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V.V. Mazepus

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ON THE DESCRIPTION OF ROTATIONAL EXITATIONS IN ODD-MASS NUCLEI BY THE PROJECTION METHOD

V.V. Mazepus

Institute of Nuclear Physics
630090, Novosibirsk 90, USSR

A b s t r a c t

The projection onto the eigenspace of the angular momentum operator is carried out for well-deformed odd-mass nuclei, the space of trial wavefunctions being more extensive than that of the usual projection approach. This projection method is shown to lead to the standard particle-plus-rotor model but not to the cranking model. The comparison with the approximate projection method is made.

There are two main models for the description of rotational bands in well-deformed odd-mass nuclei. The first one, the selfconsistent cranking model (CM), seems to reproduce the properties of odd-mass nuclei sufficiently well [I], but because of the semiclassical character its applicability to small angular momenta is still open to discussion. The other one is the particle-plus-rotor model (PRM) [2]. It is generally accepted [2, 3] to be able to reproduce Coriolis-distorted bands only if the strength of the Coriolis interaction is considerably reduced.

Several years ago an attempt was made to substantiate the CM within the framework of the approximate projection method [I, 4], but it should be noted that this method itself must be first substantiated. On the other hand, the PRM has been proved to be a correct solution at not too large angular momenta in a simple microscopic model [5], whereas the CM is not valid in this case.

In the present paper the projection onto the eigenspace of the angular momentum operator ($\vec{J}$) is carried out for well-deformed nuclei at not too large J, the space of trial states being more extensive than that of the usual projection approach. It will be shown that this improved projection method leads to the standard PRM but not to the CM. Moreover, the approximate projection method [I] will be found to disagree with the results of the consistent projection.

I. The Projection Method

I start from the variational principle (see Ref. [I])

$$\delta\mathcal{E}_J = 0 \quad , \quad \mathcal{E}_J = \frac{\langle\Phi|HP^J|\Phi\rangle}{\langle\Phi|P^J|\Phi\rangle} , \qquad (I.I)$$

where

$$P^J = A_J \sum_{KK'} (-)^{K-K'} \int d\Omega \, D^J_{KK'}(\Omega) \, R(\Omega) ,$$
$$R(\Omega) = e^{i\alpha J_z} e^{i\beta J_y} e^{i\gamma J_z} , \qquad (I.2)$$

$$D^J_{KK'}(\Omega) = \langle JK|R(-\Omega)|JK'\rangle, \quad \Omega = \{\alpha,\beta,\gamma\},$$

and $\mathcal{E}_J$ does not include the normalization coefficient A_J.

Following Refs. [I, 6] the trial functions $|\Phi\rangle$ are required to fulfil the symmetry condition

$$e^{i\pi J_x}|\Phi\rangle = i(-)^{J-\frac{1}{2}}|\Phi\rangle, \tag{I.3}$$

which is analogous to the symmetrization of the Bohr-Mottelson wavefunction of an odd-mass nucleus [2].

The condition (I.3) allows one to move the limits of integration over β in Eq. (I.2):

$$\langle\Phi|HP^J|\Phi\rangle = A_J\sum_{KK'}(-)^{K-K'}\int_0^{2\pi}d\alpha d\gamma\, e^{-iK\alpha-iK'\gamma} \times\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\beta\,|\sin\beta|\,d^J_{KK'}(\beta)\langle\Phi|HR(\Omega)|\Phi\rangle, \tag{I.4}$$

where by definition

$$D^J_{KK'}(\Omega) = e^{-iK\alpha-iK'\gamma}\, d^J_{KK'}(\beta).$$

It should be noted that definition (I.2) of the eigenfunctions of the symmetric top $D^J_{KK'}$ differs from that of Refs. [I, 4].

2. Trial wavefunctions

If the projection method is applied to even-mass nuclei, a correct result is given by the trial wavefunctions of the Hartree-Fock-Bogolyubov-type

$$|even\rangle = e^{d_{12}\alpha_1^+\alpha_2^+}|0\rangle \tag{2.I}$$

(summation over indices repeated), where α_1^+ is the operator of the quasiparticle creation in a Nilsson state $|1\rangle$ and $|0\rangle$ is the quasiparticle vacuum. Within the approximation of well-deformed nucleus the antisymmetric matrix d_{12} defined by Eq. (I.I) coincides with that of the CM [4]. In the first order perturbation theory after neglecting the variation of the selfconsistent potentials due to the perturbation it takes the form

$$d_{12} = \omega\frac{(J^{20}_x)}{E_1+E_2}, \quad \omega = \frac{\sqrt{J(J+1)}}{\Theta_0}, \tag{2.2}$$

where $E_1 = \sqrt{\varepsilon_1^2+\Delta^2}$ is the quasiparticle energy, Θ_0 is the cranking moment of inertia [II, I2] and $(J^{20}_x)_{12}$ is defined by the quasiparticle representation of the angular momentum operators:

$$J_\lambda = (J^{11}_\lambda)_{12}\alpha_1^+\alpha_2 + (J^{20}_\lambda)_{12}\alpha_1^+\alpha_2^+ + (J^{20}_\lambda)^*_{12}\alpha_2\alpha_1, \quad \lambda = x,y \tag{2.3}$$

(see Appendix A, Eqs. (A.I9 - A.2I)).

In the case of odd particle number one usually sets

$$|odd\rangle = f_1\alpha_1^+ e^{d_{23}\alpha_2^+\alpha_3^+}|0\rangle \tag{2.4}$$

with f_1 and d_{23} varied [4]. Under certain assumptions one obtains for d_{23} a solution of the Eq. (2.2)-type in which the quantity ω is defined by a more complicated consistency condition [7]. Such a solution means the odd-mass system to rotate with the classical angular frequency ω, and it is quite probably that just the trial functions (2.4) impose the semi-classical behaviour on the system. Thus a question arises whether the solution of the Eq. (2.4)-kind is stable with respect to any extension of the trial state space, or some extension of that changes the solution qualitatively. It will be seen that just the second case takes place if the projection method is used.

I assume the trial state space to consist of the wavefunctions

$$|\Phi\rangle = \alpha_1^+\left(e^{-\Omega^\lambda \mu^\lambda_{34}\alpha_3^+\alpha_4^+}\right)_{12} f_2|0\rangle \tag{2.5}$$

where Ω^λ_{12} ($\lambda = x,y$) are single particle matrices. If

$$\Omega^{\lambda}_{12} = \omega\, \delta_{\lambda x}\, \delta_{12}$$

then the wavefunction (2.5) coincides with that of Eq. (2.4). Thus the trial space chosen contains all the states of the Eq. (2.4)-type. A physical sense of the wavefunctions (2.5) consists in the fact that due to the quantum character of rotation the angular frequency is no longer a c-number.

So far as only the case of small angular momenta is investigated, the expansion in powers of Ω^{λ} in Eq. (2.5) may be made:

$$|\Phi\rangle = f_1 \alpha^+_1 |0\rangle + g^{\lambda}_1 \alpha^+_1 \Sigma^{\lambda} |0\rangle + z^{\lambda\lambda'}_1 \alpha^+_1 \Sigma^{\lambda} \Sigma^{\lambda'} |0\rangle \qquad (2.6)$$

with

$$\Sigma^{\lambda} = \mu^{\lambda}_{12}\, \alpha^+_1 \alpha^+_2 \; .$$

Eq. (2.5) yields

$$f^+ z^{\lambda\lambda'} = \overset{+}{z}{}^{\lambda\lambda'} f = \frac{1}{4}\left(\overset{+}{g}{}^{\lambda} g^{\lambda'} + \overset{+}{g}{}^{\lambda'} g^{\lambda}\right), \qquad (2.7)$$

where the notation

$$a^+ b \equiv \sum_1 a^*_1 b_1$$

is used.

It can be shown that the vectors $z^{\lambda\lambda'}_1$ are excluded from $\mathcal{E}_J$ by means of Eqs. (2.7), therefore the independent trial parameters in Eq. (2.6) are f_1 , g^{λ}_1 and μ^{λ}_{12} .

To satisfy the condition (I.3) it should be assumed that

$$e^{i\pi j_x} \mu^{\lambda} e^{i\pi j_x} = \xi_{\lambda} \mu^{\lambda} \qquad \text{(no summation over } \lambda \text{)}$$

$$e^{i\pi j_x} f = i(-)^{J-\frac{1}{2}} f \qquad (2.8)$$

$$e^{i\pi j_x} g^{\lambda} = i \xi_{\lambda} (-)^{J-\frac{1}{2}} g^{\lambda} .$$

Here $\xi_x = 1$, $\xi_y = -1$ and j_{λ} are the single particle angular momentum matrices:

$$J_{\lambda} = \sum_{12} (j_{\lambda})_{12}\, a^+_1 a_2$$

with a^+_1 being the usual fermion-creation operators (for the connection of j_{λ} , J^{11}_{λ} and J^{20}_{λ} see Appendix A).

Besides it is assumed that matrices μ^{λ}_{12} have the transitions with $\Delta K = \pm 1$ only. This assumption as well as Eqs.(2.8) will be shown to be selfconsistent and does not require any additional Lagrange multipliers.

3. Approximations

In order to perform all calculations analytically, the Hamiltonian is set to be the ordinary [redacted] QQ+PP-model. In the representation of quasiparticles defined for the nearest even-mass nucleus it has the form

$$H = \sum_1 E_1 \alpha^+_1 \alpha_1 + H^{40}_{1234} \alpha^+_1 \alpha^+_2 \alpha^+_3 \alpha^+_4 + h.c.$$
$$+ H^{31}_{1234} \alpha^+_1 \alpha^+_2 \alpha^+_3 \alpha_4 + h.c. + H^{22}_{1234} \alpha^+_1 \alpha^+_2 \alpha_4 \alpha_3 . \qquad (3.I)$$

Here $E_1 = \sqrt{\varepsilon^2_1 + \Delta^2}$ is the quasiparticle energy, ε is the Nilsson deformed field and Δ stands for a gap parameter. The quantities H^{40} , H^{31} , H^{22} have been listed in Appendix A.

To obtain $\mathcal{E}_J$ it is necessary to calculate the expectation values $\langle\Phi| HR(\Omega)|\Phi\rangle$ and $\langle\Phi| R(\Omega)|\Phi\rangle$ for the state (2.6). This highly difficult problem will be solved approximately with the following fairly strong assumptions made:

(i) the nucleus is a well deformed one and therefore

$$\langle 0| J^2_x |0\rangle = \langle 0| J^2_y |0\rangle \equiv \mathcal{D} \gg 1 ,$$

where $|0\rangle$ is the quasiparticle vacuum $(\alpha_1|0\rangle = 0)$;

(ii) the total angular momentum is not too large:

$$\frac{J}{\Theta_0} \ll E_1 \sim \Delta \; ;$$

(iii) the energy difference between Nilsson levels connected by the angular momentum matrices j_{λ} is small compared to Δ (it is the most interesting case of the strong Coriolis mixing):

$$(E_1 - E_2)\, j_{12} \ll \Delta j_{12}\,,$$

$$E_1 = \bar{E} + E_1' \;,\quad \bar{E} \sim \Delta\,,\quad E_1' \ll \bar{E}. \tag{3.2}$$

In fact I assume

$$\frac{\bar{E}}{\mathcal{D}} \sim \frac{1}{\Theta_0} \sim E_1' \quad \text{and} \quad J \sim 1\,. \tag{3.3}$$

The quantity $\langle H R(\Omega)\rangle$ is calculated up to terms of the order of Θ_0^{-1} inclusive, and $\langle R(\Omega)\rangle$ up to terms of the order of $\mathcal{D}^{-1}$. While calculating the expectation values certain sums of the products of $(j_\lambda)_{12}$ arise. The order of these sums is defined by their coherence. For instance,

$$\Big|\sum_{1234} (J_x^{20})_{12} (J_x^{20})_{23} (J_x^{20})^*_{43} (J_x^{20})^*_{14}\Big| \ll \Big[\sum_{12} |(J_x^{20})_{12}|^2\Big]^2 = \frac{1}{4}\mathcal{D}^2, \tag{3.4}$$

$$\Big|\sum_{1234} f(E_{12}) (J_x^{20})_{12} (J_x^{20})_{23} (J_x^{20})^*_{43} (J_x^{20})^*_{14}\Big| \ll$$

$$\ll \sum_{12} f(E_{12}) |(J_x^{20})_{12}|^2 \cdot \frac{\mathcal{D}}{2}\,,\quad E_{12} \equiv E_1 + E_2\,, \tag{3.5}$$

etc.

Some of the arising coherent sums are expressed in terms of the inertial parameter of Peierls and Yoccoz, Rouhaninejad and Yoccoz and Onishi (Refs. [8, 9, 10])

$$\Theta_Y = \frac{\langle 0|\, J_y^2 \,|0\rangle}{\langle 0|\, J_y^2 (H - \langle H\rangle) |0\rangle}\;, \quad \langle H\rangle = \langle 0|H|0\rangle\,.$$

In the QQ+PP-model this parameter is defined by the static quadrupole moment of the even-mass nucleus:

$$\frac{\mathcal{D}^2}{\Theta_Y} \simeq \frac{3}{2}\,\varkappa\, Q_0^2 = \sum_{12} E_{12}\, |(J_x^{20})_{12}|^2\,, \tag{3.6}$$

where $\varkappa$ is the quadrupole - quadrupole coupling constant (see Appendix A). It is assumed (Ref. [4]) that

$$\Theta_Y \sim \Theta_0\,.$$

Finally the sum

$$n = \sum_{12} \frac{|(J_x^{20})_{12}|^2}{E_{12}^2} \tag{3.7}$$

arises in intermediate calculations. It is estimated as

$$n \sim \frac{1}{2\Delta} \cdot \frac{1}{4}\,\Theta_0\,, \tag{3.8}$$

so far as

$$\Theta_0 = 4 \sum_{12} \frac{|(J_x^{20})_{12}|^2}{E_{12}}\,. \tag{3.9}$$

4. Calculation of $\langle H R(\Omega)\rangle$ and $\langle R(\Omega)\rangle$

It is convenient to denote

$$\Theta_1 = 4 \sum_{12} \mu^x_{12} (J_x^{20})^*_{12}$$

$$\Theta_2 = 4 \sum_{12} E_{12} |\mu^x_{12}|^2$$

$$\mathcal{D}_1 = 2 \sum_{12} E_{12}\, \mu^x_{12} (J_x^{20})^*_{12} \tag{4.1}$$

$$n_1 = \sum_{12} |\mu^x_{12}|^2\,.$$

The estimations for these sums are

$$\Theta_1 \sim \Theta_2 \sim \Theta_0\;,\quad \mathcal{D}_1 \sim \mathcal{D}\;,\quad n_1 \sim n \sim \frac{\Theta_0}{\Delta}\,.$$

If one designates (see Eq. (2.6))

$$|\Phi\rangle = |F\rangle + |G\rangle + |Z\rangle + \ldots \tag{4.2}$$

then

$$\langle \Phi | R(\Omega) | \Phi\rangle \simeq \langle F|R|F\rangle + \langle G|R|F\rangle + \text{Conj.}$$

$$+ \langle Z|R|F\rangle + \text{Conj.} + \langle G|R|G\rangle\,. \tag{4.3}$$

Eq. (A.I9) yields

$$e^{i\gamma J_z}|\Phi\rangle = |F(\gamma)\rangle + |G(\gamma)\rangle + |Z(\gamma)\rangle + \dots$$
$$\simeq f_1(\gamma)\alpha_1^+|0\rangle + g_1^\lambda(\gamma)\alpha_1^+\Sigma^\lambda|0\rangle + Z_1^{\lambda\lambda'}(\gamma)\alpha_1^+\Sigma^\lambda\Sigma^{\lambda'}|0\rangle \quad (4.4)$$

with

$$f_1(\gamma) = \left(e^{i\gamma j_z} f\right)_1, \quad g_1^\lambda(\gamma) = e^{i\gamma (j_z)_{11}}\left(g_1^\lambda \cos\gamma + \xi_\lambda g_1^{\bar\lambda} \sin\gamma\right),$$
$$Z_1^{\lambda\lambda'}(\gamma) = e^{i\gamma (j_z)_{11}}\left[Z_1^{\lambda\lambda'}\cos^2\gamma + \left(\xi_\lambda Z_1^{\bar\lambda\lambda'} + \xi_{\lambda'} Z_1^{\lambda\bar\lambda'}\right)\sin\gamma\cos\gamma + \xi_\lambda\xi_{\lambda'} Z_1^{\bar\lambda\bar\lambda'}\sin^2\gamma\right], \quad (4.5)$$

where $\bar\lambda$ is determined according to $\bar x \equiv y$ and $\bar y \equiv x$.

The operation of $e^{i\beta J_y}$ on the quasiparticle vacuum and the quasiparticle operators is defined as follows:

$$e^{i\beta J_y}\alpha_1^+ e^{-i\beta J_y} = u_{12}\alpha_2^+ + v_{12}\alpha_2 \quad (4.6)$$

with

$$u_{12} = \delta_{12} + i\beta (J_y^{11})_{21} - \frac{\beta^2}{2}(J_y^{11})_{23}(J_y^{11})_{31} + 2\beta^2 (J_y^{20})^*_{13}(J_y^{20})_{32} + O(\beta^3) \quad (4.7)$$

and

$$v_{12} = 2i\beta (J_y^{20})^*_{12} - \beta^2 (J_y^{11})_{31}(J_y^{20})^*_{32} - \beta^2 (J_y^{11})_{32}(J_y^{20})^*_{31} + O(\beta^3). \quad (4.8)$$

Then

$$e^{i\beta J_y}|0\rangle = \mathcal{N}(\beta)\, e^{c_{12}(\beta)\alpha_1^+\alpha_2^+}|0\rangle \quad (4.9)$$

where

$$c_{12}(\beta) = i\beta (J_y^{20})_{12} + \frac{(i\beta)^2}{2}\left[(J_y^{11})_{13}(J_y^{20})_{32} - (J_y^{11})_{23}(J_y^{20})_{31}\right] + O(\beta^3), \quad (4.10)$$

$$\mathcal{N}(\beta) = e^{-\frac{\beta^2}{2}\mathcal{D}}\left(1 + \sigma\mathcal{D}\beta^4 + \dots\right), \quad e^{-\frac{\beta^2}{2}\mathcal{D}} \equiv \mathring{\mathcal{N}}(\beta), \quad (4.11)$$

σ includes uncoherent sums and is of the order of 1. Eqs. (4.10–11) are derived by iterating equations for c and $\mathcal{N}$ obtained by means of differentiation of Eq. (4.9) with respect to β.

Taking into account the axiality of the problem one can write down for the most coherent terms

$$e^{i\beta J_y}\Sigma^\lambda e^{-i\beta J_y} \simeq \Sigma^\lambda + \frac{i\beta}{2}\delta_{\lambda y}\,\Theta_1 . \quad (4.12)$$

Using Eqs. (4.3–12) one obtains

$$\langle\Phi|R(\Omega)|\Phi\rangle \simeq \mathring{\mathcal{N}}(\beta)\Big\{(1+\sigma\mathcal{D}\beta^4)f^+(-\alpha)f(\gamma) + i\beta f^+(-\alpha)J_y^{11}f(\gamma)$$
$$+ f^+(-\alpha)[J_y^2]f(\gamma)\frac{(i\beta)^2}{2} + \frac{i\beta}{2}f^+(-\alpha)g^y(\gamma)\Theta_1 + \frac{i\beta}{2}\overset{+}{g}{}^y(-\alpha)f(\gamma)\Theta_1^*$$
$$+ f^+(-\alpha)J_y^{11}g^y(\gamma)\frac{(i\beta)^2}{2}\Theta_1 + \overset{+}{g}{}^y(-\alpha)J_y^{11}f(\gamma)\frac{(i\beta)^2}{2}\Theta_1^* + 2n_1\overset{+}{g}{}^\lambda(-\alpha)g^\lambda(\gamma) \quad (4.13)$$
$$+ \overset{+}{g}{}^y(-\alpha)g^y(\gamma)\frac{(i\beta)^2}{4}\Theta_1^*\Theta_1 + f^+(-\alpha)Z^{yy}(\gamma)\frac{(i\beta)^2}{4}\Theta_1^2$$
$$+ \overset{+}{Z}{}^{yy}(-\alpha)f(\gamma)\frac{(i\beta)^2}{4}\Theta_1^{*2}\Big\},$$

where

$$f^+(-\alpha)f(\gamma) \equiv \sum_1 f_1^+(-\alpha)f_1(\gamma), \quad f^+(-\alpha)J_y^{11}f(\gamma) \equiv \sum_{12} f_1^+(-\alpha)(J_y^{11})_{12}f_2(\gamma)$$

etc.; $[J_y^2]_{12} \equiv (J_y^{11})_{13}(J_y^{11})_{32} + 4(J_y^{20})_{13}(J_y^{20})^*_{32}$.

Eq. (4.13) contains all the terms up to the order of $\mathcal{D}^{-1}$. For the estimation of these terms it should be noted that due to the multiplier $\mathring{\mathcal{N}}(\beta)$ the integration over β leads to additional smallnesses connected with powers of β:

$$\beta^0 \sim 1, \quad \beta \sim \beta^2 \sim \frac{1}{\mathcal{D}}, \quad \beta^3 \sim \beta^4 \sim \frac{1}{\mathcal{D}^2} \quad \text{etc.}$$

The evaluation of $< HR(\Omega)>$ as well as (4.13) requires highly cumbersome calculations but is sufficiently trivial. Using equalities of Appendix B one gets ($H' \equiv H - \bar{E}$):

$$
\begin{aligned}
\langle\Phi|H'R|\Phi\rangle \simeq \mathring{N}(\beta)\Big\{ & \frac{\mathcal{D}^2}{2\Theta_Y}\left(1-\frac{\eta}{\mathcal{D}}+\sigma\mathcal{D}\beta^4\right)(i\beta)^2 f^+(-\alpha)f(\gamma) \\
& + f^+(-\alpha)J_y^{11}f(\gamma)\frac{(i\beta)^3}{2}\frac{\mathcal{D}^2}{\Theta_Y} + f^+(-\alpha)[J_y^2]f(\gamma)\frac{(i\beta)^4}{4}\frac{\mathcal{D}^2}{\Theta_Y} \\
& + f^+(-\alpha)E'f(\gamma) + 4\bar{E}f_1^+(-\alpha)(J_y^{20})_{13}(J_y^{20})^*_{32}f_2(\gamma)(i\beta)^2 \\
& + f^+(-\alpha)g^y(\gamma)\left[\frac{i\beta}{2}\mathcal{D}_1+\frac{(i\beta)^3}{4}\frac{\mathcal{D}^2}{\Theta_Y}\Theta_1\right] + \overset{+}{g}{}^y(-\alpha)f(\gamma)\left[\frac{i\beta}{2}\mathcal{D}_1^*+\frac{(i\beta)^3}{4}\frac{\mathcal{D}^2}{\Theta_Y}\Theta_1^*\right] \\
& + f^+(-\alpha)J_y^{11}g^y(\gamma)\left[\frac{(i\beta)^2}{2}\mathcal{D}_1+\frac{(i\beta)^4}{4}\frac{\mathcal{D}^2}{\Theta_Y}\Theta_1\right] \\
& + \overset{+}{g}{}^y(-\alpha)J_y^{11}f(\gamma)\left[\frac{(i\beta)^2}{2}\mathcal{D}_1^*+\frac{(i\beta)^4}{4}\frac{\mathcal{D}^2}{\Theta_Y}\Theta_1^*\right] \\
& + \overset{+}{g}{}^\lambda(-\alpha)g^\lambda(\gamma)\Big[\frac{1}{2}\Theta_2-\frac{\Theta_Y}{4\mathcal{D}^2}\mathcal{D}_1^*\mathcal{D}_1+(i\beta)^2 n_1\frac{\mathcal{D}^2}{\Theta_Y} \\
& + \frac{(i\beta)^2}{2}\delta_{\lambda y}\frac{\Theta_1^*\mathcal{D}_1+\Theta_1\mathcal{D}_1^*}{2}+\frac{(i\beta)^4}{8}\delta_{\lambda y}\frac{\mathcal{D}^2}{\Theta_Y}\Theta_1^*\Theta_1\Big] \\
& + f^+(-\alpha)z^{\lambda\lambda}(\gamma)\left[\frac{\Theta_Y}{4\mathcal{D}^2}\mathcal{D}_1^2+\frac{(i\beta)^2}{2}\mathcal{D}_1\Theta_1\delta_{\lambda y}+\frac{(i\beta)^4}{8}\frac{\mathcal{D}^2}{\Theta_Y}\Theta_1^2\delta_{\lambda y}\right] \\
& + \overset{+}{z}{}^{\lambda\lambda}(-\alpha)f(\gamma)\left[\frac{\Theta_Y}{4\mathcal{D}^2}\mathcal{D}_1^{*2}+\frac{(i\beta)^2}{2}\mathcal{D}_1^*\Theta_1^*\delta_{\lambda y}+\frac{(i\beta)^4}{8}\frac{\mathcal{D}^2}{\Theta_Y}\Theta_1^{*2}\delta_{\lambda y}\right]\Big\},
\end{aligned}
\qquad (4.14)
$$

where η as well as σ stands for an uncoherent combination and is of the order of 1.

5. Integration over the Euler angles

Let us set the normalization multiplier A_J in Eq. (1.4) to be equal to $\mathcal{D}^2/8\pi^2$. In this case

$$
\langle\Phi|P^J|\Phi\rangle = \sum_{KK'}(-)^{K-K'}\int_0^{2\pi}\frac{d\alpha\, d\gamma}{4\pi^2}e^{-iK\alpha-iK'\gamma}\frac{\mathcal{D}}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\beta\,|\sin\beta|\,d^J_{KK'}(\beta)\langle\Phi|R|\Phi\rangle = 1+O\left(\frac{1}{\mathcal{D}}\right) \qquad (5.1)
$$

(analogously for $\langle\Phi|H'P^J|\Phi\rangle$).

The integration over α and γ is not a problem, as the dependence of $f(\alpha), g^\lambda(\alpha), z^{\lambda\lambda'}(\alpha)$ on α is quite simple (Eqs. (4.5)). So far as the quantities $\langle \binom{H'}{1} R(\Omega)\rangle$ contain the rapidly decreasing factor $\mathring{N}(\beta)$ one may replace the integration limits in the integrals over β by $\pm\infty$ and expand $\sin\beta$ and $d^J_{KK'}(\beta)$ in powers of β:

$$
d^J_{KK'}(\beta) = \langle JK|e^{-i\beta J_y}|JK'\rangle = \delta_{KK'} - i\beta\langle JK|J_y|JK'\rangle + \dots \qquad (5.2)
$$

To obtain the terms of the order of $\mathcal{D}^{-1}$ in $\langle P^J\rangle$ one should retain the powers of β up to 2 while for the terms of the order of Θ^{-1} in $\langle H'P^J\rangle$ one needs the powers of β up to 4.

Now the integration is trivial. Denoting

$$
g^{(\pm)} = g^x \pm i g^y \qquad (5.3)
$$

and using the conditions (2.7-8) one gets

$$
\begin{aligned}
\langle\Phi|P^J|\Phi\rangle \simeq\ & 1+\frac{\sigma'}{\mathcal{D}}-\frac{J(J+1)}{2\mathcal{D}}+\frac{1}{2\mathcal{D}}f^+\sqrt{J(J+1)-j_z^2+j_z}\,J_+^{11}f + \text{conj.} \\
& -\frac{1}{2\mathcal{D}}f^+[\vec{J}^2]f+\frac{\Theta_1}{2\mathcal{D}}f^+\sqrt{J(J+1)-j_z^2+j_z}\,g^{(+)}+\text{conj.} \\
& -\frac{\Theta_1}{2\mathcal{D}}f^+J_\lambda^{11}g^\lambda+\text{conj.}+\overset{+}{g}{}^\lambda g^\lambda\left(2n_1-\frac{(\Theta_1+\Theta_1^*)^2}{8\mathcal{D}}\right),
\end{aligned}
\qquad (5.4)
$$

$$
\begin{aligned}
\langle\Phi|H'P^J|\Phi\rangle \simeq & -\frac{\mathcal{D}}{\Theta_Y}+\frac{\eta'}{\Theta_Y}+\frac{J(J+1)}{\Theta_Y}-\frac{1}{\Theta_Y}f^+j_z^2f \\
& -\frac{1}{\Theta_Y}f^+\sqrt{J(J+1)-j_z^2+j_z}\,J_+^{11}f + \text{conj.} \\
& +\frac{1}{\Theta_Y}f^+[\vec{J}^2]f-\frac{4\bar{E}}{\mathcal{D}}f_1^*(J_\lambda^{20})_{13}(J_\lambda^{20})^*_{32}f_2+f^+E'f
\end{aligned}
$$

$$+\frac{1}{2}\left(\frac{\mathcal{D}_1}{\mathcal{D}}-\frac{2\Theta_1}{\Theta_Y}\right)f^{+}\sqrt{J(J+1)-j_z^2+j_z}\,g^{(+)}+\text{conj}.$$
$$-\frac{1}{2}\left(\frac{\mathcal{D}_1}{\mathcal{D}}-\frac{2\Theta_1}{\Theta_Y}\right)f^{+}J^{11}_{\lambda}g^{\lambda}+\text{conj}.+\overset{+}{g}{}^{\lambda}g^{\lambda}\Big[\frac{1}{2}\Theta_2$$
$$-\frac{\Theta_Y}{4\mathcal{D}^2}\mathcal{D}_1^*\mathcal{D}_1-\frac{2n_1\mathcal{D}}{\Theta_Y}+\frac{\Theta_1^*\Theta_1}{2\Theta_Y}-\frac{\Theta_1^*\mathcal{D}_1+\Theta_1\mathcal{D}_1^*}{4\mathcal{D}} \tag{5.5}$$
$$+\frac{\Theta_Y}{8\mathcal{D}^2}(\mathcal{D}_1^2+\overset{*}{\mathcal{D}}{}_1^2)+\frac{1}{4\Theta_Y}(\Theta_1^2+\overset{*}{\Theta}{}_1^2)-\frac{1}{4\mathcal{D}}(\mathcal{D}_1\Theta_1+\mathcal{D}_1^*\Theta_1^*)\Big]$$

with

$$[\vec{J}^2]_{12}\equiv(J^{11}_{\lambda})_{13}(J^{11}_{\lambda})^*_{32}+4(J^{20}_{\lambda})_{13}(J^{20}_{\lambda})^*_{32}\;;\quad \lambda=x,y;$$
$$J^{11}_{\pm}=J^{11}_{x}\pm iJ^{11}_{y}\;;\qquad \sigma'\sim\eta'\sim 1.$$

In Eqs. (5.4-5) the normalization

$$f^{+}f=1 \tag{5.6}$$

is used.

Omitting unessential additive constants one obtains from Eqs. (5.4-5)

$$\mathcal{E}_J\simeq\frac{J(J+1)-f^{+}j_z^2f}{2\Theta_Y}-\frac{1}{2\Theta_Y}f^{+}\left(\sqrt{J(J+1)-j_z^2+j_z}\,J^{11}_{+}+J^{11}_{-}\sqrt{J(J+1)-j_z^2+j_z}\right)f$$
$$+\frac{1}{2\Theta_Y}f^{+}[\vec{J}^2]f-\frac{4\bar{E}}{\mathcal{D}}f_1^*(J^{20}_{\lambda})_{13}(J^{20}_{\lambda})^*_{32}f_2+f^{+}E'f$$
$$+\frac{1}{2}\left(\frac{\mathcal{D}_1}{\mathcal{D}}-\frac{\Theta_1}{\Theta_Y}\right)f^{+}\sqrt{J(J+1)-j_z^2+j_z}\,g^{(+)}+\frac{1}{2}\left(\frac{\mathcal{D}_1^*}{\mathcal{D}}-\frac{\Theta_1^*}{\Theta_Y}\right)\overset{+}{g}{}^{(+)}\sqrt{J(J+1)-j_z^2+j_z}\,f \tag{5.7}$$
$$-\frac{1}{2}\left(\frac{\mathcal{D}_1}{\mathcal{D}}-\frac{\Theta_1}{\Theta_Y}\right)f^{+}J^{11}_{-}g^{(+)}-\frac{1}{2}\left(\frac{\mathcal{D}_1^*}{\mathcal{D}}-\frac{\Theta_1^*}{\Theta_Y}\right)\overset{+}{g}{}^{(+)}J^{11}_{+}f$$
$$+\overset{+}{g}{}^{(+)}g^{(+)}\Big[\frac{1}{2}\Theta_2-\frac{\Theta_Y}{4\mathcal{D}^2}\mathcal{D}_1^*\mathcal{D}_1+\frac{\Theta_1^*\Theta_1}{4\Theta_Y}-\frac{1}{4\mathcal{D}}(\Theta_1^*\mathcal{D}_1+\mathcal{D}_1^*\Theta_1)$$
$$+\frac{\Theta_Y}{8\mathcal{D}^2}(\mathcal{D}_1^2+\overset{*}{\mathcal{D}}{}_1^2)+\frac{1}{8\Theta_Y}(\Theta_1^2+\overset{*}{\Theta}{}_1^2)-\frac{1}{4\mathcal{D}}(\mathcal{D}_1\Theta_1+\mathcal{D}_1^*\Theta_1^*)\Big].$$

6. Variation with respect to g and μ and the effective Hamiltonian

The variation of Eq. (5.7) with respect to $\overset{+}{g}{}^{(+)}$ yields

$$g^{(+)}\Big[\frac{1}{2}\Theta_2+\frac{\Theta_Y}{8\mathcal{D}^2}(\mathcal{D}_1-\mathcal{D}_1^*)^2+\frac{1}{8\Theta_Y}(\Theta_1+\overset{*}{\Theta}_1)^2-\frac{1}{4\mathcal{D}}(\mathcal{D}_1+\mathcal{D}_1^*)(\Theta_1+\Theta_1^*)\Big]$$
$$+\frac{1}{2}\left(\frac{\mathcal{D}_1^*}{\mathcal{D}}-\frac{\Theta_1^*}{\Theta_Y}\right)\left(\sqrt{J(J+1)-j_z^2+j_z}-J^{11}_{+}\right)f=0. \tag{6.I}$$

As Eq. (2.6) includes g^{λ} in its product with μ^{λ} only, one can normalize g^{λ} arbitrarily. Let us set

$$g^{(+)}=\frac{1}{\Theta_0}\left(\sqrt{J(J+1)-j_z^2+j_z}-J^{11}_{+}\right)f. \tag{6.2}$$

Then due to Eqs. (2.8)

$$g^{(-)}=-i(-)^{J-\frac{1}{2}}e^{i\pi j_x}g^{(+)}$$
$$=\frac{1}{\Theta_0}\left(\sqrt{J(J+1)-j_z^2-j_z}-J^{11}_{-}\right)f. \tag{6.3}$$

Thus the quantum angular frequency obtained by the minimization of the projected energy in the class of the trial wavefunctions (2.5) is not a c-number but a matrix with an essentially non-diagonal structure

$$\Omega^{(+)}=\Omega^{x}+i\Omega^{y}=\frac{1}{\Theta_0}\left(\sqrt{J(J+1)-j_z^2+j_z}-J^{11}_{+}\right). \tag{6.4}$$

It should be noted that within the approximations assumed the angular frequency of the CM corresponds to averaging Eq. (6.4):

$$\omega\simeq\frac{1}{\Theta_0}f^{+}\left(\sqrt{J(J+1)-j_z^2}-J^{11}_{+}\right)f.$$

By varying Eq. (5.7) with respect to $\overset{*}{\mu}{}^{x}$ and taking into account Eqs. (4.I) and (6.2) one obtains

$$\mu^{x}_{12}=(J^{20}_{x})_{12}\left[\frac{\Theta_Y}{4\mathcal{D}^2}(\mathcal{D}_1-\mathcal{D}_1^*)+\frac{1}{4\mathcal{D}}(\Theta_1+\Theta_1^*)-\frac{\Theta_0}{2\mathcal{D}}\right]+ \tag{6.5}$$

$$+\frac{(J_x^{20})_{12}}{E_{12}}\left[\frac{1}{2\mathcal{D}}(\mathcal{D}_1+\mathcal{D}_1^*)-\frac{\Theta_1+\Theta_1^*}{2\Theta_Y}+\frac{\Theta_0}{\Theta_Y}\right]$$

$$\equiv (J_x^{20})_{12}\,A+\frac{(J_x^{20})_{12}}{E_{12}}\,B \qquad (E_{12}=E_1+E_2).$$

From the definitions (4.I) and (6.5) a system of equations for A and B arises, only one equation being independent by virtue of Eq. (3.6):

$$B=1. \tag{6.6}$$

Thus

$$\mu_{12}^{x}=\frac{(J_x^{20})_{12}}{E_{12}}+A\,(J_x^{20})_{12} \tag{6.7}$$

with some arbitrary A. The quantity A cannot be found from the variational principle because $\mathcal{E}_J$ does not depend on it. As is obvious from Eq. (6.7) the matrix μ_{12}^{x} contains the transitions with $\Delta K=\pm 1$ only, in accordance with the primary assumptions.

By inserting Eqs. (6.2) and (6.7) into Eq. (5.7) one gets

$$\mathcal{E}_J=f^+\mathcal{H}^J f,$$

where $\mathcal{H}^J$ is the effective Hamiltonian of the odd-mass nucleus:

$$\mathcal{H}_{12}^J=E_1\delta_{12}+\left(\frac{2}{\Theta_0}+\frac{2}{\Theta_Y}-\frac{4\bar{E}}{\mathcal{D}}\right)(J_\lambda^{20})_{13}(J_\lambda^{20})^*_{32}+\frac{1}{2\Theta_0}(\vec{j}_\perp^{\,2})_{12}\,\xi_{12}^{(-)}$$

$$-\frac{1}{2\Theta_0}\left(\sqrt{J(J+1)-j_z^2+j_z}\,J_+^{11}+J_-^{11}\sqrt{J(J+1)-j_z^2+j_z}\right)_{12} \tag{6.8}$$

$$+\frac{J(J+1)-(j_z^2)_{11}}{2\Theta_0}\,\delta_{12}$$

with

$$\vec{j}_\perp^{\,2}=j_x^2+j_y^2\,,\qquad \xi_{12}^{(-)}=u_1u_2+v_1v_2\,,$$

(u_1 and v_1 are the coefficients of the Bogolyubov transformation; see Appendix A). In the derivation of Eq. (6.8) the

identity

$$\xi_{13}^{(-)}\xi_{32}^{(-)}-\eta_{13}^{(-)}\eta_{32}^{(-)}=\xi_{12}^{(-)} \quad \text{(no summation over the index 3!)}$$

has been used.

By virtue of the conditions (3.2) the quantities $\bar{E}$ determined by different ways are approximately equal:

$$\frac{\sum_{12}|(J_x^{20})_{12}|^2}{2\sum_{12}\frac{|(J_x^{20})_{12}|^2}{E_{12}}}\simeq\frac{\sum_{12}E_{12}|(J_x^{20})_{12}|^2}{2\sum_{12}|(J_x^{20})_{12}|^2}\simeq\bar{E}. \tag{6.9}$$

Therefore

$$\left|\frac{1}{\Theta_0}+\frac{1}{\Theta_Y}-\frac{2\bar{E}}{\mathcal{D}}\right|\ll\frac{1}{\Theta_0},$$

and the second term in the right-hand part of Eq. (6.8) may be neglected.

Thus

$$\mathcal{H}_{12}^J\simeq E_1\delta_{12}+\frac{1}{2\Theta_0}(\vec{j}_\perp^{\,2})_{12}\,\xi_{12}^{(-)}-\frac{1}{2\Theta_0}\Big(\sqrt{J(J+1)-j_z^2+j_z}\,J_+^{11}+$$

$$+J_-^{11}\sqrt{J(J+1)-j_z^2+j_z}\Big)_{12}+\frac{1}{2\Theta_0}\delta_{12}\left(J(J+1)-(j_z^2)_{11}\right). \tag{6.I0}$$

The unitary transformation $\exp(i\pi j_x)$ leaves the Hamiltonian (6.I0) invariant. This is obvious if one chooses the Nilsson wavefunctions $|Nn_z\Lambda\Omega\rangle$, for which

$$e^{i\pi j_x}|Nn_z\Lambda\Omega\rangle=i(-)^N|Nn_z\Lambda,-\Omega\rangle, \tag{6.II}$$

as single particle states $|1\rangle$.

According to the conditions (2.8) one should select only those eigenstates of the Hamiltonian (6.I0) for which

$$e^{i\pi j_x}f=i(-)^{J-\frac{1}{2}}f. \tag{6.I2}$$

If one chooses as a basis the superpositions of the Nilsson wavefunctions

$$|Nn_z\Lambda K) = \frac{1}{\sqrt{2}}\left(|Nn_z\Lambda K\rangle + (-)^{J-\frac{1}{2}+N}|Nn_z\Lambda,-K\rangle\right), \quad K>0 \qquad (6.13)$$

which satisfy the condition (6.12), one obtains from Eq. (6.10) the ordinary form of the PRM-Hamiltonian (see Refs. [2, 3]):

$$(\alpha K|\mathcal{H}^J|\alpha' K) = E_{\alpha K}\delta_{\alpha\alpha'} + \frac{1}{2\Theta_0}(\vec{j}_\perp^{\,2})_{\alpha K,\alpha' K}\xi^{(-)}_{\alpha K,\alpha' K} + \frac{1}{2\Theta_0}[J(J+1)-K^2]\delta_{\alpha\alpha'} + \frac{1}{2\Theta_0}(-)^{J+\frac{1}{2}}(J+\tfrac{1}{2})a_{\alpha\alpha'}\delta_{K,\frac{1}{2}}$$

$$(\alpha, K+1|\mathcal{H}^J|\alpha' K) = -\frac{1}{2\Theta_0}\sqrt{J(J+1)-K(K+1)}\,\langle\alpha, K+1|j_+|\alpha' K\rangle\,\xi^{(-)}_{\alpha K+1,\alpha' K} \qquad (6.14)$$

where $a_{\alpha\alpha'}$ are the decoupling factors and α, α' denote sets of the Nilsson quantum numbers $Nn_z\Lambda$.

Thus the projection method in the class of the trial wave-functions (2.5) leads to the effective Hamiltonian coinciding with that of the PRM.

7. Comparison with other approaches

Let us restrict the trial space by that of the CM, i.e. assume

$$\Omega^{\lambda}_{12} = \omega\,\delta_{\lambda x}\delta_{12}, \quad \mu^{x}_{12} = \frac{(J^{20}_x)_{12}}{E_{12}}. \qquad (7.1)$$

Inserting Eq. (7.1) into Eq. (5.7) and varying with respect to ω one obtains:

$$\Theta_0\omega + f^+J^{11}_x f = f^+\frac{\sqrt{J(J+1)-j_z^2+j_z}+\sqrt{J(J+1)-j_z^2-j_z}}{2}f, \qquad (7.2)$$

$$\mathcal{E}_J = \frac{J(J+1)-f^+j_z^2f}{2\Theta_Y} + f^+Ef + \frac{1}{2\Theta_Y}f^+[\vec{J}^2]f - \frac{4\bar{E}}{\mathcal{D}}f_1^*(J^{20}_\lambda)_{13}(J^{20}_\lambda)^*_{32}f_2 - \frac{1}{2\Theta_Y}f^+\Big(\sqrt{J(J+1)-j_z^2+j_z}\,J^{11}_+ + J^{11}_-\sqrt{J(J+1)-j_z^2+j_z}\Big)f - \left(\frac{\Theta_0}{\Theta_Y}-1\right)\omega f^+\frac{\sqrt{J(J+1)-j_z^2+j_z}+\sqrt{J(J+1)-j_z^2-j_z}}{2}f + \left(\frac{\Theta_0}{\Theta_Y}-1\right)\omega f^+J^{11}_x f + \frac{\Theta_0}{2}\left(\frac{\Theta_0}{\Theta_Y}-1\right)\omega^2. \qquad (7.3)$$

The qualitative distinction of the effective Hamiltonian corresponding to Eq. (7.3) from that of Eq. (6.10) consists in the fact that Eq. (7.3) contains the unphysical inertial parameter Θ_Y. However in virtue of the approximation (6.9) the numerical difference is not large because Θ_Y is close to Θ_0 (for the numerical estimation see Ref. [4]).

In Ref. [1] an approximate formula for $\mathcal{E}_J$ was proposed:

$$\mathcal{E}_J = \langle H\rangle - \frac{\langle(H-\langle H\rangle J_y^2\rangle}{\langle J_y^2\rangle} + \frac{\langle(H-\langle H\rangle)J_x\rangle}{\langle J_x^2\rangle-\langle J_x\rangle^2}\left[\sqrt{J(J+1)-\langle J_z^2\rangle} - \langle J_x\rangle\right] + \frac{\langle(H-\langle H\rangle)J_y^2\rangle}{2\langle J_y^2\rangle^2}\left[\sqrt{J(J+1)-\langle J_z^2\rangle}-\langle J_x\rangle\right]^2. \qquad (7.4)$$

The authors assume this formula to be valid for the HFB-wave-functions. Within the approximations made use of in the present paper it follows from Eq. (7.4) that

$$\mathcal{E}_J \simeq f^+Ef + \frac{1}{2}\Theta_0\omega^2 + \frac{1}{\Theta_Y}f^+[J_y^2]f - \frac{8\bar{E}}{\mathcal{D}}f_1^*(J^{20}_\lambda)_{13}(J^{20}_\lambda)^*_{32}f_2 + \omega\left[\sqrt{J(J+1)-f^+j_z^2f} - f^+J^{11}_x f - \Theta_0\omega\right] + \frac{1}{2\Theta_Y}\left[\sqrt{J(J+1)-f^+j_z^2f} - f^+J^{11}_x f - \Theta_0\omega\right]^2, \qquad (7.5)$$

if the CM-wavefunctions are chosen as trial states.

The most essencial difference between Eqs. (7.3) and (7.5) consists in the fact that in place of the correct terms

$$\frac{1}{2\Theta_Y} f^+[\vec{J}^2] f - \frac{4\bar{E}}{\mathcal{D}} f_1^*(J_\lambda^{20})_{13}(J_\lambda^{20})_{32}^* f_2$$

Eq. (7.5) contains

$$\frac{1}{\Theta_Y} f^+[J_y^2] f - \frac{8\bar{E}}{\mathcal{D}} f_1^*(J_y^{20})_{13}(J_y^{20})_{32}^* f_2 + \frac{1}{2\Theta_Y}(f^+ J_x^{11} f)^2 .$$

It should be pointed out that non-quadratic terms in Eq. (7.5) do not agree with the general structure of Eq. (I.I). Also the other differencies, though not so crucial, show Eq. (7.4) to be insufficiently correct.

It is difficult to indicate the cause of these distinctions as the detailed derivation of Eq. (7.4) in the odd case is most likely still unpublished. The possible cause may consist in the fact that as it is seen from Ref. [4] the approximate expression for $\langle\Phi|HR(\Omega)|\Phi\rangle$ was restricted to the terms of the order of β^2 only while Eq. (4.I4) shows the necessity to take into account the terms of the order of β^3 and β^4 .

Also it should be noted that in contrast to the statement of Ref. [I] Eq. (7.5) is not equivalent to the CM. The CM-equations can be obtained from Eq. (7.4) only if the second term in the right-hand part of Eq. (7.4) is neglected. In this case Eq. (7.4) yields

$$\mathcal{E}_J \simeq -\frac{1}{2}\Theta_0\omega^2 + \omega\sqrt{J(J+1) - f^+ j_z^2 f} + f^+ E f - \omega f^+ J_x^{11} f \qquad (7.6)$$

with the consistency condition

$$\sqrt{J(J+1) - f^+ j_z^2 f} = f^+ J_x^{11} f + \Theta_0 \omega .$$

These equations coincide with those of the CM at small angular momenta [7] .

Conclusion

The results of the present paper are reduced to three statements:

(i) the approximate formula (7.4) for the "projected energy" from Ref. [I] is inconsistent with the straightforward calculation of $\mathcal{E}_J$ at small angular momenta and seems to be incorrect in the odd case. Moreover it does not imply the CM-equations, in contradiction with Ref. [I] ;

(ii) the extension of the trial state space of the projection method leads to the qualitative change of the results. The restriction of the trial wavefunctions to states of the Hartree-Fock-Bogolyubov-type does not allow one to eliminate the unphysical inertial parameter Θ_Y ;

(iii) the straightforward calculation shows that at not too large angular momenta the projection method leads to the ordinary PRM equations, this result obtained for the class of the trial wavefunctions to be more extensive than that of the self-consistent CM.

It should be stressed, however, that the last statement is based upon the calculations in which the projection of the angular momentum but not of the particle number was carried out. Of course, this approach is not quite correct. Besides the approximations (3.2-3) do not correspond entirely to the real situation in the deformed odd-mass nuclei; also they obscure the distinction between the realistic moment of inertia Θ_0 and the unphysical inertial parameter Θ_Y . On the other hand in a simple model without pairing forces [5] the projection method leads to the PRM which in this case is shown to be an exact solution at not too large angular momenta. The question is if the projection onto the eigenspace of the particle number operator can lead to equations differing of those of the PRM. To answer this question and support the conclusions of the present paper the simultaneous projection of the angular momentum and the particle number should be performed and the approximation (3.2) be abandoned.

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Appendix A

The Hamiltonian of the QQ+PP-model is

$$H = T - G P^{+} P - \varkappa Q^{+}_{\mu} Q_{\mu} , \qquad (A.I)$$

where

$$T = \sum_{12} t_{12} a^{+}_{1} a_{2} , \quad P = \sum_{1>0} a_{1} a_{\tilde{1}} , \quad Q_{\mu} = \sum_{12} (q_{\mu})_{12} a^{+}_{1} a_{2} ,$$

$$\mu = 0, \pm 1, \pm 2 ,$$

a^{+}_{1} is the operator of the creation of a fermion in a state $|1\rangle$ (the set of the states $|1\rangle$ forms the Nilsson deformed basis) and t_{12} is the spherical single particle Hamiltonian reconed from a chemical potential.

In the representation of quasiparticles connected with a_1 and a^{+}_{1} by the Bogolyubov transformation

$$a^{+}_{1} = u_{1} \alpha^{+}_{1} + v_{1} \alpha_{\tilde{1}}$$
$$a_{\tilde{1}} = u_{1} \alpha_{\tilde{1}} - v_{1} \alpha^{+}_{1} \qquad (A.2)$$

(the symbol "tilde" denotes T-conjugation; $|\tilde{\tilde{1}}\rangle = -|1\rangle$, $a_{\tilde{\tilde{1}}} = -a_{1}$): the Hamiltonian, pairing and quadrupole operators have the form

$$H = \mathring{H} + H^{11}_{12} \alpha^{+}_{1} \alpha_{2} + H^{20}_{12} \alpha^{+}_{1} \alpha^{+}_{2} + h.c. + H^{40}_{1234} \alpha^{+}_{1} \alpha^{+}_{2} \alpha^{+}_{3} \alpha^{+}_{4} + h.c.$$
$$+ H^{31}_{1234} \alpha^{+}_{1} \alpha^{+}_{2} \alpha^{+}_{3} \alpha_{4} + h.c. + H^{22}_{1234} \alpha^{+}_{1} \alpha^{+}_{2} \alpha_{4} \alpha_{3} , \qquad (A.3)$$

$$P = \mathring{P} + P^{11}_{12} \alpha^{+}_{1} \alpha_{2} + P^{20}_{12} \alpha^{+}_{1} \alpha^{+}_{2} + P^{02}_{12} \alpha_{2} \alpha_{1} , \qquad (A.4)$$

$$Q_{\mu} = \delta_{\mu 0} Q_{0} + (Q^{11}_{\mu})_{12} \alpha^{+}_{1} \alpha_{2} + (Q^{20}_{\mu})_{12} \alpha^{+}_{1} \alpha^{+}_{2} + (Q^{02}_{\mu})_{12} \alpha_{2} \alpha_{1} , \qquad (A.5)$$

where

$$\mathring{P} = -\sum_{1>0} u_{1} v_{1} , \; P^{11}_{12} = u_{1} v_{1} \delta_{12} , \; P^{20}_{12} = -\tfrac{1}{2} v^{2}_{1} \delta_{1\tilde{2}} , \; P^{02}_{12} = \tfrac{1}{2} u^{2}_{1} \delta_{1\tilde{2}} , \qquad (A.6)$$

$$Q_0=\sum_1 (q_0)_{11} v_1^2 \,,\; (Q^{11}_\mu)_{12}=(q_\mu)_{12}\,\xi^{(+)}_{12} \,,\; (Q^{20}_\mu)_{12}=-\tfrac{1}{2}(q_\mu)_{1\bar{2}}\,\eta^{(+)}_{12} \,,\; (Q^{02}_\mu)_{12}=\tfrac{1}{2}(q_\mu)_{1\bar{2}}\,\eta^{(+)}_{12} \,, \tag{A.7}$$

$$\mathring{H}=\mathring{T}-G\Big(\mathring{P}^2+2\sum_{12}|P^{20}_{12}|^2\Big)-\varkappa\Big(Q_0^2+2\sum_{12\mu}|(Q^{20}_\mu)_{12}|^2\Big) \,, \tag{A.8}$$

$$H^{11}_{12}=T^{11}_{12}-G\big[\mathring{P}(P^{11}_{12}+\overset{*}{P}{}^{11}_{21})+\overset{*}{P}{}^{11}_{31}P^{11}_{32}+4\overset{*}{P}{}^{20}_{32}P^{20}_{13}\big] \tag{A.9}$$

$$-\varkappa\big[Q_0\big((Q^{11}_0)_{12}+(Q^{11}_0)^*_{21}\big)+(Q^{11}_\mu)^*_{31}(Q^{11}_\mu)_{32}+4(Q^{20}_\mu)_{32}(Q^{20}_\mu)_{13}\big] \,,$$

$$H^{20}_{12}=T^{20}_{12}-G\big[\mathring{P}(P^{20}_{12}+\overset{*}{P}{}^{02}_{12})+2\overset{*}{P}{}^{11}_{31}P^{20}_{32}\big] \tag{A.10}$$

$$-\varkappa\big[Q_0\big((Q^{20}_0)_{12}+(Q^{02}_0)^*_{12}\big)+2(Q^{11}_\mu)^*_{31}(Q^{20}_\mu)_{32}\big] \,,$$

$$\mathring{T}=\sum_1 t_{11}v_1^2 \,,\; T^{11}_{12}=t_{12}\,\xi^{(+)}_{12} \,,\; T^{20}_{12}=-\tfrac{1}{2}t_{1\bar{2}}\,\eta^{(+)}_{12} \,, \tag{A.11}$$

$$H^{40}_{1234}=-G\overset{*}{P}{}^{02}_{12}P^{20}_{34}-\varkappa(Q^{02}_\mu)^*_{12}(Q^{20}_\mu)_{34} \quad \text{(antisymm. over (I234))}, \tag{A.12}$$

$$H^{31}_{1234}=-G\big(\overset{*}{P}{}^{02}_{12}P^{11}_{34}+\overset{*}{P}{}^{11}_{41}P^{20}_{23}\big)-\varkappa\big((Q^{02}_\mu)^*_{12}(Q^{11}_\mu)_{34} \tag{A.13}$$

$$+(Q^{11}_\mu)^*_{41}(Q^{20}_\mu)_{23}\big) \quad \text{(antisymm. over (I23))}$$

$$H^{22}_{1234}=-G\big(\overset{*}{P}{}^{20}_{34}P^{20}_{12}+\overset{*}{P}{}^{02}_{12}P^{02}_{34}-\overset{*}{P}{}^{11}_{41}P^{11}_{23}\big) \tag{A.14}$$

$$-\varkappa\big((Q^{20}_\mu)^*_{34}(Q^{20}_\mu)_{12}+(Q^{02}_\mu)^*_{12}(Q^{02}_\mu)_{34}-(Q^{11}_\mu)^*_{41}(Q^{11}_\mu)_{23}\big)$$

(antisymm. over (I2) and (34)).

The matrix elements $\delta_{1\bar{2}}$ are defined by

$$\delta_{1\bar{2}}=\{a_1 a^+_{\bar{2}}\} \,,\quad \{AB\}\equiv AB+BA$$

and obey the identities

$$\delta_{1\bar{2}}=\delta_{\bar{2}1}=\delta^*_{1\bar{2}}=-\delta_{\bar{1}2} \,, \tag{A.15}$$

$$\sum_2 \delta_{\bar{1}2}f_2=f_{\bar{1}} \quad \text{for any } f_1 \,.$$

The quantities $\xi^{(\pm)}_{12}$ and $\eta^{(\pm)}_{12}$ are connected with the Bogolyubov coefficients u_1 and v_1 :

$$\xi^{(\pm)}_{12}=u_1u_2\mp v_1v_2 \,,\quad \eta^{(\pm)}_{12}=u_1v_2\pm v_1u_2 \,. \tag{A.16}$$

If as usual

$$u_1=\sqrt{\tfrac{1}{2}\Big(1+\frac{\varepsilon_1}{E_1}\Big)} \,,\quad v_1=\sqrt{\tfrac{1}{2}\Big(1-\frac{\varepsilon_1}{E_1}\Big)} \,, \tag{A.17}$$

where ε_1 is the Nilsson energy reckoned from a chemical potential, $E_1=\sqrt{\varepsilon_1^2+\Delta^2}$, Δ is a gap parameter, then

$$\mathring{P}=-\frac{\Delta}{G} \,,\quad \sum_{1>0}\frac{1}{E_1}=\frac{2}{G} \,,$$

whence neglecting the exchange terms one obtains

$$H^{11}_{12}=E_1\delta_{12} \,,\quad H^{20}_{12}=0 \,. \tag{A.18}$$

Finally the angular momentum operator

$$\vec{J}=\sum_{12}(\vec{j})_{12}a^+_1a_2$$

can be written in the term

$$J_z=\sum_1 (j_z)_{11}\alpha^+_1\alpha_1 \,, \tag{A.19}$$

$$J_\lambda=(J^{11}_\lambda)_{12}\alpha^+_1\alpha_2+(J^{20}_\lambda)_{12}\alpha^+_1\alpha^+_2+(J^{20}_\lambda)^*_{12}\alpha_2\alpha_1 \,, \tag{A.20}$$

where

$$(J^{11}_\lambda)_{12}=j^\lambda_{12}\,\xi^{(-)}_{12} \,,\quad (J^{20}_\lambda)_{12}=-\tfrac{1}{2}j^\lambda_{1\bar{2}}\,\eta^{(-)}_{12} \quad (\lambda=x,y) \,. \tag{A.21}$$

Appendix B

Some useful identities for Q^{20}_{μ}, Q^{11}_{μ} and J^{20}_{λ} can be stated.

By using the commutation $j_{\pm} = j_x \pm i j_y$ with $\varepsilon = t - 2\varkappa Q_0 q_0$ one obtains

$$(Q^{20}_{\pm1})_{12} = \frac{1}{2\sqrt{6}\,\varkappa Q_0} E_{12} (J^{20}_{\pm})_{12} \qquad \text{(B.I)}$$

with

$$(Q^{20}_{\mu})_{12} = (-)^{\mu} (Q^{02}_{-\mu})^{*}_{12}\,, \quad E_{12} = E_1 + E_2\,,$$

and

$$(Q^{11}_{\pm1})_{12} = \frac{1}{2\sqrt{6}\,\varkappa Q_0} (E_1 - E_2)(J^{11}_{\pm})_{12}\,. \qquad \text{(B.2)}$$

Starting from

$$Q_0 = \sum_{1} (q_0)_{11} v_1^2$$

and substituting

$$q_0 = \frac{1}{\sqrt{6}} [j_{\pm}\, q_{\mp1}]$$

it is easy to show that

$$\sum_{12} (Q^{20}_{\mu})_{12} (J^{20}_{\pm})^{*}_{12} = \frac{\sqrt{6}}{4} Q_0\, \delta(\mu \mp 1)\,, \qquad \text{(B.3)}$$

$$\sum_{12} (Q^{02}_{\mu})_{12} (J^{20}_{\pm})_{12} = -\frac{\sqrt{6}}{4} Q_0\, \delta(\mu \pm 1) \qquad \text{(B.4)}$$

and

$$\sum_{12} (Q^{20}_{\mu})_{12} (J^{20}_{y})^{*}_{12} = i \frac{\sqrt{6}}{8} Q_0\, \mu\, \delta(\mu^2 - 1)\,, \qquad \text{(B.3a)}$$

$$\sum_{12} (Q^{02}_{\mu})_{12} (J^{20}_{y})_{12} = -i \frac{\sqrt{6}}{8} Q_0\, \mu\, \delta(\mu^2 - 1)\,. \qquad \text{(B.4a)}$$

From Eq. (B.I) it follows

$$\sum_{12} \frac{(J^{20}_{x})^{*}_{12}}{E_{12}} (Q^{20}_{\mu})_{12} = \frac{\mathcal{D}}{4\sqrt{6}\,\varkappa Q_0} \delta(\mu^2 - 1)\,, \qquad \text{(B.5)}$$

$$\sum_{12} \frac{(J^{20}_{x})^{*}_{12}}{E_{12}} (Q^{02}_{\mu})^{*}_{12} = -\frac{\mathcal{D}}{4\sqrt{6}\,\varkappa Q_0} \delta(\mu^2 - 1) \qquad \text{(B.6)}$$

with

$$\mathcal{D} = 2 \sum_{12} (J^{20}_{x})^{*}_{12} (J^{20}_{x})_{12} = \sum_{12} (J^{20}_{+})^{*}_{12} (J^{20}_{+})_{12}\,. \qquad \text{(B.7)}$$

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