accuracy. Further, in section 4 the results of the full two-loop calculation are listed and some related points are considered. In conclusion, results are summed up.

2. Operators and their renormalization

Effective weak lagrangian is written in the following form:

$$\mathcal{L}_{wk} = \sqrt{2}\, G_F \cos\theta_c \sin\theta_c \sum_i C_i(m_W, m_c, \alpha_s(\mu), \mu)\, \mathcal{O}^i_\mu \qquad (4)$$

Here $\alpha_s(\mu) = g^2(\mu)/4\pi$ - effective strong coupling constant at a point μ . The subscript μ near an operator means that its matrix elements are taken over a state with virtuality μ without accounting gluonic corrections coming from the region above μ .

We consider decays $|\Delta S|$ = 1 of ordinary hadrons composed of the light quarks u, d, s . Then the main contribution comes from the following (of dimension 6) operators:

$$\mathcal{O}_1 = (\bar d \gamma_\mu s_L)(\bar u \gamma^\mu u_L) - (\bar u \gamma_\mu s_L)(\bar d \gamma^\mu u_L),$$

$$\mathcal{O}_2 = (\bar d \gamma_\mu s_L)(\bar u \gamma^\mu u_L) + (\bar u \gamma_\mu s_L)(\bar d \gamma^\mu u_L) + 2(\bar d \gamma_\mu s_L)(\bar d \gamma^\mu d_L) + 2(\bar d \gamma_\mu s_L)(\bar s \gamma^\mu s_L)$$

$$\mathcal{O}_3 = (\bar d \gamma_\mu s_L)(\bar u \gamma^\mu u_L) + (\bar u \gamma_\mu s_L)(\bar d \gamma^\mu u_L) + 2(\bar d \gamma_\mu s_L)(\bar d \gamma^\mu d_L) - 3(\bar d \gamma_\mu s_L)(\bar s \gamma^\mu s_L)$$

$$\mathcal{O}_4 = (\bar d \gamma_\mu s_L)(\bar u \gamma^\mu u_L) + (\bar u \gamma_\mu s_L)(\bar d \gamma^\mu u_L) - (\bar d \gamma_\mu s_L)(\bar d \gamma^\mu d_L)$$

$$\mathcal{O}_5 = (\bar d \gamma_\mu \lambda_a s_L)(\bar u \gamma^\mu \lambda^a u_R + \bar d \gamma^\mu \lambda^a d_R + \bar s \gamma^\mu \lambda^a s_R)$$

$$\mathcal{O}_6 = (\bar d \gamma_\mu s_L)(\bar u \gamma^\mu u_R + \bar d \gamma^\mu d_R + \bar s \gamma^\mu s_R) \qquad (5)$$

Here $\psi_L = \frac{1+\gamma_5}{2}\psi$, λ_a - $SU(3)_c$-generators, $Sp\lambda_a\lambda_b = 2\delta_{ab}$. Further $t_a = \frac{1}{2}\lambda_a$ will be also introduced. Operators $\mathcal{O}_{3,4}$ correspond to the representation $\{27\}$ of the flavour group $SU(3)_f$ and therefore they are being renormalized separately from the other ones corresponding to the representation 8 of this group. Below we shall omit them. Four remaining operators correspond to the $|\Delta T| = \frac{1}{2}$ change of isospin.

The structure of $\mathcal{L}_{wk}$ (4) at $\mu = m_W$ is fixed by the born graph (fig. 1):

$$\begin{pmatrix} C_1 \\ C_2 \\ C_5 \\ C_6 \end{pmatrix} = \begin{pmatrix} -1 \\ 1/5 \\ 0 \\ 0 \end{pmatrix} + O(\alpha_s(m_W)) \qquad (6)$$

Weak amplitude renormalization at $m_c < \mu < m_W$ occurs due to the scattering graphs like fig. 2(a). In the operator expansion language it is reduced to the renormalization of $\mathcal{O}_i$ by the graphs of the type of fig. 2(б). Strictly speaking, at $\mu > m_c$ the full number of operators increases because the terms in which operator u is replaced by c come into effect (at $\mu < m_c$ matrix elements of these terms are suppressed by the powers of μ^2/m_c^2). However, since u - and c -quark--containing terms are renormalized independently by the scattering graphs of fig. 2(b), we may confine ourself to dealing with the cited above set of operators O_1, O_2, O_5, O_6 understanding under $C(\mu)$ in (3), as in (6), the column of coefficients of them. Computed with the help of scattering graphs anomalous dimension matrix is

$$\gamma_1 = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 7 & 3/2 \\ 0 & 0 & 16/3 & 0 \end{pmatrix} \qquad (7)$$

In the same region $b_1 = 25/3$ (four quark flavours including c -quark bring the contribution into the effective charge). At $\mu < m_c$ annihilation graphs of fig. 3(a) come into effect too since the GIM-cancellation between u -and c --quark-containing loops is not complete. Therefore, O_2 are renormalized by the diagrams of fig. 3(b) as well. Anomalous dimension matrix is modified:

$$\gamma_2 = \gamma_1 + \begin{pmatrix} -2/9 & 10/9 & 4/3 & 0 \\ 1/3 & -5/9 & -2/3 & 0 \\ 1/6 & -5/6 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (8)$$

Now $b_2 = 9$ (c -quark doesn't give a logarithmic contribution into the effective charge in this region).

3. Example of computing two-loop diagram

Now we shall treat the diagram of fig. 4. Using this example insufficiency of the argumentation of [4-5] will be also considered.

We have the integral:

$$\int \gamma_\nu \frac{1}{\hat{\ell}-\hat{q}} \gamma_\omega \frac{1}{\hat{p}-\hat{\ell}-\hat{k}-m_1} \gamma_\mu \frac{1}{\hat{p}-\hat{\ell}-m_1} \gamma^\nu \frac{1}{\hat{p}-m_1} \gamma^\omega \frac{d\ell dp}{\ell^2[(p-r)^2-m_W^2]} \qquad (9)$$

Here $m_1 = 0$ or m_c - the mass of $u(c)$ - quark (the masses of the light quarks are neglected in comparison with their virtuality m). First, the quadratic in the gluonic momentum k part is picked out:

$$\frac{1}{(\hat{p}-\hat{\ell}-m_1)-\hat{k}} = \frac{1}{\hat{p}-\hat{\ell}-m_1} + \frac{1}{\hat{p}-\hat{\ell}-m_1}\hat{k}\frac{1}{\hat{p}-\hat{\ell}-m_1} + \frac{1}{(\hat{p}-\hat{\ell}-m_1)-\hat{k}}\hat{k}\frac{1}{\hat{p}-\hat{\ell}-m_1}\hat{k}\frac{1}{\hat{p}-\hat{\ell}-m_1} \qquad (10)$$

In addition, the momentum r in the W -boson propagator will be neglected (the account of it would lead to terms $\sim m_1^2/m_W^2$ relative to the main result). We arrive at the integral of the following form:

$$\int \frac{(p-\ell)_\alpha (p-\ell)_\beta (p-\ell)_\gamma (p-\ell)_\delta p_\varepsilon \ell_\rho}{\underbrace{\underset{\alpha_2}{\ell^2(\ell-q)^2}\,\underset{\alpha_1}{[(p-\ell-k)^2-m_1^2][(p-\ell)^2-m_1^2]^3}\,\underset{\alpha_3}{(p^2-m_1^2)}}_{x}} \, \frac{d\ell dp}{\underset{\alpha^W}{(p-r)^2-m_W^2}} \qquad (11)$$

Combining the propagators with the help of Feinman parameters $\alpha^W, \alpha_{1,2,3}$ depicted under them, we take the integral over dp. We are interested in the term arising from the structure $p_\alpha p_\beta p_\gamma p_\delta p_\varepsilon$ in the numerator and proportional to $T_{\alpha\beta\gamma\delta} = g_{\alpha\beta}g_{\gamma\delta} + g_{\alpha\gamma}g_{\beta\delta} + g_{\alpha\delta}g_{\beta\gamma}$. Up to this tensor structure and a numerical factor it is equal to

$$\frac{\alpha_1^2}{(\alpha_1+\alpha_2)(1-\alpha_1-\alpha_2)^2} \frac{\ell_\varepsilon \ell_\rho}{\underbrace{\left[\ell^2 + 2\frac{\alpha_2}{\alpha_1+\alpha_2}k\ell + \frac{\alpha_2(1-\alpha_2)k^2 - m_\alpha^2}{(\alpha_1+\alpha_2)(1-\alpha_1-\alpha_2)}\right]^2}_{x} \underbrace{(\ell-q)^2}_{1-x} \underbrace{\ell^2}_{1-x}}, \qquad (12)$$

$$m_\alpha^2 = m_1^2(1-\alpha^W) + m_W^2 \alpha^W$$

Then we combine the denominators once more with the help of the Feinman parameters x, 1-x pointed out in (12) (taking q = 0 for simplicity). After integration over $d\ell$ we pick out the coefficient of $g_{\varepsilon\rho}$; up to a numerical factor we get the expression for it:

$$\frac{\alpha_1^2}{(\alpha_1+\alpha_2)(1-\alpha_1-\alpha_2)^2} \frac{x(1-x)}{x\frac{\alpha_2(1-\alpha_2)k^2 - m_\alpha^2}{(\alpha_1+\alpha_2)(1-\alpha_1-\alpha_2)} - x^2\frac{\alpha_2^2}{(\alpha_1+\alpha_2)^2}k^2} =$$

$$= -\frac{\alpha_1^2}{1-\alpha_1-\alpha_2}\,\frac{1-x}{m_\alpha^2+\alpha_2(1-\alpha_2)m^2-x\alpha_2^2\frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}m^2} \qquad (13)$$

where in the euclidean region we set $k^2 = -m^2$. Now the result of interest is produced, first, by integration over α^W:

$$\frac{\alpha_1^2(1-x)}{1-\alpha_1-\alpha_2}\int_0^{1-\alpha_1-\alpha_2}\frac{d\alpha^W}{m_\alpha^2+m^2\left[\alpha_2(1-\alpha_2)-\alpha_2^2\frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}x\right]} =$$

$$=\frac{\alpha_1^2(1-x)}{1-\alpha_1-\alpha_2}\,\frac{1}{m_W^2}\,\ln\frac{(1-\alpha_1-\alpha_2)(1-m_1^2/m_W^2)+m_1^2/m_W^2+m^2/m_W^2\cdot\left[\alpha_2(1-\alpha_2)-\alpha_2^2\frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}x\right]}{m_1^2/m_W^2+m^2/m_W^2\cdot\left[\alpha_2(1-\alpha_2)-\alpha_2^2\frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}x\right]}, \qquad (14)$$

then over α_1 with the log-log gaining its value at $\alpha_1=1-\alpha_2$ in accordance with the formula $\int_0^{z_0}\frac{dz}{z}\ln(1+Az)\approx\frac{1}{2}\ln^2 Az_0$, where $Az_0 \gg 1$:

$$\frac{1}{m_W^2}\int_0^{1-\alpha_2}\frac{\alpha_1^2(1-x)}{1-\alpha_1-\alpha_2}d\alpha_1\ln\left[\frac{1-\alpha_1-\alpha_2}{m_1^2/m_W^2+m^2/m_W^2\cdot\alpha_2(1-\alpha_2)}+1\right]=$$

$$=\frac{1}{m_W^2}(1-x)(1-\alpha_2)^2\cdot\frac{1}{2}\ln^2\frac{1-\alpha_2}{\frac{m_1^2}{m_W^2}+\frac{m^2}{m_W^2}\alpha_2(1-\alpha_2)}. \qquad (15)$$

It is obvious now that the difference between the u - and c - containing diagrams (m_1 = 0 and m_c respectively) contains the term of the form

$$\frac{1}{m_W^2}(1-x)(1-\alpha_2)^2\left[\frac{1}{2}\ln^2\frac{m_W^2}{m^2}-\frac{1}{2}\ln^2\frac{m_W^2}{m_c^2}\right], \qquad (16)$$

where integrations over remaining Feinman parameters are trivial and yield the numerical coefficient of $[\frac{1}{2}\ln^2 m_W^2/m^2 - \frac{1}{2}\ln^2 m_W^2/m_c^2] = \ln m_W^2/m^2\cdot\ln m_c^2/m^2 - \frac{1}{2}\ln^2 m_c^2/m^2$.

Log-log terms of another kind arise from this diagram as a result of picking out the bilinear in k , q its part. For doing so we leave the linear in k term in the decomposition (9), taking the linear in q part from another propagator:

$$\frac{1}{\hat{l}-\hat{q}}=\frac{1}{\hat{l}}+\frac{1}{\hat{l}-\hat{q}}\hat{q}\frac{1}{\hat{l}} \qquad (17)$$

We come to the following integral:

$$\int\frac{(p-l)_\alpha(p-l)_\beta(p-l)_\gamma p_\delta l_\varepsilon l_\rho}{\underbrace{l^2}_{1-x-y}\underbrace{(l-q)^2}_{y}\underbrace{\underbrace{[(p-l)^2-m_1^2]^3}_{\alpha_1}\underbrace{(p^2-m_1^2)}_{\alpha_2}}_{x}}\,\frac{dl\,dp}{\underbrace{p^2-m_W^2}_{\alpha^W}} \qquad (18)$$

Using the parameters $\alpha^W, \alpha_{1,2}$ one integrates over dp , extracting the term $\sim T_{\alpha\beta\gamma\delta}$, then one integrates over dl with the help of parameters $x, y, 1-x-y$ and picks out the term $\sim g_{\varepsilon\rho}$. For the coefficient of the tensor structure $g_{\varepsilon\rho}T_{\alpha\beta\gamma\delta}$ we have up to a numerical factor:

$$\frac{\alpha_1^2(1-x-y)}{x[m_W^2\alpha^W+m_1^2(1-\alpha^W)]+m^2(y-y^2)(\alpha_1-\alpha_1^2)} \qquad (19)$$

Here we set $q^2=-m^2$.
Further, one integrates over α^W :

$$\int\frac{\alpha_1^2(1-x-y)\,d\alpha^W}{x[m_W^2\alpha^W+m_1^2(1-\alpha^W)]+m^2(y-y^2)(\alpha_1-\alpha_1^2)}=$$

$$=\frac{1}{m_W^2}\,\frac{\alpha_1^2(1-x-y)}{x}\ln\frac{(1-\alpha_1)(1-\frac{m_1^2}{m_W^2})+\frac{m_1^2}{m_W^2}+\frac{m^2}{m_W^2}\frac{(y-y^2)(\alpha_1-\alpha_1^2)}{x}}{\frac{m_1^2}{m_W^2}+\frac{m^2}{m_W^2}\frac{(y-y^2)(\alpha_1-\alpha_1^2)}{x}}, \qquad (20)$$

then over x in accordance with the formula

$$\int_0^{x_0}\frac{dx}{x}\ln\frac{A/x+B}{A/x+C}\approx\frac{1}{2}\ln^2\frac{B}{A}x_0-\frac{1}{2}\ln^2\frac{C}{A}x_0, \qquad (21)$$

valid for $\frac{B}{A}x_0, \frac{C}{A}x_0 \gg 1$.

We get to the log-log accuracy:

$$\frac{1}{m_W^2}\alpha_1^2(1-y)\Big[\frac{1}{2}\ln^2\frac{1-\alpha_1+m_1^2/m_W^2}{m^2/m_W^2\,(y-y^2)(\alpha_1-\alpha_1^2)} - \frac{1}{2}\ln^2\frac{m_1^2}{m^2(y-y^2)(\alpha_1-\alpha_1^2)}\Big] \approx \frac{1}{m_W^2}\alpha_1^2(1-y)\Big[\frac{1}{2}\ln^2\frac{m_W^2}{m^2} - \frac{1}{2}\ln^2\frac{m_1^2}{m^2}\Big] \tag{22}$$

where $\ln^2 m_1^2/m^2$ is replaced by constant and omitted at $m_1 = m_u \approx 0$ and is equal to $\ln^2\frac{m_c^2}{m^2}$ at $m_1 = m_c$. Then $u-c$-cancellation yields

$$\frac{1}{m_W^2}\alpha_1^2(1-y)\frac{1}{2}\ln^2\frac{m_c^2}{m^2}, \tag{23}$$

where the integrations over α_1, y are trivial and permit us to find the numerical coefficient of log-log.

It is obvious that this second type of the leading logs (23) in annihilation diagrams arises just in the way proposed in [5]. According to [5] multiple logs arise as a result of integrations first over α^W then over parameters entering $C(\alpha)$ where $C(\alpha)$ is some function of Feinman parameters in the denominator $\mathcal{D}$ of an expression obtained as a result of performing momentum quadratures:

$$\mathcal{D} = (\alpha^W m_W^2 + \sum_i \alpha_i^q m_q^2)C(\alpha) + F(m^2, \alpha) \tag{24}$$

But every parameter entering $C(\alpha)$ is just that which is put in correspondence with the m_W-containing denominator of the result of integration over some loop before integrating over the next loop in the course of successive parametrization and taking momentum quadratures. In our case it is parameter x in (18). However, the powers of logs may also arise, as we have seen, as the result of successive integration over parameters introduced when taking momentum integral over a certain loop containing W-boson (parameters α^W, α_1 in (11)). These parameters cannot enter the multiplier of m_W^2 all together.

4. Full calculation

Full result for an annihilation one-gluon-reducible diagram is got by picking out all the terms quadratic in the external momenta. In table I each diagram is confronted with its formal expression in terms of the external momenta, mean-square value of which being $\sim(-m^2)$. The gluonic propagator is $d_{\mu\nu} = -\frac{1}{k^2}\left(g_{\mu\nu} + (d-1)\frac{k_\mu k_\nu}{k^2}\right)$; also brief notations $\ln m_W^2$, m_c^2 for $\ln m_W^2, m_c^2/m^2$ are used. In order to extract the diagramatic contribution into any physical operator one must move on quark-mass-shell neglecting quark masses. As for the terms describing moving off shell, they correspond to gauge-nontrivial operators which are not interesting for us.

It should be also said about the logarithmic divergences in loops of the type of $\ln\Lambda^2/p^2$ where Λ is cut off parameter, p is the momentum flowing through the loop. All such the divergences can be included into renormalization of the charge entering the expressions for the one-loop diagrams. If one implies under $\alpha_s = g^2/4\pi$ the effective charge at the point m then Λ must be replaced by m (or by m_c in the case of closed loop with c-quark, fig. 5).

In table II listed are the values of the irreducible annihilation diagrams. As for the scattering diagrams, they yield the result proportional to $\frac{1}{2}\ln^2 m_W^2$ with the exception of fig. 5(a,b) where N_f = 3 light quarks and c-quark propagate in fermion loop; corresponding contribution into $\mathcal{L}_{eff}$ is

$$\left(\frac{g^2}{16\pi^2}\right)^2 2\sqrt{2}\,G_F \sin\theta_c\cos\theta_c(-4)(\bar{u}\gamma_\mu t^a s_L)(\bar{d}\gamma_\mu t^a u_L)\cdot \left[N_f\cdot\frac{1}{2}\ln^2 m_W^2 + \frac{1}{2}\ln^2\frac{m_W^2}{m_c^2}\right] \tag{25}$$

Computing irreducible diagrams is more easy than it is for gluon-reducible ones because of the needless to extract bilinear in the momenta terms. Therefore, we shall straight list the values of the summary contributions into $\mathcal{L}_{eff}$ of three groups of scattering diagrams using Feinman gauge of the gluonic propagator $d_{\mu\nu} = -g_{\mu\nu}/k^2$.

1. Diagrams where only one gluon carries colour between fermions (of the type of fig. 5 excluding fig. 5):

$$\left(\frac{g^2}{16\pi^2}\right)^2 2\sqrt{2}\, G_F \sin\theta_c \cos\theta_c \cdot 28N\, (\bar{u}\gamma_\mu t^a s_L)(\bar{d}\gamma_\mu t^a u_L)\cdot \frac{1}{2}\ln^2 m_W^2 \qquad (26)$$

2. Colour is carried by two gluons, with their lines not crossing each other (of the type of fig. 6(a)):

$$\left(\frac{g^2}{16\pi^2}\right)^2 2\sqrt{2}\, G_F \sin\theta_c \cos\theta_c \cdot 48\, (\bar{u}\gamma_\mu t^a t^b s_L)(\bar{d}\gamma_\mu t^a t^b u_L)\cdot$$
$$\cdot \frac{1}{2}\ln^2 m_W^2 \qquad (27)$$

3. The same, but the lines cross each other (of the type of fig. 6(b)):

$$\left(\frac{g^2}{16\pi^2}\right)^2 2\sqrt{2}\, G_F \sin\theta_c \cos\theta_c (-12)(\bar{u}\gamma_\mu t^a t^b s_L)(\bar{d}\gamma_\mu t^b t^a u_L)\cdot \qquad (28)$$
$$\cdot \frac{1}{2}\ln^2 m_W^2$$

Let's transform the expressions obtained to the form containing the operators $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_5, \mathcal{O}_6$. For doing this we'll use the Fierz identity in $SU(N)_c$: $t^a_{ik} t^a_{\ell m} = \frac{1}{2}\delta_{im}\delta_{\ell k} - \frac{1}{2N}\delta_{ik}\delta_{\ell m}$ and for the γ -matrices as well, taking into account anticommutativity of the fermion operators. As a result, we get the necessary relationships:

$$\sum_{q=u,d,s} (\bar{d}\gamma_\mu t^a s_L)(\bar{q}\gamma_\mu t^a q) = -\frac{1}{4}\frac{N+1}{N}\mathcal{O}_1 + \frac{1}{4}\frac{N-1}{N}\mathcal{O}_2 + \frac{1}{4}\mathcal{O}_5$$
$$\sum_{q=u,d,s} (\bar{d}\gamma_\mu t^a t^b s_L)(\bar{q}\gamma_\mu t^a t^b q_L) = \frac{1}{8}\left(1+\frac{1}{N}\right)^2\mathcal{O}_1 + \frac{1}{8}\left(1-\frac{1}{N}\right)^2\mathcal{O}_2$$
$$\sum_{q=u,d,s} (\bar{d}\gamma_\mu t^a t^b s_L)(\bar{q}\gamma_\mu t^b t^a q_R) = \frac{1}{4}\left(\frac{N}{2}-\frac{1}{N}\right)\mathcal{O}_5 + \frac{1}{4}\frac{N^2-1}{N^2}\mathcal{O}_6$$
$$(\bar{u}\gamma_\mu t^a s_L)(\bar{d}\gamma_\mu t^a u_L) = \frac{1}{4}\frac{N+1}{N}\mathcal{O}_1 + \frac{1}{20}\frac{N-1}{N}\mathcal{O}_2 \qquad (29)$$
$$(\bar{u}\gamma_\mu t^a t^b s_L)(\bar{d}\gamma_\mu t^a t^b u_L) = -\frac{1}{8}\left(1+\frac{1}{N}\right)^2\mathcal{O}_1 + \frac{1}{40}\left(1-\frac{1}{N}\right)^2\mathcal{O}_2$$
$$(\bar{u}\gamma_\mu t^a t^b s_L)(\bar{d}\gamma_\mu t^b t^a u_L) = \frac{1}{8}\left(N-\frac{2}{N}-\frac{1}{N^2}\right)\mathcal{O}_1 +$$
$$+\frac{1}{40}\left(N-\frac{2}{N}+\frac{1}{N^2}\right)\mathcal{O}_2$$

Collecting together the results obtained and taking into account that $\ln^2 m_W^2/m_c^2 = \ln^2 m_W^2 - 2\ln m_W^2 \ln m_c^2 + \ln^2 m_c^2$ we get for the coefficients C_i in formula (4) at N = 3:

$$\begin{pmatrix} c_1 \\ c_2 \\ c_5 \\ c_6 \end{pmatrix} = \begin{pmatrix} -1 \\ 1/5 \\ 0 \\ 0 \end{pmatrix} + \frac{g^2(m)}{16\pi^2}\left[\begin{pmatrix} -3\frac{N+1}{N} \\ -\frac{3}{5}\frac{N-1}{N} \\ 0 \\ 0 \end{pmatrix}\ln^2 m_W^2 + \right.$$
$$\left. + \begin{pmatrix} \frac{1}{3}\frac{N+1}{N} \\ -\frac{1}{3}\frac{N-1}{N} \\ -\frac{1}{3} \\ 0 \end{pmatrix}\ln m_c^2\right] + \left(\frac{g^2(m)}{16\pi^2}\right)^2\left[\begin{pmatrix} \frac{11}{2}N - 3 - \frac{13}{N} - \frac{9}{2N^2} \\ \frac{1}{5}\left(\frac{11}{2}N - 5 - \frac{5}{N} + \frac{9}{2N^2}\right) \\ 0 \\ 0 \end{pmatrix}\cdot\right.$$
$$\cdot \ln^2 m_W^2 + 2\cdot\begin{pmatrix} \frac{N+1}{N}\,\frac{2N+1}{2N} \\ \frac{N-1}{N}\,\frac{2N-5}{10N} \\ -\frac{1}{2N} \\ 0 \end{pmatrix}\ln m_W^2 \ln m_c^2 +$$
$$\left. + \begin{pmatrix} \frac{N+1}{N}\left(-\frac{11}{18}N - \frac{1}{2} + \frac{1}{9N}\right) \\ \frac{N-1}{N}\left(\frac{11}{18}N + \frac{3}{10} - \frac{1}{9N}\right) \\ \frac{1}{9}N + \frac{25}{18N} \\ -\frac{N^2-1}{N^2} \end{pmatrix}\ln^2 m_c^2\right] = \qquad (30)$$
$$= \begin{pmatrix} -1 \\ 1/5 \\ 0 \\ 0 \end{pmatrix} + \frac{g^2(m)}{16\pi^2}\left[\begin{pmatrix} -4 \\ -2/5 \\ 0 \\ 0 \end{pmatrix}\ln m_W^2 + \begin{pmatrix} 4/9 \\ -2/9 \\ -1/3 \\ 0 \end{pmatrix}\ln m_c^2\right] +$$
$$+ \left(\frac{g^2(m)}{16\pi^2}\right)^2\left[\begin{pmatrix} 26/3 \\ 31/15 \\ 0 \\ 0 \end{pmatrix}\ln^2 m_W^2 + \begin{pmatrix} 28/9 \\ 2/45 \\ -1/3 \\ 0 \end{pmatrix}\ln m_W^2 \ln m_c^2 + \right.$$
$$\left. + \begin{pmatrix} -248/81 \\ 566/405 \\ 43/54 \\ -8/9 \end{pmatrix}\ln^2 m_c^2\right]$$

where the terms of the first order of g^2 resulting from one-loop computations are also written for clearness. The same result will be if formula (3) expressed in terms of $\alpha_s(m) = g^2(m)/4\pi$ is expanded up to the terms $\sim \alpha_s^2$.

5. Conclusion

Consider above the two-loop diagrams may be divided into one-gluon-reducible and irreducible ones. Whereas the law of formation of leading logs is rather manifest in irreducible diagrams in any order of perturbation theory (these logs can be obtained by the successive integration over the loops), this is not so for the reducible diagrams because of the necessity to extract the quadratic or bilinear in external momenta terms. As a by-product, non-gauge-invariant structures occur describing moving the external particles off mass shell. Though they vanish by equations of motion, it is not quite evident in what way the contributions from particular diagrams into the physical operator are added together to reproduce the result required by renormalization group equations. Furthermore, in addition to $\ln^2 m_W^2/m^2$ and $\ln^2 m_c^2/m^2$, as we have seen, the terms like $\left(\ln^2 m_W^2/m^2 - \ln^2 m_W^2/m_c^2\right) = 2\ln\frac{m_W^2}{m^2}\ln\frac{m_c^2}{m^2} - \ln^2\frac{m_c^2}{m^2}$ also arise. While the former two have the form typical for the successive integrations over the loops, it cannot be said about the latter one. Therefore no wonder that verifying the result based essentially on the general principles of the theory (renormalizability, absence of adding non-gauge-invariant structures in matrix elements of physical operators between the physical states and absence of mixing of operators posessing different symmetry properties) turned to be such a non-trivial one.

In conclusion, the author is grateful to Dr. Vainshtein A.I. for statement of the problem and numerous helpful discussions.

References

1. A.I.Vainshtein, V.I.Zakharov, M.A.Shifman, Zh. Eksp. Teor. Fiz. 72 (1977) 1275.
2. M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, Phys. Rev. 18D ((1978) 2583.
3. M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, Nucl. Phys. B120 (1977) 316 and Soviet Phys., JETP, 45 (1977) 670.
4. C.T.Hill, G.G.Ross, Nucl. Phys. B171 (1980) 141.
5. C.T.Hill, G.G.Ross, Phys. Lett. 91B (1980) 234.

Table I

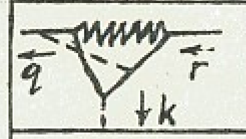
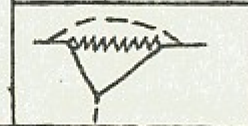
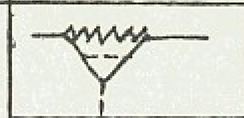
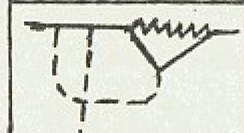
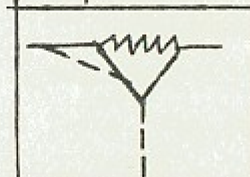
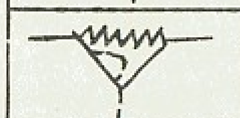
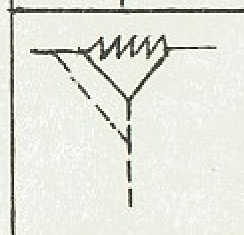
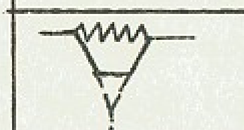
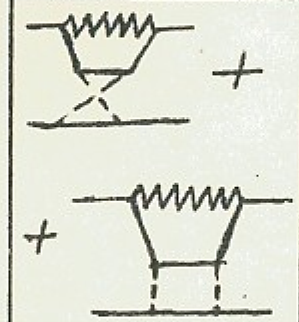
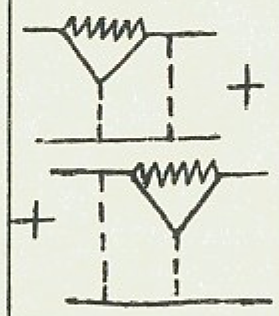
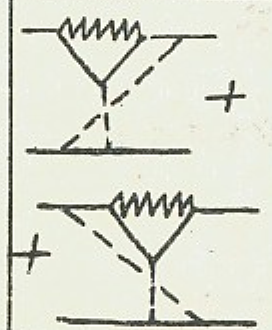
Diagram	Colour factor	Diagram without colour factor is equal to $i\frac{g^3}{(16\pi^2)^2}\cdot 2G_F\sqrt{2}\sin\theta_c\cos\theta_c\cdot\left[\frac{1}{2}(\ln^2 m_W^2-\ln^2 m_W^2/m_c^2)(k^2\gamma_\mu-\hat{k}k_\mu)\cdot A+\frac{1}{2}\ln^2 m_c^2\cdot B\right]$ where $A=$	$B=$
	$-\frac{1}{2N}t^a$	$2+\frac{2}{3}d$	$\frac{1}{3}(\gamma_\mu\hat{k}\hat{q}+\hat{k}\hat{q}\gamma_\mu+\hat{q}\gamma_\mu\hat{k})+\frac{8}{9}q^2\gamma_\mu+\frac{4}{9}\hat{q}q_\mu+(d-1)\left[\frac{1}{2}\hat{q}\gamma_\mu\hat{k}+\frac{1}{6}q^2\gamma_\mu+\frac{1}{3}\hat{q}q_\mu\right]$
	$-\frac{1}{2N}t^a$	$-\frac{2}{3}d$	$-\frac{2}{3}d(k^2\gamma_\mu-\hat{k}k_\mu)$
	$-\frac{1}{2N}t^a$	$-\frac{2}{3}d$	$\frac{2}{3}d(k^2\gamma_\mu-\hat{k}k_\mu)$
	$-\frac{1}{2N}t^a$	0	$\frac{2}{9}\hat{r}\gamma_\mu\hat{q}-\frac{4}{9}(qr)\gamma_\mu-\frac{1}{9}\hat{q}\gamma_\mu\hat{r}+\frac{1}{9}q^2\gamma_\mu-\frac{4}{9}\hat{q}q_\mu+\frac{1}{9}r^2\gamma_\mu-\frac{4}{9}\hat{r}r_\mu$
	$\left(\frac{N}{2}-\frac{1}{2N}\right)t^a$	0	$-\frac{1}{3}(\gamma_\mu\hat{k}\hat{q}+\hat{k}\hat{q}\gamma_\mu+\hat{q}\gamma_\mu\hat{k})-\frac{8}{9}q^2\gamma_\mu-\frac{4}{9}\hat{q}q_\mu+\frac{2}{3}d(k^2\gamma_\mu-\hat{k}k_\mu)-(d-1)\left[\frac{1}{2}\hat{q}\gamma_\mu\hat{k}+\frac{1}{6}q^2\gamma_\mu+\frac{1}{3}\hat{q}q_\mu\right]$
	$\left(\frac{N}{2}-\frac{1}{2N}\right)t^a$	0	$-\frac{2}{3}d(k^2\gamma_\mu-\hat{k}k_\mu)$
	$\frac{N}{2}t^a$	0	$\frac{13}{18}r^2\gamma_\mu+\frac{1}{9}\hat{r}r_\mu-\frac{1}{18}k^2\gamma_\mu+\frac{2}{9}\hat{k}k_\mu+\frac{8}{9}\hat{k}r_\mu-\frac{4}{9}\hat{r}k_\mu-\frac{22}{9}(kr)\gamma_\mu+\frac{5}{6}\gamma_\mu\hat{k}\hat{r}+(d-1)\left[\frac{1}{6}q^2\gamma_\mu+\frac{1}{2}\hat{q}\gamma_\mu\hat{k}+\frac{1}{3}\hat{q}q_\mu+\frac{1}{6}(\hat{k}k_\mu-k^2\gamma_\mu)\right]$
	$\frac{N}{2}t^a$	0	$\frac{34}{9}k^2\gamma_\mu-\frac{37}{9}\hat{k}k_\mu+\frac{1}{3}d(k^2\gamma_\mu-\hat{k}k_\mu)$
Resulting contribution into the coefficient of the operator $\sum_q(\bar{d}\gamma_\mu t^a s_L)\cdot(\bar{q}\gamma_\mu t^a q)$ in $\mathcal{L}_{eff}$ (all possible symmetries of the diagrams are taken into account)	$\left(\frac{g^2}{16\pi^2}\right)^2 2G_F\sqrt{2}\sin\theta_c\cos\theta_c\left[\left(-\frac{2}{N}\right)\cdot\frac{1}{2}\left(\ln^2 m_W^2-\ln^2\frac{m_W^2}{m_c^2}\right)+\left(\frac{13}{9}N-\frac{4}{9N}\right)\cdot\frac{1}{2}\cdot\ln^2 m_c^2\right]$		

Table II

Diagram	Contribution into $\mathcal{L}_{eff}$ is $2G_F\sqrt{2}\left(g^2/16\pi^2\right)^2\cdot\sin\theta_c\cos\theta_c\cdot A$ where $A=$
	$N\sum_q(\bar{d}\gamma_\mu t^a s_L)(\bar{q}\gamma^\mu t^a q)\cdot\frac{1}{2}\ln^2 m_c^2\cdot\left(1+\frac{d-1}{2}\right)$
	$4\sum_q(\bar{d}\gamma_\mu t^a t^b s_L)(\bar{q}\gamma^\mu t^a t^b q_L)\cdot\frac{1}{2}\ln^2 m_c^2+\frac{d-1}{2}\sum_q(\bar{d}\gamma_\mu t^a t^b s_L)(\bar{q}\gamma^\mu t^a t^b q)\cdot\frac{1}{2}\ln^2 m_c^2$
	$-4\sum_q(\bar{d}\gamma_\mu t^a t^b s_L)(\bar{q}\gamma^\mu t^b t^a q_R)\cdot\frac{1}{2}\ln^2 m_c^2-\frac{d-1}{2}\sum_q(\bar{d}\gamma_\mu t^a t^b s_L)(\bar{q}\gamma^\mu t^b t^a q)\cdot\frac{1}{2}\ln^2 m_c^2$

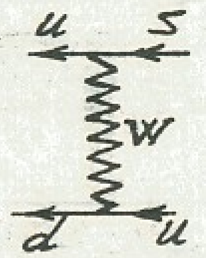


fig.1

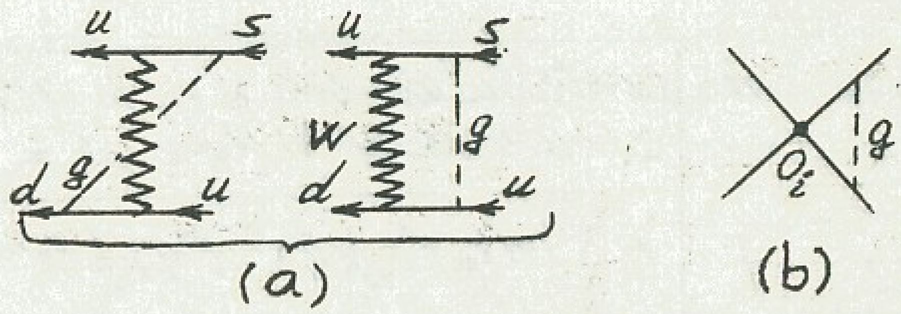


fig.2

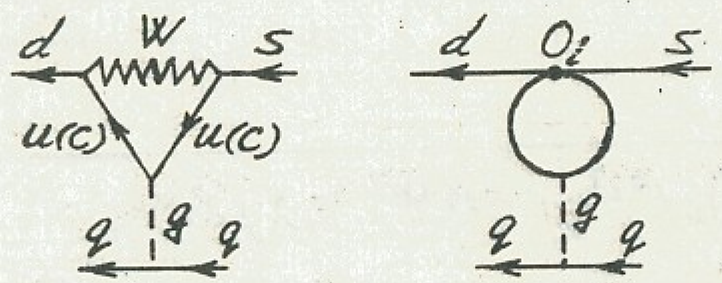


fig.3

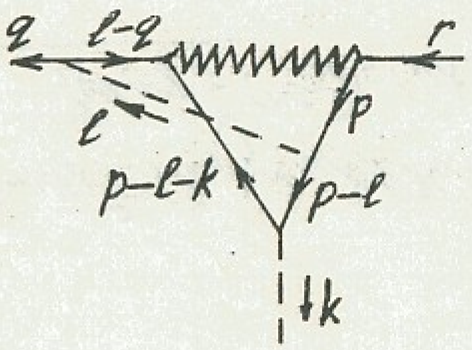


fig.4

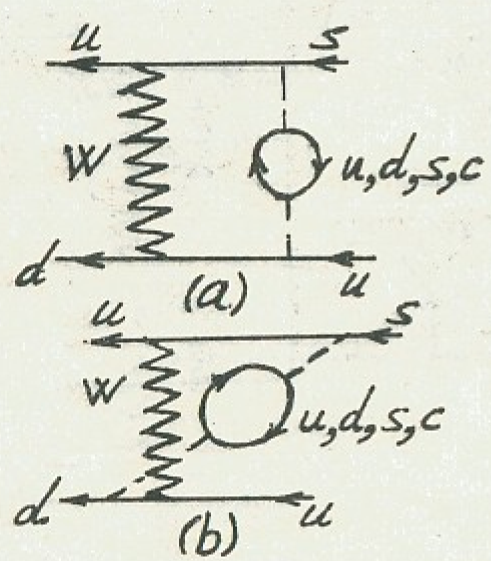


fig.5

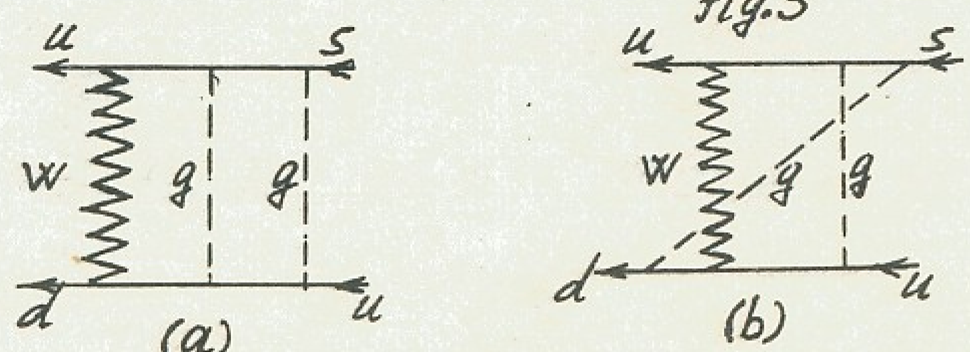


fig.6

В.М.Хацимовский

О СУММИРОВАНИИ ГЛЮОННЫХ ПОПРАВОК К АМПЛИТУДАМ СЛАБЫХ НЕЛЕПТОННЫХ ПРОЦЕССОВ

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