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HADRON-NUCLEUS COLLISIONS.

I. RELATIVISTIC CASCADE MODELS.

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ABSTRACT

The hypothesis is verified, that the main process in secondary hadron production is the relativistic intranuclear cascade. Both the eikonal and quasieikonal models are shown to be unable to reproduce the data on A-dependence of spectra for secondary  $K, \bar{p}$ .

An additional hypothesis on projectile structure (additive quark model) improves considerably the common agreement with data, although the character of remaining discrepancies proves the additive quark model to be a rather crude approximation. New experiments are proposed in order to clear up the applicability limits and accuracy of this model.

At last, the influence of possible low-energy intranuclear interaction of secondary  $K, \bar{p}$  on their spectra is estimated quantitatively.

## 1. INTRODUCTION

Common interest to hadron-nucleus collisions is induced by possibility of secondary intranuclear interactions of both projectile and newly produced particles (secondaries). So, one may treat the nucleus to be some kind of analyzer [1], a microscopic "bubble chamber" [2]. Varying its dimensions (atomic weight A) one can study:

- (i) an inner structure of projectile in both projectile [3-5] and high- $p_t$  [6,7] regions,
- (ii) a space-time development of multiparticle production processes [2,3,5,8-12], which contribute mainly into central region of spectra,

- (iii) nuclear effects like fluctons [13], short-distance correlations of nucleons [14], etc., which may dominate in the nucleus fragmentation region (in particular, in the so-called "cumulative" region, which is kinematically forbidden for production on free nucleons).

It should be admitted, that such a program is still rather far from completion. Let us briefly summarize some difficulties, arising in study of the "main" problem, the space-time development of hadron-nucleus collision.

The main point is, that in most of experiments (e.g. the emulsion [15] and [16,17] ones) there are measured some general characteristics only, like the total multiplicity of secondaries, their distribution in pseudorapidity and so on. It is seen from studies [8-12], that such data are not crucial to the intrinsic dynamics of hA-collision and can be easily reproduced by many of models, including even those based on quite different theoretical concepts<sup>1)</sup>.

Moreover, any discrepancy arising between the models and data in question is not too large and, probably, can always be explained either by crudness of theoretical models, or by in-

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<sup>1)</sup> Compare, for instance, the hydrodynamical [18,9] and parton [3,5] models.

fluence of (rather unknown) background processes<sup>2</sup>, e.g. projectile and nucleus fragmentations, absorption and rescattering of secondaries by the rest of nucleus, etc. Let us stress, that such contributions are maximal in spectra of pions, being the main component of secondaries.

From this point of view the most promising tool is expected to be the low-mass nonresonant dilepton production. Their spectra obtained in hN-collisions are quite similar to those of secondary hadrons, but in hA-collision they may leave the collision region without extra interactions. Moreover, one has the dilepton mass as an additional parameter. Changing it, one can study transition from soft hadronic processes to the hard ones.

Unfortunately, there no data on dilepton production in the both central and nuclear fragmentation regions, and corresponding theoretical considerations seem to be too early.

Very interesting information can be obtained from spectra of massive secondary hadrons (K,  $\bar{p}$ , etc.). Let us stress, that they are more informative, than those of pions. Firstly, the corresponding predictions are more model-dependent (at least, they are more sensitive to the kinematics of model). Secondly, the contributions of fragmentation processes are expected to be suppressed; moreover, any multiplication of heavy particles in low-energy cascading is negligible. So, one can simplify the whole problem, dividing it into two: the first one on the particle production in the "main" process of hA-collision, and the second one on the subsequent interactions of secondaries with the rest of nucleus.

Existing models of the "main" process in hA-collision fall naturally into two groups: those of cascade and collective models.

The former contains the models, reducing the process of hA-collision to a set of hN-subcollisions, in each of those

2) See, for instance, discussion [19]. Note, that estimates for background processes are rather unreliable and require additional assumptions.

secondaries are produced independently. Alternative possibility is presented by the latter of collective models, assuming either the collision process is collective from the very beginning, or it results in formation of collective system, which contribute in production of secondaries. In cascade models an increase of A results in more branching cascade, while in collective models it results in the growth of mass and size of collective system.

In this paper we restrict ourselves by the analysis of cascade models. Our analysis of collective models will be given in the next paper [20].

Let us briefly summarize developments of cascade approach. Its parent representatives are the cascade evaporation model [21] and Glauber optical model [22]. Their relativistic generalisations on the basis of both parton and reggeon phenomenology give rise to the parton-hadron cascade model (HPCM) [23] and also to the eikonal [11] and quasieikonal [12] models. The idea [24], that hadron is the system of spatially separated quarks lay the foundation for the additive quark model (AQM) [3-5, 10]. There are known at least two versions of AQM: the quark-parton cascade model (QPCM) [5, 10], being the generalization of HPCM, and the model [3], assuming (quasi)eikonal dynamics for the quark-nucleus interaction.

The aim of this work is the verification of basic concepts, underlying the cascade approach. The main hypothesis, that high energy hadron-nucleus interaction is reducible to hN-interactions, is tested in the framework of (quasi)eikonal model. Besides this, we verify the hypothesis on quark additivity in both projectile fragmentation and central regions. At last, we estimate the influence of intranuclear interactions for secondary K,  $\bar{p}$ .

The main characteristic under our study is the A-dependence of spectra. So, we often use the data parametrization

$$\left( \frac{E \frac{dG}{d^3p} \Big|_{hA_1}}{\left( \frac{E \frac{dG}{d^3p} \Big|_{hA_2}}{\left( \frac{A_1}{A_2} \right)^{\alpha_{12}(y, p_t)}} \right)} \right) = (A_1/A_2)^{\alpha_{12}(y, p_t)} \quad (1.1)$$

where  $y, p_t$  are rapidity and transverse momentum of final particle. Besides this, we use also  $\alpha_{12}(y, p_t)$ :

$$\left( E \frac{dN}{d^3p} \Big|_{hA_1} \right) / \left( E \frac{dN}{d^3p} \Big|_{hA_2} \right) = (A_1/A_2) \alpha_{12}(y, p_t) \quad (1.2)$$

Obviously,  $\tilde{\alpha}_{12}(y, p_t) \cong \alpha_{12}(y, p_t) - 2/3$ .

In section 2 predictions of the simplest, parameter-free version of eikonal and quasieikonal models [11,12] are shown to be inconsistent with data.

The extra hypothesis on the quark structure of projectile (sect.3), the additive quark model [3-5] improves considerably agreement with data (sects.4-5). However, our study shows also, that this model has serious problems even in the projectile fragmentation region, where its applicability may be expected to be the best. Some crucial experiments being needed are also discussed.

At last, let us some comments on the intranuclear interactions of secondaries, considered in sect.6. In many works (e.g. [3,11,12]) such interactions are completely ignored, or secondaries are supposed to be produced behind the nucleus. However, at present accelerator energies the formation length<sup>3)</sup>

$$L_f \sim p/m^2 \lesssim 2 \cdot R_{\text{nucleus}}$$

for most of particles, produced in central region, as well as for those produced in the nucleus fragmentation region.

Since the momenta of secondary pions are not high, their interactions with intranuclear nucleons may have rather complex resonance behaviour, and corresponding estimates of low-energy cascading are difficult.

Estimates can be essentially simplified for heavy particles  $K, \bar{p}$ , etc. Our numerical Monte-Carlo simulation of cascade shows that their intranuclear interactions may lead to considerable distortion of their spectra. In particular, consider-

<sup>3)</sup> Even the most "optimistic" estimates [3] shows that  $m^2 \sim \sim 0.2 \text{ GeV}^2$ .

able broadening of their  $p_t$ -distribution in nucleus fragmentation region provides a good test, whether these particles are produced inside the nucleus, or behind it.

In summary and conclusions we stress the main results, as well as some questions, which require additional study.

## 2. SIMPLEST CASCADE MODELS

In simplest models of relativistic intranuclear cascade one of two main mechanisms is usually considered:

(a) multiple intranuclear interactions of projectile (leading particle cascade model), or

(b) intranuclear hadron-parton cascade of soft secondaries. In eikonal models [11,12] the main role is ascribed to the cascading by the projectile, while in the parton-hadron cascade model [23] one considers mainly the cascading of secondaries. In fact, one can see from refs. [11,12,23,25] that at present accelerator energies  $E_0 \sim 10^2 \text{ GeV}$  contributions of both processes to the pion production are comparable.

A quite different situation is for production of heavy particles (e.g.  $K, \bar{p}$ , etc.). Since the energy of secondaries (mainly, pions) is not too high, the heavy particle production appears to be kinematically suppressed, and the main contribution should be expected from the rescatterings of projectile. So, we restrict ourselves by <sup>the</sup> consideration of eikonal models.

2.1 The main assumptions of eikonal models can be summarized in the following form [8,11,12]:

(i) the process of high energy hadron-nucleus collision consists of a set of independent subcollisions of projectile with nucleons of nucleus;

(ii) the spectra of secondaries produced in each of subcollisions are the same as that produced in collisions of projectile with free nucleon;

(iii) particles are formed outside the nucleus their interactions with nucleus should be neglected.

The latter assumption <sup>is</sup> warranted only for very fast particles. It may be incorrect for slow particles, produced in the

nucleus fragmentation region and, probably, in the central region. Below we consider both of the extreme possibilities:

$$L_f = 0 \text{ and } L_f = \infty.$$

It follows from (i)-(iii) [8,11,12]:

$$E \frac{dN}{d^3p} = \sum_V P_A(V) \sum_{E_1, \dots, E_V} W_V(E_0; E_1, \dots, E_V) \times \prod_{i=1}^V E \frac{dN}{d^3p} \Big|_{hN; E_i^0 = E_i} \quad (2.1)$$

where  $P_A(V)$  is the probability of  $V$  intranuclear subcollisions, which can be estimated [12] by the optic formulae, using the realistic intranuclear density distribution. The function  $W_V(E_0; E_1, \dots, E_V)$  being a "free parameter" of model is the conditional probability, that subcollisions occur at energies  $E_1, \dots, E_V$ .

In the eikonal model itself [11] all of the subcollisions are ordered by their effective energy<sup>4)</sup>:

$$E_1 = E_0, \quad E_2 = E_1(1-k), \quad \dots \quad (2.2)$$

where  $k \sim 1/2$  is the inelasticity coefficient of subcollisions.

In the quasieikonal model [12] equipartition of energy is assumed:

$$W_V(E_0; E_1, \dots, E_V) \propto \delta(E_0 - \sum_{i=1}^V E_i) \quad (2.3)$$

Taking in eq.(2.1) for  $(E \frac{dN}{d^3p} \Big|_{hN})$  the parametrization of

data [27-32] one obtains predictions having no free parameters.

Note, that using this parametrization one overestimates the contribution of projectile fragmentation<sup>only</sup> by the factor of  $\langle V \rangle$ . Therefore such estimates are valid<sup>only</sup> in the region, where fragmentation contributions are negligible, otherwise the eq.(2.1) gives the upper limit.

4) Subcollisions cannot be ordered in usual temporal sense because of uncertainties relation (see, e.g. [26]).

2.2 The A-dependence of spectra. Let us consider its qualitative behaviour. In eikonal model the first subcollision provides the same multiplicity as in common  $hN$ -collision at the same projectile energy  $E_0$ . So, one should expect  $\tilde{\alpha}(y, p_t) \geq 0$  in the whole kinematical region (excepting the region, where the products of projectile<sup>fragmentation</sup> dominate). In fast part of spectra contribution of next less energetic subcollisions is negligible and  $\tilde{\alpha} \rightarrow 0$ .

In contrast, in quasieikonal models the characteristic energy of any subcollision is  $E_i \sim E_0/\langle V \rangle$ , therefore the fast part of spectra is dominated by contribution of peripheral hadron-nucleus collisions with  $\langle V \rangle \sim 1$ . Their probability decrease as  $A^{-1/3}$  and one has  $\tilde{\alpha} \rightarrow -1/3$ .

In slow part of spectra any of subcollisions contributes by the same order of magnitude, so for crude estimate we have here  $\tilde{\alpha} \sim 1/3$  in both of the models. In order to obtain more accurate estimates one should take into account dependence of subcollision spectra on subcollision energy in the slow region too. In particular, parametrizing  $E \frac{dN}{d^3p} \Big|_{hN, E_0} \propto (E_0)^\beta (y, p_t)$ ,

in quasieikonal model one has an estimate  $\tilde{\alpha} \sim 1/3(1-\beta)$ . For kaons in the target fragmentation region  $\beta \sim 0$  (scaling behaviour), hence  $\tilde{\alpha}_K \sim 1/3$  here. For antiprotons  $\beta \sim 0.5-1$  even in the nucleons fragmentation region<sup>5)</sup> (scaling is violated) therefore one may expect  $\tilde{\alpha}_{\bar{p}} < \tilde{\alpha}_K$ .

2.3. Comparison with data. The quantitative estimates (see Appendix for details) are presented in Figs.1,2. They reproduce qualitatively the main trend<sup>(of data)</sup> in dependence of  $\tilde{\alpha}$  on the kind of final particle, but in central region they are considerably below the data [33,34] (let us remind, that our estimates provide the upper limit!).

It should be, however, noted the following point. The both of models may be applied to the soft processes only. The question arises: are really soft the processes, which correspond to the plotted in Figs.1,2 points [33,34] for  $p_t = 0.5-1 \text{ GeV}/c$ ?

5) In particular, this fact can be treated as an evidence, that contributions of nucleon fragmentation in antiproton spectra are small.

One may expect [36], that in  $p_t$ -distribution the transition from soft to hard hadronic processes take place at  $p_t \sim 0.7-1$  GeV/c. Moreover, the data [33,34] show that in the region of  $p_t = 0.5-1$  GeV/c  $\tilde{\alpha}$  is essentially independent of  $p_t$  and it grows rapidly at  $p_t > 1$  GeV/c. Thus, one may hope that the data [33,34] are relevant to the soft processes too, but more detailed and accurate data at lower  $p_t$  are very desirable.

Let us discuss briefly the intranuclear interactions of secondary  $K, \bar{p}$ . The rescattering of secondaries decelerate them and increase the portion of slow ones, while the absorption decrease their number. For  $K^-, \bar{p}$  absorption dominates, and we have the damping of whole spectra, while for  $K^+$  there is no absorption and deceleration decrease the number of fast particles and increase the number of slow ones (for further details, see sect.6). Note, that intranuclear interactions make disagreement between the models and data to be more dramatic.

Thus, we conclude, that both of the models are unable to reproduce the data [33,34] in central region.

For fast secondaries the predictions of eikonal and quasi-eikonal models are above and below the data [27], respectively. In principle, one can describe the data, choosing in eq.(2.1) corresponding distribution on subcollision energy  $E_1$ , but such attempts require more careful treatment for projectile fragmentation processes, which are the next topic of our consideration.

### 3. THE QUARK STRUCTURE OF PROJECTILE

In the previous section we ignore completely the inner quark structure of projectile  $h$ . First of all, the projectile structure should be taken into account in the region of its fragmentation. Besides this, we consider below the influence of projectile structure on the spectra of secondaries in both central and nucleus fragmentation regions.

The simplest nontrivial assumption is known as the additive quark model (AQM) [3-5,10]. It suggests the hadron to be

a system of loosely bounded "constituent" valence quarks. Such a quark is assumed to be "dressed". From the parton point of view these assumptions mean, that in the reference frame, where the hadron is relativistic, each of (constituent) quarks has his own parton ladder. At present accelerator energies these ladders are spatially separated - accordingly to [37] their mixing may occur at  $E_0 \gtrsim 10^4$  GeV.

By other words, the projectile is a jet of independent hadronlike objects (constituent quarks); moreover, the breaking of any constituent quark into partons as a result of its inelastic interaction with target does not break the parton ladders of others.

In the framework of AQM the whole process of multiparticle production may be divided into three subsequent stages [3-5,10]

(i) the pair inelastic interactions of constituent quarks, belonging to each of the colliding primary hadrons (or nuclei). The "wounded" quarks break into partons, the others fly, being "spectators".

(ii) the dressing of partons, leading to formation of newly produced constituent quarks;

(iii) Formation of secondary hadrons. The mutual recombination of newly produced quarks gives rise to the hadron production in central region, while the recombination of spectators with newly produced quarks as well as the spectators fragmentation contributes to the products of fragmentation of primary hadrons (or nuclei).

The cross section of quark-quark interaction is not large:  $\sigma_{qq} \sim 3-4$  mb. Therefore the typical inelastic hadron-hadron collision involves only one quark from each of primary hadrons. The mutual screening of quarks is estimated [24] to be small, and for the integrated inelastic cross sections additivity relations are valid [38]:

$$\sigma_{h_1 h_2} = \sum_i^{n_1} \sum_j^{n_2} \sigma_{ij} = \sum_i^{n_1} \sigma_{q_i h_2} = \sum_j^{n_2} \sigma_{h_1 q_j} \quad (3.1)$$

where  $n_{1,2}$  are the numbers of quarks in each of primary hadrons  $h_{1,2}$ . For  $(u, d, \bar{u}, \bar{d})$ -quarks cross sections are approximately equal, and for nonstrange hadrons we have:

$$\sigma_{h_1 h_2} = n_1 n_2 \sigma_{qq} = n_1 \sigma_{qh_2} = n_2 \sigma_{qh_1} \quad (3.2)$$

In hadron-nucleus collision the length of free path can be comparable to the size of nucleus. In this case the additivity in eqs.(3.1-2) is violated:

$$V_h \equiv \frac{n_h \cdot \sigma_{qA}}{\sigma_{hA}} > 1, \quad (3.3)$$

where  $V_h$  is the mean number of quarks, which are broken into partons as result of hadron-nucleus collision. Obviously, for heavy nuclei  $V_h \rightarrow n_h$ , being the total valence quark number.

Let us consider also

$$V_A \equiv \frac{A \cdot \sigma_{qN}}{\sigma_{qA}} > 1, \quad (3.4)$$

which characterizes the mean number of nucleons, which interact with "wounded" quark<sup>6)</sup>.

The greater atomic weight  $A$  is, the greater both the number  $V_h$  of "wounded" quarks and the number  $V_A$  of intranuclear interactions of each of them are, hence the higher multiplicity is in central and nucleus fragmentation regions, corresponding decrease of spectators number results in decrease of multiplicity in projectile fragmentation region.

Such a picture has been proposed in ref. [39], its consequences are considered in refs. [3-5, 10, 40]. Let us below analyze the most important ones.

#### 4. PROJECTILE FRAGMENTATION

The main assumption is here that the process of projectile fragmentation may be divided into two subsequent independent

<sup>6)</sup> Let us remind, that in quasieikonal model [12] an analogous quantity  $\bar{V} \equiv A \sigma_{hN} / \sigma_{hA}$  characterizes the mean number of projectile intranuclear interactions.

stages [3-5, 10]:

- (i) the passing of projectile through nucleus (some of quarks are absorbed at this stage), and
- (ii) fragmentation of spectators set  $\{q_i\}$  into the final hadron  $h'$ :

$$\sigma_A(h \rightarrow h') = \sum_{\{q_i\}} \sigma_A(h \rightarrow \{q_i\}) \cdot F_A(\{q_i\} \rightarrow h') \quad (4.1)$$

The cross section  $\sigma_A(h \rightarrow \{q_i\})$  to have a set of spectators  $\{q_i\}$  can be calculated [3, 40] using the standard formulae for passing the particles through matter. The probability factor  $F_A(\{q_i\} \rightarrow h')$  characterizes the transition process  $\{q_i\} \rightarrow h'$  and can be estimated using eq.(4.1) from data on fragmentation cross sections  $\sigma_A(h \rightarrow h')$ . On the other hand, if one knows this factor from independent estimates, one has predictions for  $\sigma_A(h \rightarrow h')$ .

A lot of predictions [3, 4] arises, if one takes an additional assumption, that  $F_A(\{q_i\} \rightarrow h')$  is independent of  $A$ <sup>7)</sup>. Under this assumption the  $A$ -dependence for any item in eq.(4.1) is given by known factor  $\sigma_A(h \rightarrow \{q_i\})$ , while  $F_A(\{q_i\} \rightarrow h')$  depends only on  $X \equiv p_{h'}/p_0$ , where  $p_0, p_{h'}$  are the momenta,

carried by projectile and final hadrons, respectively. Therefore one may estimate  $F_A(\{q_i\} \rightarrow h')$  at any fixed  $A$ , in order to predict  $\sigma_A(h \rightarrow h')$  for arbitrary  $A$ .

It is obvious a priori, that the validity of such a model is violated at small  $X$ , where one should expect a considerable contribution from  $A$ -dependent density of newly produced quarks. One of the criteria for this contribution to be small may be the validity of scaling for spectra of fragmentation  $h'$ , because the density of newly produced quarks depends not only on  $A$ , but the energy of projectile.

In many practical cases eq.(4.1) takes a very simple

<sup>7)</sup> This assumption is also used in [41]. Some attempts to estimate such  $A$ -dependence can be found in refs. [5, 10, 40].



form.

4.1 The simplest examples. Let us assume for simplicity, that among the quarks (antiquarks) of projectile  $h$  there no strange ones, and therefore  $\sigma_A(h \rightarrow \{q_i\})$  is independent of the kinds of "final" quarks (antiquarks). It is very convenient to introduce cross sections to have any one, two, ... "final" (anti)quarks:

$$\sigma_A(M \rightarrow q) = 2 \sigma_A(M \rightarrow q_1) = 2 \sigma_A(M \rightarrow \bar{q}_2)$$

where  $q_1$  and  $\bar{q}_2$  are respectively quark and antiquark of projectile meson  $M$ . For baryon  $B$  one has:

$$\sigma_A(B \rightarrow q) = 3 \sigma_A(B \rightarrow q_1) = \dots$$

$$m \quad \sigma_A(B \rightarrow qq) = 3 \sigma_A(B \rightarrow q_1 q_2) = \dots$$

Let us consider the following cases:

(a)  $h$  is meson ( $M$ ) and  $h'$  is any hadron, both  $M$  and  $h'$  have one common (anti)quark  $q_1$ . From eq.(4.1) it follows [3,4]

$$\sigma_A(M \rightarrow h') = 1/2 \cdot \sigma_A(M \rightarrow q) \cdot F(q_1 \rightarrow h') \quad (4.2)$$

In this case one has the  $A$ -dependence to be universal for any  $X$  and kind of  $h'$ .

(b)  $h$  is barion ( $B$ ) and  $h'$  is meson ( $M'$ ),  $B$  and  $M'$  have one common quark  $q_1$ . One has:

$$\begin{aligned} \sigma_A(B \rightarrow M') &= 1/3 \sigma_A(B \rightarrow q) \cdot F(q_1 \rightarrow M') + \\ &+ 1/3 \sigma_A(B \rightarrow qq) \sum_{i=2,3} F(q_1 q_i \rightarrow M') \quad (4.3) \end{aligned}$$

Since both the characteristic time of spectators mutual interaction and the time of their fragmentation (or recombination) into hadrons are the same order of magnitude, the presence of extra spectator  $q_i$  (in the second term) may influence the process  $q_1 \rightarrow M'$ . If this influence is neglected, one has [4]:

$$\sigma_A(B \rightarrow M') = 1/3 \left[ \sigma_A(B \rightarrow q) + 2 \sigma_A(B \rightarrow qq) \right] \cdot F(q_1 \rightarrow M') \quad (4.3')$$

As well in previous case of eq.(4.2) one has the  $A$ -dependence to be universal for any  $X$  and  $M'$ .

(c)  $h$  and  $h'$  are both baryons ( $B, B'$ ), having only one common quark  $q_1$ . This case is identical to the previous one.

(d)  $h$  and  $h'$  are both baryons ( $B, B'$ ), having a common pair of quarks  $q_1 q_2$  [3,4]:

$$\begin{aligned} \sigma_A(B \rightarrow B') &= 1/3 \left\{ \sigma_A(B \rightarrow q) \sum_{i=1,2} F(q_i \rightarrow B') + \right. \\ &\left. + \sigma_A(B \rightarrow qq) \left[ F(q_1 q_2 \rightarrow B') + \sum_{i=1,2} F(q_i q_3 \rightarrow B') \right] \right\} \quad (4.4) \end{aligned}$$

The simplest way to test the above assumptions is the verification of eq.(4.2). In this way one should check up the factorization of dependences on  $A$  and  $X$ . This way is most direct one to clarify the question, whether  $F_{(A)}(q \rightarrow h')$  is independent of  $A$ .

Unfortunately, such a factorization is absent for sums in eqs.(4.3-4), and their verification requires some knowledge on  $F(\{q_i\} \rightarrow h')$ . So, let us take an additional assumption, that function  $F^{(B)}(q \rightarrow h')$ , which enters in eqs.(4.3-4), coincides with that  $F^{(M)}(q \rightarrow h')$ , entering in eq.(4.2) after a rescaling of Feynman variable  $X$  ( $\equiv P_{h'}/P_h$ ):

$$F^{(B)}(q \rightarrow h'; X) = F^{(M)}(q \rightarrow h'; \frac{3}{2}X) \quad (4.5)$$

The rescaling reflects the fact, that both the functions should be taken at the same  $X^* = X/\langle X_q \rangle$ , where  $\langle X_q \rangle$  is the portion of projectile momentum, carried by each of spectator quarks<sup>8)</sup>.

Taking this input, one can estimate by the eqs.(4.2-5) both the quark and diquark fragmentation functions at some

<sup>8)</sup> This relation seems to be quite reasonable in the region  $X^* \lesssim 1$ , while at  $X^* \gtrsim 2$  it fails completely. If one estimates both the left and right hand sides of eq.(4.5), using the quark counting rules [12], one can see, that it is valid within the accuracy 40-50 per cent in the region  $X^* \lesssim 1.5$ :

$$\frac{4}{3} (1-X^*/3)^3 \approx (1-X^*/2)$$

fixed  $A$ , using a sufficiently complete set of data on  $\sigma_A(h \rightarrow h')$ . In particular, at  $A=1$  one has from the additivity relations (see eqs.(3.1-2)):

$$\sigma_{A=1}(M \rightarrow q) = \sigma_{MN},$$

$$\sigma_{A=1}(B \rightarrow qq) = \sigma_{BN}, \quad \sigma_{A=1}(B \rightarrow q) \approx 0.$$

Therefore one can estimate the quark fragmentation function directly from the meson-nucleon reactions, as well as the diquark one from the barion-nucleon reactions. Further, assuming these functions to be independent of  $A$  and substituting them into eqs.(4.3-4) one obtains predictions for any  $A$  <sup>9)</sup>. Below we perform such an analysis for the processes  $PA \rightarrow \Lambda^0, K_S, K^+$ , using the data [43-46].

4.2.  $\Lambda^0$  production. From eqs.(4.2-4) one has:

$$\sigma_A(\pi \rightarrow \Lambda^0) = \frac{1}{2} \sigma_A(\pi \rightarrow q) F(d \rightarrow \Lambda^0) \quad (4.6)$$

$$\sigma_A(p \rightarrow \Lambda^0) = \sigma_A(p \rightarrow q) F(d \rightarrow \Lambda^0) + \frac{1}{3} \sigma_A(p \rightarrow qq) [F(uu \rightarrow \Lambda^0) + 2F(ud \rightarrow \Lambda^0)] \quad (4.7)$$

where symmetry  $F(u \rightarrow \Lambda^0) = F(d \rightarrow \Lambda^0)$  is used.

The probabilities of quark and diquark fragmentation into  $\Lambda^0$  are estimated from data [43,44] ( $\pi Be \rightarrow \Lambda^0 + X$ ,  $p_0 = 200$  Gev/c and  $pBe \rightarrow \Lambda^0 + X$ ,  $p_0 = 300$  Gev/c), assuming  $\sigma_{qN} = 9$ mb (corresponding values of  $\sigma_A(h \rightarrow \{q\})$  are given in Table 1). The results are plotted in Fig.3.

The predictions for  $\sigma_A(p \rightarrow \Lambda^0)$ ,  $A=64, 206$  are shown in Fig.4. Let us note, that in the region  $X \lesssim 0.5-0.6$  they are systematically below the data [44], and corresponding values for  $\alpha_{Pb/Be}$  are below the data too. The main point is here, that the discrepancy is also large in the region  $X \lesssim 0.3$ ,

9) Let us add, that in this way one can take into account any influence of resonance contributions on the spectra of final particles.

where the eq.(4.5) is expected to be valid.

Another disagreement, may be, the most dramatic one, arises, if one compares with data the predictions for  $\alpha_{Pb/Cu}$ . Let us stress, that it takes place in the whole region of  $X$ , including the region  $X \gtrsim 0.5-0.6$ , where one-spectator contribution is negligible, hence it independent of the question, whether the assumption on eq.(4.5) is valid. Note, that the experimental value of  $\alpha_{Pb/Cu}$  can be reproduced, if only the only parameter of the model  $\sigma_{qN}$  has an unacceptable low value  $\sim 6-7$ mb.

4.3.  $K_S$  and  $K^+$  production. For  $K_S$  one has analogously:

$$\sigma_A(\pi \rightarrow K_S) = \frac{1}{2} \sigma_A(\pi \rightarrow q) F(d \rightarrow K_S) \quad (4.8)$$

$$\sigma_A(p \rightarrow K_S) = \frac{1}{3} \sigma_A(p \rightarrow q) F(d \rightarrow K_S) + \frac{2}{3} \sigma_A(p \rightarrow qq) F(ud \rightarrow K_S). \quad (4.9)$$

The fragmentation functions, estimated from the same data [43,44] ( $\pi Be \rightarrow K_S + X$ ,  $p_0 = 200$  Gev/c and  $pBe \rightarrow K_S + X$ ,  $p_0 = 300$  Gev/c) are presented at Fig.3. It appears to be rather unexpected, that at  $X \gtrsim 0.3$  the quark fragmentation function drops more slowly, than the diquark one. Moreover, in the same region the corresponding predictions (Fig.5) for  $\sigma_A(p \rightarrow K_S)$ , as well as for  $\alpha$  are considerably above the data [44].

The most likely explanation is that AQM fails just for the process  $\pi^- \rightarrow K_S$  in the region  $X^* \gtrsim 1$ . This guess is supported by the existence of some break in spectrum at  $X^* \sim 1$  (see Fig.3), as well as by the fact, that at  $X^* > 1$  this spectrum is at variance with the quark counting rules [42]. It is commonly accepted, (see, e.g. [42,47]), that triple reggeon contribution dominates here.

For the process  $\pi^+ \rightarrow K^+$  the break arises at  $X^* \sim 1.5$  [45]. The corresponding quark fragmentation function (Fig.3), averaged over the projectile proton

$$F(q \rightarrow K^+) = \frac{1}{3} F(d \rightarrow K^+) + \frac{2}{3} F(u \rightarrow K^+)$$

is estimated from data [45] on the processes  $\pi^+ \rightarrow K^+$ , assuming  $F(d \rightarrow K^+) \cong F(\bar{d} \rightarrow K^-)$ . The corresponding diquark function  $F(qq \rightarrow K^+)$  is estimated from the data [46].

The main observation is that the quark and diquark functions coincide within errors in the whole region  $0.5 < X \lesssim 1.5$ . It excludes completely the hypothesis [4], that each of surviving spectators fragments into final hadrons independently, because in this case one should expect  $F(qq \rightarrow h) = 2F(q \rightarrow h)$ . The recombination picture [3], appears to be more realistic one, since it predicts  $F(qq \rightarrow h) = 5/4 \cdot F(q \rightarrow h)$ .

However, such a proportionality of the quark and diquark fragmentation functions leads to A-dependence of  $K^+$ -meson spectra to be universal at any X. It corresponds to  $\alpha_{Pb/Be} \approx 0.61$ , while the data [27-29] agree with this value only at  $X \lesssim 0.3$  and for higher X they tend to a lower values  $\sim 0.5$ . Our predictions for  $G_A(p \rightarrow K^+)$ , plotted in Fig.6 agree with data [27-29] at  $X \sim 0.2-0.3$  and overestimate (at least, by the factor of 1.5-2) those for heavy nucleus (Pb, Cu) in the region  $X \sim 0.4-0.5$ .

However, the situation is not yet clear to come to final conclusions because of two problems. The first one is that the data [27-29] are taken at too low energies ( $E_0 = 19, 24$  and  $70$  GeV), while the fragmentation functions correspond to  $E_0 = 200-300$  GeV. The second problem being the most serious one is that our additional assumption, expressed by the eq.(4.5) can fail at  $X > 0.3$  (or  $X^* > 1$ ); if so, the whole procedure becomes meaningless in this region.

4.4. Possible developments. The main problem to verify AQM for baryon-induced reactions (eqs.4.3-4) is the necessity to have an independent estimates for quark fragmentation function. A very attractive way to estimate this function is the study of electroproduction processes at low  $Q^2$ . Concretely, one should extract both the quark distribution inside the projectile baryon and the quark fragmentation function and convolute them. Unfortunately, the distribution of constituent quark is not yet a solved problem, as well as there no data on the quark fragmentation functions at low  $Q^2$ .

Nevertheless, let us illustrate this idea by the analogous convolution for quark-parton functions, extracted from high  $Q^2$  electroproduction and  $e^+e^-$ -annihilation processes:

$$H(q \rightarrow K^+; X) = \sum_q \int_X^1 dz \cdot C_{p \rightarrow q}(z) \cdot D_{q \rightarrow K^+}(z/X)$$

where  $C_{p \rightarrow q}$  is the quark-parton distribution [48] inside the proton and  $D_{q \rightarrow K^+}$  is the corresponding fragmentation function [49]. There is surprising similarity in shape between the considered above function  $F(q \rightarrow K^+)$  and  $H(q \rightarrow K^+)$  (see, Fig.3), although the former describes the soft fragmentation processes for constituent quarks, while the latter describes the hard fragmentation processes for quark-partons. The difference in normalization reflects the fact, that the constituent quarks carry the whole momentum of projectile, while the quark-partons carry only the half of it. There is no doubts, that this question deserves more careful study.

Finally, let us summarize this section. The most important and direct test of AQM is the verification of eq.(4.2) for meson-nucleus reactions. Firstly, one should clear up, whether the A-dependence of final particles spectra is universal at any X. Secondly, one should check, that this A-dependence is the same for any final hadron  $h'$ , containing one spectator quark. In this way one can verify the main hypothesis, that the production process consists of two independent subsequent stages, as well as the assumption, that the quark fragmentation function is independent of A.

Because of lack of data for meson-induced reactions we have performed the less transparent but independent test of the model for baryon-induced reactions, using the eqs.(4.3-5). We have found that there are some problems for  $\Lambda^0$ -production; for meson production some problems can arise because of triple reggeon contribution, being important at  $X \sim 1$ . Besides that, we have also found, that in the region, where AQM works well, the recombination picture [3,5,10] for final hadron formation processes is more preferable, than the fragmentation one [4]. At last, we conclude that more detailed data are very desirable.

le in order to clear up the accuracy and applicability limits of additive quark model for fragmentation processes.

### 5. PARTICLE PRODUCTION IN THE CENTRAL REGION

Accordingly to AQM, in the central region secondaries are produced as a result of decays of  $V_h$  "wounded" quarks. If the dynamics of quark-nucleus interactions is quasieikonal (the multiladder processes dominate, as it is assumed, e.g. in ref. [3]),<sup>10</sup> each of the "wounded" quarks has  $V_A$  intranuclear interactions. The total number of "production center" is in AQM the same, as the number  $V$  of those in the "structureless" quasieikonal model (QEM) [12], considered in the sect.2 :

$$V_h \cdot V_A = \frac{n_h \cdot \sigma_{qA}}{\sigma_{hA}} \cdot \frac{A \cdot \sigma_{qN}}{\sigma_{qA}} = \frac{A \sigma_{qN}}{\sigma_{hA}} = \bar{V} \quad (5.1)$$

Let us stress the most important specific features:

(i) In the structureless QEM an effective subcollision energy  $E_i$  decrease with  $A$  as  $E_0/\bar{V}$ , while in AQM  $E_i \sim E_0/V_A$ . It means that AQM predicts both the narrowing of spectra and their shift in rapidity toward the nucleus fragmentation region <sup>to be</sup> smaller, than those predicted by QEM. Note, that these effects should be more noticeable mainly in spectra of heavy particles ( $K^-, \bar{p}, \varphi, \dots$ ), since at present energies the pion spectra are considerably contributed by products of nucleus fragmentation.

(ii) In the structureless QEM in each of production centers hadrons are produced, while in AQM we have quarks, which recombine into hadrons later on.

10) Another variant of AQM, the model of quark-parton cascade [5,10] requires a set of additional assumptions and therefore it is not considered here.

Let us discuss the latter feature in more details. There are two possible ways for quarks to recombine into hadrons, either each of recombining quarks belongs only to the same production center or the quarks, produced in different production centers can recombine too.

If the former possibility takes place, the AQM predictions can be expressed through the spectra of secondaries produced in hN-collisions, exactly in the same way, as it is done in QEM (see sect.2)<sup>11)</sup>:

$$E \frac{dN}{d^3p} = V_h \sum_V P_{qA}(V) \sum_{E_1 \dots E_V} W(E_0; E_1 \dots E_V) \sum_{i=1}^V E \frac{dN}{d^3p} \Big|_{E_0=E_i}^{hN} \quad (5.2)$$

where  $P_{qA}(V)$  is the probability for  $V$ -repeated quark-nucleus interaction

For the latter possibility any quantitative prediction is essentially model-dependent (such a model is considered, e.g. in ref. [10]). It depends on unknown fusion probabilities, space-time picture of processes, etc. Qualitatively, such a "collectivization" of quarks from different production centers "moves away" the kinematical bounds and results in increase of the yields of (heavy) particles in comparison to those predicted by eq.(5.2). Moreover, the collectivization of quark, produced by the different constituent quarks of projectile must violate the Anisovich relations [39] for the spectra of particles produced in the both central and nucleus fragmentation regions:

$$E \frac{d\sigma}{d^3p}(BA \rightarrow h+X) / E \frac{d\sigma}{d^3p}(MA \rightarrow h+X) = 3/2 \quad (5.3)$$

here  $B$  and  $M$  are the projectile baryon and meson respectively,  $h$  stands for the final particle. The main point is that in eq.(5.3) all effects of intranuclear low energy cascading

<sup>11)</sup> It is assumed, that  $E \cdot dN/d^3p|_{hN} = E \cdot dN/d^3p|_{qN}$ . This relation is valid only in the region, where the contribution of  $h$  and  $q$  fragmentation is negligible. In particular, one may expect that in pN-collisions this relation is valid for  $K^-, \bar{p}$ .

should cancel. Let us stress, that eq.(5.3) should be verified in the region, where the quantum numbers of quarks, contained by B and M are unessential; besides that both of the spectra should be taken at the same quark energy, i.e.  $E_0^{(B)} = \frac{3E_0^{(M)}}{2}$ .

The current data [17] on the spectra of unidentified charged secondaries (mainly, pions) are compatible within their accuracy with the absence of such of "collectivization". However, the most crucial is the verification of eq.(5.3) for the spectra of heavy secondaries, since they are expected to be more sensitive to the kinematical restrictions.

In this work we suppose that any collectivity is absent. The corresponding predictions, following from the eq.(5.2) are given in Fig.7.

If one neglects completely the interactions of secondaries with the rest of nucleus, the predictions (curve 1) is in fair agreement with data [27,33-35], especially in the region of  $y_{lab} \sim 2-4$ , where the structureless QEM underestimates the data.

If one takes into account the intranuclear interactions of secondaries, the disagreement is drastically destroyed (curve 2). The model fails even if one assumes the formation length  $L_f$  to be [3,5,10]

$$L_f \sim P/m^2,$$

where P is the momentum of final particle and  $m^2$  takes the lowest generally accepted value  $\sim 0.25 \text{ GeV}^2$  [3].

So, the most important question arises, whether the secondaries are formed inside the nucleus or behind it. If they are formed inside the nucleus, the above agreement should be considered as an accidental one.

To answer this question at least for slow particles one should study the  $p_t$ -slope parameter B:  $E \cdot dN/d^3p \propto \exp(-Bp_t^2)$ . If there are no intranuclear interactions for secondaries, both  $B_{hA}(y)$  and  $B_{hN}(y)$  must coincide. In contrast, if these interactions does really take place the slope parameter  $B_{hA}(y)$  differs drastically from  $B_{hN}(y)$  in the region  $y_{lab} \lesssim 1$  (see Fig.8).

## 6. INTRANUCLEAR INTERACTIONS OF SECONDARIES

6.1. Pions. The main contribution into the multiplicity of secondaries comes from the light particles production, namely, the pions production. The momenta of pions, produced in the central region ( $y_{lab} \sim 1-3$ ) are rather small ( $p_{lab} \sim 0.2-3 \text{ GeV}/c$ ) and fall into the region of  $\pi N$ -resonance interaction. Corresponding processes are very complicated and their quantitative description can hardly be reliable. Let us summarize the most important ones.

Elastic scattering processes (including the charge exchange) in the region of resonances ( $\Delta(1236), N(1470), \dots$ ;  $p \sim 0.2-0.7 \text{ GeV}/c$ ) are nearly isotropic. They shift the particles toward the nucleus fragmentation region in the both rapidity and pseudorapidity scales. Typical cross sections [50] are:  $\sigma(\pi N \rightarrow \Delta) \approx 60-70 \text{ mb}$ ,  $(\pi N \rightarrow N(1470)) \approx 30 \text{ mb}$  and correspond to the free path length <sup>12)</sup>:

$$L_{el} = (n \sigma_{el})^{-1} \sim 1-2 \text{ fm} \quad (6.1)$$

where  $n = 0.17 \text{ fm}^{-3}$  is the nuclear density. For the fast pions ( $p \gtrsim 2-3 \text{ GeV}/c$ ) the elastic scattering is dominated by the forward cone and its influence becomes unimportant.

The inelastic multiplication of pions arises at  $p \gtrsim 0.6 \text{ GeV}$ . The main processes ( $\pi N \rightarrow \pi \Delta \rightarrow \pi \pi N$ , etc.) have cross sections  $\sigma \sim 15-20 \text{ mb}$ , which correspond to the free path  $L \sim 3-2 \text{ fm}$ .

The competing process is the pion absorption by the nucleons conglomerates:  $\pi + (NN) \rightarrow N + N$ . No reliable quantitative estimates for such processes are known. The data (see compilation [50]) on the deuteron desintegration  $\pi^+ d \rightarrow p + p$  show that at  $p = 0.25 \text{ GeV}/c$   $\sigma \sim 12 \text{ mb}$  and drops rapidly

<sup>12)</sup> All of the estimates, presented below are based on the cross sections, measured for interactions of particles with free nucleons. It is unknown, whether this cross sections are renormalized inside the nucleus.

for higher  $p$  (e.g. at  $p \approx 0.38$  GeV/c one has  $\sigma \sim 3$  mb).

All the facts listed above show that if the secondary pions are formed inside the nucleus, their intranuclear interactions must be essential. Since any reliable estimate for these processes is a very complicated unsolved problem, the pion spectra are not a good tool to study the main mechanism of multiparticle production in high energy hadron-nucleus collisions.

6.2. Heavy particles. More reliable information on the "main" process in high-energy hadron-nucleus collision can be obtained from the spectra of heavy particles ( $K, \bar{p}$ , etc.).

Firstly, they are produced with higher momentum ( $p \sim 1-10$  GeV/c). Therefore, their interactions inside the nucleus are mainly nonresonant and can be easily estimated.

Secondly, the thresholds of their production are considerably higher than those for pions, hence the probability of their production in the intranuclear interactions of secondaries is very small and can be neglected.

Let us consider the most important processes:

(a) Elastic scattering. In the region of  $p \sim 1-10$  GeV/c one has  $d\sigma/dt \propto \exp(Bt)$ , where  $B \sim 12-15$  and  $6-8$  GeV<sup>-2</sup> for antiprotons and kaons respectively (corresponding references can be found in ref.50). After the elastic scattering the (longitudinal)rapidity remains practically unchanged, while the transverse momentum grows<sup>13)</sup>:  $p_t^2 \rightarrow p_t^2 + 1/B$ .

(b) Inelastic "deceleration" (inelastic scattering without annihilation of incoming particle). This process has a threshold at  $p \sim 0.9$  and  $1.7$  GeV/c for  $K$  and  $\bar{p}$  respectively. It results in both the growth of transverse momentum  $p_t^2 \rightarrow p_t^2 + 1/b$ , (where  $b^{-1} \sim 0.15-0.3$  GeV<sup>-2</sup>) and shift of particle in the longitudinal rapidity  $y \rightarrow y - \Delta y$ . The data [51] show, that within the accuracy, being quite acceptable for us one can take the particle with the quantum numbers of the incoming one

<sup>13)</sup> Let us note, that at  $E_0 \sim 10^2$  GeV the secondaries in question are initially distributed in a narrow cone beam with  $\langle p_L \rangle \sim 5$  GeV/c and  $\langle p_t \rangle \sim 0.5$  GeV/c.

to be distributed uniformly over the interval  $(0,1)$  of  $X_{cm} = p_{cm}/(p_{cm})_{max}$ .

(c) Absorption. For  $K^+$  it is absent at all, for  $K^-$  and  $\bar{p}$  its cross section grows rapidly at low momenta. While the inelastic "deceleration" increases the number of slow particles, the absorption decreases it.

We estimate the influence by the Monte-Carlo cascade simulation. Fig.9 demonstrates the evolution of typical  $K, p$  spectra versus the path in nuclear matter ( $l = 0. - 6$  fm).

Since for  $K^+$  there no absorption, the main effect is their deceleration, which results in specific distortion of their spectra. This distortion is quite similar to the kinematical shift of spectra, considered in many models of hadron-nucleus collision (see, e.g. [9-12, 18]), but in contrast to the latter it is always accompanied by the considerable growth of  $\langle p_t \rangle$  in the slow particles region.

For  $K^-$  the whole influence of both deceleration and absorption results in the damping of whole spectra with the shape being practically unchanged. It is not the case for  $\bar{p}$ , which spectrum is at first reformed to some universal shape and then it damps without any further changes in shape.

Of course, the Fig.9 has only an illustrative use, because (i) the spectra of particles, produced in  $hA$ -interactions differ from those in  $pp$ -collisions; (ii) the intranuclear path depends on the impact parameter of primary particle, as well as the production point and direction of secondary one; (iii) there is some formation time being different for slow and fast particles, therefore the slow and fast parts of spectra must be deformed in different ways.

Nevertheless, one should stress the most general predicted feature, namely, the considerable growth of  $\langle p_t \rangle$  for slow particles. It has an obvious origin:  $\langle p_t^2 \rangle \sim \langle n_1 \rangle \langle p_{scatt}^2 \rangle$ , where  $\langle n_1 \rangle$  is the mean number of particle intranuclear interactions and  $\langle p_{scatt}^2 \rangle$  is the characteristic value of transverse momentum transferred in each of interactions. Since the cross sections are large for slow particles, both  $\langle n_1 \rangle$  and

$\langle p_t^2 \rangle$  are also large.

Thus the experimental study of  $\langle p_t^2 \rangle$  in the nucleus fragmentation region can provide a good answer to question: "Where are the particles formed, inside the nucleus or behind it?"

## 7. SUMMARY

We have considered the most popular cascade variants of the space-time picture for hadron-nucleus interactions. We have tried to restrict ourselves by the analysis of the most important hypothesis underlying each of the models considered here.

In the sect.2 assuming the (quasi)eikonal picture [11,12] of hadron-nucleus interaction we have shown that the process of  $K, \bar{p}$ -production in  $hA$ -collision cannot be reduced to a set of common  $hN$ -interactions, in each of those the particles are produced independently.

The main hypothesis of additive quark model (AQM) is that the projectile hadron can be considered as a loosely bounded system of hadronlike constituent quarks. In sect.3 we summarize the whole picture assumed by AQM [3-5,10].

In projectile fragmentation region (sect.4) AQM relates the spectra of projectile fragments to the probabilities  $F(\{q_i\} \rightarrow h)$  describing the transition of spectator set  $\{q_i\}$  to the final hadron  $h$ . The crucial question arise, whether these probabilities are independent of the nucleus atomic weight  $A$ . If they are, a lot of predictions arises (see, e.g. [3-4]). In this way there are two possibilities: either one extracts these probabilities from the  $A$ -dependence of final particle spectra, or one uses an additional assumption and extracts these probability at some fixed  $A$  in order to predict the  $A$ -dependence. The former possibility is considered in ref. [41], the latter one is considered in our work (it has been also considered in many different ways in refs. [3,4]).

The only additional assumption to be accepted in our work is the eq.(4.5), which identifies the one-spectator fragmentation function for baryon projectile with that for meson projectile.

With this assumption we verify the model, using the current data [27-29,43-46]. For  $\Lambda^0$ -production the model is in very crude agreement with data [43,44]. The character of discrepancies shows, that the  $A$ -dependent corrections to the probabilities  $F(\{q_i\} \rightarrow h)$  can be important (such corrections are considered in refs. [5,10,40]; all they are quite model-dependent). Let us stress, that any dependence of these probabilities on  $A$  reduces considerably the predictive power of AQM.

For  $K$ -meson production the model appears to be in reasonable agreement with data only at  $X \lesssim 0.3$ , while at higher  $X$  the verification becomes difficult, since the eq.(4.5) can be invalid in this region. Some troubles arise also because of triple reggeon contribution.

In the region, where AQM works satisfactory, we have found that the fragmentation hadron formation is the recombination process [3], rather than the quark fragmentation one [4] (a similar result by the independent analysis have been also obtained in ref.41).

Finally, let us stress, that the most important prediction of AQM, expressed by the eq.(4.2) is not yet tested because of the lack of data. So, one may conclude that more data are needed in order to clear up the applicability region and accuracy of AQM.

Central region. The simplest version of AQM has been considered, that assumes the quasieikonal dynamics [3] for the quark-nucleus interaction. In this model hadrons are formed via recombination of newly produced quarks [3,5,10,39]. In order to obtain some basic quantitative predictions we assume in addition, that there are no recombination of quarks, produced in different  $qN$ -interactions (where  $q$  is the projectile quark). By the other words, we neglect any "collectivization" process.

Let us stress, that any "collectivity" increases the portion of heavy particles in comparison with the predicted one by our model. Moreover, the collectivization of quarks, produced by the different projectile quark must violate the Anisovich relations [39]. Note, that this violation is to be maximal for heavy particles production.

Our model is found to be in a good agreement with data, if only one neglects completely the intranuclear interactions of

secondaries. Any attempt to take into account these interactions leads to the fatal disagreement with data.

Up to now it is not clear, whether the secondaries are formed inside the nucleus or behind it. In particular, if the pions are produced inside the nucleus, their spectra are considerably affected by the low-energy intranuclear interactions. In sect.6 we summarize the main processes of low-energy intranuclear cascading and estimate quantitatively their influence on the spectra of heavy particles ( $K, \bar{p}$ ). We find also, that for slow heavy particles ( $\gamma \lesssim 1-2$ ) their intranuclear interactions result in the considerable growth of  $\langle p_t \rangle$  in comparison with that for particles, produced in  $hN$ -interactions. Thus, the data on heavy particle production are very desirable.

### 8. CONCLUSIONS

In this work we restrict ourselves by consideration of the most popular simplest models, assuming the cascade nature for the process of hadron-nucleus interaction.

Firstly, we analyze the "structureless" eikonal models [11,12] and find that they are unacceptable in central region.

Secondly, we add the hypothesis on quark additivity and find that it improves the agreement in both projectile fragmentation and central regions. However, the remaining discrepancies prove the additive quark model to be a very crude approximation. In order to clear up its accuracy, as well as its applicability limits a number of new experiments are required. Note, that the most of them may be carried out at the present accelerators in Serpukhov, Batavia and CERN.

A number of interesting and important questions like the energy dependence of spectra, particle correlations, etc. appear to be out of our consideration. The main reason is the lack of necessary data, as well as the poor accuracy of the available ones. A very interesting information can be also obtained from the both experimental and theoretical study of the low-mass dilepton production, especially in the central and nucleus fragmentation regions, where for hadronic secondaries

there is a problem to separate the main mechanism of multiparticle production from the subsequent interactions of secondaries with the rest of nucleus.

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APPENDIX

All the estimates for EM, QEM and AQM (eqs.(2.1),(5.2)) are made by the Monte-Carlo simulation for intranuclear cascade. For the nuclear density Wigner-Dawson distribution is assumed. Our method allows us to include in natural way intranuclear interactions of secondaries.

In order to estimate particle yields in pA-interactions, their spectra in pN-interactions are required. We use the parametrization

$$E \frac{dN}{d^3p} = \exp \left[ C_1 + C_2 Y + (C_3 + C_4/E^*) L + (C_5 + C_6/E^{*2}) L^2 + B(m_t - m) \right],$$

$$Y = \ln(S/M_p^2), \quad L = \ln(1 - E_{cm}/E^*),$$

$$E^* = (S - M_x^2 + m^2)/(2 \cdot S^{1/2}),$$

$$m_t = (p_t^2 + m^2)^{1/2};$$

where  $M_p$  stands for nucleon mass,  $m, p_t$  and  $E_{cm}$  are the mass, transverse momentum and energy of produced particle in CM reference frame for nucleon of target and incoming proton;  $S = 2M_p E_0$ , where  $E_0$  is the lab energy of incoming proton. All the parameters  $C_1 - C_6, M_x, B$  are determined by the least squares fit to data [27-32], where any difference in pp- and pBe-spectra is ignored. The results of this fit are given in table 2.

The low-energy cascade of secondaries is simulated using hN-cross sections, parametrized in the following form:

$$K^+N: \quad \begin{aligned} \sigma_{el} &= \begin{cases} 13 \text{ mb}, & p < 0.8 \text{ GeV/c} \\ 23.4/(p+1) \text{ mb}, & 0.8 \text{ GeV/c} < p < 10 \text{ GeV/c} \end{cases} \\ \sigma_{in} &= \begin{cases} 0, & p < 0.8 \text{ GeV/c} \\ 15 - 7.5/(p-0.3) \text{ mb}, & 0.8 \text{ GeV/c} < p < 10 \text{ GeV/c} \end{cases} \end{aligned}$$

$$K^-N: \quad \begin{aligned} \sigma_{el} &= 15/p \text{ mb}, \quad 0.25 \text{ GeV/c} < p < 4 \text{ GeV/c} \\ \sigma_{in} &= \begin{cases} 0, & p < 0.8 \text{ GeV/c} \\ 6 - 1.2/(p-0.6) \text{ mb}, & 0.8 \text{ GeV/c} < p < 10 \text{ GeV/c} \end{cases} \end{aligned}$$

$$K^-N: \quad \sigma_{abs} = \begin{cases} 13/p \text{ mb}, & p < 1 \text{ GeV/c} \\ 13/p^{1/2} \text{ mb}, & 1 \text{ GeV/c} < p < 10 \text{ GeV/c} \end{cases}$$

$$\bar{p}N: \quad \begin{aligned} \sigma_{el} &= 44 p^{-1/2} \text{ mb}, \quad 0.3 \text{ GeV/c} < p < 20 \text{ GeV/c} \\ \sigma_{abs} &= \begin{cases} 50/p \text{ mb}, & 0.3 \text{ GeV/c} < p < 0.6 \text{ GeV/c} \\ 60 p^{-1/2} \text{ mb}, & 0.6 \text{ GeV/c} < p < 20 \text{ GeV/c} \end{cases} \\ \sigma_{in} &= \begin{cases} 0, & p < 1.6 \text{ GeV/c} \\ 20 \text{ mb}, & 1.6 \text{ GeV/c} < p < 20 \text{ GeV/c} \end{cases} \end{aligned}$$

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FIGURE CAPTIONS

- Fig.1. The dependence of  $\tilde{\alpha} = d \ln(E \frac{dN}{d^3p}) / d \ln A$  on the rapidity  $y$ , (quasieikonal model,  $E_0=70$  Gev). The curves 1,2 correspond to  $L_F = \infty, 0$  respectively. The points corresponds to data:  $\circ, \Delta$  [33,34] for  $p_t = 0.5-1$  Gev/c;  $\bullet$  are the same data for  $p_t > 1$  Gev/c;  $+$  [27] and  $*$  [35].
- Fig.2. The same as in Fig.1 for eikonal model.
- Fig.3. The probabilities  $F(\{q_i\} \rightarrow h')$  versus  $X^* = X/\langle X_q \rangle$ . The curves 1,2 correspond to the estimates from the parametrization of data [45,46], while the points are estimated directly from the corresponding data points. The curve 3 is described in subsect.4.4.
- Fig.4.  $\Lambda^0$ -production. The full points are the AQM predictions, the open ones are the data [44].
- Fig.5.  $K_S^0$ -production. The full points are the AQM predictions, the open ones are the data [44].
- Fig.6.  $K^+$ -production. The curves are the AQM predictions. The points correspond to the data:  $\bullet$  [27],  $\theta_{lab}=6$  mrad,  $E_0=67$  Gev/c;  $\circ$  [28],  $\theta_{lab}=17$  mrad,  $E_0=24$  Gev;  $*$  [29],  $\theta_{lab}=20$  mrad,  $E_0=19.2$  Gev. All the data, as well as the predictions are taken at the same  $p_t(x)$ .
- Fig.7. The AQM predictions in the central region. All the notations and data are the same, as in Fig.1. The dashed line correspond to the AQM predictions at  $L_F \sim P/m^2$ ,  $m^2 = 0.25$  Gev<sup>2</sup>.

- Fig.8. The dependence of the  $p_t^2$ -slope parameter versus longitudinal rapidity as it is expected in AQM because of intranuclear interactions of secondaries. The solid curve is for  $L_F = \infty$  and coincides with those for pp-collisions at the same  $E_0$  (= 70 Gev in our case). The dashed ones correspond to  $L_F = 0$ ,  $A = 27$  and  $184$  respectively.

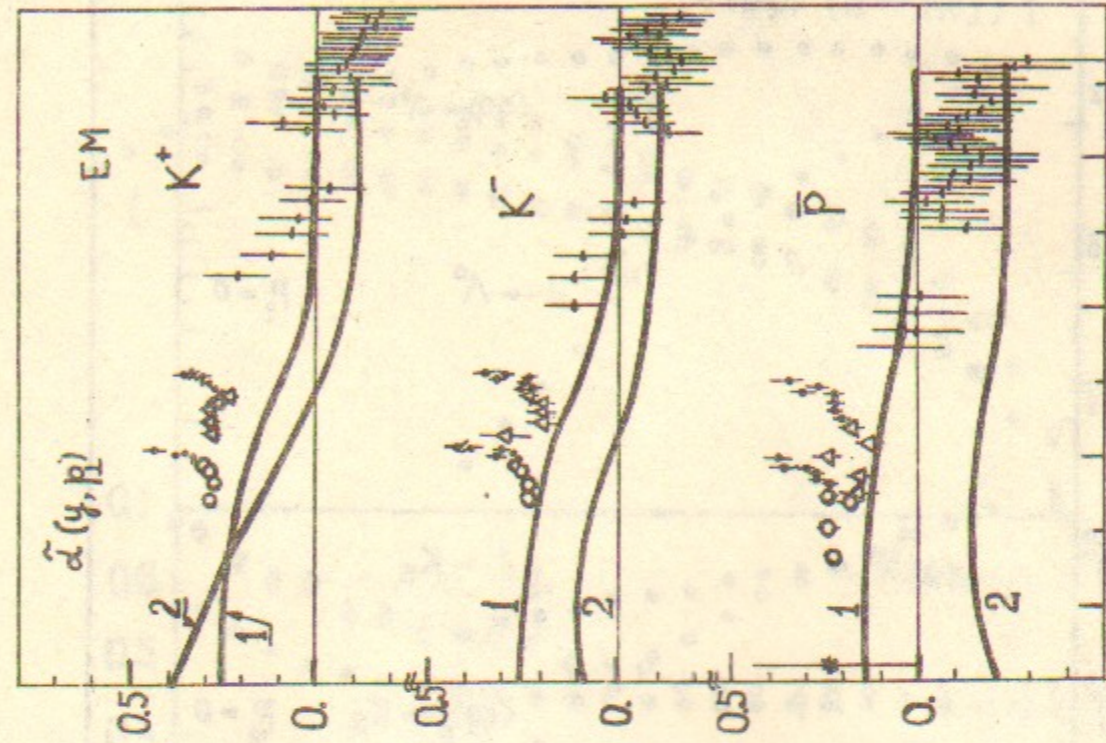
- Fig.9. The evolution of spectra and  $p_t^2$ -slope parameters  $B$  versus the thickness of nuclear matter: 1 - 0 fm, 2-5 from 1.5 to 6 fm with the step 1.5 fm.

Table 1. The cross sections  $\sigma_A(h \rightarrow \{q_i\})$  versus A.  
(It is assumed, that  $\sigma_{qN} = 9\text{mb}$ ).

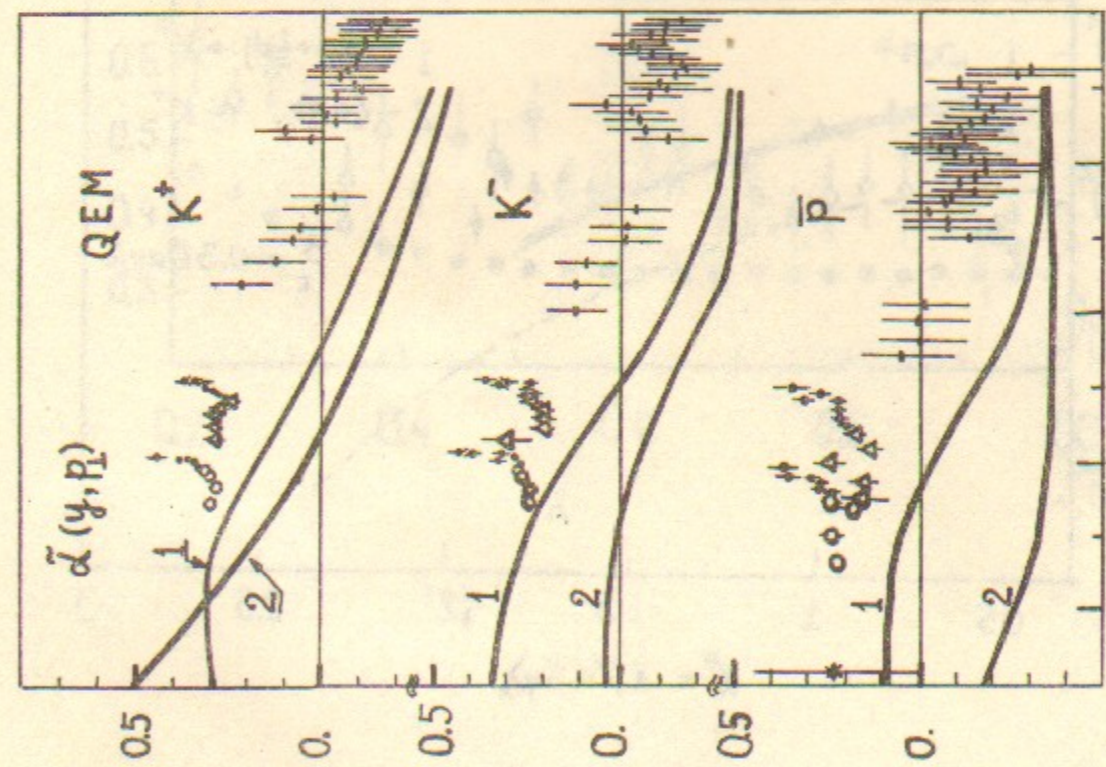
A	$\sigma_A(p \rightarrow q)$ mb	$\sigma_A(p \rightarrow qq)$ mb	$\sigma_A(\pi \rightarrow q)$ mb
9	33.3	136.5	113.2
12	47.7	163.8	141.0
64	264.0	376.3	426.8
183	564.8	532.9	731.8
206	604.6	549.4	769.3

Table 2.

	$c_1$	$c_2$	$c_3$	$c_4$ Gev	$c_5$	$c_6$ Gev <sup>2</sup>	$M_X$ Gev	B Gev <sup>-2</sup>
$K^+$	-3.23	0.16	5.64	-8.21	0.933	-4.96	2.30	-4.41
$K^-$	-3.96	0.27	9.92	-14.9	0.997	-14.5	1.75	-4.53
$\bar{p}$	-4.94	0.367	12.6	-18.2	1.22	-15.1	2.21	-5.45



1 2 3 4  
FIG. 2 y



1 2 3 4  
FIG. 1 y

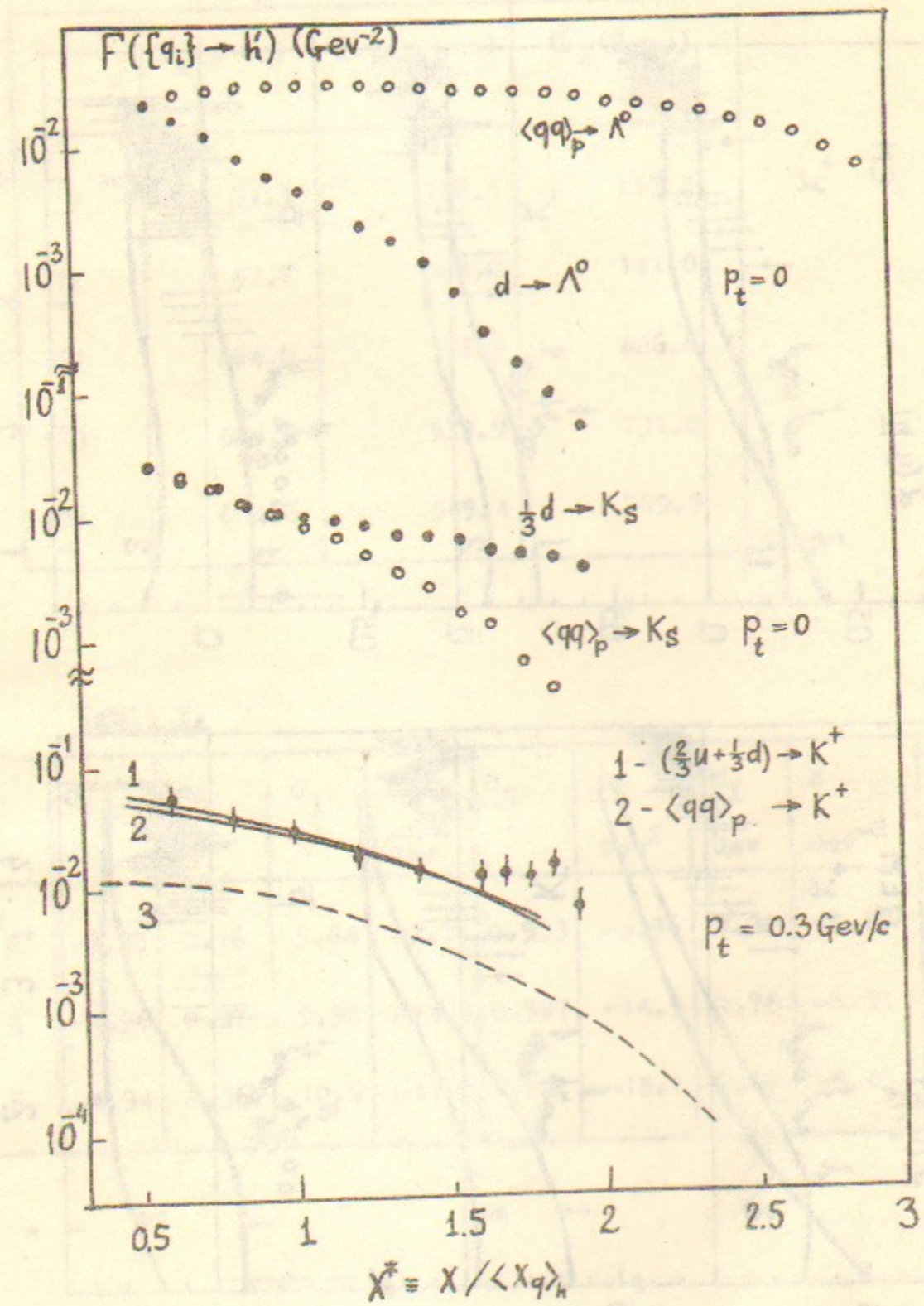


Fig. 3

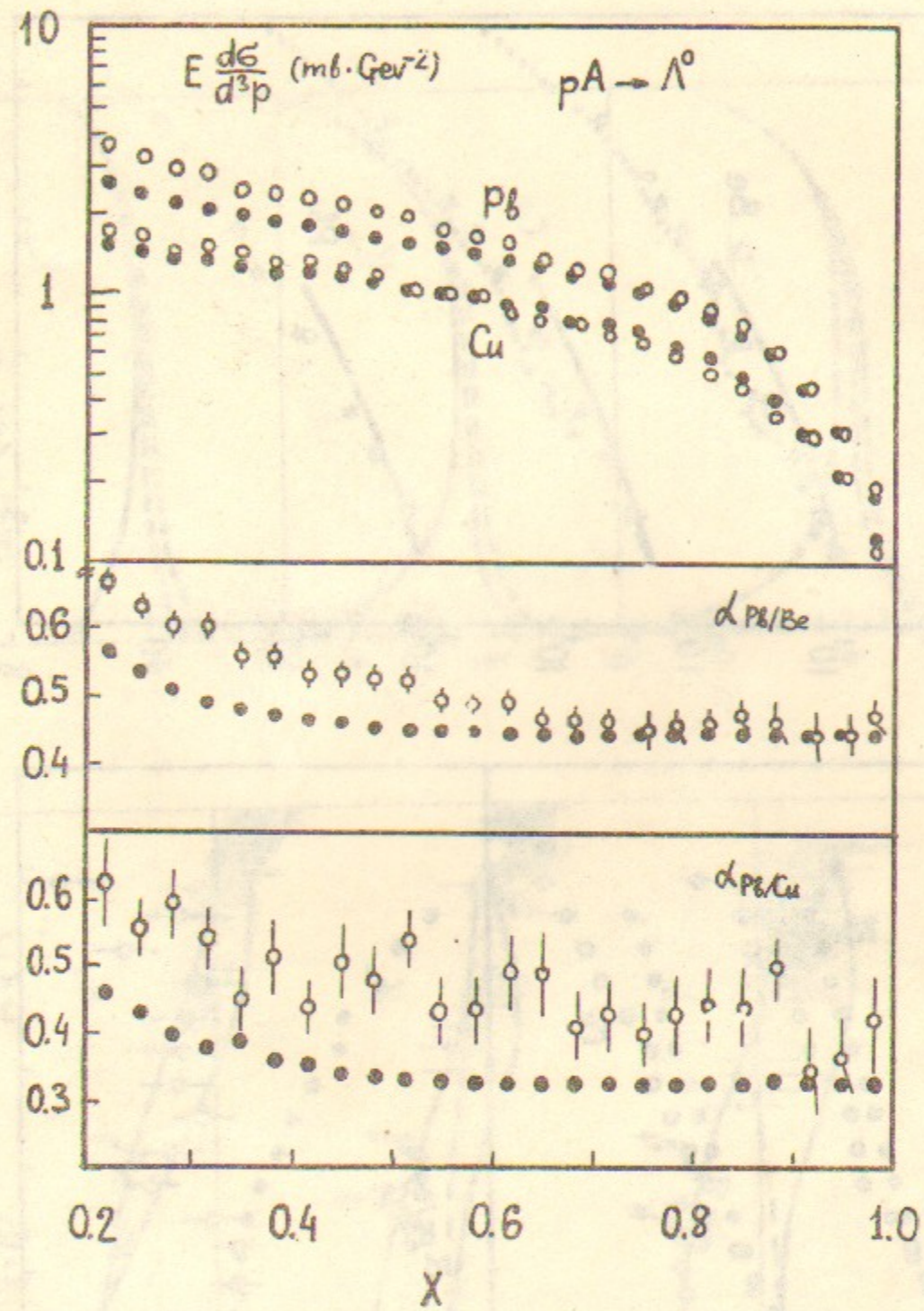


Fig. 4.

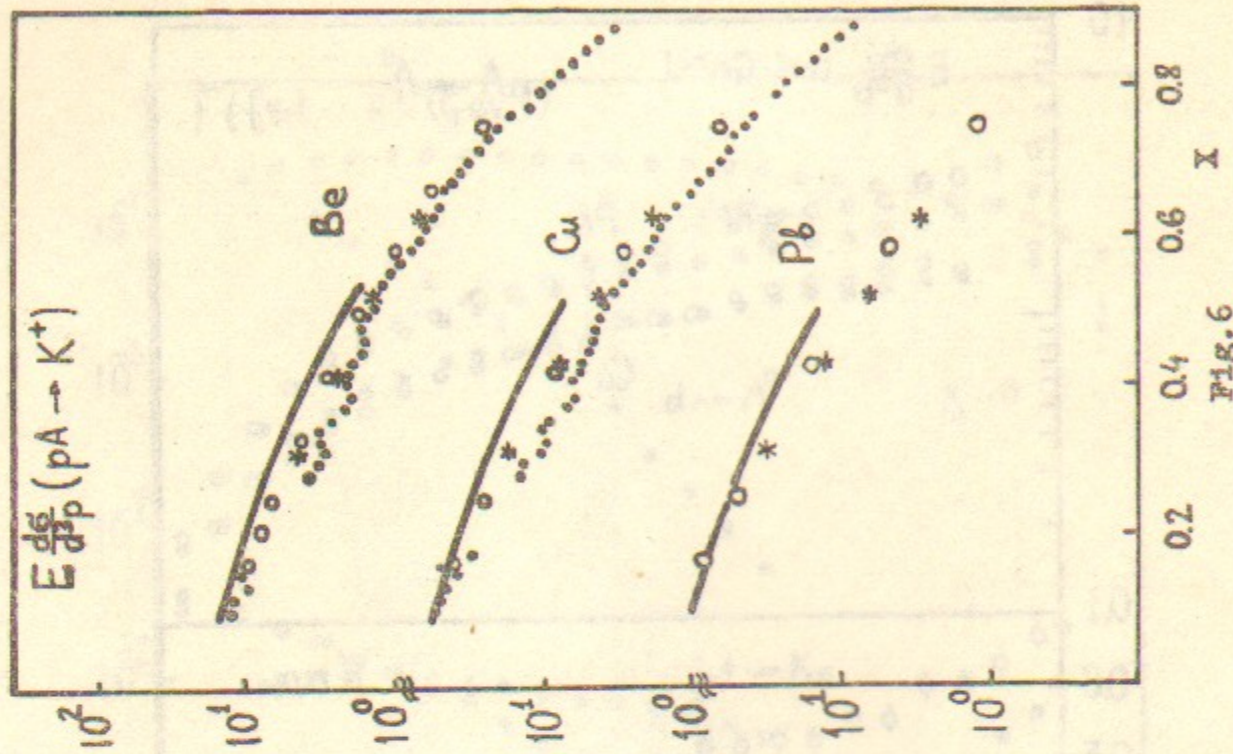


Fig.6

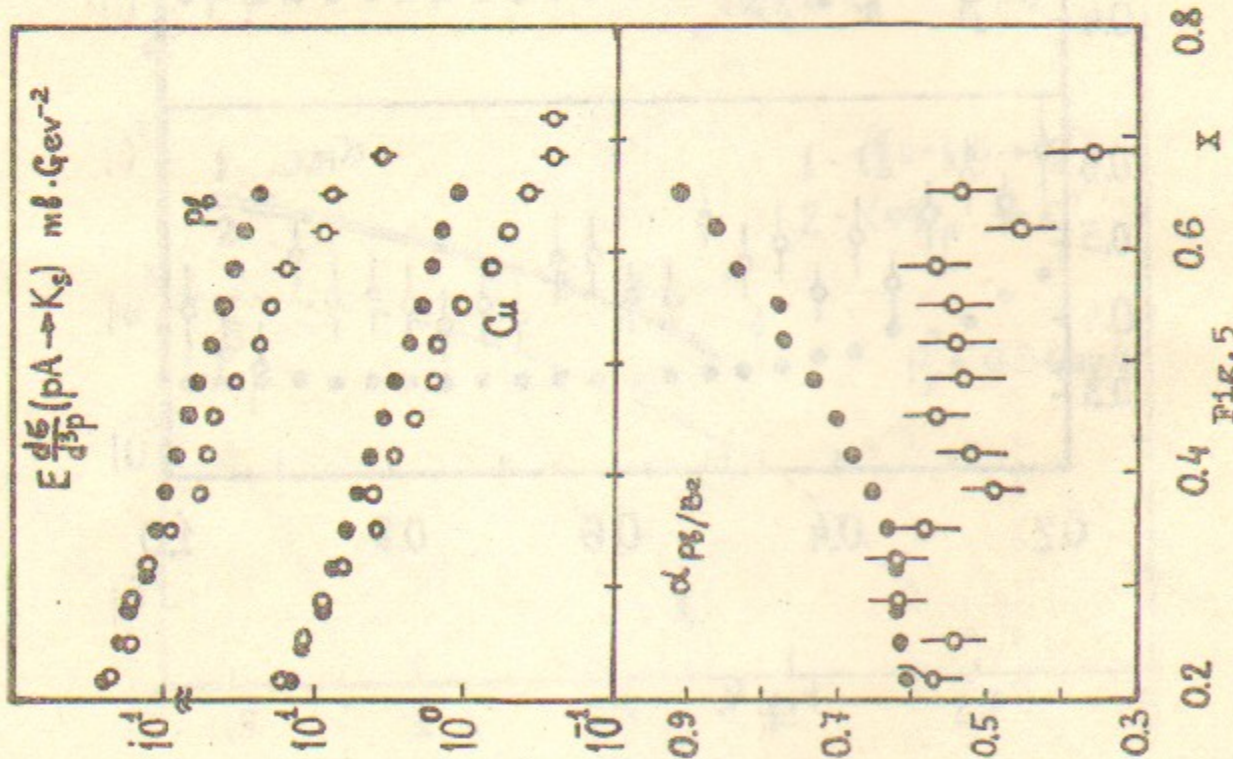


Fig.5

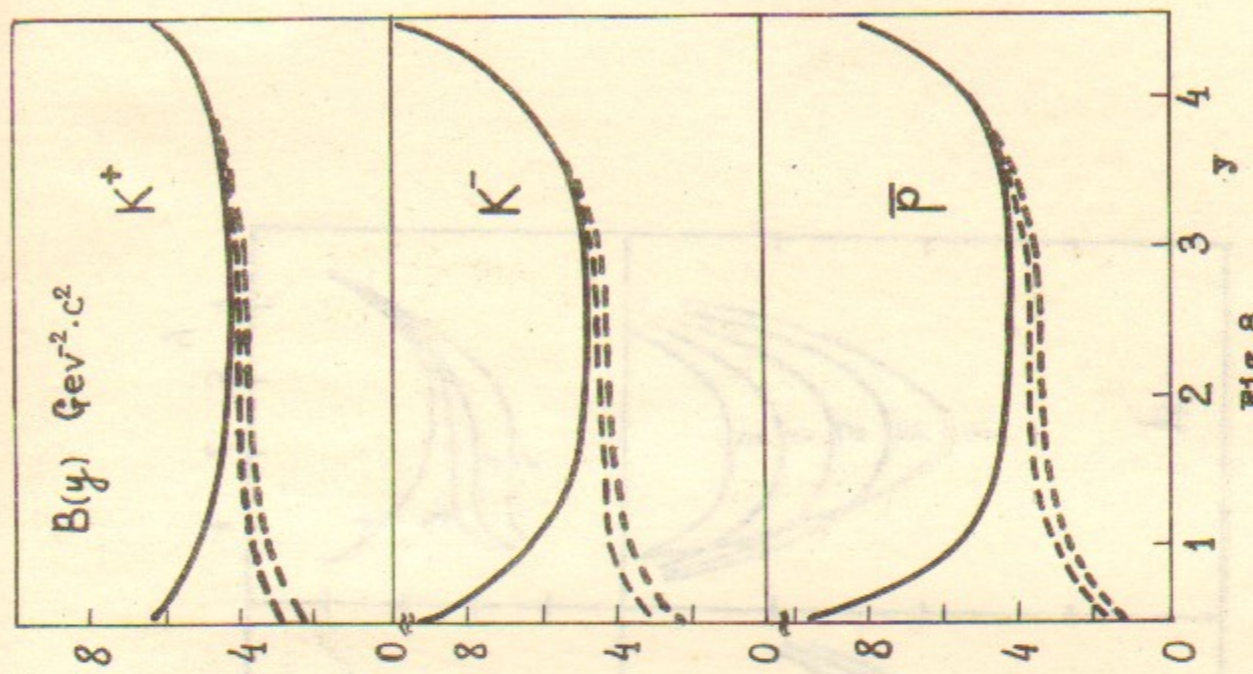


Fig.8

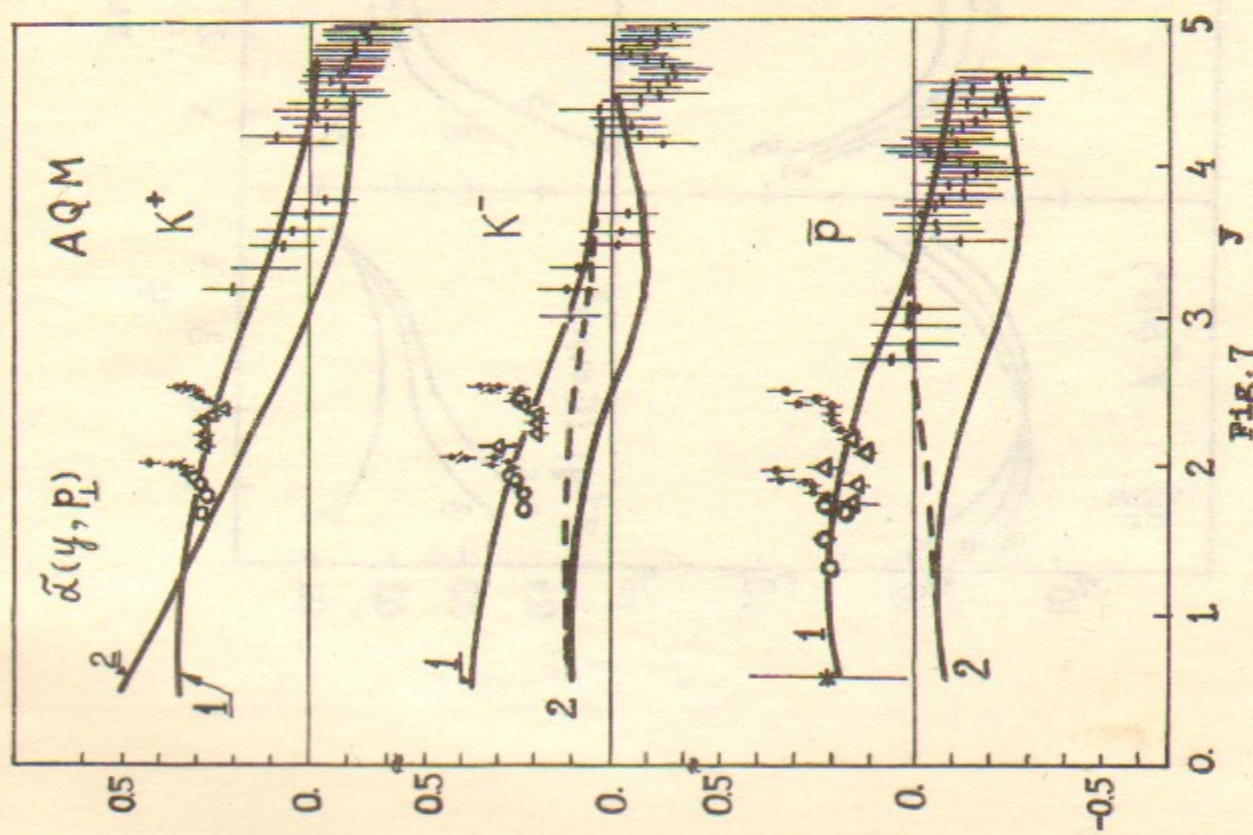


Fig.7

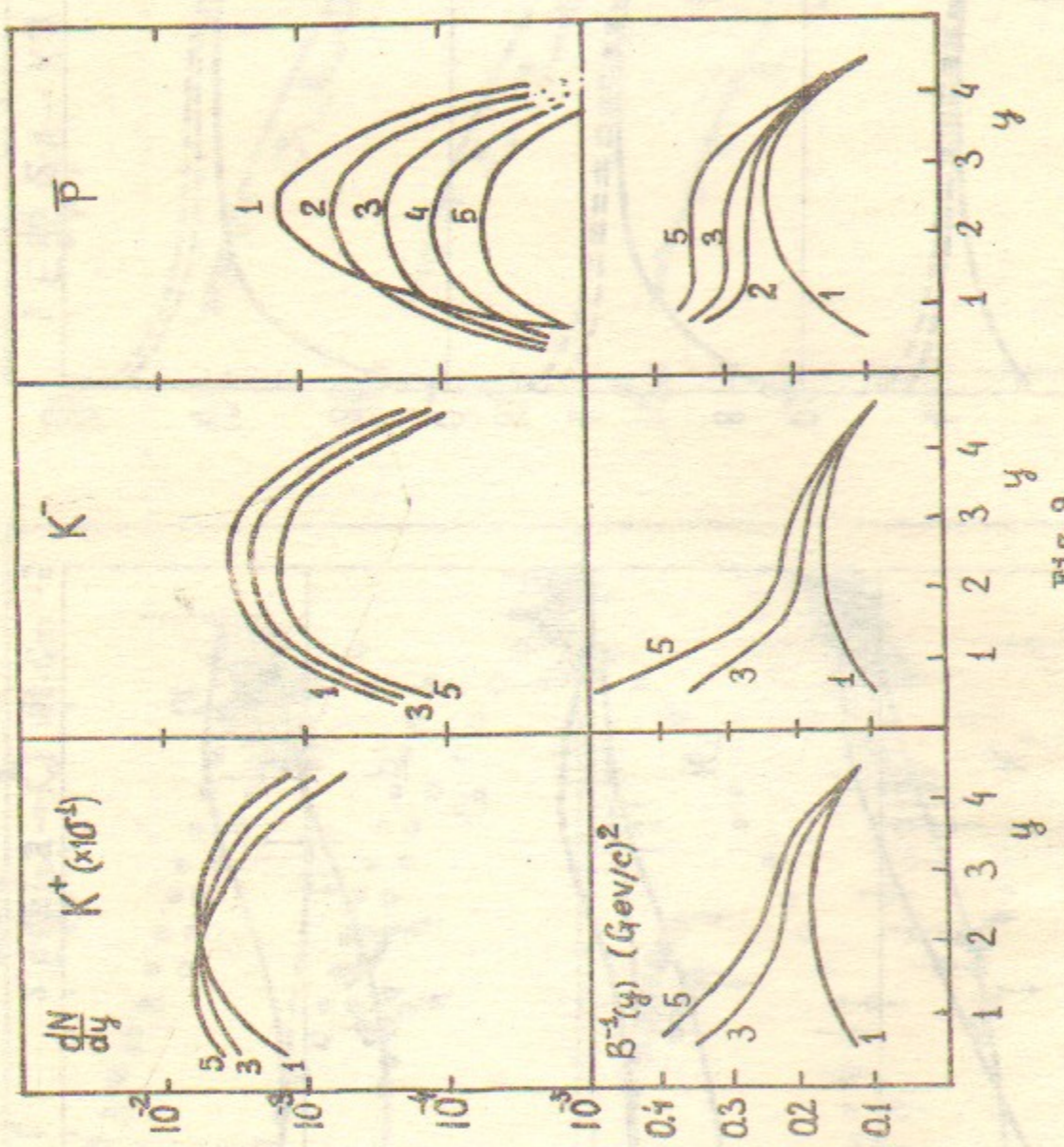


FIG. 9