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EFFECT OF VACUUM FLUCTUATIONS ON CROSS SECTIONS OF HARD PROCESSES IN QCD

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A b s t r a c t

Some arguments in favour of the applicability of the operator expansion for the correlators of heavy quark currents in the physical region are given. The power correction, proportional to $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, to the total cross section of e^+e^- annihilation into hadrons, which contains the heavy quark and antiquark, is obtained. The summation of the leading infinite subsequence of non-perturbative corrections is carried out in the non-relativistic approximation. The concept of a constituent gluon is discussed. From the analysis it follows the necessity of existing at least two types of vacuum fluctuations - soft and relatively hard ones - in the non-perturbative QCD vacuum.

The expectation that quantum chromodynamics (QCD) will become a theory of strong interactions is supported, in many respects, by the comparison of theory with experiment in hard processes. In such processes, for some reason or other, short distances much less than the confinement radius $R_c \sim 1/\mu$ prove to be essential. Hard processes in QCD are described now by means of the operator expansion /1/ as well as the other similar methods /2/, and by means of the asymptotic freedom /3/. Unlike the asymptotic freedom, the status of the operator expansion in QCD is much less clear. For some of scalar models and Abelian gauge theories the applicability of the operator expansion has been proved at least in the framework of perturbation theory /4/.

However, the situation becomes more complicated as far as the non-Abelian gauge theories are concerned. Vacuum in these theories has a complicated topological structure due to the existence of such solutions of classical Euclidean field equations as, for example, instantons /5/. The usual sence of the operator expansion consists in the possibility of factorizing the contributions from large ($1/\mu$) and short ($1/Q$) distances. But the scale of Euclidean solutions (of the BPST-instanton type) can be as small as possible. Their existence results in the following: starting from the operators of rather large dimensions, $n_{cr} \geqslant 11 + 7N_f/3$ (with due regard for the dynamical generation of the light quark masses), or $n_{cr} \geqslant 11 + N_f/3$ (if the light quark masses are fixed, m_q = const) /6/, the matrix elements of these operators are saturated not only by large-scale fluctuations, but the instanton solutions with $\rho \ll 1/\mu$ as well /7/. One can avoid this difficulty in a rather formal way: one should consider small-size vacuum fluctuations (VF) separately, without including their contribution to the operator expansion.

If the operator expansion is applied to a process with large external Euclidean momenta, $q^2 = -Q^2 < 0$, $Q^2 \gg \mu^2$, it is possible to single out the hard parton stage of the process at which the partons have a large virtual mass $K^2 (Q^2 > K^2 \gg \mu^2)$ and, consequently, propagate over short distances only. When VF of small size are omitted from an analysis, it is accepted to suppose that it is possible to expand the propagators of partons in series in $1/K^2$. The important information about the low-energy region may be obtained if one makes use of a few terms of this series, and then, using the dispersion relations, one tries to bring to accordance the Euclidean amplitude with the integral over the physical momenta $q^2 > 0$, proceeding from the trial spectral density of the necessary type /8/.

But the Euclidean sum rules are low-sensitive to the high-energy region. In view of this, very important becomes the question on the extension of the operator expansion to the neighbourhood of the physical region. The study of the time-like region of Q^2 provides direct information about the duality interval and many other details of the spectral density. In the present paper our attention will be paid mainly to the processes with heavy quarks.

1. Let us consider, for example, the e^+e^- annihilation into hadrons containing the heavy quark and antiquark of flavour f. The produced quark and antiquark are almost on the mass shell and can propagate over large distances. Therefore, at first sight, their interaction with VF is strong and cannot be neglected even at very high energies /9/. However, really, in the total cross section of $e^+e^- \to (\text{hadrons})_f$ the partial cancellation of the effects associated with large distances takes place and is exactly such that the operator expansion is valid here. This fact leads

to the power corrections to the total cross section

$$R_f(s) = 3Q_f^2 \left[\frac{v_f(3-v_f^2)}{2} \left(1 + \frac{\alpha_s}{\pi} \chi(v_f)\right) + C_V(v_f) \frac{\langle \alpha_s G^2 \rangle M_f^4}{s^4} \right], \quad (1$$

where M_f is the mass of the quark of flavour f, $\chi(v_f) = \frac{4\pi}{3}\left(\frac{\pi}{2v_f} - \frac{3+v_f}{4}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\right)$, $v_f = \sqrt{1 - \frac{4M_f^2}{s}}$, $C_V(v_f)$ is presented in eq.(6). The first term in the right-hand part of (1) is the well-known result of perturbation theory /10/, and the second represents the VF effect in the process $e^+e^- \to (\text{hadrons})_f$.

Due to the cancellation mentioned above, the power corrections to σ_{tot}^f for production of the heavy quark and antiquark at short distances a) exist, b) are calculable, and c) quite rapidly diminish with growing q^2, starting from some $q^2 > q_0^2$. The point a) can be stated more strongly: the sum of the terms of the series of power corrections to the perturbative cross section $\sigma_{tot}^{f,0}$ should "draw" the experimentally measurable quantity, $\Delta\sigma_{tot}^f = \sigma_{tot}^f - \sigma_{tot}^{f,0}$.

The cross sections of the processes in which the initial partons (which form a parton ladder) are produced either from a point (just as in the e^+e^- annihilation) or, at distances of $\sim 1/q^2 \ll 1/M^2$ (M^2 is the invariant mass of the produced group of hadrons), are related to the simplest correlator of local or quasilocal currents - the two-point one. So, for example, for the process in the channel 0^+ (in the frame where $\vec{q} = 0$)

$$\Gamma_{tot}^S(q^2) = \frac{1}{\sqrt{q^2}} \, \mathrm{Im} \, K^S(q^2), \qquad (2)$$

where

$$K^S(q^2) = i \int d^4x \, e^{iqx} \langle \Theta | T[J^S(x) \, J^S(0)] | \Theta \rangle, \quad (3)$$

and the current $J^S=\bar{\Psi}_1\Psi_2$. Averaging, in eq.(3), over the exact QCD vacuum Θ corresponds to the functional integration over both the vacuum fields B_μ and the perturbative fluctuations a_μ ($A_\mu=B_\mu+a_\mu$). The functional integral can be divided into different sectors, each of them being connected with its VF field B_μ .

After such a separation the quarks and gluons on the diagrams corresponding to the correlator (3) turn out to be placed in the external field B_μ . The most effective description of the processes in an external field is given by operator technique /11/ (see also Ref./12/). One of the important elements of this technique is the exponential parametrization of propagators. As a result of the exponential parametrization, the integration over quark and antiquark momenta is replaced by that over invariant variables: λ (their summary proper time) and v (their relative velocity). Generally speaking, in the formal expansion over the VF field the infrared convergence of integrals becomes worse. Nevertheless, if the real times of the formation process are small compared to the typical hadron sizes $1/\mu$, then the coefficients in the expansion over the VF field are infrared stable. In the first non-vanishing order with respect to field B_μ , the difference, for example, of the correlator (3) from the perturbative correlator $K_0^S(q^2)$ is of the form

$$\Delta K^S(q^2)=\frac{1}{64}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\int_0^\infty \lambda d\lambda\int_{-1}^{1}dv\, e^{i\lambda f(v)}\times$$
$$\times\left[\frac{2}{3}(q^2+2M^2)-(1+v^2)q^2+\frac{i\lambda}{24}(1-v^2)^2q^2(q^2-M^2)\right], \tag{4}$$

where $M=m_1+m_2$, $f(v)=\frac{q^2}{4}(1-v^2)-\frac{1}{2}(m_1^2+m_2^2)-\frac{v}{2}(m_1^2-m_2^2)$.

The times of the process of formation are determined by the function $f(v)$ in the phase in eq.(4). Let us consider,

for simplicity, the case of equal masses: $m_1=m_2=m$. Then, $f(v)=\frac{q^2}{4}(v_0^2-v^2)$, $v_0=\sqrt{1-\frac{4m^2}{q^2}}$. In integration over λ the points $v=\pm v_0$ are potentially dangerous. At these points $f(v)$ vanishes, and the characteristic proper times go to infinity, as a result. Just these values of relative velocity correspond to the free propagation of the quark and antiquark over large distances and can lead to infrared singularity in $\Delta K(q^2)$. For q^2 in the non-physical region (far from the cut) and for q^2 in the physical region (at $q^2-4m^2\gg\mu^2$; $\mathrm{Im}\,q^2=0$) no essential difficulties arise for a real part of $\Delta K(q^2)$. Indeed, if one makes the integral over v near the singularities $v=\pm v_0$,regular in the sense of the principal-value, then the infrared instability may be here of a logarithmic type only. If the quark and antiquark are both heavy, there is no this instability as well, since when $m\to m+c\cdot\mu$ $(c\sim 1)$ the argument of the logarithm $(1+v_0)/(1-v_0)$ is stable. For $\mathrm{Im}\,\Delta K(q^2)$ in the physical region the situation is different: as a whole, $\mathrm{Im}\,\Delta K(q^2)$ is determined by the neighbourhood of singularities at $v=\pm v_0$ (Fig.1). If $(m^2)^2\gg\langle\alpha_s G^2\rangle$, then the integration contour can be deformed in such a way that when going round the points $v=\pm v_0$ one can keep at safe distances from singularities: $(\Delta v_0)^2\gg\langle\alpha_s G^2\rangle/q^2(q^2-m^2)$. Then, $\lambda\ll 1/\mu^2$ $(\langle\alpha_s G^2\rangle\simeq\mu^4)$. When $q^2\sim 4m^2$, the main contribution to the integral (4) comes from the region $\lambda\sim\lambda_0\sim 1/(q^2-4m^2)$, and in the case of $q^2\gg 4m^2$, the region $\lambda\sim\lambda_0\sim 1/4m^2$ contributes. That's why the perturbative corrections to eq.(4) will contain $\alpha_s(1/\lambda_0)\ll 1$. Such a deformation of the contour is impossible to carry out, if singularities $v=\pm v_0$ are too close. Thus, we require that the distance from the threshold be $q^2-4m^2\gg q^2\langle\alpha_s G^2\rangle/(q^2-m^2)^2$. The described possibility of getting round the singularities guarantee the infrared stability of the result. The present consideration is not valid for the correlators with light quarks and for the gluon correlators. It should be emphasized that the phase in eq.(4) doesn't depend on the channel; therefore, our analysis is applicable for any quantum numbers of the local currents of heavy quarks. Moreover, the contribution of large-size vacuum fluctuations with $\rho \gg 1/B$ ($B^2 \sim |gG|$, $|gG|$ is the characteristic magnitude of a field of such fluctuations) in the correlator $K(q^2)$, and thereby in σ_{tot} as well, can be taken into account exactly. We shall not dwell here on this problem, through it should be noted that in this case the phase to the lowest order in the field is equal to $f(v) - \frac{\lambda^2}{48}(1-v^2)^2\, q \cdot (gG)^2 \cdot q$. The condition of smallness of the second term in comparison with the first one results in the inequality: $\delta = q^2 (gG)^2/(q^2 - 4m^2)^3 \ll 1$, i.e. the remoteness (from the threshold) condition is satisfied. In the non-relativistic approximation (without taking into account the Coulomb interaction) just this relation guarantees the smallness of non-perturbative contributions. The expansion of correlator $K(q^2)$ up to higher powers of the VF field can be carried out in a similar way, since the basic features of its infrared structure remain unchangeable. Thus, in the case of heavy quarks for $K(q^2)$ the operator expansion near the physical region is valid. So,

$$\mathrm{Im}\, \Delta K^{S}(q^2) = \frac{\langle \alpha_s G^2 \rangle}{32\, q^2} \cdot \frac{(1+v_0^2)(3+v_0^2)}{v_0^3} \qquad (5)$$

and the coefficient $C_V(v_0)$ in eq.(1) turns out to be equal to

$$C_V(v_0) = \frac{2\pi (1+v_0^2)}{v_0^5} \qquad (6)$$

In the non-relativistic approximation the ratios $\mathrm{Im}\, \Delta K(q^2)/$

$\mathrm{Im}\, K_0(q^2)$ ($K_0(q^2)$ is the perturbative correlator) are, as one should expect, proportional to the quantity δ for all channels. An important feature of these corrections consists in that all these are proportional to the quark mass in a certain power (for channels $0^{\pm}$, $\mathrm{Im}\, \Delta K(q^2) \sim m^2$, and for the channel 1^- and the transversal part 1^+ $\mathrm{Im}\, \Delta K_{\perp}(q^2) \sim (m^2)^2$; also note that $\mathrm{Im}\, \Delta K_{\parallel}(q^2) \sim (m^2)^2$ for 1^+). This fact corresponds to the known cancellation of logarithmic corrections in the framework of perturbative QCD. Double logarithmic corrections are cancelled for all types of current due to their colorlessness. Since the anomalous dimension of the vector current is equal to zero, the power corrections in this channel are additionally suppressed in the ratio $\frac{m^2}{q^2}$ (this is valid for the transversal part of the axial correlator as well). This rule is rather common and is in intuitive agreement with the results of perturbation theory, extended to the low-energy region. It should be mentioned that the formal transition to the limit $m^2 \to 0$ cannot be applied here. If one is interested in the "exchange" by the non-perturbative gluon between partons, which are in the colored state, suppression of m^2/q^2 doesn't occur, and the interaction with VF here is stronger than with colorless currents. But at a large relative momentum of heavy quarks the interaction with VF is still suppressed by the ratio $\langle \alpha_s G^2 \rangle / q^4$ and disappear at $q^2 \to \infty$.

In the limit of large q^2 the corrections (1), (6), and (5) in themselves are small, and the operators of higher dimension than that of $\alpha_s G^2$ may be neglected. Of principal importance is the following question: at what values of $q^2 \sim q_0^2$ do the corrections of different orders become comparable in magnitude? It may happen, generally speaking, at still relatively large q^2 such that the lower corrections to the total cross section (for example, proportional to $\langle\alpha_s G^2\rangle$) are much less than the perturbative cross section. We introduce an interval of variability Δ^2 which is defined by the quantity $q_0^2-4m^2$. It should be, generally speaking, distinguished from the interval of duality. It is rather clear that the interval of variability is determined (at least for heavy quarks) by the structure of a series of power corrections. When $q^2<q_0^2$, we cannot, in principle, confine ourselves to the finite number of terms in this series, and it is necessary to sum up a certain infinite subsequence of power corrections.

2. It appears that in the non-relativistic approximation such a kind of summation is possible. The sum of the infinite subsequence of the most important power corrections is representable in the form of the multiplicative combination of the quark matrix element and the correlator of gluon vacuum fields.

A non-relativistic quark-antiquark system is characterized by two basic parameters: size $R=1/\eta$ and characteristic frequency of motion ω. Its Green function is of the form

$$G(\xi',\xi)=\langle\langle\xi'|\frac{1}{i\partial_0-H_0(\tau)+\frac{1}{2}g(\vec{r}\cdot\vec{\mathcal{E}}(\tau))\cdot t}|\xi\rangle\rangle, \tag{7}$$

where $\xi\equiv(\vec{r},\tau)$, $|\xi\rangle=|\vec{r}\rangle|\tau\rangle$, $t=t_1-t_2$,

$$H_0(\tau)=\frac{\vec{p}^2}{m}+\frac{\alpha_s}{2}t_1\cdot t_2-gB_0(\tau)\cdot T,$$

$T=t_1+t_2$; t_1,t_2 are the generators of the color group for the quark and antiquark; the second averaging is carried out over all vacuum configurations B_μ. The operator $\mathcal{V}(\vec{r},\tau)=\frac{1}{2}g(\vec{r}\cdot\vec{\mathcal{E}}(\tau))\cdot t$ is a Hamiltonian of the interaction between the quark-antiquark pair and VF in the first order of the non-relativistic approximation, $\vec{\mathcal{E}}(\tau)$ is the chromo-electric vacuum field corresponding to the field B_μ. In the second order over this interaction the

Green function differs from the perturbative one by

$$\Delta G(\xi',\xi)=\langle\langle\xi'|\Lambda\mathcal{V}\Lambda\mathcal{V}\Lambda|\xi\rangle\rangle, \tag{8}$$

where $\Lambda=1/(i\partial_0-H_0(\tau))$, $\mathcal{V}\equiv\mathcal{V}(\vec{r},\tau)$. If the initial and final states are colorless, one can omit the term $gB_0(\tau)\cdot T$ (in $H_0(\tau)$) in the external operators Λ in eq.(8) because T is the operator of the system's color charge. Since $\omega/\eta\sim\mathcal{V}\ll 1$, then the keeping of $gB_0(\tau)\cdot T$ in the intermediate operator is not the exceeding in accuracy. Indeed, if $|gB^0(\tau)|\gtrsim\omega$, the expansion over $gB_0(\tau)$ doesn't lead to an additional smallness, while $|g(\vec{r}\cdot\vec{\mathcal{E}}(\tau))|\ll\omega$ for heavy enough quarks (we assume that $\eta>|\mathcal{D}_0|,|\vec{\mathcal{D}}|$; $\mathcal{D}_\mu\equiv\partial_\mu-igB_\mu$, $|\mathcal{D}_\mu|\equiv(\langle G_{\alpha\beta}\mathcal{D}_\mu\mathcal{D}_\mu G_{\alpha\beta}\rangle/\langle G_{\alpha\beta}G_{\alpha\beta}\rangle)^{1/2}$; here there is no summation over indices μ). Moreover, if $|gB_0(\tau)|\gtrsim\omega$, then it is necessary to keep the time dependence of the fields (rather than $\vec{r}$-dependence) under the condition $|gB_0(\tau)|\gtrsim\omega$. As a result, for $\Delta G(\xi',\xi)$ (8), transformed into the energy representation, we have

$$\Delta G(\vec{r}',\vec{r};E)=\int\frac{dp_0}{2\pi}M_E(\vec{r}',\vec{r};p_0)K_\mathcal{E}(p_0), \tag{9}$$

where

$$M_E(\vec{r}',\vec{r};E)=\langle\vec{r}'|\frac{1}{H_0^s-E}\vec{r}\frac{1}{H_0^a-E+p_0}\vec{r}\frac{1}{H_0^s-E}|\vec{r}\rangle \tag{10}$$

and $K_\mathcal{E}(p_0)$ is the result of the transformation into the energy representation of the gluon correlator $K_\mathcal{E}(\tau)$:

$$K_\mathcal{E}(\tau)=\pi\alpha_s\left\langle\vec{\mathcal{E}}(0)Pe^{ig\int_0^\tau B_0(\xi)d\xi}\vec{\mathcal{E}}(\tau)Pe^{ig\int_\tau^0 B_0(\xi)d\xi}\right\rangle. \tag{11}$$

In eq.(10), operators H_0^s and H_0^a are the projections of the Hamiltonian $H_0^Q=\frac{\vec{p}^2}{m}+\frac{\alpha_s}{2}t_1\cdot t_2$ onto the singlet and octet states, respectively. The imaginary part of $\Delta G(0,0;E)$ is connected with $\Delta\sigma_{tot}=\sigma_{tot}-\sigma_{tot}^0$, where σ_{tot} is the total cross section of the quark and antiquark production at short distances and σ^0_{tot} is the same cross section but with VF not taken into account. In the case of e$^+$e$^-$ annihilation,

$$\Delta\sigma_{tot}^{e^+e^-}(q^2)= \frac{Q_f^2 e^4}{2m^2q^2}\,\mathcal{J}m\,\Delta G(0,0;E). \qquad (12)$$

The expression (9) contains the sum of the contributions corresponding to the set of an infinite number of operators $\vec{\mathcal{E}}(0)(\mathcal{D}_0^2)^n\vec{\mathcal{E}}(0)$, $n = 0,1,\ldots$. In the non-relativistic limit, taking into account the other sets of operators leads to additional powers $(\omega/m)^2\sim v^2\ll 1$ and, consequently, one can neglect their contributions. The accepted approach is, in principle, generalized to the following, in significance, operators such as $\vec{\mathcal{H}}\mathcal{D}_0^{2n}\vec{\mathcal{H}}$, $\alpha_s\bar{\Psi}\mathcal{D}_0^{2n}\Psi$, $\vec{\mathcal{E}}(0)\mathcal{D}_0^{n_1}\vec{\mathcal{E}}\mathcal{D}_0^{n_2}\vec{\mathcal{E}}\mathcal{D}_0^{n_3}\vec{\mathcal{E}}$, $\vec{\mathcal{E}}\mathcal{D}_0^{2n}\vec{D}^2\vec{\mathcal{E}}$, and the others. We would like to mention also that the non-relativistic approximation in the lowest order over interaction with VF was used in Ref./13/ for a study of the bound states of the heavy quark and antiquark.

The variable p_0 signifies an energy of the unperturbative gluon in the state $Q\bar{Q}g$. Mixing of the state $Q\bar{Q}$ with the above one is the main unperturbative effect in the accepted approximation. The correlator $K_{\mathcal{E}}(p_0)$ may be regarded as an energy distribution function of this gluon. This function strongly **varies**, starting from some $|p_0|\sim\mu_g$. The quantity μ_g may be regarded as a dynamical mass of the constituent gluon. Both the notion of a constituent gluon and that of a constituent quark are not strictly determined. In the framework of QCD the transition from current objects to the constituent ones is reasonable to connect with VF which lead to dressing the "bare" partons with a "coat" of virtual gluons and quark-antiquark pairs. The size of this "coat" can be related to that of the constituent

parton $R_{q,g}$. The mass of the constituent parton may be roughly estimated by means of the uncertainty principle : $\mu_{q,g}\sim 1/R_{q,g}$. In the earlier estimates of the constituent quark mass /14/ the important circumstance has not taken into account: non-universality of the notion of a constituent parton itself. In different processes the effective mass of the constituent parton is not, generally speaking, the same and falls down with increasing its virtuality K^2 (i.e. with decreasing those distances on which its "coat" discerns), turning into the universal current mass. There is no considerable difficulty in understanding what this non-universality is connected with. Indeed, consider, as an example, the same two-point correlator (3) but for the various quantum numbers of currents $\mathcal{J}(x)$. An interaction with VF at large Euclidean q^2 can be taken into account by means of the expansion of Green functions $G(x,y)$ of the quark and antiquark in vacuum fields $G_{\mu\nu}$; $\Psi,\bar{\Psi}$ ($\Psi,\bar{\Psi}$ refer to the light quarks), for example, in the Schwinger's gauge /11/ for the field B_μ. After averaging over all vacuum configurations in $K(q^2)$, not only the contributions corresponding to the diagrams in Figs. 2a and 2b, but those corresponding to the interference diagrams in Figs. 2c and 2d survive. It is the interference diagrams that introduce a significant dependence on quantum numbers of $\mathcal{J}(x)$ and lead to the non-universality of the constituent parton masses. Note that the vanishing of the contribution from the diagram in Fig.2a for light quarks becomes obvious if one turns to the analogy with radiation of soft gluons in perturbative QCD. Nevertheless, the possibility of introducing the constituent parton mass in terms of QCD is still remained: the contributions from the diagrams in Figs.2c and 2d may be related to the interaction of constituent partons with each other. Let us substitute the quark lines for gluon ones in Fig.2a. Then, the appearance of this diagram results in generation of the constituent gluon mass μ_g in the case of a large virtual mass K^2. When the gluon virtual mass K^2 is decreasing, the next power corrections should be taken into account.

In the fixed gauge $\xi = 1$ for the a_μ field (B_μ field is taken in the Schwinger's gauge) one obtains $\mu_g^2 = \frac{N_c}{N_c^2-1} \frac{\langle (gG)^2 \rangle}{K^2}$. Note that in the limit of large N_c, μ_g^2 is constant. If one extrapolates this expression to $K^2 \sim \mu_g^2$, then it appears that $\mu_g \sim$ 0.7 GeV. It is remarkable that the dynamical gluon mass μ_g exceeds considerably the characteristic hadron scale μ.

The fact that factorization has occurred in the expression (9) for ΔG and the quark matrix element has separated from the gluon correlator $K_\varepsilon(p_0)$ is of significance for understanding the concept of a constituent gluon. The correlator $K_\varepsilon(\tau)$ has the meaning of the gauge-invariant Green function of the gluon connected with the non-relativistic quark and antiquark in the color-octet state. This correlator is independent of the quark and antiquark dynamics. Those times at which $K_\varepsilon(\tau)$ strongly decreases, may be related to the sizes R_g of the constituent gluon. For estimation of these times, turn to formulas (12) and (9). In the experiment, a very broad interval of variability in the process $e^+e^- \longrightarrow$ charm is observed: the total cross section is greatly varied and differs from the perturbative one up to energies Q = 5÷6 GeV /15/ and, according to the data of the group MARK-1 /16/, up to Q = 7÷7.5 GeV. Thus, the value of the interval of variability Δ is about 3 GeV, as minimum. Since formulae (12) and (9) are derived in the non-relativistic approximation, whose application is grounded (particularly, on the basis of potential models) up to $Q \sim 5$ GeV, it follows necessarily that

$\mu_g \sim$ 0.8÷1.6 GeV, which agrees with our above estimate of μ_g.

Thus, in formation of the correlator $K_\varepsilon(\tau)$, essential should be the VF of relatively small size; $1/\mu_g \sim (1\,GeV)^{-1}$. It would bear in mind that a considerable contribution to $K_\varepsilon(\tau)$ also comes from the fluctuations of large size: $\rho \sim 1/\mu \gg 1/\mu_g$. But the presence of the second type of vacuum fluctuations with $\rho \sim 1/\mu_g$ turns out to be quite necessary in order to explain the large interval of variability for the total cross section of $e^+e^- \longrightarrow$ charm. Also, there exist some other arguments in favour of $\mu_g \sim$ 1 GeV. Among them there are: a) large transverse momenta $p_\perp \sim$ 1÷2 GeV in hadron collisions and in the other processes, and, apparently, these are of non-perturbative nature; b) evidence from the sum rules for the correlators of gluon currents about the large scale, 1÷2 GeV, in the gluon channels /17/; c) observation by Parisi and Petronzio /18/ of that a number of low-energy effects may be qualitatively explained, introducing the gluon mass $\mu_g \sim$ 0.8 GeV.

From these arguments we make the physical assumption about the existence of two types of fluctuations in the non-perturbative QCD vacuum: relatively hard and soft ones. The first type of VF is mainly connected with the gluon degree of freedom and is responsible, in particular, for the formation of large intervals of variability in different channels. These fluctuations are likely to bear no direct relation to the confinement. We assume that the average $\langle G^2 \rangle$ and more so the average $\langle G \mathcal{D}^{2n} G \rangle$ etc. are saturated mainly by the first-type fluctuations. A scale of the second-type VF is of the order of that of usual hadron ones, of about $\sim 1/\mu$. One could expect that the quark degree of freedom is connected with the larger sizes compared to the gluon one. In literature /19,20/ the opinion is advanced that the breakdown of chiral symmetry and the generation of the constituent quark masses occur, nevertheless, at the distances which are small as compared to the confinement radius R_c .

Since the first-type VF are of small size, $\rho \sim (1\ \mathrm{GeV})^{-1}$, their fields may, in principle, be represented by instantons because in the quasi-classical limit the instantons dominate among the other perturbative fluctuations. With an increase of the sizes ρ of instantons, their interaction with the large-scale fields grows very fast, that leads to a rapid increase of their density /20/. By estimation of the authors of Ref./21/, the instantons are strongly modified due to this interaction already at $\rho \sim (1\ \mathrm{GeV})^{-1}$. If the dynamical gluon mass μ_g is about 1 GeV, then, starting from these distances, the dependence of $\alpha_s(\rho)$ is determined by non-perturbative effects. Following the paper /18/, we assume that at $\rho > (1\ \mathrm{GeV})^{-1}$ the variation of $\alpha_s(\rho)$ because of perturbative effects comes to stop. In this case, a rough model for the instanton density may be chosen, for example, in the following form:

$$D(\rho) = D_0(\rho)\cdot\theta(\rho_c-\rho) + D\cdot\theta(\rho-\rho_c) , \qquad (13)$$

where $\rho_c \sim (1\ \mathrm{GeV})^{-1}$, $D_0(\rho)$ is the standard density of the small-size instantons /22/, D is the quantity regarded as a fitting parameter. Note that the description of the phenomena taking place at the distances under consideration, 0.3 GeV^{-1}÷2 GeV^{-1}, depends slightly on the form of $D_0(\rho)$ and is very sensitive to the choice of parameters D, ρ_c .

The contribution to the average $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ from a single instanton, if the instanton density is given by formula (13), is equal to

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = \frac{4D}{\rho_c^4} ; \qquad (14)$$

instantons with $\rho < \rho_c$ may be ignored since $D_0 \ll D$, as will be seen below. If just the instantons are assumed to saturate the average $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, with due regard for the estimate $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ = 0.012 GeV^4 (from the sum rules for charmonium /8/), we have D = $0.30\cdot 10^{-2} \gg D_0(\rho)$ at $\rho_c = (1\ \mathrm{GeV})^{-1}$. It follows that there is a rapid jump in the instanton density with our choice of $D(\rho)$ (13) at $\rho = \rho_c$. This jump may be related to the exponentially growing interaction of instantons with large-size VF. A fraction $f(\rho)$ /20/ of the 4-space occupied by the instantons and anti-instantons of sizes less than ρ is as follows:

$$f(\rho) = 2\int_0^{\rho} \frac{d\rho'}{\rho'^5} D(\rho') \left(\frac{\pi^2}{2}\rho'^4\right) = \pi^2 D\cdot\theta(\rho-\rho_c)\ln\frac{\rho}{\rho_c} \ll 1 \qquad (15)$$

that is indicative of diluteness of the instanton gas at $\rho \sim \rho_c$. But from the diluteness of the instanton gas in this situation it doesn't follows, generally speaking, the validity of the hypothesis of dominance of the vacuum intermediate state /8,21/. This hypothesis makes it possible to reduce $\langle (G^2)^n \rangle$ to $(\langle G^2 \rangle)^n$:

$$\left\langle \left(\frac{\alpha_s}{\pi} G^2\right)^n \right\rangle = (2n-1)!! \left(\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle\right)^n \qquad (16)$$

Taking into account that, for the instanton field,

$$\frac{\alpha_s}{\pi} G^2(x; x, \rho) = \frac{3}{\pi^2}\left(\frac{4\rho^2}{[\rho^2+(x-x_0)^2]^2}\right)^2,$$

and with due regard for expressions (13) and (14), we obtain, at $D = 0.3\cdot 10^{-2}$ and not too large n = 2,3,..., that the ratio

$$d_n = \frac{\left\langle \left(\frac{\alpha_s}{\pi} G^2\right)^n \right\rangle}{(2n-1)!! \left(\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle\right)^n} = \frac{3}{n(2n-1)(4n-1)(2n-1)!!}\left(\frac{12}{\pi^2 D}\right)^{n-1}$$

highly exceeds unity, that contradicts to (16).

In the dilute instanton gas approximation the correlator $K_\varepsilon(\tau)$ (11) turns out to be equal to

$$K_\varepsilon(\tau)=2\int\limits_0^\infty \frac{d\rho}{\rho^5}D(\rho)\,\mathcal{K}^0_\varepsilon\left(\frac{\tau}{\rho}\right),$$

$$\mathcal{K}^0_\varepsilon\left(\frac{\tau}{\rho}\right)=\int d\tau_0\, d^3z_0\,\frac{2\rho^4(1+\cos 2\varphi)}{[(\tau_0+\tau)^2+z_0^2+\rho^2]^2[(\tau_0-\tau)^2+z_0^2+\rho^2]^2}, \qquad (17)$$

where

$$\varphi=\frac{z_0}{\sqrt{z_0^2+\rho^2}}\left(\mathrm{arctg}\frac{\tau_0+\frac{\tau}{2}}{\sqrt{z_0^2+\rho^2}}-\mathrm{arctg}\frac{\tau_0-\frac{\tau}{2}}{\sqrt{z_0^2+\rho^2}}\right).$$

With eq.(13) taken into account, we have

$$K_\varepsilon(z)=\frac{2D}{\tau^4}\int^{\frac{\tau}{\rho_c}} dz\cdot z^3\,\mathcal{K}^0_\varepsilon(z). \qquad (18)$$

The function $\mathcal{K}^0_\varepsilon(z)$ is shown in Fig.3. Formulae (12), (9), and (17) will be analysed in detail and compared with experimental data on $e^+e^-\to$ charm with the use of the sum rules of the type as those in Ref./9/ elsewhere. Note that it follows from the previous consideration that as the quark mass grows, the general structure of $e^+e^-\to(\mathrm{hadrons})_f$ cross section becomes universal.

3. In the present paper we have made an attempt to analyse the non-perturbative effects in the physical region. If the heavy quark and antiquark are produced at the distances much smaller than $1/\mu$, the corrections of such a kind as in eq.(1) to the total cross sections turn out to be infrared stable far from the threshold. However, the corrections of this type can be compared with experiment only in the region $Q-2m\gg\mu$, wherein they are small. To describe the total cross section at lower energies it proved to be necessary to sum the infinite subsequence of power corrections, and we succeeded in making this

summation in the non-relativistic limit (see eqs.(9) and (12)).

From comparison of eqs.(9) and (12) with experimental data it is seen that the significant component of the QCD vacuum is the VF of relatively hard type which we have associated with instantons. These VF can manifest themselves in a whole number of physical phenomena described in the framework of QCD.

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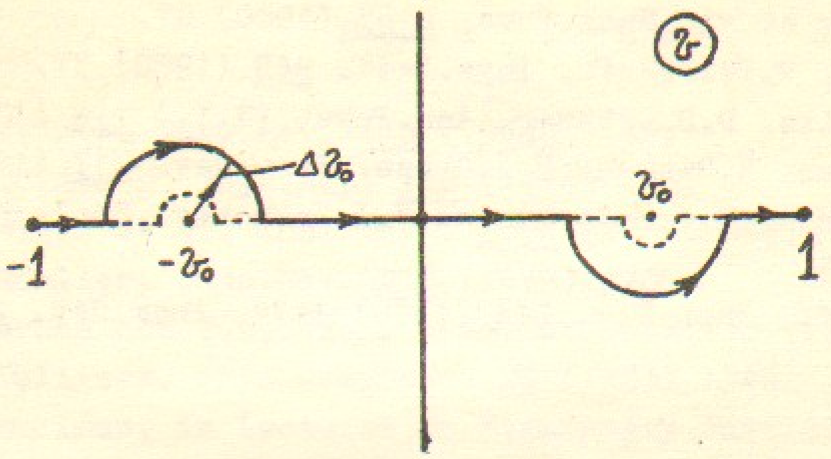

Fig.1

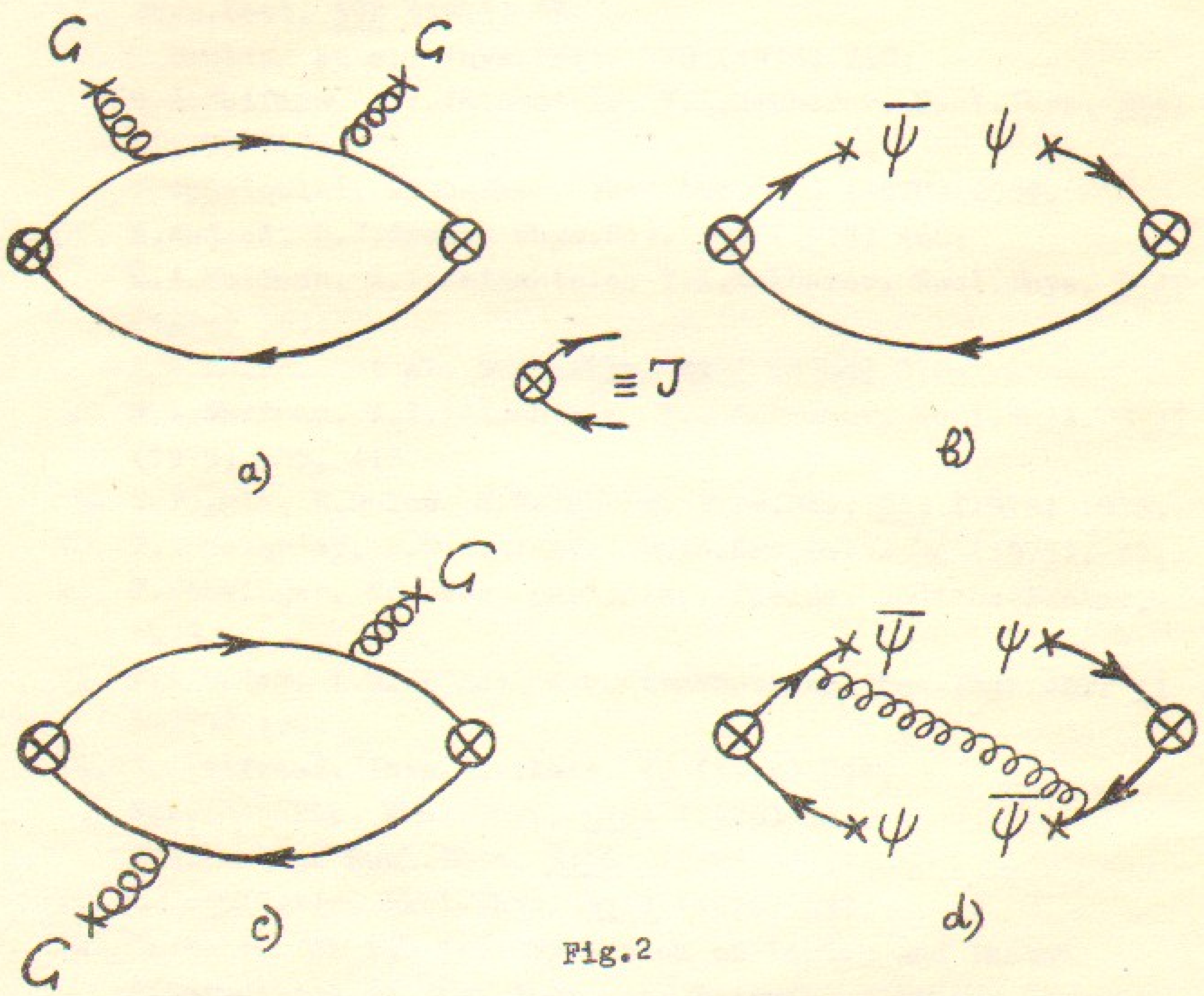

Fig.2

Fig.3