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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
СО АН СССР

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HADRONS CONTAINING A HEAVY QUARK AND QCD SUM RULES

ПРЕПРИНТ 81-67

Новосибирск

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Abstract

Masses and other parameters of mesons and barions containing one heavy (c, b, ...) quark are calculated by the method of QCD sum rules developed by Shifman, Vainshtein and Zakharov. In our case the calculations simplify strongly because heavy quark acts like static centre similar to proton in hydrogen atom. Analysis of the sum rules is more difficult, for continuum contribution is more important. Brief discussion of ordinary barions — the nucleon and isobar — is also given.

1. Introduction

The understanding of hadronic world has started with the discovery of the isotopic symmetry and, less accurate, $SU(3)_F$ symmetry of hadrons made of light u,d,s quarks. As it becomes clear now, these symmetries are not due to similarity of the quark masses, but to the fact that they are too small to be important.

Similar phenomenon takes place if some quark mass is too large to be important. Families of similar hadrons should exist, the difference between their members being only the substitution of one heavy quark by another one. An attempt to put this idea to more quantitative ground has been made in my work [1] , where the limit $M_Q \to \infty$ (M_Q is the mass of heavy quark Q) was discussed as well as $O(1/M_Q)$ corrections. In this limit hadrons with one heavy quark resemble hydrogen atom with its fixed center, and many problems of current models of hadronic structure (e.g. that of C.M. motion) are trivialised. Mesons made of one very heavy and one light quark are, in some sense, the simplest hadrons in which nontrivial QCD dynamics is essential, so their studies are of great- importance. Of course, mesons made of two very heavy quarks are simpler, but they are next to trivial.

In the present work I attack the same problem, but with the help of such powerful and fundamental method as that of QCD sum rules developed By Shifman, Vainshtein and Zakharov with colloborators. The applications of this method to charmonium [2,3] , classical mesons π, ρ [2] , gluonium [4] and barions [5,6] have significantly affected our understanding of hadronic structure. Very deep relations with QCD vacuum structure has been demonstrated, and new classification seems to appear, see [7].

Compared with these applications, the problem under consideration has its particularity. The presence of heavy quark significantly simplifies the calculations. At the same time, it is more difficult to single out the contribution of lowest states which are of main interest.

Only one work has so far been done in this direction, published only as short conference report [8] full of misprints. We use some methodical inventions such as the limit $M_Q \to \infty$ and Borel improvement of sum rules. Our analysis is more complete, we give not only upper limits, but definite predictions for D,B meson decay constants, and consider wider range of channels including the barionic ones. We also consider in section 7 some questions concerning ordinary barions.

The paper is organized as follows. Section 2 deals with general facts about two current correlators in perturbation theory, while the nonperturbative corrections are discussed in the section 3. Borel improvement and final form of mesonic sum rules are discussed in section 4. Their analysis is made in section 5. In the section 6 we come to barionic currents. In section 7 we deviate somehow from our main

topic and discuss some questions concerning the theory of ordinary hadrons, namely the nucleon and the isobar. The reason is that it in many respects resembles analysis made in section 5. We also make critical remarks about analysis made in the first works [5,6] on the subject. Then we come to barions containing heavy quarks. Section 9 contain summary and discussion of further calculations and improvements, to which we hope to return in next publications.

Before we complete the Introduction, few words about our convention for particle names. As far as no generally accepted classification is available, we follow the following principle: all names are those for strange particles with the subscript, showing which quark substitute the strange one. In particular, we discuss the following particles:

$$K_Q(0^-),\ K_Q^*(1^-),\ \varkappa_Q(0^+),\ Q_Q(1^+)$$

$$\Lambda_Q(1/2^+,0),\ \Lambda_Q^*(3/2^+,0),\ \Sigma_Q(1/2^+,1),\ \Sigma_Q^*(3/2^+,1)$$

where we have shown also quantum numbers J^P and isospin I (for barions). Note, that for barions such names are already used, for K_Q there are proper names K_c=D, K_b=B and we met no discussion of $\varkappa_Q$, Q_Q. Note, that the use of familiar names is useful, but any analogy has its limits. In particular, unlike the case of strange hadrons, no symmetry between light and heavy quarks exist, so the classification in familiar SU(3) multiplets is meaningless.

2. Correlators of mesonic currents

The basis of the method of QCD sum rules is the calculation of the correlators of certain currents in close space-time points. Due to asymptotic freedom, such calculation is possible in perturbation theory, and with the help of Wilson operator product expansion (OPE) it can also be done for nonperturbative effects. The next step is the use of dispersion relations, which connect such correlators with physical spectral density with corresponding quantum numbers. Of particular interest are the lowest states in each channel, being the hadronic states under consideration.

For mesons with one heavy quark Q and one light antiquark q we have the following local currents to be discussed

$$j^{PS} = i\bar{q}\gamma_5 Q \quad (0^-) \qquad j^{S} = \bar{q}Q \quad (0^+) \tag{1}$$

$$j^{V} = \bar{q}\gamma_\mu Q \quad (1^-) \qquad j^{A}_\mu = \bar{q}\gamma_\mu\gamma_5 Q \quad (1^+)$$

where we show also their quantum numbers J^P. Note, that if both quarks are very heavy ones, then only negative parity currents survive. Quarks u,d are very light (we never include m_q and consider massless case), but still there is great difference between positive and negative parity states.

The definition of currents becomes complete in field theory when we have specified normalisation scale for it. Change in this scale leads to renormalisation of fields,

and also of currents made of them. Such expressions are known for momenta much larger any mass $Q^2 \gg M_Q^2$, where the result is given by known anomalous dimensions

$$j^J(Q) = \left(\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right)^{d_J} j^J(\mu) \tag{2}$$

$$d_V = d_A = 0 \; ; \quad d_{PS} = d_S = -4/9$$

Here μ is the normalisation point in which the fields can be considered as free ones. For momenta smaller than M_Q we are in the nonrelativistic region without the logarithmic renormalisation, so in this region (to be considered) one may change $Q \to M_Q$ in (2). Multiplying by the factor (2) we obtain renormalisation invariant current (independent of μ). Of course, this is the quantity connected with physical spectral density.

With these comments we are ready to put M_Q to infinity averywhere outside logs, which significantly simplify the problem.

The correlator we are discussing is defined as follows

$$K(q) = i\langle 0|\int T(j(x)j^+(0))\, e^{iqx}\, dx\,|0\rangle \tag{3}$$

At large $Q^2 = -q^2$, corresponding to small x, the main contribution is given by free propagation of quarks corresponding to the diagram Fig.1(a). For its calculation we use dispersion relation to be used throughout the whole work

$$K(\Delta) = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\,K(E)\,dE}{E+\Delta} + (\overset{\text{subtr.}}{\text{polynomial}}) \qquad (4)$$

Here Δ is virtuality, or missing energy counted from the physical threshold at quark level, M_Q in our case. The so called subtraction polynomial which has no imaginary part is also shown. Up to this polynomial the spectral density of real states $\mathrm{Im}\,K(E)$ determins correlator itself.

It is quite trivial to calculate imaginary part of the contribution of loop diagram Fig.1(a):

$$\mathrm{Im}\,K_0^{PS} = \mathrm{Im}\,K_0^{S} = \frac{3}{2\pi}E^2 \qquad (5)$$

Here E is energy above threshold, and the limit $M_Q \to \infty$ is implied. For vector and axial currents one may introduce invariant structures as follows

$$K_{\mu\nu} = (q_\mu q_\nu/q^2 - g_{\mu\nu})K_\perp + (q_\mu q_\nu/q^2)K_\parallel \qquad (6)$$

and their imaginary part coincides with (5).

There are perturbative corrections, for example those of the order α_s due to diagrams Fig.1(b-d). Their calculation is although routine but combersome bisiness. Fortunately, this result can be taken from recent paper [12] and the correction factor to (5) is as follows:

$$1 + \left(3.0 + 0.6\ln\frac{M_Q}{\Delta}\right)\alpha_s(M_Q^2) \qquad (7)$$

It should be noted that in $1/M_Q$ order one finds splitting in total momentum J, but no splitting in parity P takes place in perturbation theory. As far as experimentally they are completely different, the nonperturbative effects should be important.

3. Power corrections

Account for these effects, being due to nontrivial vacuum structure in QCD, is the most important ingredient of the method developed by Shifman, Vainstein and Zakharov.

The simplest and most essential effect of the kind in our problem is the interaction with quark vacuum condensate due to diagram shown at Fig.2a. It is computed trivially and in our case ($M_Q \to \infty$) the resulting corrections to the correlator K is

$$\delta K^{PS} = \delta K_\perp^V = -\delta K^S = -\delta K_V^A = -\frac{1}{2\Delta}\langle 0|\bar{q}q|0\rangle \qquad (7)$$

Note that it really splits channels with different parity, which is needed. At the same time, it does not split say pseudoscalar and vector channels. In this point the work [8] contains an important mistake, $\delta K_\perp^V$ has the wrong sign. This mistake becomes evident in the limit $M_Q \to \infty$ we use for in this case the direction of the spin of heavy quark (the only difference between these channels) is of no importance. Clear, that K_Q and K_Q^* are degenerate in such limit.

Calculation of further corrections is more difficult for it is connected with gluon field. The most adequate method for such calculations was developed in the works [9, 10], and it is based on Schwinger formalism for treatment of processes in some arbitrary external field. We are not able to give any details here and only say that some formal coordinate basis $|x\rangle$ is introduced in which momentum operator P_μ acts as covariant derivative. In our limit $M_Q \to \infty$ corrections to (8) are given by the following expression

$$\delta K^{PS} = \frac{1}{2}\int dx \langle x|\bar{q}\,\gamma_5\left(\frac{\gamma_0+1}{-\Delta+P_0}\right)\gamma_5 q|0\rangle \tag{9}$$

It corresponds to the same diagram Fig.2a, but in some external field. Operator P_0 is small compared with c-number virtuality Δ, so in first approximation the result coincides with (8). Expanding the propagator in P_0/Δ as

$$\bar{q}\,\frac{1}{\Delta-P_0}\,q = \frac{1}{\Delta}\bar{q}q + \frac{1}{\Delta^2}\bar{q}P_0 q + \frac{1}{\Delta^3}\bar{q}P_0^2 q + \ldots \tag{10}$$

one has some serie in <u>local</u> operators $\bar{q}P_0 \ldots P_0 q$ with definite coefficients. This expression is then nothing else than OPE by Wilson.

Only zero spin operators have nonzero vacuum average values. So, one may write the first correction as follows

$$\langle 0|\bar{q}P_0^2 q|0\rangle = \frac{1}{4}\langle 0|\bar{q}P^2 q|0\rangle = -\frac{1}{16}\langle 0|\bar{q}(ig\sigma_{\alpha\beta}G^a_{\alpha\beta}t^a)q|0\rangle \tag{11}$$

where we have used the equations of motion $\hat{P}\hat{P}q = (P^2 + \frac{ig}{4}(\sigma G t))q = 0$. As a result, the following correction to current correlators arizes

$$\delta K^{PS} = \delta K^V_\perp = -\delta K^S = -\delta K^A_\perp = \frac{1}{32\Delta^3}\langle 0|\bar{q}(ig\sigma_{\alpha\beta}G^a_{\alpha\beta}t^a)q|0\rangle \tag{12}$$

Analogous but more lengthy calculations give further correction; where $\mathcal{D}_\mu G^a_{\mu\nu} = -\frac{g}{2}\bar{q}\gamma_\mu t^a q$ has been used

$$\delta K^{PS} = \frac{g^2\langle 0|(\bar{q}\gamma_\mu t^a q)^2|0\rangle}{3\cdot 2^7\Delta^4} \tag{13}$$

Corrections to other correlators are the same up to common sign, which is the same as in (8).

Another serie of corrections are due to diagram Fig.1a in external field. The first possible operator here is the so called gluon condensate $\langle 0|g^2(G^a_{\mu\nu})^2|0\rangle$. However, explicite calculation along the line discussed in details in [10] give zero result in the limit $M_Q \to \infty$, so we do not discuss this point in the present work. Further corrections are too small, because in general the loop diagrams contain small numerical factor of the type $1/4\pi^2$.

Now, what are the values of the parameters $\langle 0|\bar{q}q|0\rangle$ etc?

Stricly speaking, they depend on vacuum structure which is not so far understood and we may take them only from experiment. However, as discussed in [2] the very interesting phenomenology of such average values can be discussed in various approximations such as heavy-to-light quark matching, instanton calculus etc. Not going into this, we just list our guess for these quantities

$$\langle 0|\bar q q|0\rangle \simeq -(220\,\mathrm{MeV})^3$$
$$\langle 0|\bar q\,(ig\sigma G t)\,q|0\rangle \equiv m_0^2\langle 0|\bar q q|0\rangle\ ;\quad m_0^2 \simeq 0.5\,\mathrm{GeV}^2 \qquad (14)$$
$$\langle 0|(\bar q\gamma_\mu t^a q)^2|0\rangle = -\frac{16}{9}\left(\langle 0|\bar q q|0\rangle\right)^2$$

Note, that we have chosen slightly smaller value of $\langle 0|\bar q q|0\rangle$ than in [2] where $\langle 0|\bar q q|0\rangle = -(240\ \mathrm{Mev})^3$ for it makes the agreement better in all cases considered.

4. Sum rules

Now everyting is ready to formulate the final form of the sum rules. They are the dispersion relation (4), in which now we are going to determine spectral density from the calculated correlator.

For such a program the so called Borel improvement of the sum rules (saggested in [2]) is very useful. It is

the functional transformation defined as follows

$$Bf(\Delta) = \lim_{\substack{n\to\infty\\ \Delta\to\infty\\ \Delta/n = m = \text{fixed}}} \left[\frac{\Delta^{n+1}}{n!}\left(-\frac{d}{d\Delta}\right)^n\right] f(\Delta) \qquad (15)$$

First, being applied to dispersion relation it cancel the subtraction polynomial. Then, note the following transformation rules:

$$B\left(\frac{1}{\Delta+E}\right) = \exp(-E/m)$$
$$B\left(1/\Delta^k\right) = \frac{1}{(k-1)!}\ \frac{1}{m^{k-1}} \qquad (16)$$

The former one shows that the dispersion relation with the weight $1/(\Delta+E)$ turns into relation with exponential weight, much more sensitive to lowest states. Due to the latter relation, higher power corrections become numerically suppressed, so the theoretical predictions become more accurate.

After Borel transformation is made, the sum rules for mesonic currents look as follows

$$\frac{3m^3}{\pi^2} + P\left(\frac{\langle\bar q q\rangle}{2} - \frac{\langle\bar q(ig\sigma G t)q\rangle}{32\cdot 2!\,m^2} - \frac{g^2\langle(\bar\psi\gamma_\mu t^a\psi)^2\rangle}{384\cdot 3!\,m^3} + \ldots\right) = \qquad (17)$$
$$= \frac{1}{\pi}\int_0^\infty dE\, e^{-\frac{E}{m}}\,\mathrm{Im}\,K(E)\cdot\left(\log\frac{M_0^2}{\Lambda^2}\Big/\log\frac{M^2}{\Lambda^2}\right)^{2d_j}$$

where P in (17) is parity of the current. Note that we have divided the r.h.s. by the current anomalous dimansion. Strictly speaking, all operators should be treated in the same way, say $\langle\bar{q}q\rangle$ should be understood as $\langle\bar{q}q\rangle_\mu\cdot(\alpha_s(M_Q^2)/\alpha_s(\mu^2))^{4/9}$ However, such factors are too slowly varying to affect our results.

The last question is that up to what m this expression for correlator can be trusted. In this respect the perturbative corrections are of interest, because all power ones seem to be reasonably estimated and they converge rapidly. As it will become clear later, perturbative correction is reasonable for it becomes important approximately in the point where l.h.s. of (17) start to deviate from the r.h.s. In what follows, we consider $m > 400$ Mev as the reasonable region for the validity of the sum rule (17) with sufficient accuracy.

5. Sum rule analysis

Having fixed the sum rules for mesonic currents, we are ready to look for the information about the corresponding physical spectral density.

The lowest state in any channel is some mesonic state with the quantum numbers of the current. Due to confinement, it is separated from the quark threshold M_Q by finite gap E_2. Its contribution intu spectral density depends on the matrix elements defined in the standard way as follows:

$$\langle K_Q|\bar{Q}\gamma_\mu\gamma_5 q|0\rangle = -i f_{K_Q} P_\mu$$
$$\langle K_Q^*|\bar{Q}\gamma_\mu q|0\rangle = \epsilon_\mu \frac{M_{K_Q^*}^2}{g_{K_Q^*}}\ ;\ f_{K_Q^*} \equiv \frac{M_{K_Q^*}}{g_{K_Q^*}} \quad (18)$$

and is equal to

$$\operatorname{Im} K_2^{PS}(\Delta) = \frac{\pi}{2} f_{K_Q}^2 M_{K_Q}\,\delta(E-E_2) \quad (19)$$

Note, that the theoretical part of the sum rules has the finite limit $M_Q \to \infty$, and (19) also should be finite in this limit. Another way to see that $f_K^2 M_K \to const$ is the following: let the light quark be nonrelativistic one, so that the ordinary wave function of it exists. Then, one finds that

$$f^2 M = 12\,n\ ;\quad n \equiv |\Psi(0)|^2 \quad (20)$$

Here we have introduced new quantity, the density of light quark on the heavy one n as the important parameter of bound mesonic states. In what follows we use notation (20) for relativistic quarks as well.

Already at this level some results for resonance parameters E_2, n can be obtained due to positively defined contribution of other physical states, as it was done in [8]. This property demands that the contribution of the resonance to (17)

$$6n\exp\left(-\frac{E_2}{m}\right) \quad (21)$$

is smaller than the theoretical expression (17) for any m. For example, for negative parity case it gives the inequality

$$n \exp\left(-\frac{E_2}{0.4}\right) \lesssim 1.6 \cdot 10^{-3}\ \mathrm{GeV}^3 \quad . \qquad (22)$$

The asymptotics of spectral density at large energy is also known on general ground, it tends to that for free quarks. This fact is evident from the sum rules themselves, for the simple loop is the leading contribution at large m. Following [2] , we may write it as follows

$$\mathrm{Im}\, K(E) = 6\pi n\, \delta(E-E_2) + \frac{3}{2\pi} E^2 \theta(E-E_c) + \mathrm{Im}\,\delta K \qquad (23)$$

where the undefined $\mathrm{Im}\,\delta K(E)$ is small both for large and small E . It contains next resonances, threshold structures etc. Important, that it is separated by finite gap from E_2 and corresponds to $K_Q\pi$,... multiparticle states. All contribution apart from first resonance we call "continuum" in what follows. We also move the term with θ-function to the left side of the sums rules in order to cancel large but trivial term $O(m^3)$. Without it, the limit $m \to \infty$ can be taken, producing the so called duality relation

$$\frac{E_c^3}{2\pi^2} - \frac{1}{2}\langle 0|\bar{q}q|0\rangle = 6n + \frac{1}{\pi}\int_0^\infty \mathrm{Im}\,\delta K(E)\, dE \qquad (24)$$

Usually the relations of this kind are interpreted as the equality of quark cross section to that "eaten up" by the resonance. Note, that we have obtained the finite nonperturbative correction to such naive duality arguments.

The last step before the analysis results are reported is the choice of convenient dimensionaless variables. If we define the scale as follows

$$\varkappa \equiv \left(-\frac{\pi^2}{6}\langle 0|\bar{q}q|0\rangle\right)^{1/3} \simeq 260\ \mathrm{MeV} \qquad (25)$$

the sum rules can be written as

$$x^3\left(1-\exp\left(-\frac{\varepsilon_c}{x}\right)\right)\left(1+\frac{\varepsilon_c}{x}+\frac{\varepsilon_c^2}{2x^2}\right)\left(1+3\alpha_s+0.6\ln\frac{M}{m}\alpha_s+\ldots\right) + \qquad (26)$$
$$+ P\left(1-m_0^2/32\varkappa^2x^2\right) = \frac{2\pi^2 n}{\varkappa^3}\exp\left(-\frac{\varepsilon_2}{x}\right)$$
$$x \equiv m/\varkappa\ ;\ \varepsilon_c = E_c/\varkappa\ ;\ \varepsilon_2 = E_2/\varkappa$$

Some unimportant terms are omitted here as well as $\mathrm{Im}\,\delta K(E)$ (see below).

Analysis under discussion is just the fit of the l.h.s. to single resonance in r.h.s. by finding the most suitible parameters $\varepsilon_c, \varepsilon_2, n$. For definiteness, the accuracy of the theoretical calculations should be introduced. In the region $m > 400$ Mev we consider the main error as coming from poor (± 50%) knowledge of $\langle 0|\bar{q}(ig\sigma G t)q|0\rangle$.

As it is shown at Fig.3, the quality of the fit is excellent, and the parameters are as follows

$$E_z^K = 0.4 \pm 0.1 \text{ Gev},\ E_c^K \simeq 0.9 \pm 0.1 \text{ Gev},\ n_K \simeq 5 \cdot 10^{-3} \text{ Gev}^3 \quad (27)$$

It was found impossible to give meaningful predictions for $Im\,\delta K$. For example, the introduction of second resonance give its contribution several times smaller than that of the first one and inside error bars. Of course, it is not a proof that $Im\,\delta K$ is absent completely and we estimate its maximal possible influence on, say, n to be of the order of 30%.

The mass of B^- meson K_b= B meson is already known and it can be compared with our results (27). The mass of the b quark has been deterined accurately in proper normalisation point M_b^2 in the work [13]

$$M_b = 4.79 \pm 0.03 \text{ Gev} \quad (28)$$

which together with (27) give $M_B = 5190 \pm 100$ to be compared with experimental value $M_B \lesssim \frac{1}{2} M_{\Upsilon(4S)} = 5270$ Mev.

For $D = K_c$ meson one has to include corrections $o(1/M_c)$, for charmed quark is not heavy enough. We are planning to discuss such effects elsewhere and only comment here that the main effect is very simple recoil motion of c quark, making all charmed hadrons to be ~150 Mev heavier than estimated in $M_Q \to \infty$ case. So, $M_D \simeq 1270 + 400 + 150 \simeq 1820$ Mev.

It is interesting to verify our predictions

$$f_D \simeq 220 \text{ Mev},\quad f_B \simeq 140 \text{ Mev} \quad (29)$$

for the naive bag model calculations [11] give predictions several times larger.

We have made analysis of the same type for positive parity channel as well and were surprised to find that the accuracy of the method is in this case even better. As it is seen from Fig.4, the contribution of the continuum is relatively smaller in this case. The parameters of these mesons found from the fit are as follows

$$E_z^Q \simeq 1.2 \pm 0.2 \text{ Gev};\ E_c^Q \simeq 1.8 \pm 0.2 \text{ Gev},\ n^Q \simeq 5 \cdot 10^{-2} \text{ Gev}^3 \quad (30)$$

Rather nontrivial is very large difference between these numbers and those for negative parity states. The difference in mass is nearly 1 Gev, and the density is much larger. Such properties are not reproduced by models like MIT bag, where excited states have smaller density in larger bag. In some sense, another sign of correction mean quite different interaction with vacuum for positive parity, or another "bag".

In conclusion, we predict new resonances $\mathcal{K}_Q(0^+)$, $Q_Q(1^+)$ with masses near 2.7 Gev for charm and 6.0 Gev for beauty mesons.

6. Barionic currents

The generalization of QCD sum rule method to barions was considered so far in two papers [5,6], which differ strongly in definition of currents and corrections included. So in this section we discuss the classification of barionic currents in general, and with application to the case when one of the quarks is heavy.

We begin with the classification of two quarks into their total quantum numbers. Spinor indeces are in standard way expanded in 16 gamma matrices with the help of charge conjugation matrix C as follows

$$\psi_\alpha \psi_\beta \to \psi^T C \Gamma_i \psi$$
$$\Gamma_i = 1\,(-),\ \gamma_5\,(-),\ \gamma_\mu\,(+),\ \gamma_\mu\gamma_5\,(-),\ \sigma_{\mu\nu}\,(-) \tag{31}$$

Plus or minus in brackets mean the symmetry of matrix $(C\Gamma_i)_{\alpha\beta}$. If the two quarks are up and down ones, this sign determines the isospin of the pair

$$(+) \leftrightarrow I=1 \quad ; \quad (-) \leftrightarrow I=0 \tag{32}$$

Adding the third quark to symmetrical cases we have two possibilities

$$(u^T C \gamma_\mu u)\gamma_5 u \quad ; \quad (u^T C \sigma_{\mu\nu} u)\gamma_5 \sigma_{\mu\nu} u \tag{33}$$

Using Fiertz transformation and demanding the symmetry of all quarks we obtain unique current for isobar [5].

For proton two currents remains, for example

$$j_N^1 = (u^T C \gamma_\mu u)\gamma_\mu \gamma_5 d \ ; \ j_N^2 = (u^T C \sigma_{\mu\nu} u)\sigma_{\mu\nu}\gamma_5 d \tag{34}$$

Changing d to s we obtain currents for Σ barion.

For Λ case we need isospin-0 quark pair, so there are three possibilities

$$j_\Lambda^1 = (u^T C \gamma_5 d)s \ ; \ j_\Lambda^2 = (u^T C \gamma_\mu \gamma_5 d)\gamma_\mu s$$
$$j_\Lambda^3 = (u^T C d)\gamma_5 s \ . \tag{35}$$

So far, we did not consider $SU(3)_F$ properties. Note, that the number of currents obtained above corresponds to the following decomposition of its representations

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1 \tag{36}$$

In more familiar nonrelativistic SU(6) nomenclature only $10 \oplus 8$ survive. We do not go into this further because for very heavy quark no symmetry is expected with light ones.

On the contrary, the nonrelativistic limit $M_Q \to \infty$ strongly simlifies the problem. One has then two currents for both Σ_Q and Λ_Q types of barions

$$j_\Sigma^1 = (u^T C \vec{\gamma} u)\vec{\gamma}\gamma_5 Q \ , \ j_\Sigma^2 = (u^T C \gamma_0 \vec{\gamma} u)\gamma_0 \vec{\gamma}\gamma_5 Q \tag{37}$$

$$j_\Lambda^1 = (u^T C\gamma_5 d)Q\,,\quad j_\Lambda^2 = (u^T C\gamma_0\gamma_5 d)\gamma_0 Q \tag{37}$$

We do not discuss spin-3/2 particles for their degeneration with spin-1/2 ones is evident in $M_Q \to \infty$ limit.

Note also, that currents (33-35) have two invariant structures in the two current correlator, $\gamma_{\alpha\beta}$ and $1_{\alpha\beta}$. For spin-3/2 case there are 4 of them. However, in our non-relativistic case only one structure $\frac{1}{2}(\gamma_0+1)_{\alpha\beta}$ is present.

7. Comments on the nucleon and isobar

Strictly speaking, in this section we deviate from the main line of the present work. However, the sum rules for such currents as found in [5,6] are in fact very close to those analysed in section 5, so it is instructive to compare the results. Also we would like to make critical remarks on the method of analysis used in these papers.

As noted in the preceeding section, the total number of currents and invariant structures in their correlators is rather large. All this system of sum rules remains the subject for more detailed investigations. We discuss here the simplest ones for the nucleon and isobar, retaining only first nonperturbative corrections. So, we do not go for high accuracy, but look for consistency with previous calculation for mesons with heavy quark.

As in section 5, we introduce the following free perameters: resonance position m_2, threshold position m_c and resonance residial defined as

$$\lambda_N N_\alpha = \langle 0|\, j_\alpha \,|N\rangle$$
$$\lambda_\Delta \Delta_{\mu\alpha} = \langle 0|\, j_{\mu\alpha} \,|\Delta\rangle \tag{38}$$

Then (omitting log factors) we find from [5] such sum rules

$$m^6\left\{1-\exp\left(-\frac{m_c^2}{m^2}\right)\left(1+\frac{m_c^2}{m^2}+\frac{m_c^4}{2m^4}\right)\right\}+\frac{4}{3}a_i\langle\bar{q}q\rangle^2(2\pi)^4 =$$
$$= 2b_i(2\pi)^4\lambda_i^2\exp\left(-\frac{m_2^2}{m^2}\right) \tag{39}$$

where constants are $a_N = b_N = 1$, $a_\Delta = 5$, $b_\Delta = 5/2$.
Using dimensionless variables $x = \frac{m^2}{\mathscr{E}^2}$; $\varepsilon_i = \frac{m_i^2}{\mathscr{E}^2}$ with $\mathscr{E} = (\frac{4}{3}a_i)^{1/6}(-\langle\bar{q}q\rangle)^{1/3}$ we can reduce it to single form

$$x^3\left[1-\exp\left(-\frac{\varepsilon_c}{x}\right)\left(1+\frac{\varepsilon_c}{x}+\frac{\varepsilon_c^2}{2x^2}\right)\right]+1 = \tag{40}$$
$$= n\cdot\exp(-\varepsilon_2/x)$$

where $n = \frac{3b_i\lambda_i^2}{2\mathscr{E}^6}$, and we recognise the same sum rules as we have discussed in section 5 . Therefore, our results can be directly applied to this problem.

But before we come to this, note the following simple relation which exists between the nucleon and the isobar channels in the approximation considered:

$$m_\Delta/m_N = 5^{1/6} = 1.308 \tag{41}$$
$$\lambda_\Delta/\lambda_N = 2^{1/2}$$

In reality first relation is valid with accuracy better than 1%, which is "too good" and probably is due to some coincidence. For example, second sum rules [5] are related with another scale $(10/3)^{1/3} = 1.49$, with 13% error. Of course, the account for further corrections should improve the accuracy, but in general no simple scale relation for the nucleon and isobar channels is left.

Now let us translate the results (27) of the fit to nocleon and isobar parameters. They are as follows:

$$m_{\tau}^{N} = 990 \pm 100\,\text{Mev}\ (980)\,, \quad m_c^N = 1410 \pm 150\,\text{Mev}\ (2000)$$

$$\lambda_N^2 (2\pi)^4 \simeq 1.0\ \text{Gev}^6\ (1.9)$$

$$m_{\tau}^{\Delta} \simeq 1280 \pm 130\,\text{Mev}\ (1400)\,, \quad m_c^{\Delta} = 1820 \pm 170\,\text{Mev}\ (2400) \tag{42}$$

$$\lambda_{\Delta}^2 (2\pi)^4 \simeq 2\ \text{Gev}^6\ (4)$$

In brackets the corresponding results from [5] are given. Masses are rather close, which is mostly the effect of power 1/6 taken, but continuum thresholds and matrix elements of currents significantly deviate.

We believe this is due to inadequate analysis method used in [5,6]. Insteade of fit, their authors used some additional hypotheses that at the value of Borel parameter m equal to resonance mass continuum contribution is small and sum rules can be trated as algebraic equations. This assumtion is not justified. Continuum contribution is not negligible. When it is accounted for, equations

become meaningless for both sides of them agree well in rather wide interval of m. Also, there is no reason for this values of m to be exactly resonance mass, although it is of course of the same order of magnitude.

Our discussion of the nucleon and isobar we finish with the following observation. The continuum thresholds in both cases coincide with the second resonances known in these channels, $N'(1470)$ and $\Delta'(1690)$. For ρ - meson case, the same is true for $\rho'(1250)$. Three coincidence without exceptions - it should be due to some reason. Note also, that we know ρ' to be very badly created by current in e^+e^- data, and also N', Δ' are predicted from sum rules to be small. Very interesting possibility is that they are not radial excitations, but molecular-type threshold bound states.

In any case, it is interesting to look for primed resonances $D'(2300)$, $B'(5600)$ etc.

8. Barions with a heavy quark

The derivation of the sum rules for currents (37) is analogous to that for mesonic currents, discussed in details above. Therefore we give here directly the sum rules

$$\frac{1}{20\pi^4} \int_0^{E_c} E^5 e^{-E/m} dE \pm \frac{1}{6} (\langle \bar{q} q \rangle)^2 = \frac{1}{2} \lambda^2 \exp(-E_\tau/m)$$

$$\lambda B_\alpha = \langle 0 | j_\alpha | B \rangle \tag{43}$$

where the unite of energy is taken to be (25)

$$\varkappa \equiv \left(-\frac{\pi^2}{6}\langle 0|\bar{q}q|0\rangle\right)^{1/3} \simeq 260 \text{ Mev} \qquad (44)$$

and the sign $\pm$ corresponds to Λ and Σ.
As above, we represent the spectral density as resonance plus continuum with θ-function, which is moved into the left hand side. The resulting sum rule has the form

$$x^6\left\{1-\frac{1}{120}\exp\left(-\frac{\varepsilon_c}{x}\right)\left[\left(\frac{\varepsilon_c}{x}\right)^5+5\left(\frac{\varepsilon_c}{x}\right)^4+20\left(\frac{\varepsilon_c}{x}\right)^3+60\left(\frac{\varepsilon_c}{x}\right)^2+\right.\right.$$
$$\left.\left.+120\frac{\varepsilon_c}{x}+120\right]\pm 1\right. = \frac{\pi^4\lambda^2}{\varkappa^6}\exp\left(-\frac{\varepsilon_2}{x}\right) \qquad (45)$$

$$x \equiv m/\varkappa\ ; \quad \varepsilon_i = E_i/\varkappa$$

In this case the quark cross section grows more repidly from the threshold, and the contribution of the continuum is much more important than in the case of mesons. This is shown at Fig.5,6 , and at first sight it seems impossible to get any information about the properties of the resonance. However, it turns out that there exist well localized region of parameters, which give good fit to the sum rules. This situation is very nontrivial and presumably is not due to some coincidence. The results of the fit are the following values of parameters

$$E_2^{\Lambda} \simeq 0.7\pm 0.15 \text{ Gev}, \quad E_c^{\Lambda} \simeq 0.9 \pm 0.15 \text{ Gev}, \quad \lambda_\Lambda \simeq 2\cdot 10^{-3} \text{ Gev}^3$$
$$(46)$$
$$E_2^{\Sigma} \simeq 1.1\pm 0.2 \text{ Gev}, \quad E_c^{\Sigma} \simeq 1.4\pm 0.2 \text{ Gev}; \quad \lambda_\Sigma \simeq 6\cdot 10^{-3} \text{ Gev}^3$$

Because of very important continuum contribution in this case, it is of particular interest to check these results. However, only recently Λ_B barion was first observed with its mass equal to 5425 ±175 Mev. . Our calculation give 5500±150 Mev, which is good agreement. For Λ_c using naive expression

$$M_{\Lambda_c} = M_c + E_c^{\Lambda} + 150 \text{ Mev} \simeq 2120 \pm 150 \text{ Mev} \qquad (47)$$

we also have number close to experimental one 2273± 6 Mev. For $\Sigma_Q - \Lambda_Q$ splitting our accuracy is not good: $M_{\Sigma_Q} - M_{\Lambda_Q} = \simeq 400 \pm 250$ Mev and of course does not contradict the experimental number $M_{\Sigma_c} - \Lambda_c$ 180 Mev. At least, the sign is evidently correct.

9. Results and discussion

As we have shown above, it is possible to calculate masses and some parameters of hadrons containing heavy qu quark starting from QCD Lagrangian, OPE method and some phenomenology of physical vacuum. Some results agree reasonably well with data, and some need new experiments to be chacked. In particular, it is important to measure D, B mesons decay constant, say by $B \to \tau \nu_\tau$. Different theoretical appraaches give rather different values for it. Loosely speaking, the radius of D meson is predicted within factor of two. It is also interesting to look for scalar and axial resonances in $D\pi, B\pi$ systems, which seem to be predicted most accurately by our method. We hope there are good chances to find "primed" particles in the threshold region etc. We are not going to list here all the predictions made above, but try to formulate the main conclusion from this work.

Its main result is the extention of SVZ method [2] to new region, where it seems to work well. This statement is not trivial because in our case the sum rules are "much worse" than those discussed in [2] for π, ρ mesons, and in their work [8] SVZ themselves were careful not to go too far with them. In order to explain what we mean by this, let us discuss some classification of sum rules.

The point is how strong are power corrections due to nontrivial vacuum structure. We do not mean the value of corresponding vacuum average in absolute units, of course, for in any case it is transformed away in suitable dimensionaless variables. We mean how strongly the corresponding function changes. Note also, that in the case of ordinary hadrons the natural variable is squared mass, while in our nonrelativistic case it is the first power of energy.

With this comment, the sum rules studied are af tree types

$$\int_0^{\varepsilon_c} e^{-\varepsilon/x} \varepsilon^k d\varepsilon \pm 1 = u \exp(-\varepsilon_r/x)$$

$$\begin{aligned} (k=1) &\to \pi, \rho \ldots \\ (k=2) &\to N, \Delta, K_Q, Q_Q \ldots \\ (k=5) &\to \Lambda_Q, \Sigma_Q \end{aligned} \qquad (48)$$

With the increase in strength of corrections, continuum becomes more important, and predictions for resonance alone become more difficult.

However, as it was found by our analysis, the properties of continuum are also reasonably fixed by the sum rules, and deviations from $\theta(\varepsilon-\varepsilon_c)\cdot \mathrm{Im}\, K^0(\varepsilon)$ is not large, at least in respect to integrals with exponential weight entering the sum rules. Even for barions with heavy quark, where continuum dominates completely over the resonance contribution, the fit give quite definite and reasonable results. Note also, that in the only spectral density measured, that of electromagnetic current, θ-function approximation to reality is very good indeed. Therefore, the results under discussion may stimulate further studies and applications of QCD sum rule method.

At the end, we would like to enumerate several further calculations along the line of the present work, which can be made easily. They are $1/M_Q$ corrections which, first, increase acuuracy for charmed hadrons and, second, produce splitting between J=0,1 mesons and 1/2,3/2 barions. The next problem is the introduction of the strange quark (say, F-type mesons) and looking for the effect of the strange quark mass. One may also try to calculate electromagnetic splitting in isomultiplets, which in such problem is much simpler than for ordinary hadrons. We hope to return to these questions elséwhere.

The author acknowledge multiple useful discussions with A.I.Vainshtein, M.A.Shifman, V.I.Zakharov, B.L.Ioffe and V.L.Chernyak and also valuable help from O.V.Zhirov in computer calculations.

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Figure captions

1. Diagram of zero order in strong interaction (a) for the two current correlator, together with perturbative (b-d) corrections.
2. Born diagrams for nonperturbative corrections.
3. Borel transformed correlator (curve 1), the same quantity with continuum subtracted (dashed curve 2) and the contribution of the K_Q resonance (curve 3).
4. The same as at Fig.3, but for positive parity mesons.
5. The same as at Fig.3, but for Λ_Q .
6. The same as at Fig.3, but for Σ_Q .

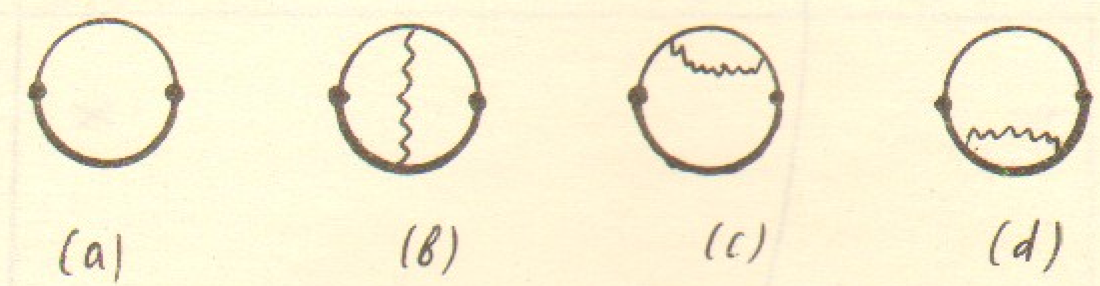

Fig. 1

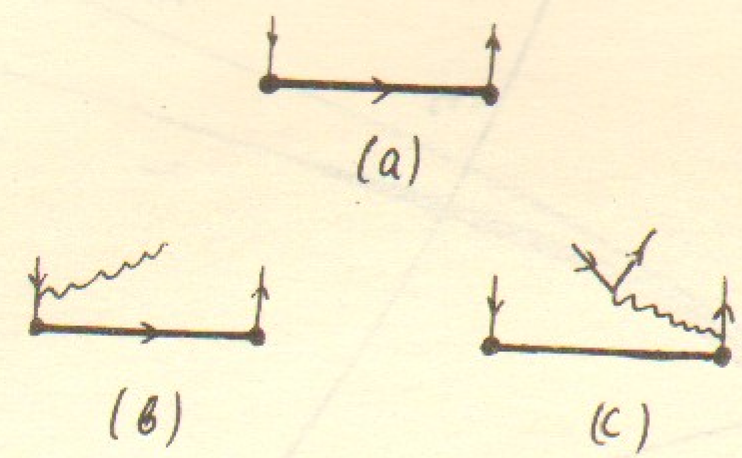

Fig.2

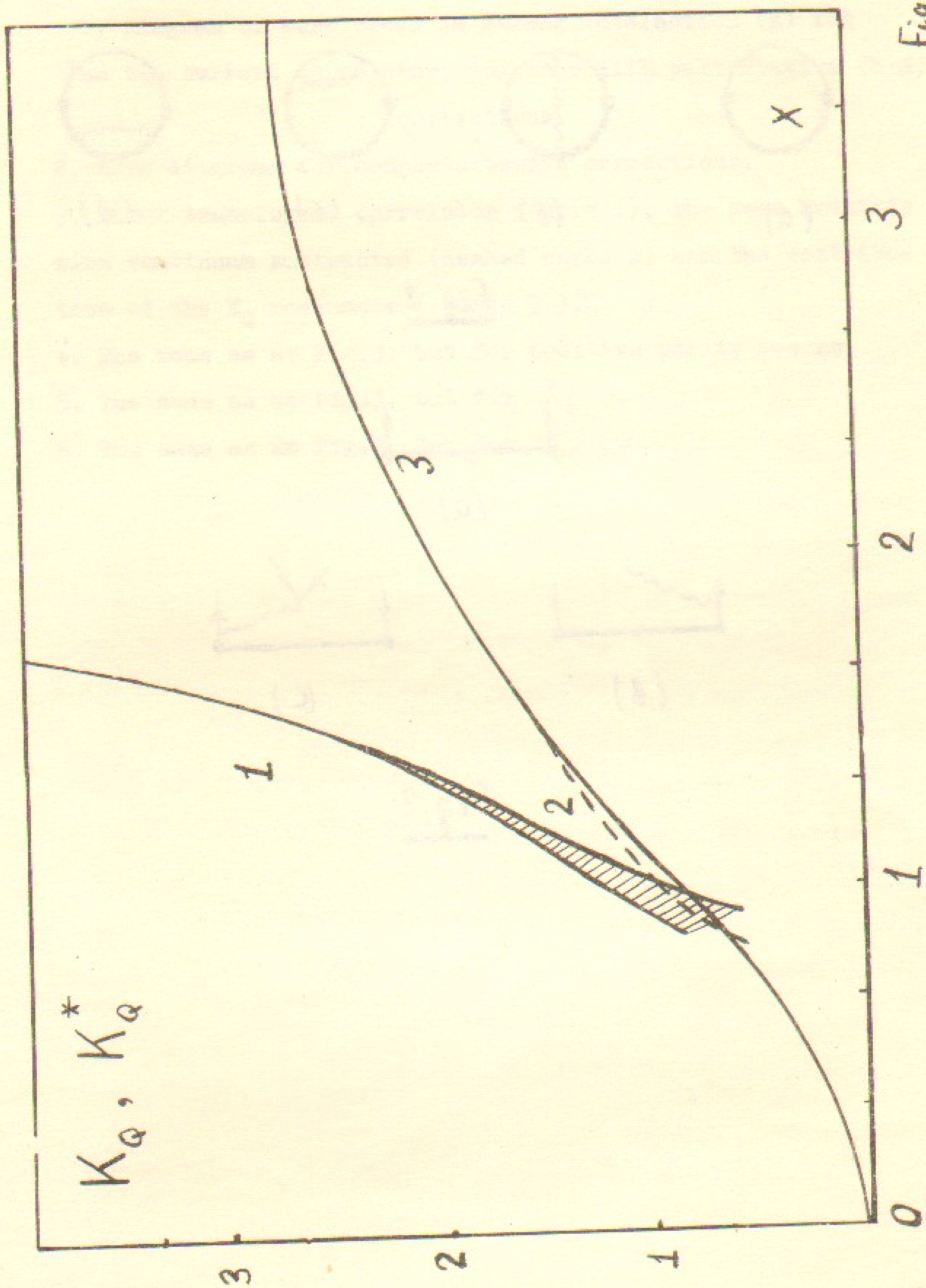

Fig. 3

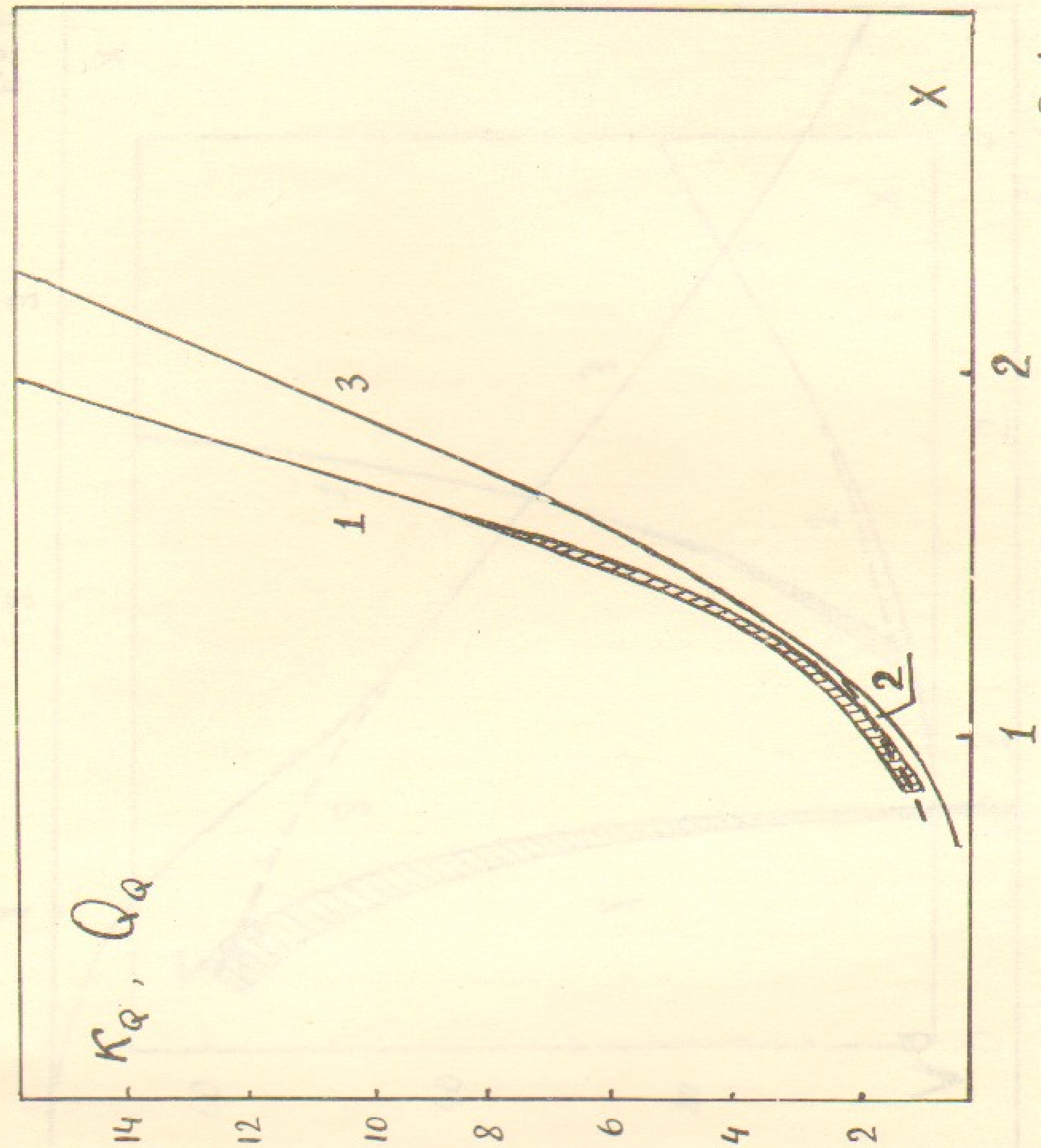

Fig. 4

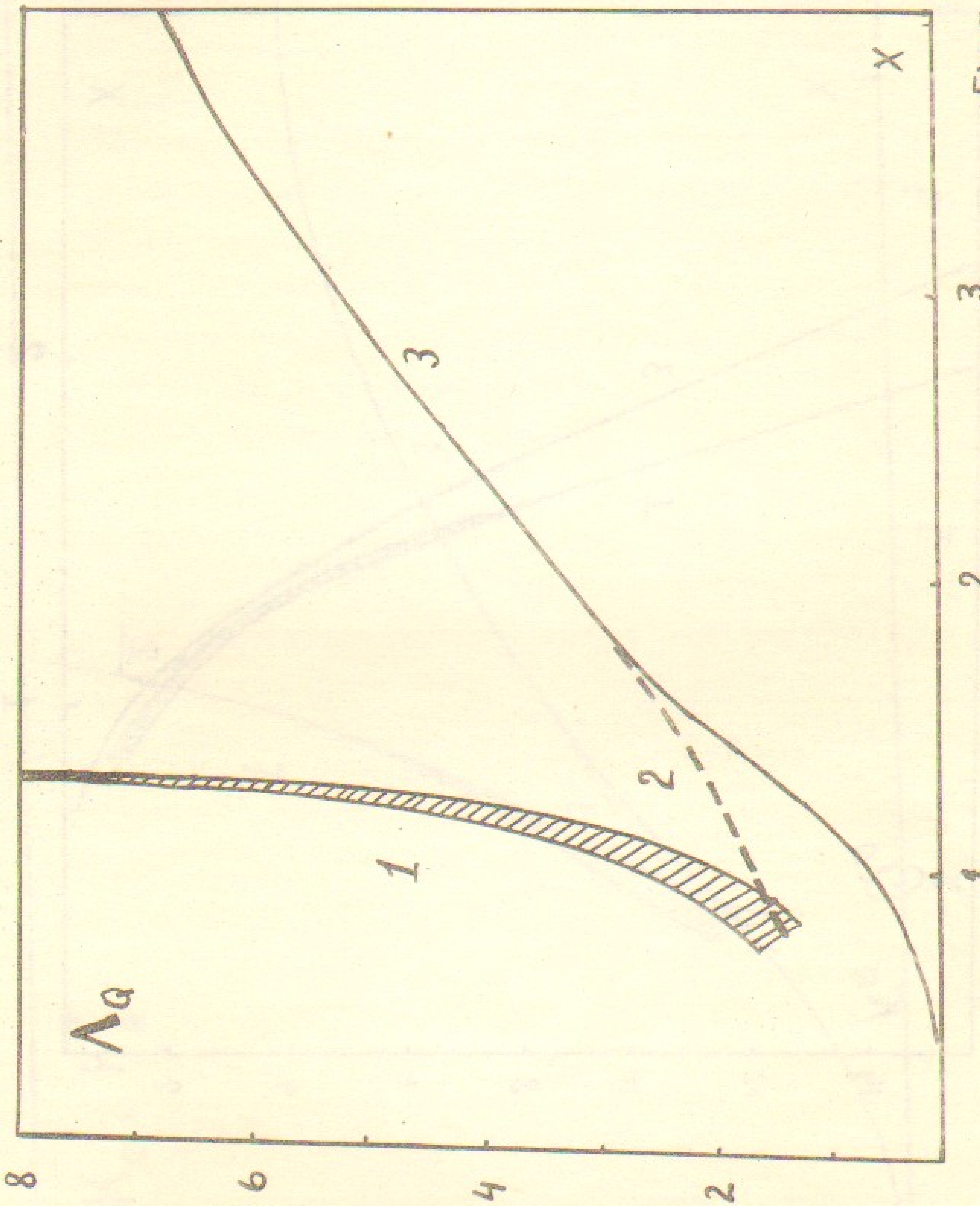

Fig.5

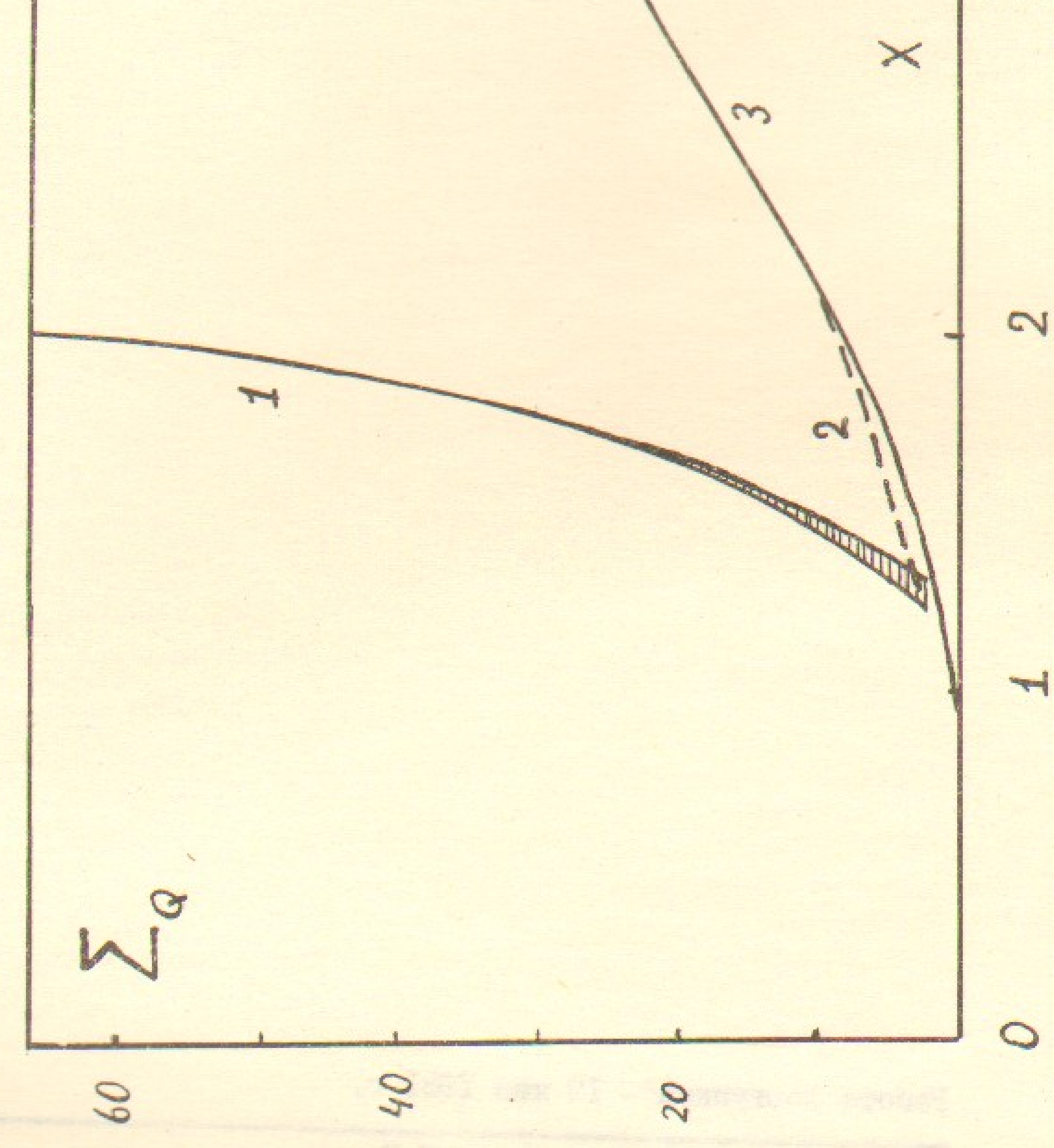

Fig.6