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TIME TRANSLATION INVARIANCE AND BLACK HOLE EVAPORATION

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Abstract

The invariance of physical processes with respect to the choise of the initial time t_o of an observer situated at the distance $r_o > 2$ m outside a collapsing star makes it impossible for him to determine the position of the collapsing star surface relative to the event horizon. The consequences on the black hole evaporation due to this invariance are discussed.

In this paper the gravitational collapse of the spherical dust cloud is considered. This model is known as a good approximation to a collapse of a real star at the last stage of the collapse when the pressure is much smaller than the gravitational force. Small nonspherical perturbations does not change the general picture of collapse [1,2]. When the surface of the star comes close to the event horizon the external fields created by the star are radiated in the form of the wave pulse with the characteristic exponentially decreasing back front [3]. Behind of the pulse the external metric become spherical*. The recording of this pulse by the external observer can be the indication that the star is at the relativistic stage of collapse (the collapse signal).

The space-time outside the star can be described either with help of the schwarzshild coordinates

$$ds^2=\left(1-\frac{2m}{r}\right)dt^2-\left(1-\frac{2m}{r}\right)^{-1}dr^2-r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right) \qquad (1)$$

or with help of any analytical continuation of the metric (1) in the region $r < 2$ m. The complete space of events is described by the kruskal metric [6,1-2]

$$ds^2=\frac{32m^3}{r}e^{-r/2m}\left(dv^2-du^2\right)-r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right) \qquad (2)$$

$$\left(\frac{r}{2m}-1\right)e^{r/2m}=u^2-v^2\,, \quad e^{t/2m}=\frac{u+v}{u-v} \qquad (3)$$

The metric inside the star in the simplest case is (the subsequent results does not depend on this metric)

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*) Scattering of waves on the space-time curvature leads to the existence of the power descenting tails of this wave pulse [4,5]. But they have nothing to do with the process of gravitational collapse.

$$ds^2 = d\tau^2 - a^2(\tau)dR^2 - a^2(\tau)R^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{4}$$

$$a(\tau) = \left[1 - \frac{3}{2}(2m)^{1/2}R_b^{-3/2}\tau\right]^{2/3}, \quad R \le R_b$$

The internal metric is joined continuously across the star surface $R = R_b$ to the external metric (see for ex.[7]). The star surface moves along the geodesic line of the dust particle; the equation for this line in the coordinates (1) is

$$t - t_0 = \frac{2}{3}(2m)^{-1/2}(r_0^{3/2} - r^{3/2}) - 2(2mr)^{1/2} + 2(2mr_0)^{1/2} - 2m\ln\frac{r^{1/2} - (2m)^{1/2}}{r_0^{1/2} - (2m)^{1/2}} + 2m\ln\frac{r^{1/2} + (2m)^{1/2}}{r_0^{1/2} + (2m)^{1/2}} \tag{5}$$

When $r \to 2m$ the schwarzshild time t does not suitable. However with the help of the formulas (3) the corresponding equation of the same geodesic line in the coordinates (u, v) which are correct for any $\tau > 0$ can be found easily. This lines are represented on the Fig. 1 for two different values of (the curves (a, a') and (b, b') on the Fig. 1).

Usually for description of the gravitational collapse the supposition that the star surface moves along the geodesic line (a, a') on the Fig. 1 is used. In this case the recording of the collapse signal by the observer situated at the distance $r_c > 2m$ means in fact that the star surface is already under the event horizon and the black hole is fprmed. But the star surface can moves along the geodesic line (b, b') too. In the last case an arbitrary long time can be between the collapse signal recorded by the observer at r_c and the real crossing of the event horizon by the star surface (see Fig. 1). Let us note that in the main the collapse signal is formed though at a small but finite distances from the event horizon r = 2 m. Therefore the time interval Δt_d between the emission of the collapse signal by the star and one record by the observer at is finite too; it is independent on the value t_0 in (5).

The geodesic line (a, a') can be transformed into (b, b') (or vice versa) by the transformation of the coordinates

$$v = v'\,ch(\Delta t/2) + u'\,sh(\Delta t/2)$$
$$u = v'\,sh(\Delta t/2) + u'\,ch(\Delta t/2) \tag{6}$$

In the schwarzshild coordinates (1) the transformation (6) leat to the time translation

$$t = t' + \Delta t \tag{7}$$

obviously that any process must be invariant with respect to the transformations (7), (6). Therefore the motions of the star surface along the geodesic lines (a, a') and (b, b') are physically indistinguishable. As well known the crossing of the event horizon by the star surface is unobservable for an external observer[1,2]. The collapse signal which was described above in fact is the last signal which can be seen by the external observer. The indistinguishability of the geodesic lines (a, a') and (b, b') permit to make the more strength statement. Namely an ancertainty principle for the gravitational collapse is correct: after the collapse signal recording by external observer it can not define by no means where is the collapsing star surface - under the event horizon or above one. A simple consequence from this principle is independence of any physical process which can be seen by external observer from the star surface position with respect to the event horizon.

Although we used the specific coordinates by Kruskal's (2),(3) in our reasonings but the final result does not depends on that. Indeed the coordinate transform (3) realize the maximum expansion of the schwarzshild solution (1) which is not locally expansive. Therefore the events space of any transformation of the coordinates (1) which has the same completeness as (3) can be put to one-one correspondence with the kruskal events space. Let us note too that the jone of the metric of expanding universe and the metric (1) is invariant with respect to the transformations (6),(7). Therefore the existence of the "absolute" time in Fridman universes does not influence on this invariance.

Let us consider now the black hole evaporation[8]. The two points of view on the origin of this radiation are known. Some authors suppose that all radiation are formed by a nonstatic part of the metric; that is on the surface of the collapsing star before it cross the event horizon [9,10]. In fact this means that it is necessary to take account of the back reaction because the energy-momentum tensor calculated in the neglection of this reaction is divergent on the event horizon. Outher authors [8,11-13] supporse that before the crossing by the star surface of the event horizon the external observer register only a small number of radiation quanta and the radiation flow becomes stationary after the formation of the singularity. This alternatives correspond to the different vacuum states of the quantum field in the metric (1) (see, for ex. [12]). In the last case the number of quanta which are recorded by an external observer depends on the position of the collapsing star surface with respect to the event horizon. That brakes the time translation invariance. In the first case the invariance is not broken.

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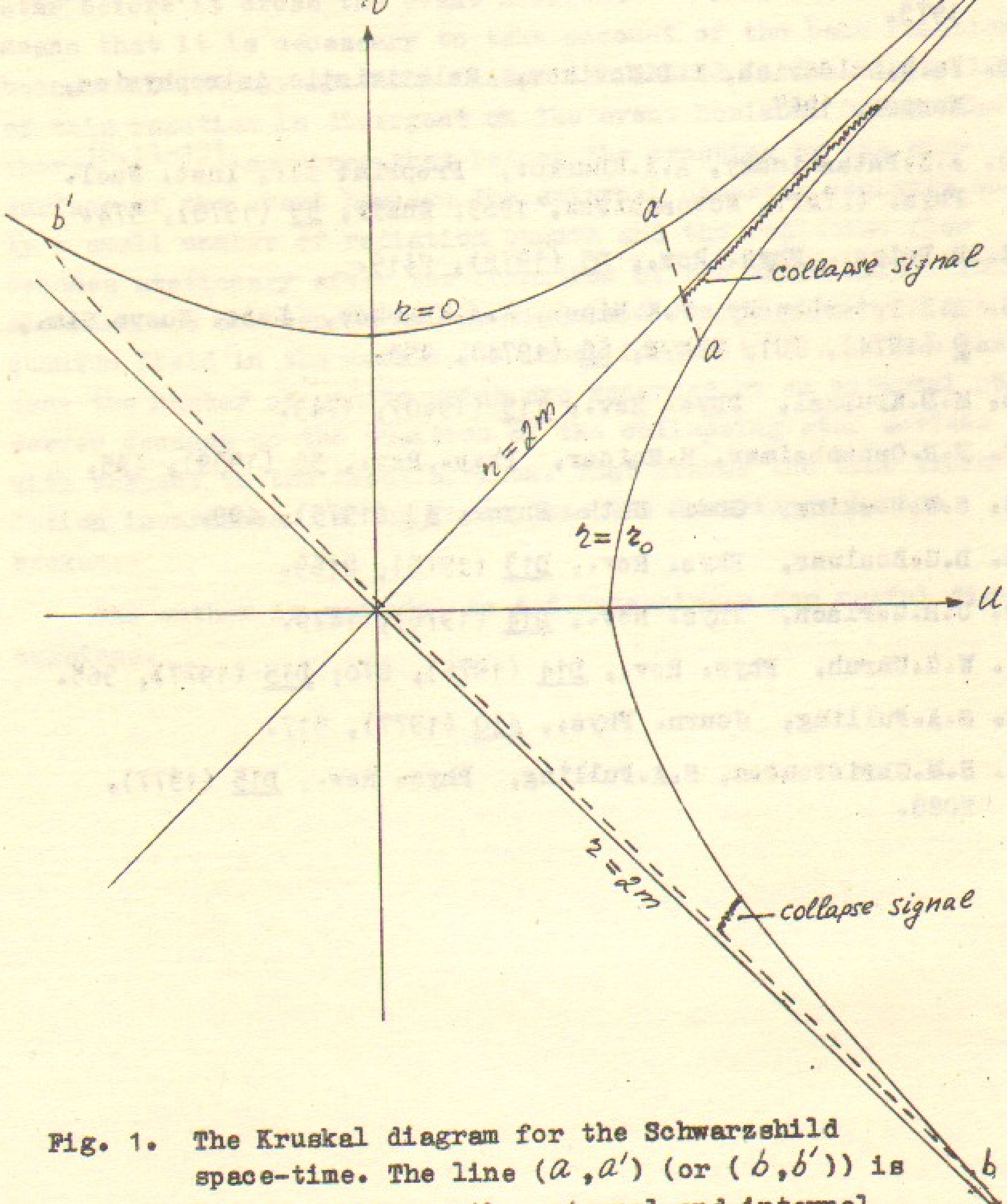

Fig. 1. The Kruskal diagram for the Schwarzshild space-time. The line (a, a') (or (b, b')) is boundare between the external and internal metrics. The proper time of the observer situated at a distance $r_0 > 2m$ is bound up with the coordinate time v by relation $e^{t/2m} = 2v^2/c_0^2 + 1 + (2v/c_0)(v^2/c_0^2+1)^2$, $c_0 = (r_0/2m - 1)\, e^{r_0/2m}$

If $|v| >> 1$, $\exp(t_0/2m) \simeq \begin{cases} 4v^2/c_0^2, & v > 0 \\ c_0^2/4v^2, & v < 0 \end{cases}$

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ИНВАРИАНТНОСТЬ ОТНОСИТЕЛЬНО ВРЕМЕННЫХ СДВИГОВ И ИСПАРЕНИЕ ЧЕРНЫХ ДЫР

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