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ESTIMATE OF THE NEUTRON ELECTRIC
DIPOLE MOMENT IN THE WEINBERG MODEL
OF C P VIOLATION

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## ESTIMATE OF THE NEUTRON ELECTRIC DIPOLE MOMENT IN THE WEINBERG MODEL OF C P VIOLATION

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## Abstract

The neutron electric dipole moment  $\mathcal{D}_n$  in the model discussed is shown to be caused mostly by the contribution of strange quarks. The obtained estimate for  $\mathcal{D}_n$  is close to the experimental bound.

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1. The aim of the present work is a detailed analysis of different mechanisms leading to the neutron electric dipole moment (EDM) in the Weinberg model of C P violation[1]. Such an analysis seems to be quite timely since the experimental bound[2,3] on the value of  $\Theta_n$  is even now close to the predictions obtained early in the frame of this model. Moreover, in the near future the neutron E D M will be measured perhaps with the accuracy better by an order of magnitude[3].

From the theoretical point of view the model[1] seems to be sufficiently realistic one. C P violation is caused here by the charged Higgs bosons exchange. The effective four-fermion C P violating interaction looks as follows

H= A (md cose de u - md sine de CL + ms sine 52 UL+ ms cose 524)

 $\times \left( m_{4} \cos \theta_{c} \, \bar{u}_{R} d_{L} + m_{u} \sin \theta_{c} \, \bar{u}_{R} d_{L} - m_{c} \sin \theta_{c} \, \bar{c}_{R} d_{L} + m_{c} \cos \theta_{c} \, \bar{c}_{R} d_{L} \right) + H.C.$ (1)

where  $\mathcal{A}$  is in fact a parameter of the model. An interest to the model[1] is caused by the following reasons. Firstly, in distinction from the Kobayashi-Maskawa model[4] it provides a natural explanation for the smallness of C P violating interaction in comparison to the usual weak one. The corresponding small parameter is  $\frac{m_e^2}{m_h^2}$  where  $\frac{m_F}{m_F}$ ,  $\frac{m_H}{m_F}$  are the typical masses of fermions and Higgs bosons. Its origin is evident: the coupling of a Higgs boson to a fermion is proportional to  $\frac{m_F}{m_F}$ , and the factor  $\frac{4m_H^2}{m_H}$  is a remnant of the Higgs field propagator. Secondly, the non-conservation of C P parity in the Higgs sector can arise, as it was shown in Ref.[5], due to spontaneous

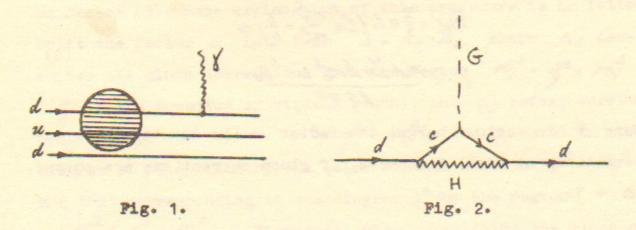
violation of C P - invariance which seems quite attractive from the theoretical point of view. Thirdly, in the model discussed there is a dynamical suppression of the amplitude of the direct

 $K_{\perp} \rightarrow 2\pi$  transition[6,7] violating the relation  $z_z \approx z_{00}$  checked experimentally with the accuracy of some per cent.

Let us discuss briefly previous estimates of the neutron E D M in the model [1]. In Refs. [1,6] the quark E D M was calculated with logarithmic accuracy (the terms of the order of unity were dropped in comparison with  $\ln \frac{m_{\star}^2}{m_e^2}$ ; here  $m_c$  is the mass of C - quark). Then, neglecting the contribution of strange quarks to  $\Theta_n$  and assuming validity of the simplest quark model, Weinberg[1] obtained  $\Theta_n \simeq 2.3 \cdot 10^{-24}$  e.cm. More careful estimates by Anselm and D'yakonov[6] led under the same assumptions to  $\Theta_n \simeq -2.8 \cdot 10^{-25}$  e.cm.

Let us discuss now the effects that influence the magnitu- \* de of neutron E D M , but previously were not taken into account.

Note first of all that all the earlier calculations refer to the contribution of quark E D M to the neutron dipole moment. However,  $\Re$  may arise also because the discussed interaction admixes to the neutron wave function a state with opposite C P - parity (see, e.g., Fig. 1). In the Weinberg model the direct four-fermion interaction between  $2\ell$  - and  $2\ell$  - quarks is small and proportional to the current mass squared  $2\ell$  . However, another mechanism of C P violation in a system of light quarks is possible, shown at the Fig. 2.



The structure of the operator arising when the diagram 2 is calculated, looks as  $g = \frac{1}{2} \int_{0}^{2} \int$ 

comparison with the direct one due to heavy intermediate

C - quark. The quark C D M contribution to the neutron dipole

moment is considered in detail in the Section 2.

In the Section 3 the estimate for  $\mathcal{D}_n$  is given with the account of strange quarks. The fact that their contribution can be essential was noted previously [6,8]. In particular, in Ref. [6] it was stressed that the S - quark EDM is

 $\frac{m_S}{m_d} \cdot clg^2 \theta_c \sim$  400 times larger than the d - quark one. Our conclusion is that just S - quarks give the main contribution to  $\Theta_R$ .

2. Before the discussion of C D M contribution we shall write down the expression for the & - quark E D M -caused by the diagrams 3:

$$\mathcal{D}_{d} = \frac{2}{3} e L \left( \ln \frac{m_{H}}{m_{e}^{2}} - \frac{3}{4} \right) \Delta$$

$$L = \frac{m_{d} \int_{m} A m_{e}^{2} \sin^{2} \theta_{e}}{16 \pi^{2}}$$
(2)

Here  $\ell$  is proton charge; the factor  $\Delta$  is due to strong interactions at small distances. If gluon corrections are absent,  $\Delta = 1$ .

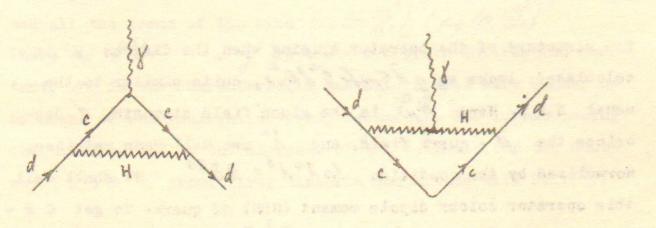


Fig. 3<sup>8</sup>
Fig. 3<sup>0</sup>

In the formula (2) we omit the contribution due to intermediate  $\mathcal{U}$  - quark that has the additional small factor  $m_u^2/m_c^2$   $c l g^2 \theta_c$ . Besides that, when calculating  $\Theta_N$ , we shall not take into account the  $\mathcal{U}$  - quark E D M which is  $m_s^2/m_c^2$  times smaller than  $\Theta_d$ .

The value of the factor  $\Delta$  was obtained in the main logarithmic approximation in the Ref.[7]. In the notations of the present work

$$\Delta \simeq \frac{0.9}{\ln \frac{m_H^2}{m^2}} \tag{3}$$

Our point of view is that one should keep both constant term along with  $\ln \frac{m_s^2}{m_s^2}$  (-3/4 in this case) and the correction factor (3). Some explanation of this procedure is as follows. Split the factor  $\Delta$  into two:  $\Delta = \Delta_1 \cdot \Delta_2$  where  $\Delta_1$  describes the gluon corrections from the region  $m_c^2 < g^2 < m_s^2$  (g is the momentum of virtual gluon), and  $\Delta_2$  refers correspondingly to the region  $m_s^2 < g^2 < m_c^2$ . The integral in the diagram  $3^b$  is defined by the quark virtual momenta  $g^2 \sim m_s^2$  and that corresponding to the diagram  $3^b$  by the region

 $m^2 < p_g^2 < m_H^2$ . Therefore, when calculating the gluon corrections in the region  $P_g^2 \le m_e^2$ , the quantity defined by these diagrams can be considered as a local operator

donn's dFnn (here Fnn is the electromagnetic field strength). Hence, the formula (2) is valid for gluon corrections in the region  $P_g^2 < m_e^2$ . But it is not literally true for gluon corrections of virtuality  $P_g^2 > m_e^2$ . Nevertheless, one can hope that an error arising in this way is not large sin-

ce  $\alpha_s$  is in this region already small, and the main contribution to  $\Delta$  is defined by the interval  $m^2 < l_g^2 < m_e^2$ .

Let us pass now to the calculation of the A - quark
C D M . The corresponding contribution to the Hamiltonian is
caused by the diagram 2 and is equal to

In this expression the factor  $\Delta g$  takes into account strong interaction effects at small distances,  $\Delta g = 1$  when the latter are switched off. This factor was calculated in fact in Ref.[7]. It is equal to

$$\Delta_{g} = 2 \mathcal{F}_{2}^{13/27} \left\{ \frac{29/25}{d_{g}(m) l_{n}} \frac{m_{s}^{2}}{m_{c}^{2}} \right\} \left\{ 0.21 \left( \frac{1}{2} 0.54 - \frac{1}{2} 0.41 \right) + 0.04 \left( \frac{1}{2} \frac{12}{2} - \frac{1}{2} 0.41 \right) + 0.01 \left( \frac{1}{2} 0.05 - \frac{1}{2} 0.41 \right) \right\} \approx 0.35$$

$$(5)$$

where

We believe that here it is reasonable again both to keep the constant along with  $ln \frac{m^2}{m_c^2}$  and to take into account the gluon corrections in the whole region of  $l_g^2$ :  $l_g^2 < l_g^2 < l$ 

Pass to the estimate of the neutron E D M . The simplest quark model leads to the following expression for the contribution of the A - quark E D M to the neutron E D M

$$\mathfrak{D}_{n}(d) = \frac{4}{3}\mathfrak{D}d = \frac{8}{9}Le\left(\ln\frac{m_{H}}{m_{e}^{2}} - \frac{3}{4}\right)\Delta \tag{6}$$

Remember that the 2 - quark E D M is negligibly small. Taking (2) into account, we obtain

$$\mathfrak{D}_{n}(d) \simeq -0.86 \cdot 10^{-25} \, e. \, cm$$
 (7)

Here and below we take  $\sin \theta_c = 0.22$ ,  $m_d = 7$  MeV and, in accordance with Refs. [5-7],  $\ln \frac{m_H^2}{m_c^2} \approx 3$ ,  $\Im M A M_c^2 = -0.3$  Gp.

To estimate the contribution to the neutron E D M due to the quark C D M (we denote this contribution  $\Theta_n(6)$ ) we use non-relativistic quark model. Some justification of such an approach is that a static characteristic of nucleon is discussed. In this approximation the interaction (4) is rewritten as the following operator acting on the  $\mathcal{A}$  - quark wave-functions:

$$H_g = -gL(\ln \frac{m_h^2}{m_e^2} - \frac{3}{2}) + \frac{3}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} \left(-\frac{\pi}{2} A_o^2\right)$$
 (8)

We omit here, by the way, a non-linear term in the gluon electric field  $G_{i0}^{a}$  since in usual units it is inversely proportional to the velocity of light. The C P - odd interaction (8)

can be rewritten as follows:

$$H_g = iL \left( \ln \frac{m_H^2}{m_e^2} - \frac{3}{2} \right) \Delta_g \sum_{\alpha} \vec{c}_{\alpha} \left[ \vec{P}_{\alpha} H_0 \right]$$
 (9)

where  $H_0 = \sum_{\kappa} \frac{\vec{p}_{\kappa}^2}{2m} \cdot g \frac{\vec{p}_{\kappa}^2}{2} f_0^{\alpha}(\vec{r}_{\kappa})$  is the non-perturbed non-relativistic Hamiltonian of the quark system. The wave-function with the account for the perturbation (9) is transformed in such a way:

$$|\tilde{n}\rangle = |n\rangle + \sum_{m} \frac{|m\rangle\langle m| H_{q}| n\rangle}{En - Eno} =$$

$$= |n\rangle + iL \left( \ln \frac{m_{H}^{2}}{m_{e}^{2}} - \frac{3}{2} \right) \Delta_{q} \sum_{m} |m\rangle\langle m| \sum_{d} \vec{\delta}_{d} \vec{P}_{d} |n\rangle =$$

$$= \int_{q} 1 + iL \left( \ln \frac{m_{H}^{2}}{m_{e}^{2}} - \frac{3}{2} \right) \Delta_{q} \sum_{d} \vec{\delta}_{d} \vec{P}_{d} \int_{q} |n\rangle$$

$$= \int_{q} 1 + iL \left( \ln \frac{m_{H}^{2}}{m_{e}^{2}} - \frac{3}{2} \right) \Delta_{q} \sum_{d} \vec{\delta}_{d} \vec{P}_{d} \int_{q} |n\rangle$$

The neutron E D M induced in this way is equal to

In the valence quarks model we find:

$$\Theta_{n}(6) = \frac{4e}{g} L \left( \ln \frac{m_{H}^{2}}{m_{e}^{2}} - \frac{3}{2} \right) \Delta_{g} \simeq -0.34 \cdot 10^{-25} e. \text{ ou}$$
 (12)

Therefore, the total contribution of non-strange quarks to the neutron E D M is

$$\partial_n (d.) + \partial_n (6) = -1.2 \cdot 10^{-25} e. cm.$$
 (13)

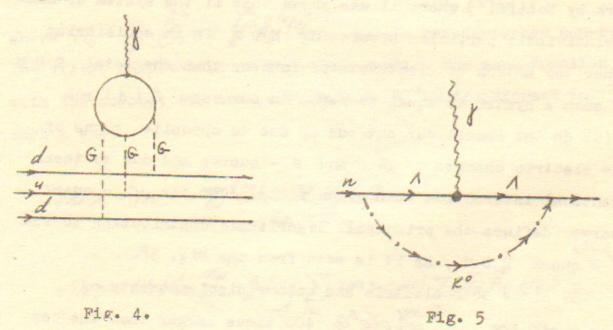
The idea of this calculation of  $D_n(G)$  is taken from the work by Schiff[9] where it was shown that if the system of non-relativistic particles possessing E D M is in equilibrium under the action of electrostatic forces, then the total E D M of such a system is equal to zero. In our case  $D_n(d)$  and  $D_n(G)$  do not cancel out, but add up due to opposite signs of the electric charges of  $\mathcal{A}$  - and  $\mathcal{C}$  - quarks and due to their identical interaction with gluon field. Just the  $\mathcal{C}$  - quark charge defines the principal, logarithmic contribution to the  $\mathcal{A}$  - quark E D M, as it is seen from the Fig. 3<sup>8</sup>.

3. Since both electric and colour dipole moments of S - quark are mg ctg & -400 times larger than the corresponding  $\alpha$  - quark moments, then even if the  $S\bar{S}$  - pairs admixture in neutron constitutes 1:2% (which does not contradict experimental data), their contribution to the neutron E D M can turn out decisive one. In the quantum chromodynamics language this contribution could be described, e.g., by the diagram 4 with minimal number of closed loops. C P - invariance is violated here in any of the vertices of the 5 - quark loop: either in a gluon one (due to CDM), or in a photon one (due to E D M). Note that the contribution of a loop with heavy quarks is much smaller than this one, as it follows already from dimensional considerations. Although a quantitative computation of the diagrams of the type 4 does not seem possible, their rough estimate is consistent with the assumption of small  $S\bar{S}$  - pairs admixture in a nucleon.

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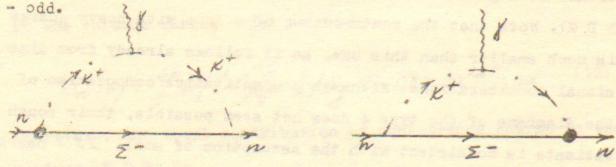
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In the phenomenological language such a mechanism is described, say, by the diagram 5 where both  $KN\Lambda$  - vertices are strong, and  $\sqrt{\phantom{a}}$  - quantum interacts with the  $\Lambda$  - hyperon E D M.

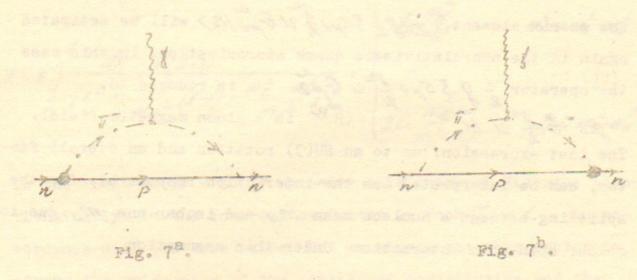


The contribution of this diagram to the neutron E D M is very large, being close to the S - quark E D M (D<sub>B</sub>). However, the fact that the strange quarks admixture in a neutron is small, means that a strong cancellation between different diagrams of this kind takes place.

One should consider separately the diagrams 6<sup>a,b</sup> where one of the KNE vertices is a strong one, and another is CP -



As it was stressed in Ref. [10], the similar graphs 7 a,b are singled out by the fact that their contribution to the neutron E D M tends to infinity in the limit of SU(2) x SU(2) symmetry ~ ln 1/m, . Previously the diagrams b,7 were calculated in Ref. [11].



From our point of view, the contribution of the diagrams of to the neutron EDM is singled out by two circumstances.

Firstly, the CP-odd KNZ vertex can arise due to the S-quark CDM and is therefore relatively large. Secondly, the contribution discussed tends logarithmically to infinity in the limit of SU(3) x SU(3) symmetry. Hence there are no ground to expect considerable cancellation of this contribution with other diagrams.

To calculate the CP - odd KNZ vertex  $(g_{KNZ})$  due to the effective Hamiltonian (cf. with (4))

we use the PCAC hypothesis:

$$g'_{KNZ} = \langle K \Sigma | H_S | n \rangle = L \frac{m_S}{m_d} e^{\frac{1}{2}g} \Theta_c \Delta_g \left( \ln \frac{m_H}{m_Z} - \frac{3}{2} \right)^{\kappa}$$

$$(15)$$

$$\kappa \langle \Sigma | \frac{1}{2} g \, \overline{S} \, G_{MV} \, \frac{1}{2} \, 2 \, G_{MV} | n \rangle \frac{1}{f_K}$$

The matrix element  $\langle z/g_z | \bar{s} \delta_{\mu\nu} \rangle_z^2 \mathcal{U} \delta_{\mu\nu} / n \rangle$  will be estimated again in the nonrelativistic quark approximation. In this case the operator  $\frac{1}{2} g_z | \bar{s} \delta_{\mu\nu} \rangle_z^2 \mathcal{U} \delta_{\mu\nu}$  is reduced to  $-g_z | \bar{s} \delta_z | \bar{s} \rangle_z^2 \mathcal{U} \delta_{\mu\nu} = 0$  (Ha is a gluon magnetic field). The last expression, up to an SU(3) rotation and an overall factor, can be interpreted as the interaction responsible for the splitting between a nucleon mass  $m_N$  and isobar one  $m_\Delta$  due to colour spin-spin interaction. Under this assumption

$$\langle z | -g s^{\dagger} \vec{\delta} \frac{1}{2} u \vec{H}^{\alpha} | n \rangle = -\frac{2}{3} m^{*} (m_{b} - m_{N})$$
 (16)

and

$$g_{KN\Sigma} = -\frac{2}{3} \frac{m^*(m_{\Delta} - m_{N})}{f_{K}} \left[ \lim_{m \to \infty} etg^2 \theta_{e} \log \left( \ln \frac{m_{N}^2}{m_{Z}^2} - \frac{3}{2} \right) \right]$$
 (17)

Here m\* ~ 300 MeV is the effective quark mass.

Direct calculation of the diagrams 6 gives the following result for the strange quarks contribution to the neutron E D M

$$\mathcal{D}_{n}(s) = \frac{-(2d-1)\sqrt{2}}{24\pi^{2}} \frac{g_{\pi NN}}{m_{N}} \frac{m^{2}}{f_{R}} e_{L} \frac{ms}{md} \times e^{2g^{2}} \frac{g_{\pi NN}}{m_{L}^{2}} \frac{m^{2}}{2} \frac{3}{2} \frac{\Delta g}{g} \cdot I$$
(18)

According to SU(3) symmetry we assume here that the strong  $\mathbb{Z} \times \mathbb{Z}$  constant is equal to  $(2\alpha-1)/2/3\pi \mathbb{Z} \times \mathbb{Z}$ . From the data on  $\beta$  - decay of  $\Sigma$  hyperon it follows that the relative weight of  $\Im$  - coupling  $\alpha \simeq 2/3$ . The factor I in (18) is equal to

$$I = 2 \int_{0}^{1} \frac{dx \cdot x(1-x)}{\left[x^{2} + \frac{m_{N}^{2}}{m_{N}^{2}}(1-x) + \frac{m_{Z}^{2} - m_{N}^{2}}{m_{N}^{2}}\right]} = \begin{cases} \ln \frac{m_{N}^{2}}{m_{K}^{2}} & m_{K}^{2} > m_{X}^{2} - m_{N}^{2} \\ 2 \ln \frac{m_{N}^{2}}{m_{Z}^{2} - m_{N}^{2}} & m_{K}^{2} < m_{Z}^{2} - m_{N}^{2} \end{cases}$$

$$(19)$$

In the limit  $SU(3) \times SU(3)$  symmetry this integral diverges logarithmically. In the concrete estimates we took  $T \simeq L$  in accordance with its true value. But we single out this factor to stress the uniqueness of the considered contribution and the fact that there are no special grounds for its cancellation with other contributions. Note that reasonable account for non-pointlike structure of the interactions changes slightly the numerical value of the integral (19).

The final estimate by the formulae (18), (19) gives

$$\mathfrak{D}_{n}(5) = +8.10^{-25} e.cm$$
 (20)

As it is seen from (19), this contribution is due to strong interactions at large distances. It is non-analytic in the breaking of  $SU(3) \times SU(3)$  symmetry and therefore cannot be obtained neither in perturbation theory in  $\alpha_s$ , nor, say, in the bag model. Nevertheless, the numerical value (20) is in agreement with the most naive estimate grounded on the assumption of the

small ( $\sim 10^{-2}$ ) admixture of strange quarks in a nucleon.

An analogous estimate of the diagrams (7) shows that the magnitude of their contribution is about 100 times smaller than (20). Therefore, the contribution of strange quarks dominates indeed the neutron E D M. With the account for (7), (12), (20) the latter is equal to

Let us discuss briefly the accuracy of this result. Firstly, the numerical value of the main parameter of the model Inflished up to a factor  $\sim 2$  only, due to the presence of two mechanisms of CP - violating  $\mathcal{K}_L$  - decays with quite comparable contributions  $[^6, ^7]$ . Secondly, the accuracy of calculation of strong interaction corrections at small distances is also not too high: the quantities  $\Delta$ ,  $\Delta_g$  (see (2)-(5)) are defined up to a factor  $\sim 1.5 \div 2$ . And at last, about our estimate of the CP - odd constant  $\mathcal{J}_{KNZ}$ . When using the considerations, analogous to those leading to the expression (17), for the estimate of the contribution of CDM - type operator to the decay  $\mathcal{K}_L \twoheadrightarrow 2\mathcal{N}$ , we come to the result that is 2.5 times smaller than simple dimensional estimates for this amplitude  $[^{12}, ^6, ^7]$ . One can hope therefore that our value (17) for  $\mathcal{J}_{KNZ}$  is not overestimated.

Hence, the neutron E D M in the Weinberg model of C P violation is due mostly to the contribution of strange quarks and constitutes about  $10^{-24}$  e·cm. This estimate is close to the best of the published bounds [3]  $\Re n < 1.4 \cdot 10^{-24}$  e·cm.

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