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THEORY OF POWER CORRECTIONS TO DEEP-INELASTI[C] SCATTERING IN QUANTUM CHROMODYNAMICS. 1. Q^{-2} EFFECTS.

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THEORY OF POWER CORRECTIONS TO DEEP-INELASTIC SCATTERING IN QUANTUM CHROMODYNAMICS;

1. Q^{-2} EFFECTS;

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Abstract

For first and second moments of structure functions of electromagnetic and weak scattering Q^{-2} corrections (Q is the momentum transfer) are found by means of operator product expansion method. Renormalizations of relevent operators of twist-4 are found to be small. Available data on scaling violation are discussed, and results are translated into average value of certain diquark operator over the nucleon. The naive quark picture with valence quarks only can not reproduce them, and some clusters of virtual quarks with dimensions about 1/3 of that for the nucleon are predicted, identified naturally with constituent quarks.

1. Introduction

1.1. Preface

The studies of lepton-hadron deep-inelastic scattering has been very fruitful during the last decade. Discovery of (approximate) Bjorken scaling has lead to Feynman parton model with its point-like fundamental constituents, quarks. Now these observations have fundamental theoretical explanation in the framework of the asymptotically free quantum chromodynamics (QCD).

With all progress in our understanding of interactions at small ($\ll$ 1 fermi) distances, we still know very little about the large scale interactions, which bind quarks inside hadrons. Experiment give us a lot of information about the properties of hadrons, but it is very difficult at the moment to connect it with the fundamental theory, so more or less phenomenological models are used.

In order to connect data with well defined quantities one needs some probe, which is in some sense point-like and well understood by itself. Deep-inelastic lepton-hadron scattering is the best one.

Last years the theory of deep-inelastic scattering has been concentrated on the calculations of logarithmic effects arising due to emission of soft gluons, see rewies [1-3]. Experimental groups have increased their accuracy and have been able to detect certain deviations from Bjorken scaling.

The works [4-7] have shown that these effects can be ascribed to logarithmic corrections. However, next order corrections [4] did not produced better agreement with data. Most suspicious is the fitted value of fundamental QCD parameter $\Lambda \simeq 0.5-0.7$ Gev, which seems to be too large. For definiteness we refer here $\Lambda_{\overline{MS}}$, for other popular renormalization schemes one has $\Lambda_{\overline{MS}} = 2.7 \Lambda_{MS} = 0.28 \Lambda_{MOM}$. Recent analysis of e^+e^- annihilation [8] has given much smaller $\Lambda \sim 0.1$ Gev, and in it power corrections dominate over logarithmic ones in the whole region where scaling violation is really seen.

For purely phenomenological data describtion it is rather difficult to tell power from log, see e.g. [9]. The importance of power effects for quark-resonance duality has been stressed in [10]. Finally, new generation of muon data experiments are in progress and they find no scaling violation at large Q^2, implying small $\Lambda \lesssim 0.1$ Gev. If so, the dominance of power effects becomes evident.

The present work is devoted to systematic studies of physicall effects connected with Q^{-2} corrections to moments of structure functions. The main methodical tool is Wilson operator expansion (OPE) which was previously used for calculations of perturbative corrections. Our application of OPE to next twist operators means the account for nonperturbative effects at large distances in their matrix elements, while their coefficients can be calculated perturbatively. The latter fact is nontrivial but possible up to high enough power corrections, see next section. The use of standard perturbatibe diagrams turns out so inadequate that so far nobody was able to carry out such programm. We use another approach based on the generalization of Schwinger formalizm for fundamental processes taking place in some external field. Most important is the explicite gauge invariance of this method at any step of the calculations. With the help of this method systematic calculation of OPE, at least for lowest operator spins, becomes possible. We also discuss in chapter 2 renormalization of the next twist operators. It turns out that for operators containing gluonic field the best method also is based on Schwinger formalism, this time for loop diagrams in external field.

Data on scaling violation in electron, muon and neutrino scattering, although collected during last decade, still are not accurate enough for precise and unequivocal determination of scale breaking parameters. We discuss in chapter 3 several examples, in particular Gross-Llewellyn-Smith sum rule, second moments of F_2 structure function and the problem of large F_L seen in SLAC-MIT experiments. The second point is most essential, for in it at least qualitative numerical estimate of the scale breaking parameters is possible.

Chapter 4 is devoted to rather detailed discussion of the relation between experimental value of scaling violation and our present understanding of hadronic structure. It is shown that the so called naive quark picture [19] is wrong for it gives predictions with the wrong sign and nearly one order of magnitude smaller. Our OPE results connect, loosely speaking, scaling violation with the square of color density averaged over nucleon. So, one can conclude that clusters of dimension 1/3 of that for nucleon are present, naturally idenified with constituent quarks.

1.2. Operator product expansion

The general idea of this method is the systematic separation of the two different scales of relevant distances. In the case of deep-inelastic scattering they are small distances of the order of $1/Q$ (Q is the momentum transfer) and large distances up to normal hadronic scale 1 fermi.

More precisely, it is the expansion of the product of two operators A(x) and B(0) in close points (x is small) into the set of some local operators $O_i(0)$. Their coefficients c_i are determined at scale x, while all information about larger scales is given by O_i. Such formal approach is very suitible for QCD in which we are able to calculate at small but not at large distances, so we can at least localize our ignorance in some definite way.

Short comment about th status of OPE. In perturbation theory it is obviously valid and is just expansion in powers of small x . Nonperturbative phenomena like instantons give more complicated coefficients, although the OPE still can be defined, see e.g. [13]. In the present work we do not discuss so distant terms as x^{11} where such phenomena first appear.

The standard OPE for the product of vector currents is as follows:

$$A_{\mu\nu} = i \int d^4x\, e^{iqx}\, T(j_\mu(x)\, j^+_\nu(0)) = \tag{1.1}$$

$$= \sum_{i,n} \left(\frac{2}{-q^2}\right)^n \Big[(g_{\mu\nu} - q_\mu q_\nu/q^2)\, q_{\mu_1} q_{\mu_2} C_L^i - \tag{1.1}$$

$$- (g_{\mu\mu_1} g_{\nu\mu_2} q^2 - g_{\mu\mu_1} q_\nu q_{\mu_2} - g_{\nu\mu_2} q_\mu q_{\mu_1} + g_{\mu\nu} q_{\mu_1} q_{\mu_2}) C_2^i -$$

$$- i\epsilon_{\mu\nu\mu_1\alpha}\, q_\alpha q_{\mu_2} C_3^i \Big]\, q_{\mu_3} \ldots q_{\mu_n}\, O^{(i)}_{\mu_1 \ldots \mu_n}$$

where summation is made over all types i of operators $O^{(i)}_{\mu_1 \ldots \mu_n}$ with Lorentz spin n, namely symmetric and traceless tensors. Three types of coeeficients C_L, C_2, C_3 correspond to three types of structure functions for unpolarized target.

If one neglects s,c... quarks and Cabbibo angle etc. he finds simple relations between crossing symmetry of (1.1) (which is $\mu \leftrightarrow \nu$, $q \leftrightarrow -q$) and isotopic properies of operators. In such approximation neutrino interact only with d quark, and antineutrino with u (we do not consider in this work neutral currents of weak interactions). Introducing crossing even and odd amplitudes as

$$A^{\pm}_{\mu\nu}(q) = A_{\mu\nu}(q) \pm A_{\nu\mu}(-q) \tag{1.2}$$

we may consider the second term either as the scattering of antiparticle, or as isotopically reversed target. Then it follows that structure functions F_L^s, F_2^s for isosinglet target correspond to operators with even spin, while F_3^s - only to operators with odd spin. For isovector case the situation is reversed.

For electromagnetic scattering crossing symmetry is obvious and all structure functions expand in operators with even spin.

Concluding this point we may say, that scattering of electrons, neutrino and antineutrino (charged currents) give in total 7 sets of C_A^i with even n and 3 with odd ones.

The next step is the connection between the two current correlator and scattering cross section. The most convenient way of doing this was proposed by Nachtmann [14] , it connects directly definite spin n with certain integrals of the measured structure function, the so called Nachtmann moments

$$M_L(n,Q^2) = \int_0^1 \frac{dx}{x^2}\, \xi^{n+1} F_L(x,Q^2)$$

$$M_2(n,Q^2) = \int_0^1 \frac{dx}{x^3}\, \xi^{n+1} F_2(x,Q^2)\left[n^2+2n+3+\right.$$

$$\left. + 3(n+1)(1+4m^2x^2/Q^2)^{1/2} + n(n+2)4m^2x^2/Q^2\right]/(n+2)(n+3) \qquad (1.3)$$

$$M_3(n,Q^2) = \int_0^1 \frac{dx}{x^2}\, \xi^{n+1} F_3(x,Q^2)\left[1+(n+1)(1+4m^2x^2/Q^2)^{1/2}\right]/(n+2)$$

here x is $(-q^2/2pq)$, $\xi = 2x/(1+\sqrt{1+4m^2x^2/Q^2})$ and m is the target mass. The connection mentioned is rather simple:

$$M_A(n,Q^2) = \sum_i C_A^i \langle\langle O^{(i)}_{\mu_1 \ldots \mu_n} \rangle\rangle \qquad (1.4)$$

where $\langle\langle O \rangle\rangle$ is the so called reduced average value of the

operator defined as

$$\langle N| O_{\mu_1 \ldots \mu_n} |N\rangle = 2\langle\langle O \rangle\rangle (p_{\mu_1} \ldots p_{\mu_n} - \text{traces}) \qquad (1.5)$$

Because moments are dimensionless numbers, the power dependence of the coefficients on Q^2 in QCD is evident from dimensional considerations. With standard introduction of the operator twist t as the dimension minus spin, one has $C_A^i \sim Q^{2-t}$. At $Q^2 \to \infty$ only t=2 (or leading twist) operators survive which are

$$L_{\mu_1 \ldots \mu_n} = i^{n-1} \bar{\psi} \gamma_{\mu_1} \mathcal{D}_{\mu_2} \ldots \mathcal{D}_{\mu_n} \psi \qquad (1.6)$$

$$G_{\mu_1 \ldots \mu_n} = i^{n-2} G_{\mu_1 \alpha} \mathcal{D}_{\mu_2} \ldots \mathcal{D}_{\mu_{n-1}} G_{\alpha \mu_n}$$

In the former one we may still use different flavors of quarks. In this expression (and below) the symmtrization over indeces and trace subtraction is not explicitely shown, but is implied.

Now we have to give more precise definition of OPE in respect to operator normalization. Usually it is made by setting all matrix elements equal to those for free quark and gluon fields at some fixed Euclidian momentum μ . By definition, all virtual momenta $Q^2 > \mu^2$ are ascribed to OPE coefficients, while $Q^2 < \mu^2$ to matrix element of operators. Of course, physical quantities are μ independent and the variation of μ leads to renormalizations of both coefficients and operators according to renormalization group equations. These phenomena are studied in the logarithmic approximation for leading twist operators in the works [1-4].

We study below power corrections, which arize from next twist operators. In order to define them proprly, one needs to account also for power (in Q^{-2}) corrections to OPE. Because integration in loop diagrams for coefficients of the leading twist operators is restricted to $\mu < p < Q$, corrections of the order $O(\mu^2/Q^2)$ appear. This μ dependence of the amplitude is canceled by $O(\mu^2)$ correction to average value of the next twist operator. Crudely speaking, it happens as follows:

$$\left(1 + c\frac{\mu^2}{Q^2}\right)_{twist-2} + \left(c\,\frac{M^2-\mu^2}{Q^2}\right)_{twist-4} = 1 + \frac{cM^2}{Q^2}$$

where c is some number, and M is internal hadronic scale of matrix element. We do not discuss such μ^2/Q^2 terms in details because the physical contribution M^2/Q^2 turns out to be so large that there exists low enough μ such that $\mu^2 \ll M^2$ and still $\alpha_s(\mu)$ is reasonably small in order to calculate coefficients in perturbative way. This statement will be discussed in section 4 in more details.

1.3. Processes in external field

This introductory section is devoted to very convenient formalism developed by Schwinger [11] for the motion of particles in the arbitrary external field. Generalised easily to nonabelian QCD it is adequate to its problems because of its explicite gauge covariance at all steps of calculations.

This is not so for standard perturbative expansion in Feynman diagrams, which is much more often used. For example, for quark-gluon operators of the type $\bar\psi G \psi$ to be discussed below one may use diagrams shown at Fig.1. However, the covariant operators contain parts of different order in coupling constant (e.g. covariant derivatives) and it is rather combersome to reconstruct them out of perturbative expressions. More important is the problem with gluon emission from external legs (e.g. diagrams a,c). The main part of such emission takes place at distances much larger than Q^{-1} and correspond not to coefficient of $\bar\psi G \psi$ but to average of $\bar\psi\psi$ which we do not need and should subtract. Only the part in which smallness of propagators is compensated in nominator (the so called contact part) should be considered. Its selection is rather inconvenient in practical calculations. The general reason is that if we want to select different distances, then coordinate representation is more natural.

The Schwinger formalism is just the coordinate one, it is based on formal set of states $|x\rangle$, eigenvalues of the coordinate operator:

$$X_\mu |x\rangle = x_\mu |x\rangle \tag{1.7}$$

Also momentum operator $\mathcal{P}_\mu$ is introduced which satisfies

$$[\mathcal{P}_\mu\, X_\nu] = i g_{\mu\nu} \tag{1.8}$$

and

$$[\mathcal{P}_\mu\, \mathcal{P}_\nu] = i g T^a G^a_{\mu\nu} \tag{1.9}$$

where T^a are color (Gell-Mann) matrices for relevent representation of $SU(3)_c$. In coordinate basis $\mathcal{P}_\mu$ acts as the covariant derivative

$$\langle x| \mathcal{P}_\mu |y\rangle = i \frac{\partial}{\partial x_\mu} \delta(x-y) + g T^a A^a_\mu(x) \delta(x-y) \tag{1.10}$$

In such representation one may write down simple formal presentation for propagators, for example for massless quarks

$$G(x,0) = \langle x| \frac{1}{\hat{\mathcal{P}}} |0\rangle \tag{1.11}$$

which can be proved by the action of the Dirac equation. The Fourier transform of (11) looks as

$$G(q) = \int d^4x\, e^{iqx} \langle x| \frac{1}{\hat{\mathcal{P}}} |0\rangle = \int d^4x \langle x| \frac{1}{\hat{\mathcal{P}}+\hat{q}} |0\rangle \tag{1.12}$$

where we have used relations

$$e^{-iqx} \mathcal{P}_\mu = (\mathcal{P}_\mu + q_\nu) e^{-iqx} \; ; \; e^{iqx}|0\rangle = |0\rangle \tag{1.13}$$

Using the propagator, one can write down expression for any process in the external field. For example, Compton scattering we are going to study is connected with the amplitude

$$T^{S,A}_{\mu\nu} = - \int dx \langle x| [\bar{\Psi} \gamma_\nu \frac{1}{\hat{\mathcal{P}}+\hat{q}} \gamma_\mu \Psi \pm (\mu \leftrightarrow \nu)] |0\rangle \tag{1.14}$$

where abstract operator Ψ is introduced which acts as

$$\Psi |x\rangle = \Psi(x) |x\rangle \tag{1.15}$$

Here $\Psi(x)$ is an arbitrary solution of the Dirac equation

$$\hat{\mathcal{D}} \Psi = 0 \tag{1.16}$$

in the same field. Such representation of the amplitude is very convenient for expansion in 1/Q

$$\frac{1}{\hat{\mathcal{P}}+\hat{q}} = \frac{1}{\hat{q}} - \frac{1}{\hat{q}} \hat{\mathcal{P}} \frac{1}{\hat{q}} + \frac{1}{\hat{q}} \hat{\mathcal{P}} \frac{1}{\hat{q}} \hat{\mathcal{P}} \frac{1}{\hat{q}} - \ldots \tag{1.17}$$

This expression, although simple, is the main point. The l.h.s. is some <u>integral nonlocal operator</u>, while the expansion in r.h.s. is made over <u>local differential operators</u> and the coefficients are simple powers of 1/Q, our small parameter. That is why this method is so useful for OPE.

2. Theory of Q^{-2} corrections

2.1. First moments; corrections to sum rules

The simplest operators relevant for description of deep-inelastic scattering are those of spin 1. According to (1.4) they are connected with first moments of the neutrino scattering structure functions. General considerations of section 1.2 are rather simple in this case in the leading order in 1/q of amplitudes (1.2):

$$A^{+}_{\mu\nu} = \frac{i}{q^2}\,\epsilon_{\mu\nu\alpha\beta}\, q_\alpha \left(\bar{u}_L \gamma_\beta u_L + \bar{d}_L \gamma_\beta d_L\right)$$
$$A^{-}_{\mu\nu} = \frac{1}{q^2}\left[\bar{u}_L \left(q_\mu \gamma_\nu + q_\nu \gamma_\mu - g_{\mu\nu}\hat{q}\right) u_L - (u \to d)\right] \qquad (2.1)$$

The first one is connected with isosinglet target only, while the second - with the isovector one. At the same time the first contains only F_3 structure function, while the second contains F_2. One should not worry that the second line is not transverse $q_\mu A^{-}_{\mu\nu} \neq 0$. Its longitudinal part is due to equal-time commutator of currents, i.e. another current. That is why for higher twist corrections this longitutinal part is always equal to zero.

The operators which enter (2.1) are made out of left-handed spinors because such are the properties of weak interaction. However, one can substitute $\bar{\psi}_L \psi_L \Rightarrow \frac{1}{2}\bar{\psi}\psi$ for unpolarised target.

The operators in (2.1) are just currents and their average over nucleons are known, so the classical sum rules [15-17] appear. Now we are going to calculate corrections to them.

Let us expand amplitudes (1.14) further, up to terms of the second order in P_μ. For brevity we (for some time) will consider only one type of quarks and omit $\int dx \langle x| \ldots |0\rangle$ *)

$$\delta T^{S,A}_{\mu\nu} = -\bar{\psi}\gamma_\nu \frac{1}{\hat{q}}\hat{P}\frac{1}{\hat{q}}\hat{P}\frac{1}{\hat{q}}\gamma_\mu \psi \pm (\nu \leftrightarrow \mu) \qquad (2.2)$$

Odd number of gamma matrices can always be reduced to one, so in fact we have to deal with the rank-3 tensor

$$T_{\mu_1\mu_2\mu_3} = \bar{\psi}\, P_{\mu_1} P_{\mu_2} \gamma_{\mu_3} \psi \qquad (2.3)$$

The spin-3 part of this tensor refer to the leading twist operator which we are not interested in now, while the spin-1 one give corrections we look for. In general (2.3) can be written as

$$T_{\alpha\beta\gamma} - T^{spin\,3}_{\alpha\beta\gamma} = g_{\alpha\beta} X_\gamma + g_{\beta\gamma} Y_\alpha + g_{\alpha\gamma} Z_\beta + i\epsilon_{\alpha\beta\gamma\delta} U_\delta \qquad (2.4)$$

Taking all traces and convolutions with $\epsilon_{\alpha\beta\gamma\delta}$ one finds the system of linear equations for unknown $X_\alpha, Y_\alpha, Z_\alpha, U_\alpha$. Using the equations of motion one can find that

$$\bar{\psi}\hat{P} P_\alpha \psi = \bar{\psi} P_\alpha \hat{P}\psi = 0$$
$$\bar{\psi} P^2 \gamma_\alpha \psi = \frac{g}{2}\bar{\psi}\tilde{G}^a_{\alpha\beta} t^a \gamma_\beta \psi \qquad (2.5)$$
$$\epsilon_{\alpha\beta\gamma\delta}\bar{\psi} P_\alpha P_\beta \gamma_\gamma \psi = i g \bar{\psi}\tilde{G}^a_{\gamma\delta} t^a \gamma_\gamma \psi$$

*) For example, in such condenced notations $\bar{\psi}\gamma_\alpha\psi P_\beta$ means $\int dx \langle x|\bar{\psi}\gamma_\alpha\psi P_\beta|0\rangle$ which is zero as the integral of the total derivative.

and finally

$$T^{spin-1}_{\alpha\beta\gamma} = \frac{5}{36} g_{\alpha\beta} O_\gamma - \frac{1}{36}\left(g_{\alpha\gamma} O_\beta + g_{\beta\gamma} O_\alpha\right) + \frac{i}{12}\epsilon_{\alpha\beta\gamma\delta} O_\delta \tag{2.6}$$

where the operator O_α is defined as $\left(\tilde{G}^a_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\gamma\delta} G^a_{\gamma\delta}\right)$

$$O_\alpha = g\bar{\Psi}\tilde{G}^a_{\alpha\beta} t^a \gamma_\beta \gamma_5 \Psi \tag{2.7}$$

Note that it is *unique* positive parity spin-1 twist-4 operator because another combination is identically zero:

$$\bar{\Psi} G^a_{\alpha\beta}\gamma_\beta t^a \Psi = -\frac{2i}{g}\bar{\Psi}\left[\mathcal{P}_\alpha \mathcal{P}_\beta\right]\gamma_\beta \Psi = 0 \tag{2.8}$$

Using (2.6) one may find corresponding corrections to the amplitudes

$$\begin{aligned} T^A_{\mu\nu} &= \frac{2i}{q^2}\epsilon_{\mu\nu\alpha\beta} q_\alpha L_\beta + \frac{8i}{9q^4}\cdot\epsilon_{\mu\nu\alpha\beta} q_\alpha O_\beta \\ T^S_{\mu\nu} &= \frac{2}{q^2}\left(q_\mu L_\nu + q_\nu L_\mu - g_{\mu\nu} Lq\right) - \frac{8}{9q^6}\left(g_{\mu\nu} q^2 - q_\mu q_\nu\right)(qO) \end{aligned} \tag{2.9}$$

It is now trivial to introduce two types of quarks and rewrite (2.9) as the following corrections to moments:*)

$$M_1^{\nu, NS}(1, Q^2) = -\frac{1}{2}\left[1 - \frac{4}{3\beta \ln Q^2/\Lambda^2} - \frac{4}{9Q^2}\langle\langle O_\alpha^{NS}\rangle\rangle + \ldots\right] \tag{2.10}$$

$$M_2^{\nu, NS}(1, Q^2) = -1$$

*) Singlet and nonsinglet moments are defined as $M^{S,NS} = \frac{1}{2}\left(M_p \pm M_n\right)$.

$$M_3^{\nu, S}(1, Q^2) = 3\left(1 - \frac{2}{\beta \ln(Q^2/\Lambda^2)} - \frac{4}{27Q^2}\langle\langle O_\alpha^S\rangle\rangle + \ldots\right) \tag{2.10}$$

The three relations in (2.10) (including first terms only) are known as Bjorken, Adler and Gross-Llewellyn-Smith sum rules by the names of their authors, see works [15-17]. Logarithmic corrections are given according to first order perturbative calculations [4], here and below the coefficient of Gell-Mann-Low function is $\beta = 11 - 2/3\, n_{fl.}$, the number of flavours.

Last terms are the power corrections we have calculated above. The notations used are as follows:

$$O_\mu^{S(NS)} = g\tilde{G}^a_{\mu\nu}\left(\bar{u} t^a \gamma_\nu \gamma_5 u \pm \bar{d} t^a \gamma_\nu \gamma_5 d\right) \tag{2.11}$$

So, the accurate comparison of (2.10) with data can give us the correlator of gluonic field with quark axial current.

One more comment about the Adler sum rule which is valid without any corrections. This statement follows very simply from our approach, but in fact it can be traced from the original analysis [16].

2.2. Second moments

Second moments are interesting because of several reasons. First, the diquark operators come into play starting from spin-2 operators, and we believe them to be rather important in producing Q^{-2} scaling violation. Second, this is the first of even moments, which are connected not only with weak interaction, but electromagnetic as well, so more accurate data are available.

There is no principal difference in the calculations compared to the case of the first moments considered above, although they become more lengthy. For bravity, we start with the case of one type of quarks ψ and simple vector current $\bar{\psi}\gamma_\mu\psi$, adding quark charges and isotopic factors later.

The starting point now is the amplitude (1.14), expanded up to terms of the third power in $\mathcal{P}_\mu$. Naturally, the central object is reducible rank-4 tensor

$$T_{\alpha\beta\gamma\delta} = \bar{\psi}\,\mathcal{P}_\alpha \mathcal{P}_\beta \mathcal{P}_\gamma \gamma_\delta \psi \tag{2.12}$$

from which the spin-2 part should be extracted. Agair, one may write it in general form

$$\begin{aligned} T_{\alpha\beta\gamma\delta} - T^{spin\text{-}4}_{\alpha\beta\gamma\delta} &= g_{\alpha\beta} Z_{\gamma\delta} + g_{\beta\delta} Z_{\alpha\gamma} + g_{\alpha\gamma} Y_{\beta\delta} \\ &+ g_{\beta\delta} X_{\alpha\gamma} + g_{\alpha\delta} W_{\beta\gamma} + g_{\gamma\delta} W_{\beta\delta} + i\epsilon_{\sigma\beta\gamma\delta} V_{\sigma\alpha} + \\ &+ i\epsilon_{\alpha\sigma\gamma\delta} U_{\sigma\beta} + i\epsilon_{\alpha\beta\sigma\delta} t_{\gamma\sigma} \end{aligned} \tag{2.13}$$

where $t \div Z$ are some unknown operators of spin 2. They can all be determined if all traces and convolutions with $\epsilon_{\alpha\beta\gamma\delta}$ are found. Note that one possible structure $i\epsilon_{\alpha\beta\gamma\sigma} S_{\sigma\delta}$ is not given in (13), for it is not independent due to the identity:

$$\epsilon_{\mu\beta\gamma\delta}\, g_{\alpha\nu} + \epsilon_{\mu\gamma\delta\alpha}\, g_{\beta\nu} + \epsilon_{\mu\delta\alpha\beta}\, g_{\gamma\nu} + \epsilon_{\mu\alpha\beta\gamma}\, g_{\delta\nu} = 0 \tag{2.14}$$

The solution of this system of linear equations is the following:

$$\begin{aligned} T^{spin\text{-}2}_{\alpha\beta\gamma\delta} = \frac{1}{64}\Big[& g_{\alpha\beta}\left(2\mathcal{D} + \tfrac{1}{2}A - 3B\right)_{\gamma\delta} + \\ & + g_{\alpha\gamma}\left(4\mathcal{D} - \tfrac{5}{2}A - 3B\right)_{\beta\delta} + g_{\alpha\delta}\left(-2\mathcal{D} + \tfrac{1}{2}A + B\right)_{\beta\gamma} + \\ & + g_{\beta\gamma}\left(2\mathcal{D} + \tfrac{1}{2}A - 3B\right)_{\alpha\delta} + g_{\beta\delta}\left(4\mathcal{D} - \tfrac{1}{2}A + B\right)_{\alpha\gamma} + \\ & + g_{\gamma\delta}\left(-2\mathcal{D} + \tfrac{1}{2}A + B\right)_{\alpha\beta} + i\epsilon_{\sigma\beta\gamma\delta}\left(\tfrac{1}{2}A - 5B - \mathcal{D}\right)_{\alpha\sigma} + \\ & + i\epsilon_{\alpha\beta\sigma\delta}\left(\tfrac{1}{2}A - 5B - \mathcal{D}\right)_{\gamma\sigma} + i\epsilon_{\alpha\sigma\gamma\delta}\left(A - 2B - 2\mathcal{D}\right)_{\beta\sigma} \Big] \end{aligned} \tag{2.15}$$

where 3 spin-2, twist-4 operators are introdured:

$$A_{\mu\nu} = (-2g)\bar{\psi}\left(\mathcal{D}_\alpha G^a_{\alpha\nu}\right) t^a \gamma_\mu \psi = g^2 (\bar{\psi}\gamma_\mu t^a\psi)(\bar{\psi}\gamma_\nu t^a \psi) \tag{2.15}$$

$$B_{\mu\nu} = g\bar{\Psi}\{\overleftrightarrow{G}^{a}_{\mu\alpha} i\mathcal{D}_{\nu}\}_{+} t^{a}\gamma_{\alpha}\gamma_{5}\Psi$$

$$\mathcal{D}_{\mu\nu} = g\bar{\Psi}(\mathcal{D}_{\mu}G^{a}_{\nu\alpha})t^{a}\gamma_{\alpha}\Psi \qquad (2.15)$$

where symmetrisation and trace subtraction is not explicitely shown. Note that for operator $A_{\mu\nu}$ we have used the equations of motion

$$\mathcal{D}_{\alpha}G^{a}_{\alpha\beta} = -\frac{g}{2}\bar{\Psi}\gamma_{\beta}t^{a}\Psi \qquad (2.16)$$

and have rewritten it in the four-fermion (or diquark) form. Using these results we find the amplitudes

$$T^{S}_{\mu\nu} = \left(-\frac{4}{Q^4}\right)\left(L_{\mu\nu}Q^2 - q_{\mu}q_{\alpha}L_{\alpha\nu} - q_{\nu}q_{\alpha}L_{\alpha\mu} + g_{\mu\nu}\cdot q_{\alpha}q_{\beta}L_{\alpha\beta}\right) + \left(-\frac{1}{8Q^6}\right)\Big[q_{\mu}q_{\alpha}(5A-2B)_{\nu\alpha} + q_{\nu}q_{\alpha}(5A-2B)_{\mu\alpha} + g_{\mu\nu}q_{\alpha}q_{\beta}(-3A+14B)_{\alpha\beta} + (q_{\mu}q_{\nu}q_{\alpha}q_{\beta}/Q^2)\cdot(-2A-12B)_{\alpha\beta} + Q^2(-5A+2B)_{\mu\nu}\Big] \qquad (2.17)$$

$$T^{A}_{\mu\nu} = \frac{4i}{Q^4}\epsilon_{\mu\nu\alpha\beta}q_{\alpha}q_{\gamma}\left(L_{\beta\gamma} - \frac{3}{32Q^2}A_{\beta\gamma} + \frac{7}{16Q^2}B_{\beta\gamma}\right)$$

Here for completeness we also presented the leading twist operator $L_{\mu\nu}$ (1.6) , the quark stress tensor. Note also that these expressions are automatically transverse, namely $q_{\mu}A_{\mu\nu} = q_{\nu}A_{\mu\nu} = 0$. The nontrivial and still unexplained phenomenon is that the operator $\mathcal{D}$ does not in fact contribute to final results (17) although it is present in intermediate expressions.

Our results are so far not complete because there exists another diagram shown at Fig.3 and connected with the interference of deep-inelastic scattering over different quarks of the nucleon. It is not connected with external gluonic field in the needed approximation, so the result can easily be obtained with ordinary Feinmann diagrams:

$$\delta T^{S}_{\mu\nu} = \frac{1}{Q^6}\left(C_{\mu\nu}Q^2 - q_{\mu}q_{\alpha}C_{\alpha\nu} - q_{\nu}q_{\alpha}C_{\alpha\mu} + g_{\mu\nu}q_{\alpha}q_{\beta}C_{\alpha\beta}\right) \; ; \quad \delta T^{A}_{\mu\nu} = 0 \qquad (2.18)$$

$$C_{\mu\nu} \equiv g^2(\bar{\Psi}\gamma_{\mu}\gamma_{5}t^{a}\Psi)(\bar{\Psi}\gamma_{\nu}\gamma_{5}t^{a}\Psi)$$

With such correction we have completed the calculation.

The final form of our results for two quark flavours in terms of the structure function moments is the following:

$$
\begin{aligned}
M_L^{v,S}(2,Q^2) &= \frac{18}{5} M_L^{e,S}(2,Q^2) = \langle\langle \frac{1}{16Q^2} A^S + \frac{3}{8Q^2} B^S \rangle\rangle \\
6 M_L^{e,NS}(2,Q^2) &= \langle\langle \frac{1}{16Q^2} A^{NS} + \frac{3}{8Q^2} B^{NS} \rangle\rangle \\
M_2^{v,S}(2,Q^2) &= \langle\langle L^S + \frac{5}{32Q^2} A^S - \frac{1}{16Q^2} B^S + \frac{1}{4Q^2} C^{vS} \rangle\rangle \\
\frac{18}{5} M_2^{e,S}(2,Q^2) &= \langle\langle L^S + \frac{5}{32Q^2} A^S - \frac{1}{16Q^2} B^S + \frac{1}{4Q^2} C^{eS} \rangle\rangle \\
6 M_2^{e,NS}(2,Q^2) &= \langle\langle L^{NS} + \frac{5}{32Q^2} A^{NS} - \frac{1}{16Q^2} B^{NS} + \frac{1}{4Q^2} C^{NS} \rangle\rangle \\
M_3^{v,NS}(2,Q^2) &= \langle\langle - L^{NS} - \frac{3}{32Q^2} A^{NS} + \frac{7}{16Q^2} B^{NS} \rangle\rangle
\end{aligned}
\tag{2.19}
$$

with the operators defined as

$$
\begin{aligned}
B^{S(NS)}_{\alpha\beta} &= g\bar{u}\{\tilde{G}^a_{\alpha\sigma}, iD_\beta\}_+ t^a \gamma_\sigma \gamma_5 u \pm (u \to d) \\
A^{S(NS)}_{\alpha\beta} &= g^2[\bar{u}\gamma_\alpha t^a u \pm \bar{d}\gamma_\alpha t^a d][\bar{u}\gamma_\beta t^a u + \bar{d}\gamma_\beta t^a d] \\
C^{eS}_{\alpha\beta} &= g^2(\bar{u}\gamma_\alpha\gamma_5 t^a u)(\bar{u}\gamma_\beta\gamma_5 t^a u - \frac{4}{5}\bar{d}\gamma_\beta\gamma_5 t^a d) + (u \to d) \\
C^{vS}_{\alpha\beta} &= g^2[\bar{u}\gamma_\alpha t^a(1+\gamma_5)d][\bar{d}\gamma_\beta t^a(1+\gamma_5)u] \\
C^{NS}_{\alpha\beta} &= g^2(\bar{u}\gamma_\alpha t^a\gamma_5 u)(\bar{u}\gamma_\beta\gamma_5 t^a u) - (u \to d)
\end{aligned}
\tag{2.20}
$$

Summurizing this section, we have found corrections to all 7 second moments of electromagnetic and weak (charged current) deep-inelastic scattering. Note, that the relation

$$M_L^{v,S}(2,Q^2) = \frac{18}{5} M_L^{e,S}(2,Q^2) \tag{2.21}$$

turns out to be valid not only in the leading twist approximation, but in the next twist one as well. Because of this, one has 4 singlet operators for 3 different relations and they can not be determined uniquely even from the complete set of data. For isovector case, one has 3 to 3 relation, so all averages can in principle be detemined.

2.3. Anomalous dimensions. Spin 1.

As discussed in section 1.2, in order to study Q-dependence of moments one has to normalize operators in some fixed normalization point μ . As it is well known, in the leading log approximation the corresponding coefficients are related by

$$C(\mu) = \exp\left[\hat{M} \frac{1}{b} \ln \frac{\alpha_s(\mu)}{\alpha_s(Q)} \right] C(Q) \tag{2.22}$$

where the indices of C_i and mixing matrix $\hat{M}_{ij}$ are suppressed. Matrix $\hat{M}_{ij}$ can be found already from one-loop calculations.

However, the logarithmic dependence due to (2.22) is of little importance for next-to-leading twist operators, for their contribution can be observed in rather limited region Q^2=1–10 Gev^2 where such logs change very slow. The main motivation of the discussion of such effects in our work is the need to use as small μ as possible in order to get rid off effects of the order μ^2/Q^2 , see section 1.2 .

This section gives an example of the application of the method used throughout this work, formal description of the motion in arbitrary external field, to calculations of logarithmic renormalization of operators, containing gluon field. In particular, the twist-4 spin-1 operator (2.7) *is considered*

$$O_\alpha = g\bar\Psi \tilde G^a_{\alpha\beta} t^a \gamma_\beta \gamma_5 \Psi$$

The gluonic field should be devided into classical external field $A^{a(cl)}_\mu$ and quantum field a^a_μ

$$A^a_\mu = A^{a(cl)}_\mu + a^a_\mu \tag{2.23}$$

so the operator O_α becomes equal to *the sum*

$$O_\alpha = g\tilde G^{a(cl)}_{\alpha\beta}\, \bar\Psi t^a \gamma_\beta \gamma_5 \Psi + g\epsilon_{\alpha\beta\gamma\delta}\mathcal{D}_\gamma a^a_\delta \cdot (\bar\Psi t^a \gamma_\beta\gamma_5\Psi) + \frac{g^2}{2}\epsilon_{\alpha\beta\gamma\delta} f^{abc} a^b_\gamma a^c_\delta \bar\Psi t^a \gamma_\beta\gamma_5\Psi \tag{2.24}$$

where the covariant derivative $\mathcal{D}_\gamma$ contains $A^{a(cl)}_\mu$. Let us formally add this term to gluon Lagrangian *as small perturbation*

$$\mathcal{L} = i\bar\Psi \hat{\mathcal{D}}\Psi - \frac{1}{2}(\mathcal{D}_\mu a^a_\nu)^2 + g a^a_\mu a^c_\nu f^{abc} G^{b(cl)}_{\mu\nu} + \frac{g}{2}\bar\Psi\gamma_\mu t^a \Psi a^a_\mu + \mathcal{D}_\mu\bar\eta^a \mathcal{D}_\mu \eta^a + \epsilon_\alpha O_\alpha \tag{2.25}$$

Feynmann gauge is chosen here, for which additional term $\frac{1}{2}(\mathcal{D}_\mu a_\mu)^2$ and the ghost one $\mathcal{D}_\mu\bar\eta\, \mathcal{D}_\mu\eta$ are added to original Lagrangian. The resulting propagators are:

$$\overline{a^a_\mu(x) a^b_\nu(y)} = \langle x | \left[\frac{-i}{\mathcal{P}^2 g_{\mu\nu} - 2g G_{\mu\nu}}\right]^{ab} | y\rangle$$
$$\overline{\eta^a(x)\bar\eta^b(y)} = \langle x|\left[\frac{i}{\mathcal{P}^2}\right]^{ab}|y\rangle \tag{2.26}$$

The operator of momentum $\mathcal{P}_\mu$ for gluonic fields acts as follows

$$\langle x | [\mathcal{P}_\mu]^{ab} | y\rangle = i\left(\frac{\partial}{\partial x_\mu}\delta^{ab} + gA^{ab}_\mu\right)\delta(x-y)$$
$$A^{ab}_\mu = f^{acb} A^c_\mu, \quad G^{ab}_{\mu\nu} = f^{acb} G^c_{\mu\nu} \tag{2.27}$$

The Lagrangian (2.25) contains only one term $\frac{g}{2}\bar\Psi \hat a^a t^a \Psi$ which is not bilinear in fields, so corrections due to operator O_α can be expanded in powers of the interaction of a_μ with quark fields. Diagrams shown at Fig. 4 a,b are of zero order in this interaction (crosses mean vertexes created by operator O_α). The corresponding expressions are

$$M^{(a)}_\alpha = -igTr\left(\tilde G^a_{\alpha\beta} t^a \gamma_\beta\gamma_5 \frac{1}{\hat{\mathcal{P}}}\right) \tag{2.28}$$
$$M^{(b)}_\alpha = -i\frac{g^2}{2}\epsilon_{\alpha\beta\gamma\delta} f^{abc}\, Tr\left(\bar\Psi t^a\gamma_\beta\gamma_5\Psi \left[\frac{1}{\mathcal{P}^2 g_{\delta\gamma} - 2gG_{\delta\gamma}}\right]^{cb}\right)$$

Note that in these expressions trace is taken over all variables including coordinate ones.

In the first order in interaction with quarks one has diagrams shown at Fig. 4 c,d which correspond to

$$M^{c+d}_\alpha = \frac{g^2}{2}\epsilon_{\alpha\beta\gamma\delta}\, Tr\left(\bar\Psi\gamma_\beta\gamma_5 t^a \frac{1}{\hat{\mathcal{P}}}\gamma_\nu t^b \Psi \cdot \left[\frac{1}{\mathcal{P}^2 g_{\nu\delta} - 2gG_{\nu\delta}}\mathcal{P}_\gamma\right]^{ba}\right) + h.c. \tag{2.29}$$

and in second order there appear diagrams e,f,g, expressions for them are the same as in standard perturbation theory.

Strictly speaking, one should also include the renormalization of the external field $G_{\alpha\beta} \to Z^{1/2} G_{\alpha\beta}$. However, in the formalism used the source of Q_μ is conserved and the same constant Z determines renormalization of the charge g, so that the product $g G_{\alpha\beta}$ is not renormalized. Therefore, including g in the operator we may ignore field renormalization.

We look for logarithmic renormalization, so because of dimensional considerations one should look for terms of the same dimension as the original operator O_μ. Starting with $M_\alpha^{(a)}$, we may rewrite it as follows

$$M_\alpha^{(a)} = -ig\int d^4x\, \tilde{G}^a_{\alpha\beta}(x)\, Tr_{\substack{(spin)\\(color)}}\left[t^a\gamma_\beta\gamma_5\langle x|\frac{1}{\hat{P}}|x\rangle\right] \quad (2.29)$$

The axial vector $Tr[t^a\gamma_\beta\gamma_5\langle x|\frac{1}{\hat{P}}|x\rangle]$ expressed in terms of gluon field strength can be $\mathcal{D}_\gamma \tilde{G}^a_{\gamma\beta}$, but this combination vanishes identically. So, (29) is in fact zero.

For $M_\alpha^{(b)}$ we expand the propagator in $G_{\delta\gamma}$. Zero order term $g_{\delta\gamma}/P^2$ does not contribute because of convolution with $\epsilon_{\alpha\beta\gamma\delta}$. First order term is equal to

$$M_\alpha^{(b)} = ig^2 f^{abc}\int d^4x\, \bar{\Psi}\gamma_\beta\gamma_5 t^a\Psi\langle x|\left\{\frac{1}{P^2}\tilde{G}_{\alpha\beta}\frac{1}{P^2}\right\}^{cb}|x\rangle \quad (2.30)$$

and the next ones can not give log. Due to this, we may ignore commutators in (30) and extract $\tilde{G}_{\alpha\beta}$. What is left is easily found in ordinary momentum representation

$$\langle x|\left[\frac{1}{P^4}\right]^{ab}|x\rangle = \delta^{ab}\int\frac{d^4P}{(2\pi)^4}\frac{1}{P^4} = \delta^{ab}\frac{i}{16\pi^2}\ln\left(\frac{Q^2}{\mu^2}\right) \quad (2.31)$$

and finally

$$M_\alpha^{(b)} = -\frac{g^2}{16\pi^2}\ln\left(\frac{Q^2}{\mu^2}\right)\int dx\, O_\alpha(x) \quad (2.32)$$

The calculation of $M_\alpha^{(c+d)}$ is more lengthy, so we only outline the main steps. The fermionic propagator can be rewritten as

$$\frac{1}{\hat{P}} = \frac{1}{P^2 + \frac{ig}{4}\sigma_{\mu\nu}G^a_{\mu\nu}t^a}\,\hat{P} \quad (2.33)$$

The contribution of terms linear in $G^a_{\mu\nu}$ from the expansion of fermionic and gluonic propagators are calculated as above. Terms of zero order in $G_{\mu\nu}$ should be rewritten in the form in which all momenta P_μ and $1/P^2$ are, say, in its right part, and all coordinate dependent parts are in its left part. The following expansion in commutators is of use here

$$\frac{1}{A}B = B\frac{1}{A} + [B,A]\frac{1}{A^2} + [[B,A],A]\frac{1}{A^3} + \ldots \quad (2.34)$$

Remember, that P_μ between fermi fields is in fundamental representation of color group, while ouside them it is in adjoint representation, e.g. ΨP_μ means $t^a\Psi[P_\mu]^{ab}$.

The last step is the calculation of expressions of the type $\langle x|(P^2)^{-m}P_{\mu_1}\ldots P_{\mu_n}|x\rangle$. They are invariant with respect to substitution $P_\mu \to P_\mu + K_\mu$, where K_μ is the arbitrary c-number vector, and this can be used for their calculation. For example, expanding $\langle x|(P+K)^{-2}|x\rangle$ in K_μ one has

$$\langle x|\frac{1}{P^2}P_\mu\frac{1}{P^2}|x\rangle = 0 \quad (2.35)$$

Transforming this relation as follows

$$0 = \langle x| \frac{1}{P^2} P_\mu \frac{1}{P^2} |x\rangle = \langle x| \frac{1}{P^4} P_\mu + \frac{1}{P^6}[P^2, P_\mu] + \frac{1}{P^8}[P^2[P^2 P_\mu]] + \dots |x\rangle$$

one finds the following result

$$\langle x| \left\{ \frac{1}{P^2} P_\mu \right\}^{ab} |x\rangle = -\frac{g}{16\pi^2} \ln\frac{Q^2}{\mu^2} \left\{ D_\alpha G_{\alpha\mu} \right\}^{ab} \qquad (2.36)$$

Another expression found in similar way is

$$\langle x| \left\{ \frac{1}{P^4} P_\mu P_\nu \right\}^{ab} |x\rangle = -\frac{ig}{16\pi^2} \ln\frac{Q^2}{\mu^2} \left\{ G_{\mu\nu} \right\}^{ab} \qquad (2.37)$$

Using them we are able to find the contribution of diagrams c,d:

$$M_\alpha^{(c+d)} = \frac{1}{6}\left(7N_c + \frac{8}{N_c}\right) \frac{g^2}{16\pi^2} \ln\frac{Q^2}{\mu^2} \int d^4x\, O_\alpha(x) \qquad (2.38)$$

The final result for the renormalization of the operator O_α is

$$M_\alpha = -\frac{4}{3}\left(N_c - \frac{1}{N_c}\right) \frac{g^2}{16\pi^2} \ln\frac{Q^2}{\mu^2} \int d^4x\, O_\alpha(x) \qquad (2.39)$$

which corresponds to the anomalous dimension of this operator

$$C_{O_\alpha}(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_s(Q)}\right]^{-\frac{4}{3b}\left(N_c - \frac{1}{N_c}\right)} C_{O_\alpha}(Q) \qquad (2.40)$$

This result is not in agreement with that given in paper [21],

where also mixing of O_α with the nonexisting operator (2.8) is considered.

In conclusion let us emphasize the methodical virtue of the formalism used in this section, much more economical than the standard perturbation theory. The reason is that only local and gauge invariant objects appear at each stages of calculations.

2.4. Logarithmic mixing of spin-2 operators

Spin-2 twist-4 operators are rather numerous, including diquark and quark-gluon operators (some of them we have already met in section 2.2) and also pure gluonic operators. The discussion of their mixing matrix we begin with the following statement: this matrix is triangular for diquark operators do not mix with others. It follows from the fact that pairing of two quark lines in a loop give ordinary polarization operator for vector current which is proportional to $g_{\mu\nu}K^2 - K_\mu K_\nu$, where K_μ is the gluon 4-momentum. This fact imply the operator $D_\mu G^a_{\mu\nu}$ which, by convention used, we write as $(-\frac{g}{2})\bar\psi\gamma_\nu t^a\psi$ by means of the equations of motion and refer as the diquark operator.

So, we start with the classification of the diquark operators. Note that we try to follow the notations of the work [18] where spin-zero case is considered as close as possible. First we enumerate all possible structures. Lorentz spin 2 can be created in tree ways:

$$\bar\psi_L\gamma_\mu\psi_L\,\bar\psi_L\gamma_\nu\psi_L + (L \to R)$$
$$\bar\psi_L\gamma_\mu\psi_L\,\bar\psi_R\gamma_\nu\psi_R \quad , \qquad \bar\psi_L\sigma_{\mu\alpha}\psi_R\,\bar\psi_R\sigma_{\nu\alpha}\psi_L \qquad (2.41)$$

There are two color scalars

$$(\bar\Psi t^a \Psi)(\bar\Psi t^a \Psi) \quad , \quad (\bar\Psi\Psi)(\bar\Psi\Psi) \tag{2.42}$$

and four different flavor structures

$$\begin{array}{ll} (\bar\Psi\Psi)(\bar\Psi\Psi) & (\bar\Psi\lambda^a\Psi)(\bar\Psi\lambda^a\Psi) \\ (\bar\Psi\lambda^a\Psi)(\bar\Psi\Psi) & (\bar\Psi\lambda^a\Psi)(\bar\Psi\lambda^b\Psi) \end{array} \tag{2.43}$$

The direct product give 24 combinations. Fortunately, they are not all independent because of Fiertz transformations. For spin indices it is

$$(\bar\Psi^1_L \gamma_\mu \Psi^2_L)(\bar\Psi^3_L \gamma_\nu \Psi^4_L) = -(\bar\Psi^1_L \gamma_\mu \Psi^4_L)(\bar\Psi^3_L \gamma_\nu \Psi^2_L) \tag{2.44}$$

$$(\bar\Psi^1_L \gamma_\mu \Psi^2_L)(\bar\Psi^3_R \gamma_\nu \Psi^4_R) = -\frac{1}{2}(\bar\Psi^1_L \sigma_{\mu\lambda} \Psi^4_R)(\bar\Psi^3_R \sigma_{\nu\lambda} \Psi^2_L)$$

There are in fact 6 independent singlet and 6 nonsinglet operators, which are listed below. For brevity, $\bar\Psi_L \gamma_\mu t^a \Psi_L \Rightarrow (t^a)_L$ etc.

$$\left[\begin{array}{ll} S_1 = \mathbb{1}_L \otimes \mathbb{1}_R & S_2 = (t^a)_L \otimes (t^a)_R \quad (1\otimes 1) \\ S_3 = \mathbb{1}_L \otimes \mathbb{1}_L + (L\to R); & S_4 = (t^a)_L \otimes (t^a)_L + (L\to R) \quad (1\otimes 1) \end{array}\right. \tag{2.45}$$

$$\left[\; S_5 = (\lambda^a t^b)_L \otimes (\lambda^a t^b)_R \; ; \; S_6 = (\lambda^a)_L \otimes (\lambda^a)_R \quad (8\otimes 8)\right.$$

$$\left[\begin{array}{l} N_1 = (\lambda^a)_L \otimes \mathbb{1}_R + (L\leftrightarrow R) \; ; \; N_2 = (\lambda^a t^c)_L \otimes (t^c)_R + (L\leftrightarrow R) \\ \qquad N_3 = (\lambda^a)_L \otimes \mathbb{1}_L + (L\to R) \qquad (8\otimes 1 \oplus 1\otimes 8) \end{array}\right.$$

$$\left[\; N_4 = (\lambda^a)_L \otimes (\lambda^b)_R \qquad N_5 = (\lambda^a t^c)_L \otimes (\lambda^b t^c)_R \qquad (8\otimes 8)\right. \tag{2.45}$$

$$\left[\; N_6 = (\lambda^a)_L \otimes (\lambda^b)_L + (L\to R) \qquad (27\otimes 1 \oplus 1\otimes 27)\right.$$

Brackets in the right side contain representations of right-handed and left-handed flavor groups. These representations are not mixed by renormalization. For example, because of this the operator N_6 is not mixed at all and is renormalized multiplicatively.

The calculation of the one-loop logarithmic renormalization is made with standard perturbative diagrams shown at Fig.5. In Lorentz gauge the renormalization of external line and vector current is zero separately, which helps in the calculations.

For the operator N_6 the result is most simple:

$$C^{N_6}(\mu) = \left[1 - \frac{4}{3}\frac{\alpha_s}{4\pi}\ln\left(\frac{Q^2}{\mu^2}\right)\right] C^{N_6}(Q) \tag{2.46}$$

and therefore*)

$$C^{N_6}(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_s(Q)}\right]^{-\frac{4}{27}} C^{N_6}(Q) \tag{2.47}$$

Two doublets $(S_6, S_5), (N_4, N_5)$ have identical mixing matrices

*) This result also disagrees with that of [21], even in number of operators.

$$\hat{M}_{8\otimes 8} = \begin{pmatrix} 0 & -\frac{16}{9} \\ -\frac{1}{2} & -\frac{7}{3} \end{pmatrix} \tag{2.48}$$

Singlet matrix is the largest one, it mixes S_1, S_2, S_3, S_4:

$$\hat{M}_{1\otimes 1} = \begin{pmatrix} 0 & -\frac{16}{9} & 0 & 0 \\ -\frac{1}{2} & -\frac{13}{3} & \frac{2}{3} & -\frac{40}{9} \\ 0 & 0 & 0 & \frac{16}{9} \\ 0 & -1 & \frac{5}{6} & -\frac{26}{9} \end{pmatrix} \tag{2.49}$$

We remind that in principle it is only part of larger matrix, including operators $B_{\mu\nu}$, $\mathcal{D}_{\mu\nu}$ (2.15) and gluonic one

$$\mathcal{E}_{\mu\nu} = g^3 G_{\mu\alpha} G_{\alpha\beta} G_{\beta\nu} \tag{2.50}$$

However, the triangular form of this matrix shows that at least the eigenvalues of (49) are true anomalous dimensions.

The main result of these calculation is that mixing is very weak for diquark operators. For the operator $B_{\mu\nu}$ which enter OPE directly we have checked that their mixing with S_i is also numerically small.

3. Data analysis

Although studies of scaling violations in deep-inelastic scattering has been made in experiments with electron, muon and neutrino beams during last decade, the accuracy of most data are insufficient for quantitative unequivocal separation of power corrections under discussion. In this section we discuss the parameters of scaling violation in lowest moments, which are obtained with the help of some plausible assumptions.

3.1. Gross-Llewellyn-Smith sum rule

Among first moments, only that for singlet $M_3^S(1,Q^2)$ is available from existing data, they are shown at Fig.6 and taken from the work [5]. Perturbative corrections with two values of Λ, namely 0.7 and 0.1 Gev, is shown by the dashed lines. Data seem to exclude the first possibility. Interesting, that this very case, no power correction and large Λ, is advocated in this paper.

Error bars are too large, but if any scaling violation is present, it probably has sign opposite to those predicted by the pertubation theory.

For illustration, we have plotted also the curve with some power term as well (solid line). Using formula (2.10) connecting this correction in terms of the reduced matrix element of the operator O_α^S (2.11), we may say that this line corresponds to

$$\langle\langle O_\alpha^s \rangle\rangle = -1\ Gev^2 \tag{3.1}$$

Of course, this is not better than order-of-magnitude guess. Further experimental work is evidently needed in order to clarify this point.

3.2. Second moments of F_2^s

In this case the situation is somehow better, because higher statistics electron and muon data can be used. Another particularity of this case becomes evident from our results (2.19): the only quark-qluon operator which contributes OPE of currents has coeffecient much smaller than in F_3,F_L and is probably inessential. This give us the possibility to concentrate below on diquark operators.

Combined BEBC-GGM data for $\nu,\bar{\nu}$ beams in the form of Nachtmann moments are available from [5] , while similar analysis for combined SLAC electron and FNAL muon experiments are available from [7] . For our purposes it is rather inconvenient to plot these moments directly versus Q^2 for large but unimportant Q-independent part of these moments becomes dominant, with scaling violations hardly seen within the accuracy of general normalization of data points. We propose to take derivative and use the following index of scaling violation

$$i_A(n,Q^2) = -\frac{d \ln M_A(n,Q^2)}{d \ln Q^2} \tag{3.2}$$

which is insensitive to normalization differences of different data sets. This quantity is shown at Fig. 7-9 for the second moment, sixth moment and all moments at Q^2=3.7 Gev^2 , respectevely. After this derivative is taken, errors become large and the plots do not look very atrractive. But this is the real situation with our experimental knowledge of scaling violation.

It is evident from these figures, that second moments have much smaller effect than higher ones, so the accuracy is poor in this case by itself. However, very smoth dependence on momentum number n suggests some extrapolation from high n to n=2 in order to make more definite estimates of scale breaking parameter. The simple expression for moments

$$M_2^\nu(n,Q^2)\ \left(\text{or } \frac{18}{5}M_2^e(n,Q^2)\right) = \langle\langle L_{\alpha\beta}\rangle\rangle + \frac{\alpha_2 n}{2Q^2} + \frac{\beta_2 n^2}{4Q^4} \tag{3.3}$$

and indices

$$i_2(n,Q^2) = \frac{\alpha_2 n}{2\langle\langle L\rangle\rangle Q^2} + \frac{n^2(\beta_2 - \alpha_2^2/2\langle\langle L\rangle\rangle)}{4\langle\langle L\rangle\rangle Q^4} \tag{3.4}$$

were used with the parameters found to be equal to

$$\alpha_2 \simeq 0.15\, Gev^2 \; ; \; \beta_2 \simeq -0.06\, Gev^2 \tag{3.5}$$

The corresponding curves are shown at Fig.7-9 as the dashed *lines*. In our analysis we have also included perturbative (logarithmic) scaling violation due to renormalization of the leading twist operators. The QCD parameter Λ was taken to be 100 Mev and its effect alo is shown by the dotted curves. By itself, it can not explain scaling violation observed.

Other people [9,10] has also attempted similar power fits, and find similar parameters. Among the first ones is the work [10], where more detailed studies of quark-resonance duality was made. Again, their parameter α_2 is close to ours.

Note the negative sign of the parameter β_2. It is needed for two purposes: first, to reproduce rapid decrease of all moments below $Q^2 = 1$ Gev^2, and second, to slow down n-dependence as it is seen in Fig.9.

In what follows we refer (3.5) as to "experimental" values of scale breaking parameters. Their accuracy is presumably inside 50%.

3.3. σ_L/σ_T puzzle

The most difficult for experimental investigations is the measurements of structure function F_L. The available data of SLAC-MIT groups [12] *show up*

surprisingly large effect, for which no rational explanations have been proposed. Therefore, such effect is referred as the " σ_L/σ_T puzzle".

As far as x-dependence of ratio σ_L/σ_T is very weak, it can also be considered (with some accuracy) to be the ratio of second moments we are mainly interested in. The data poins from [12] and also some neutrino data *(see [20])* are shown at Fig.10.

The contribution of perturbative corrections to leading twist operators is given by

$$\left.\frac{M_L(2,Q^2)}{M_2(2,Q^2)}\right|_{perturb} = \frac{\alpha_s(Q^2)}{9\pi}\left(4 + \frac{\langle\langle G_{\mu\nu}\rangle\rangle}{\langle\langle L_{\mu\nu}\rangle\rangle}\right) \tag{3.6}$$

where $\langle\langle G_{\mu\nu}\rangle\rangle$ is the average of gluon stress tensor and $\langle\langle G_{\mu\nu}\rangle\rangle \simeq \langle\langle L_{\mu\nu}\rangle\rangle \simeq 1/2$. This contribution with Λ =100 Mev is shown by the dotted line, and is evidently too small.

As we show below, diquark operator contribution *(see 4.3)* goes into right direction (the dashed curve) but also is not sufficient. As we understand it now, the most reasonable interpretation of these puzzle is the interaction with quark condensate to be studied in the second part of this work. The empirical $1/Q^4$ curve is shown at Fig. 10 by the solid line. This interpretation can be checked in the following way: the predicted effect should be much larger for electromagnetic scattering than for weak ones. This is a good problem for experiments.

4. Constituent versus current quarks

4.1. Naive quark picture

In this section we discuss the relation between the magnitude of some average values of next twist operators, as found from our OPE formulae and experimental data, with some current ideas about hadronic sructure.

There are some models like the nonrelativistic quark models, MIT bag etc. which produce not only reasonable classification of hadronic states, but also produce mass spectra and other parameters (like magnetic moments) in qualitative agreement with experiments. However, they all operate with some objects, called constituent quarks, which are treated as meaningful dynamical degrees of freedom of the system. From the field theoretical point of view such description is very nontrivial, because each valence quark creates the cloud of virtual gluons and light quark pairs, and no reason is known so far for such clouds to interact relatively weakly. Some suggestions on this will be discussed below.

The most naive point of view is the idea that constituent quarks and the true (or current) quarks of the field theory are nearly the same. This is true for global properties like charge, color and flavor, but completely wrong in other respects. First, the constituent quark is treated as massive object (with mass $\sim$ 0.3 Gev), while current quarks u,d are nearly massless, which is the

origin of the important property of strong interactions, nearly exact chiral symmetry. Therefore, in order constituent quark idea makes sense the breakdown of chiral symmetry should happen. We return to this point later.

Also the very naive point of view expressed above contradicts some facts concerning the important role of glue in hadronic structure. For example, data on leading twist spin-2 operators tell us, that nearly one half of the nucleons momentum is carried by gluons.

An interesting attempt to explane the last point was made in the works [19]. Loosely speaking, they propose valence quarks to posess some cloud of virtual particles, but have tryed to calculate its properties perturbatively. More precisely, it was proposed that the naive three quark picture of the nucleon takes place in some low normalization point $\mu_0 \sim 1\ \text{fermi}^{-1}$. This suggestion has been shown to work, for example, for stress tensors: after transformation to low μ_0, the quarks really carry nearly all energy and momentum.

The aim of the present section is to estimate the average values of the next twist operators in the same manner and then confront the results with data. In advance we may say, that the result is rather disappointing: both sign and magnitude of the effect is wrong. Further discussion of this will be continued below.

We start with the diquark operators, for their connection with the naive quark picture is most simple. As it was shown in section 2.4, the mixing of these operators while transformed to low normalisation point μ_0 from $\mu \simeq Q$ is rather small and can be neglected. Using the properties of tree-quark wave function, we may strongly reduce the number of independent operators, classified in section 2.4. Starting with color, one may see that

$$\langle N|(\bar\psi_1 t^a \psi_2)(\bar\psi_3 t^a \psi_4)|N\rangle = \left(-\frac{8}{3}\right)\langle N|(\bar\psi_1 \psi_2)(\bar\psi_3 \psi_4)|N\rangle \quad (4.1)$$

The factor (-8/3) has very simple meaning. The length of "color spin" squared $(t^a)^2 = 16/3$ and the extra factor (-1/2) is nothing else that $\cos(120^\circ)$, the angle between three vectors added to zero in total. This sign will become important below.

The isospin matrices $\vec\tau$ can be reduced as follows:

$$\langle N|(\bar\psi_1 \vec\tau \psi_2)(\bar\psi_3 \vec\tau \psi_4)|N\rangle = (P_1 - 3P_0)\langle N|(\bar\psi_1 \psi_2)(\bar\psi_3 \psi_4)|N\rangle \quad (4.2)$$

with P_1, P_0 to be the probability to have diquarks with isospin 1 and 0. Nonrelativistic wave function tell us, that $P_0 = P_1 = 1/2$, and this is probably not far from reality.

What is left now is the spin structure, with two possibilities:

$$n_{LL} = \langle\langle(\bar\psi_L \gamma_\mu \psi_L)(\bar\psi_L \gamma_\nu \psi_L) + (L \to R)\rangle\rangle \quad (4.3)$$
$$n_{LR} = 2\langle\langle(\bar\psi_L \gamma_\mu \psi_L)(\bar\psi_R \gamma_\nu \psi_R)\rangle\rangle$$

These quantities have the dimension of mass squared and their physical meaning can be explained as follows: it is the probability to find quarks with similar or opposite chiralities in the same point in the plane transverse to momentum of the proton in infinite momentum frame. Longitudinal distance is dropped due to Lorentz contraction in this frame.

This observation leads to some simplified order-of-magnitude estimates of numerical value of (4.3). Let us neglect correlation of chirality and position of three valence quarks in the nucleon and consider them as being homogeneously distributed in some disk of radius R. Then the following estimate is obtained

$$n_{LL} \simeq n_{LR} \simeq \frac{1}{2} \cdot 3 \cdot \frac{1}{\pi R^2} \simeq 0.01 \text{ Gev}^2 \quad (4.4)$$

First factor 1/2 is due to chirality, second factor 3 is the number of quark pairs, and πR^2 is the disk area. Numerical value in (4.4) is obtained by the connection of R with mean square radius $R^2 = 2\langle r^2\rangle$, and the use of its experimental value.

Now we are able to give average value of any operator and, using our results of section 2.2 neglecting mixing and quark-gluon operators one finds the following results for scale breacking parameters α_2 (defined in (3.3))

$$\alpha_2^v \simeq -1.09\, n_{LL} + 0.25\, n_{LR} \simeq -0.007 \text{ Gev}^2 \quad (4.5)$$
$$\alpha_2^e \simeq 0.07\, n_{LL} - 0.90\, n_{LR} \simeq -0.008 \text{ Gev}^2$$

"Experimental" value of $\alpha_2^v \simeq \alpha_2^e$ found in section 3.2 is 0.15 Gev^2, so the disagreement in sign and magnitude is evident.

Rather peculiar example is the case of first moments in which we have only one quark-gluonic operator without any mixing possible. Its estimate in the naive quark picture can be given as follows. Gluonic field (being produced by other quarks) effects on the quark struck by the scattering. Inspection of the operator $C_\mu = \tilde{G}_{\mu\nu} \bar{\psi} \partial_\nu \partial_\gamma \psi$ shows that in the rest frame of the nucleon it is the naive Hamiltonian for perturbative spin splitting of hadrons, say the nucleon and the isobar. This results in the following estimate

$$\langle\langle O_\alpha \rangle\rangle \simeq - m_N (m_\Delta - m_N) \simeq - 0.3\ GeV^2 \qquad (4.6)$$

Comparing this with (3.1) we observe that now the sign agrees, but the value is 3 times smaller.

So, the conclusion of this section is that the naive quark picture is in contradiction with data on scaling violation.

4.2. The sign of the diquark contribution

We start this section with the proof that diquark operators under consideration are positively definite. This is evident for the operator

$$O_{\mu\nu} = (\bar{\psi}\gamma_\mu \psi)(\bar{\psi}\gamma_\nu \psi) - \frac{1}{4} g_{\mu\nu} (\bar{\psi}\gamma_\alpha \psi)(\bar{\psi}\gamma_\alpha \psi) \qquad (4.7)$$

for $\langle \vec{p}=0 | O_{00} | \vec{p}=0 \rangle$ matrix element can be written as the sum of full squares:

$$\langle \vec{p}=0 | \left[\frac{3}{4} (\bar{\psi}\gamma_0 \psi)^2 + \frac{1}{4} (\bar{\psi}\vec{\gamma}\psi)^2 \right] | \vec{p}=0 \rangle \qquad (4.8)$$

It is clear then that the introduction of γ_5 in each fermionic bracket, or t^a, or λ^a can not influence this general statement. Looking then at OPE formulae found in section 2.2, one may conclude that the contribution of diquark operators to scaling violation is positive, in agreement with data.

This conclusion seems to contradict the explicite estimates of the preceeding section, where the negative effect was found. Is here the violation of general positivity statement made above really present ?

The answer is negative: no violation takes place here. It is very instructive to clarify this point, for it also sheds light on what physical contribution is missing in the naive quark picture.

Note first, that minus sign appears in (4.1) due to color matrices, for the valence quarks have <u>different</u> colors. But where is the positive contribution from diquarks of <u>the same</u> color, which guarantees total positivity ? Even if there are no such quarks in the wave function, there exists the low momentum region of the loop diagram, which also should correspond to matrix element of the diquark operator. We did not included it in our naive quark calculation. Is this a mistake ?

The answer is again negative: the calculation was right. Remember our discussion at the end of section 1.2. The corresponding part of the loop diagram depends on the normalization point, and it should cancel power correction to the coefficient of the leading twist operator of the order μ^2/Q^2 . The existence of this additional contribution explains why there is no contradiction between the positivity of diquark operators and the negative sign of the scale breaking parameter in the naive quark approximation.

This consideration shows clearly where the naive quark picture goes wrong: it is in the assumption that low momentum part of the loop-type contribution can be estimated perturbatively. In order to explain data one should introduce large nonperturbative contribution to this average value, so that it is not canceled by much smaller correction to coefficient of the leading twist operator.

It is very important that the diagonal loop-type contribution has peculiar features, leading to observable predictions. Its flavor structure is the same as for the single quark operators, so it implies the validity of the well known relations

$$M_2^{\nu,S}(2,Q^2) = \frac{18}{5} M_2^{e,S}(2,Q^2) ; \quad M_2^{\nu,NS}(2,Q^2) = 6 M_2^{e,NS}(2,Q^2) \qquad (4.8)$$

not only for leading twist operators, but for next-to-leading ones as well. Data discussed in section 3.2 seem to support this result for $\alpha_2^{eS} \simeq \alpha_2^{\nu,S}$ indeed. Unfortunately, general accuracy of data is not high and this point needs further studies.

4.3. Constituent quark structure

As we have shown above, the perturbative approach to this problem gives completely wrong results, both in sign and magnitude. Much larger nonperturbative effect is present, which seems to be diagonal in flavor (or loop-type). In this section we are going to discuss what its physical origin may be.

First, one has to connect data with OPE results (2.19) simplifying the flavor structure in this "additive quark" manner:

$$\alpha_2 = \langle\langle \frac{5}{32} (\bar\Psi \gamma_\mu \Psi)(\bar\Psi \gamma_\nu \Psi) + \frac{1}{4} (\bar\Psi \gamma_\mu \gamma_5 \Psi)(\bar\Psi \gamma_\nu \gamma_5 \Psi) \rangle\rangle \simeq 0.15\,\mathrm{Gev}^2 \qquad (4.9)$$

This result imply very large diquark densities n_{LL} , n_{LR} (4.3), of the order of 0.1 Gev^2 which is at least one order of magnitude larger than those estimated for naive three quark picture of the nucleon. Two limiting pictures can be proposed in order to explain this result. The first one is most streightforward, it assumes that virtual quark pairs in the nucleon are uncorrelated but very numerous, in order to provide so large diquark density. The second possibility is that virtual pairs are not numerous, but very strongly correlated with valence quark positions, so that their relative density (which is in fact measured) is very high.

Already on the basis of our present knowledge of the leading twist physics we may give strong arguments against the former possibility. The reason is that the density of quark pairs is known by F_2 and F_3 structure function difference and it is by no means large. Therefore, we are left with the latter one.

So, let the constituent quark consists of the valence one together with correlated (in space) quark-antiquark pair distributed in some region of dimension $R_{c.q.}$. Then, in order to obtain the needed diquark density one needs that

$$R_{c.q.} \sim 1/(0.5 \div 1 \text{ Gev}) \qquad (4.10)$$

or constituent quark is rather small indeed. If so, many empirical observations about the hadronic structure obtain natural explanation.

First, being much smaller than the hadron itself, the constituent quark becomes reasonable dynamical object, as it was in fact considered for long time in hadronic spectroscopy. Second, the so called additive quark model is explained, as it was proposed on the basis of hadronic cross sections [22], quark penetration through heavy nuclei [23] etc. Note, that these models use quark-quark cross section of the order of

$$\sigma_{qq} \simeq \frac{1}{9}\,\sigma_{NN} \qquad (4.11)$$

quite in agreement with (4.10).

Concluding this point we may say, that many phenomenological facts point toward the existence of some substructure in the nucleon, with its scale several times smaller than the nucleon dimension. We would like to note, that our arguments, although consistent with earlier observations, are more strict and less model dependent.

Now, what can be the possible origin of two scales in ordinary hadrons ? While the hadronic dimensions are naturally identified with the scale of color confinement, we are going to connect the dimensions of the constituent quarks with another, also poorely understood phenomenon in QCD, the spontaneus breakdown of chiral symmetry leading to its nonzero mass. The

relation between chiral symmetry breaking and instanton physics was discussed in works [24,25]. Although the problem is far from being solved, some large numerical factors producing new scale really seem to appear in it. To give an example, the expression of effective mass is [25]

$$m_{eff} = \frac{2\pi^2}{3}\langle 0|\bar{\psi}\psi|0\rangle\, \rho^2$$
$$\langle 0|\bar{\psi}\psi|0\rangle \simeq -(0.24 \text{ Gev})^3 \qquad (4.12)$$

where ρ is the instanton radius. For needed $m_{eff} \simeq 0.3$ Gev rather small $\rho \lesssim 1/500$ Mev instantons are sufficient. Similar observation was also made in [24]. Such dimensions are again close to constituent quark dimensions proposed above, see (4.10).

Summarizing the present chapter we may say, that data on scaling violation together with our OPE results give such values for certain diquark operators over the nucleon, which imply the existence of quark clusters of size nearly 1/3 of that of the nucleon. They are naturally identified with constituent quarks. The arising physical picture explains many phenomenological observations concerning hadronic structure and interactions.

5. Summary and discussion

The present paper is the first one devoted to systematic OPE calculations of power corrections to deep-inelastic scattering. So it is clear that the first questions we came across were the methodical ones. The standard perturbative diagrams are very inadequate for OPE calculations, for any diagram contains the contribution from all distances, large and small. Also they are not gauge invariant by itself, only in sum.

We believe Schwinger formalism used in our work is highly effective and very adequate for such calculations. We hope for its use in other related problems and for further calculations of power corrections.

Which suitible method, one has to face then the complexity of the problem by itself. The widespread opinion that power corrections are very complicated has caused some pessimism about the use of further studies of deep-inelastic scattering. It is true, that in principle many different effects cause power corrections, but the point is that in reality few of them are impor tant. And we believe that deep-inelastic scattering still remains the best way to look for more and more information about hadronic structure.

However, not too much can be found from the present data. We have discussed them in chapter 3, and in fact the only reliable estimate with, say, 50% accuracy can be made for scaling violation parameter for $F_2^S(x,Q^2)$. But even this information turns out to be very important. It can not be explained by the naive quark picture in which nucleon is made of three valence quark. More dense quark cloud - the constituent quark -- should be present inside the nucleon, with

dimensions about 1/3 of that for the nucleon. This result is of great importance for hadronic physics in general. Although such picture of the nucleon was susspected from data on hadronic collisions [22,23], and hadronic spectroscopy, it was always based on some model-dependent arguments. In our work the only basis is QCD itself and experimental data, so the conclusion is much more strict. We call for more attention of experimentalist to this question, for better data at moderate Q^2=1-10 Gev^2. It is proper time now to come from the question "whether QCD is right?" to "How the nucleon is made?"

Now, in summary, what is the physical reason for scaling violating Q^{-2} effects? As far as we understand them now it is the <u>interference between scattering over one and two current quarks inside the same constituent quark.</u> Interference with scattering accompanied by absorbtion or emission of gluon with large ($\sim Q$) momentum can also contribute, but its role is not so far clear.

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Figure captions

1. Diagrams contributed to coefficients for $\bar{\Psi} G \Psi$ type operators.

2. Compton effect diagrams.

3. Diagrams for Compton scattering on two quarks.

4. Diagrams for renormalization of quark-gluon operators.

5. Diagrams for renormalization of diquark operators.

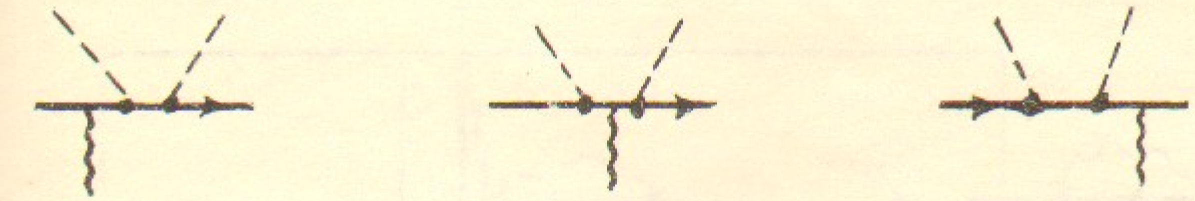
6. Deviations from GLS sum rule [17] as measured by BEBC-GGM groups [5]. The dashed lines are perturbative corrections with Λ given at Figure, the solid line includes also power correction of magnitude (3.1).

7. Index of scaling violation (3.2) for the second moment of F_2^S for weak [5] and electromagnetic [7] deep-inelastic scattering. Dotted line is perturbative contribution with Λ =0.1 Gev, the dashed one includes power corrections (3.4).

8. The same as at Fig.7, but for the sixth moment.

9. The same as at Fig.7, but as a function of moment number n at fixed Q^2=3.7 Gev^2.

10. Ratio of integrated over x longitudinal and transverse structure functions for MIT-SLAC data [12] and neutrino data (taken from compillation [20]). Dotted line is the perturbative contribution with Λ=0.1 Gev, dash-dotted one is that of diquark twist-4 contribution, and the solid line is empirical Q^{-4} fit.



(a) (b) (c)

Fig 1

Fig 2

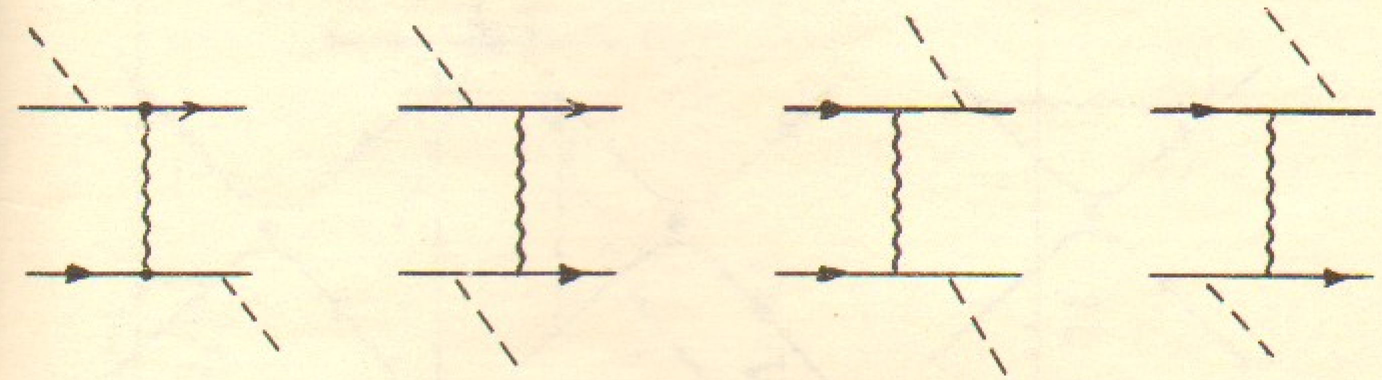


Fig.3

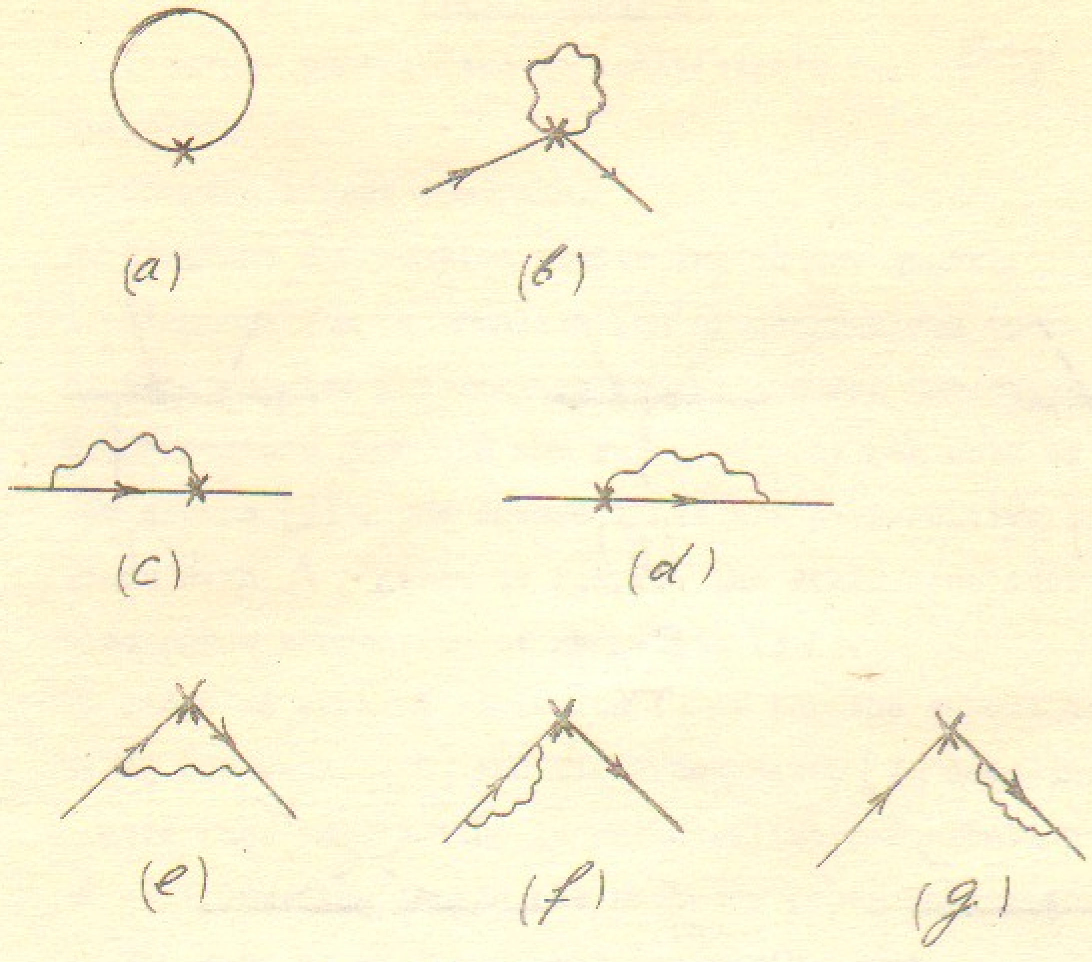


Fig. 4

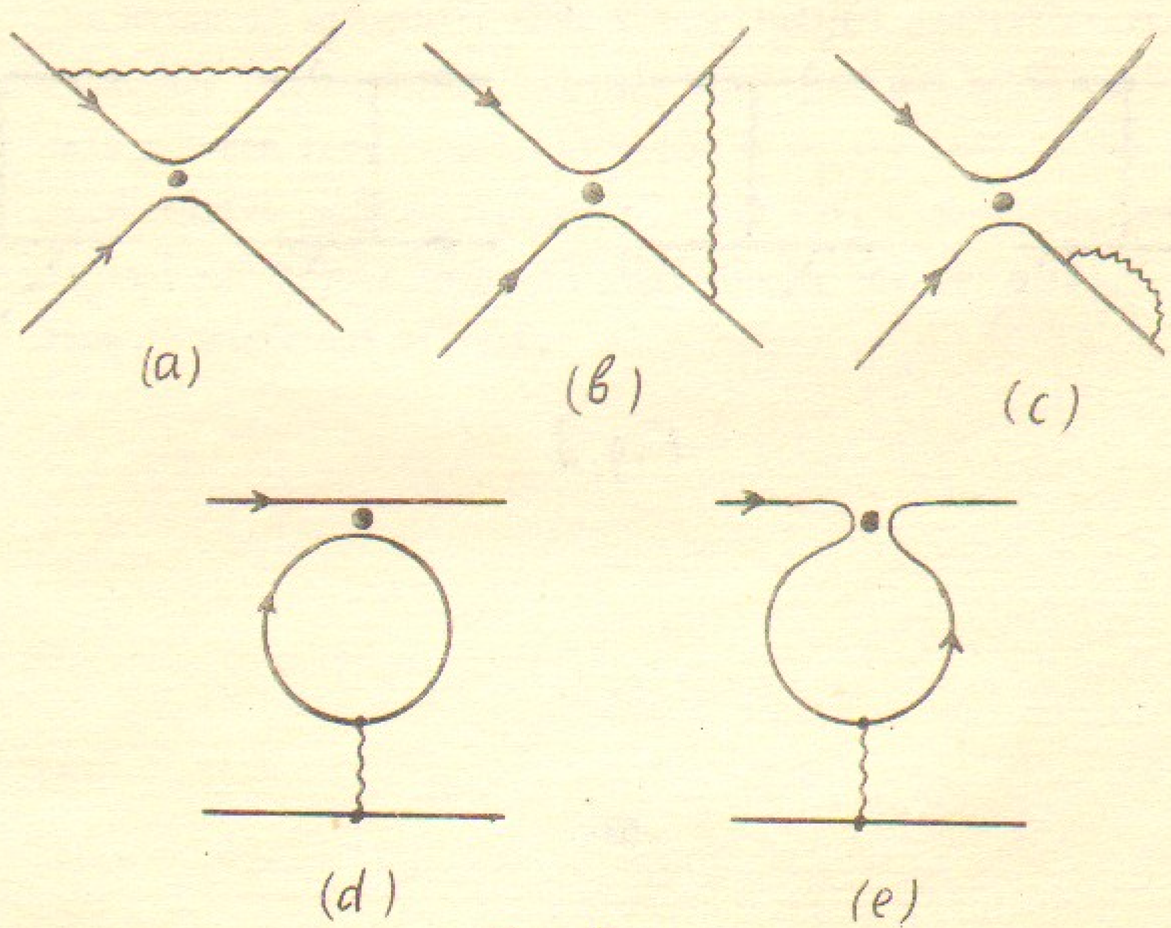


Fig. 5

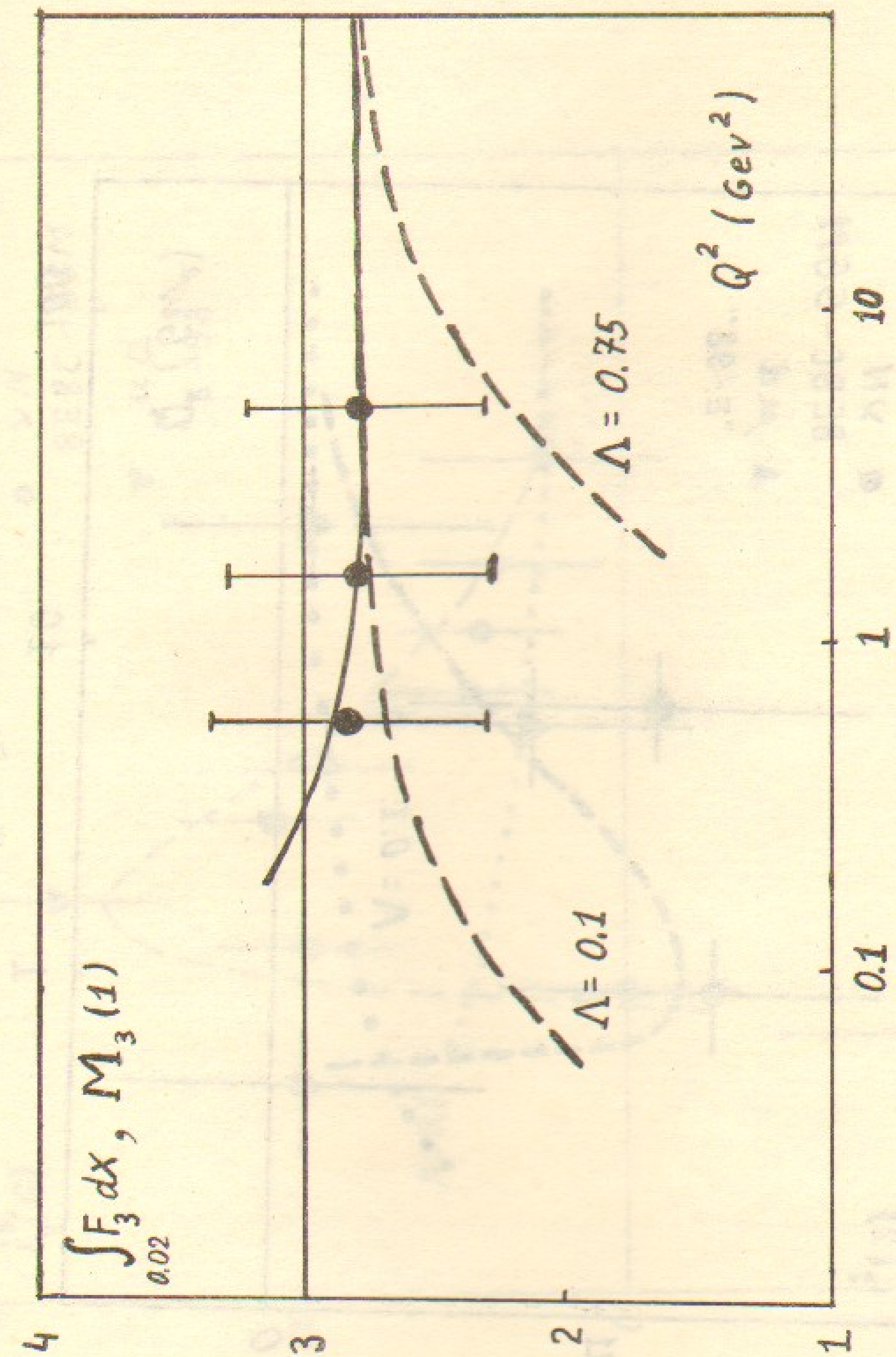
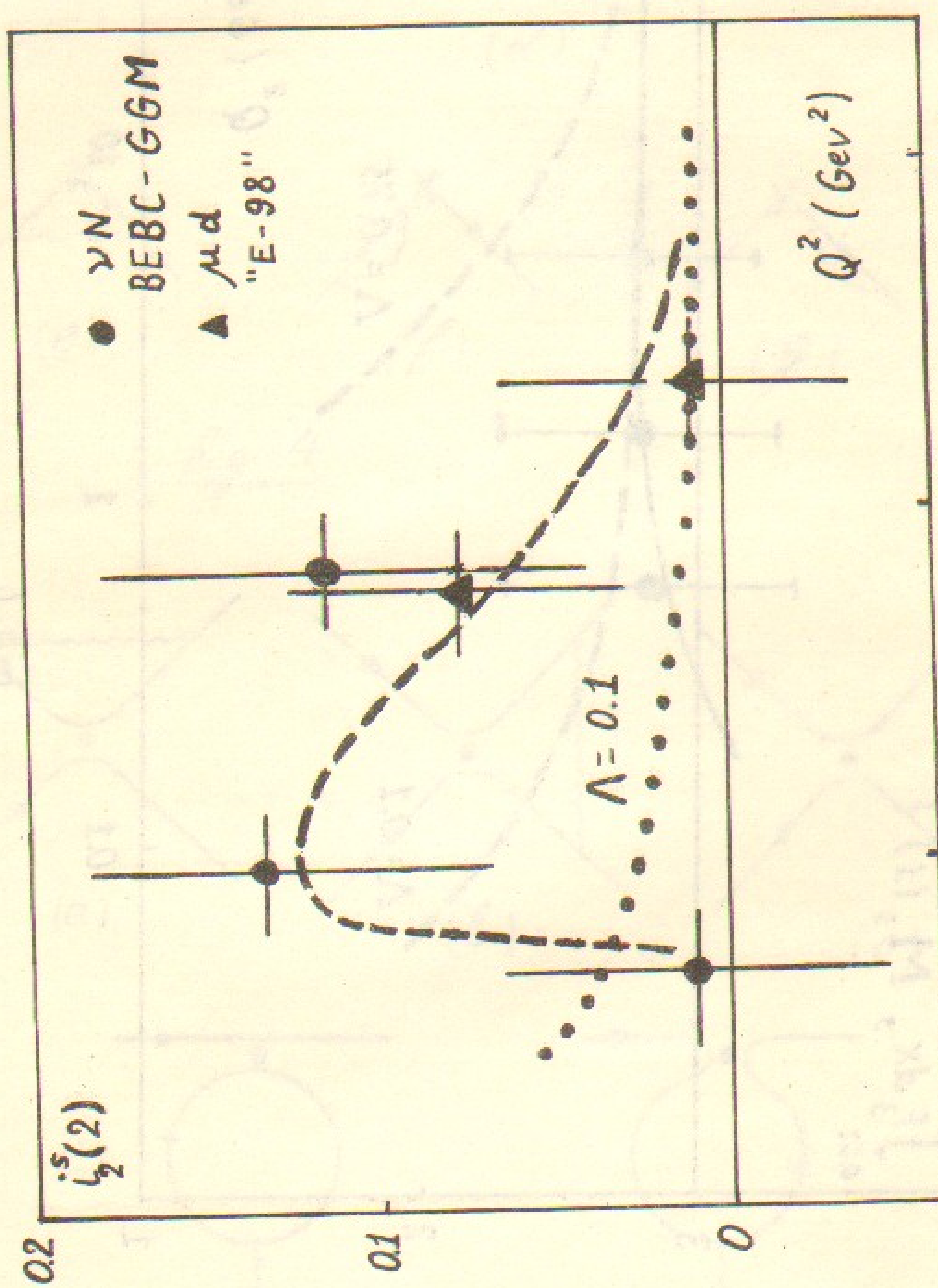
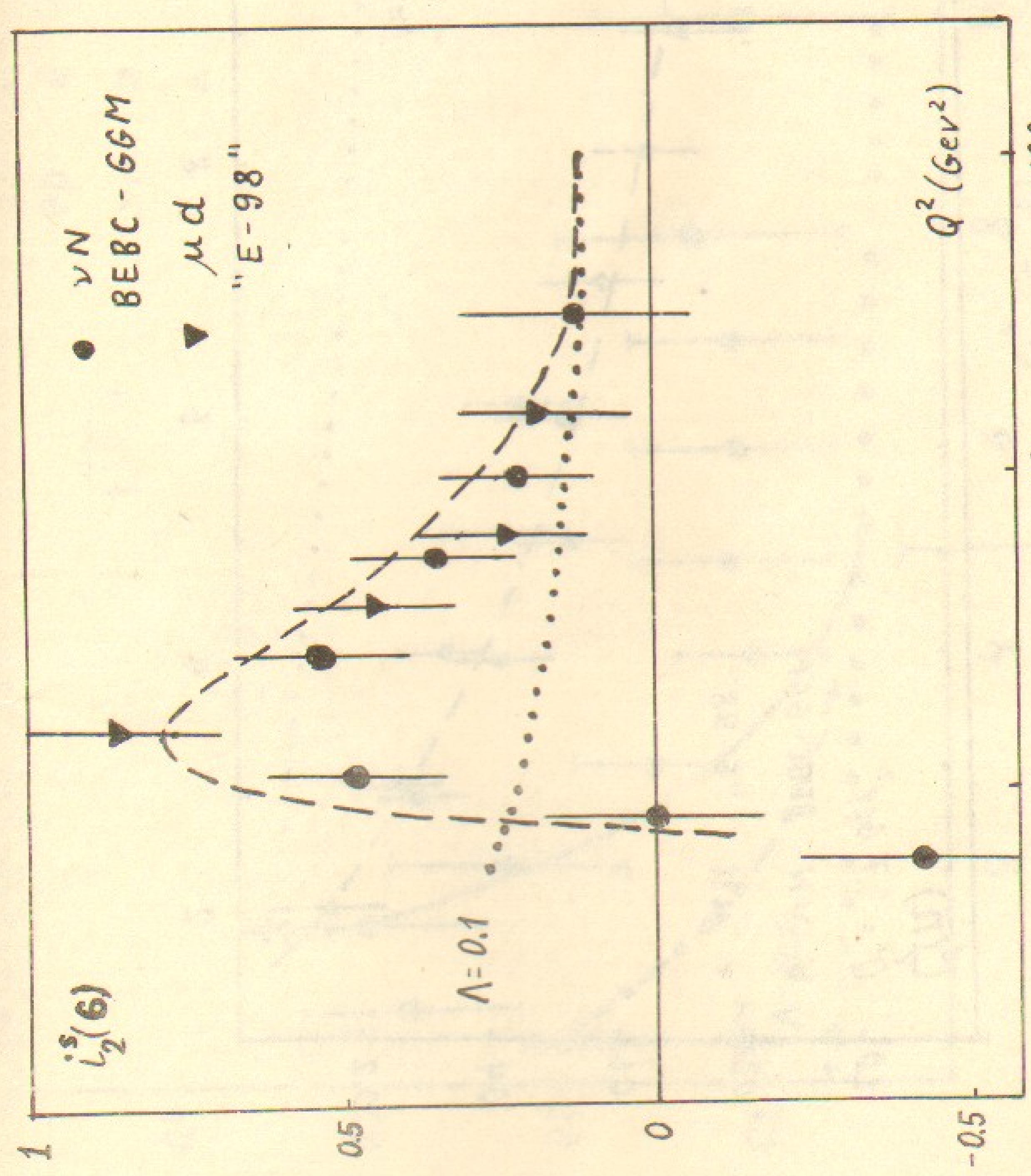


Fig. 6



$i_2^s(2)$
νN
BEBC-GGM
μd
"E-98"
Λ= 0.1
Q^2 (Gev^2)
0.2
0.1
0
1
10
100

Fig.7



$i_2^s(6)$
νN
BEBC-GGM
μd
"E-98"
Λ= 0.1
Q^2 (Gev^2)
1
0.5
0
-0.5
1
10
100

Fig.8

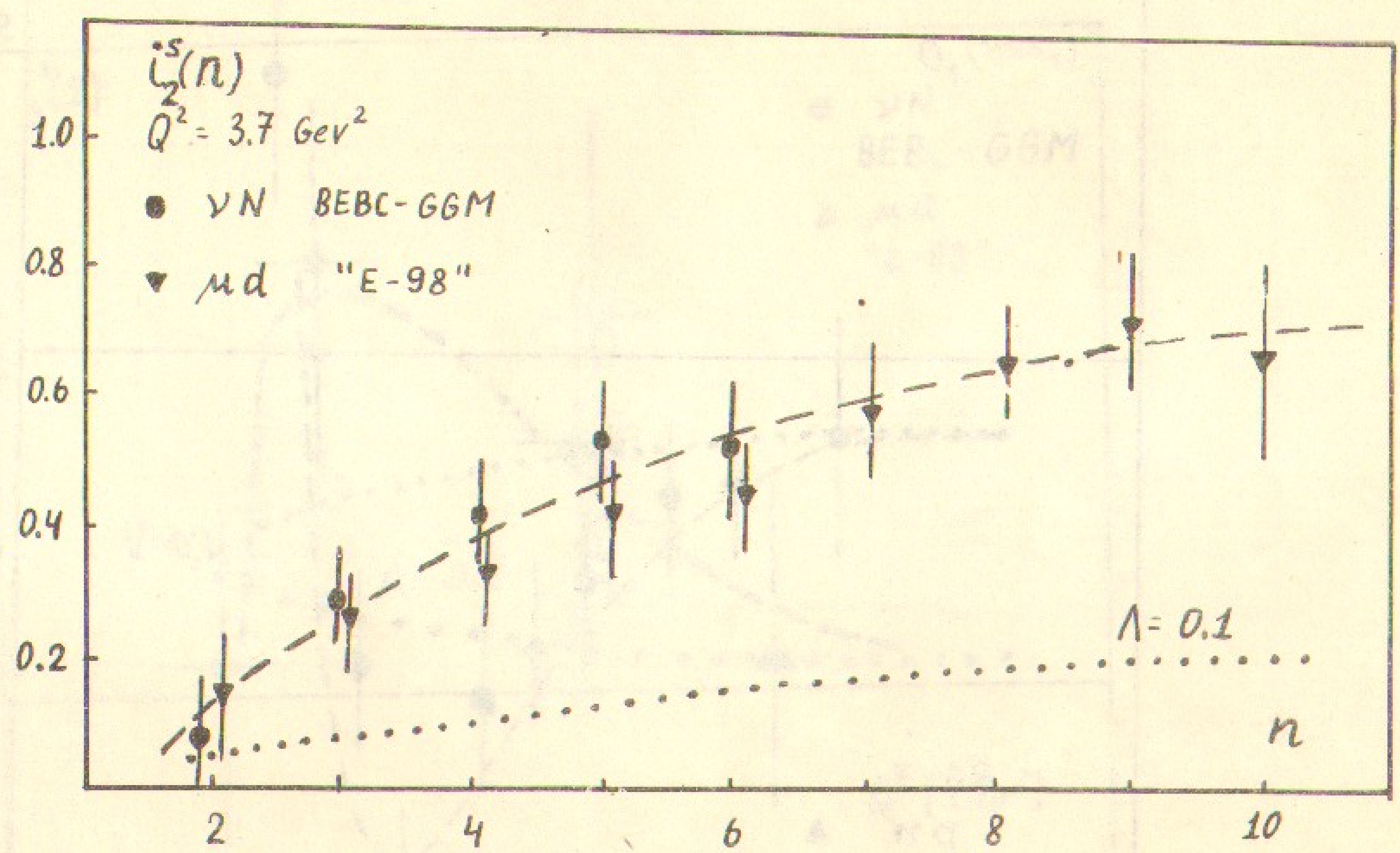

$\overset{s}{\zeta}_2(n)$
$Q^2 = 3.7\ Gev^2$
● νN BEBC-GGM
▼ μd "E-98"
1.0
0.8
0.6
0.4
0.2
$\Lambda = 0.1$
n
2
4
6
8
10

Fig.9

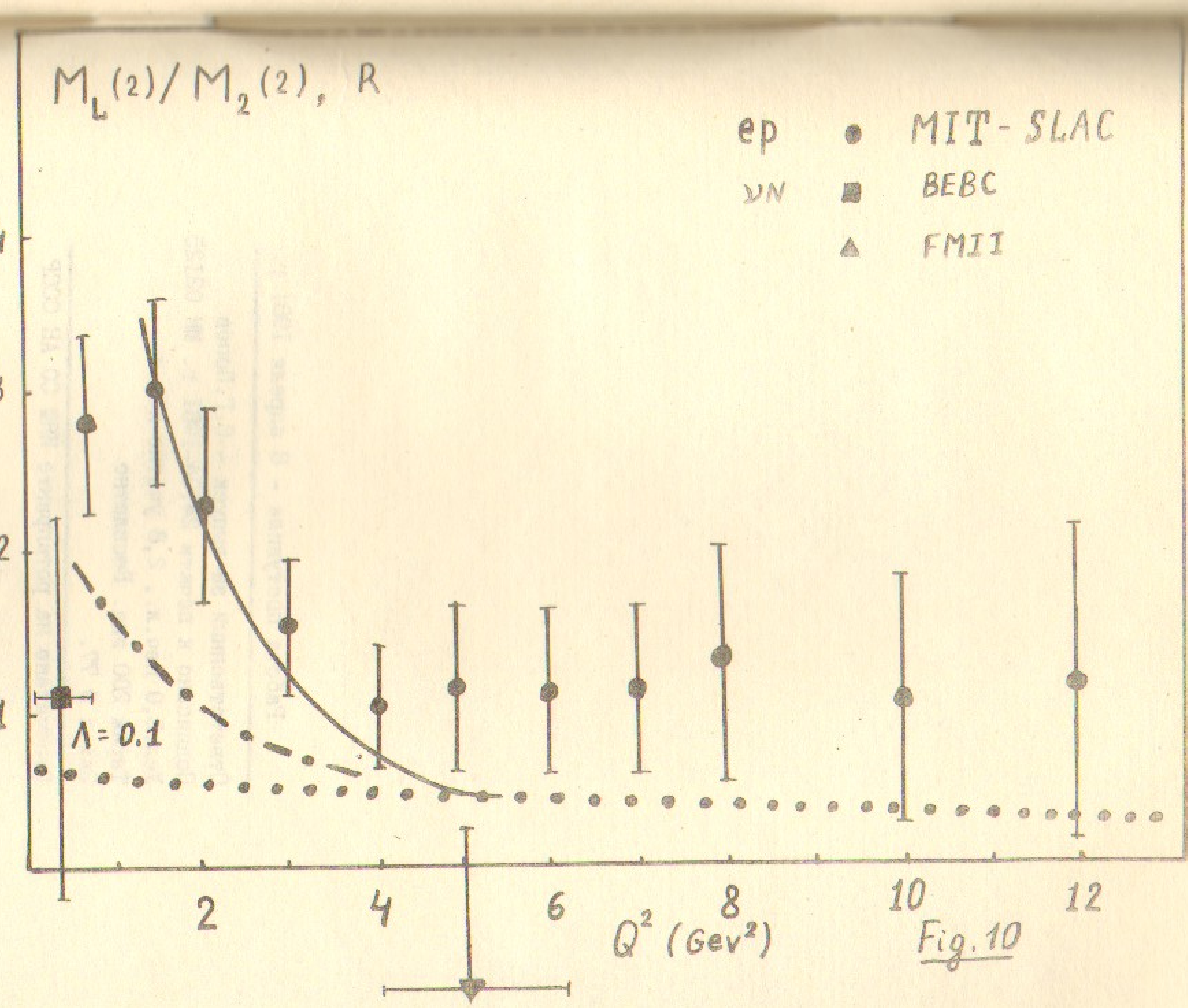

$M_L(2)/M_2(2)$, R
ep ● MIT-SLAC
νN ■ BEBC
▲ FMII
0.4
0.3
0.2
0.1
$\Lambda = 0.1$
2
4
6
8
10
12
Q^2 (Gev^2)

Fig.10