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A b s t r a c t

We present the formulae for radiative corrections to cross sections of QED processes $e^{\pm}e^{-} \to e^{\pm}e^{-}$, $e^{+}e^{-} \to \mu^{+}\mu^{-}, \gamma\gamma$ due to virtual and soft photon emission in the case of transversely polarized initial $e^{\pm}e^{-}$ beams. The differential cross section for the emission of an additional hard photon have been obtained. The comparison of QED contributions with those of the Z^{0} - boson exchange is given.

1. Radiative corrections (r.c.) play an important role in studying of $e^{\pm} e^{-}$ collisions as well as the Q E D processes with hard photons emission [1]. The case when initial leptons are polarized contains more opportunities not only to verify Q E D themselves but to look for the neutral current contribution which is of order of $\gamma = G\varepsilon^2/(\pi\alpha\sqrt{2}) \sim 3 \cdot 10^{-4}\,\varepsilon^2$ (GeV) (G - the Fermi weak coupling constant, $\alpha = 1/137$, 2ε - the total cm. energy). This quantity reaches the magnitude of 10-15% for now accessible range of energies (PEP, PETRA), and may be enhanced when polarized leptons are used. The studying of Q E D processes is interesting for various reasons. In the first place it is still an interesting question to see whether Q E D remains valid at high energies. Secondly, the Q E D reactions, since they are in principle calculable, provide an excellent tool to test the experimental apparatus and to measure the beam luminosity. Thirdly they are the background to hadronic processes.

In this paper we present the results of calculations of r.c. to Q E D processes with the transversely polarized $e^{\pm} e^{-}$ colliding beams. Moreover for the energy range $m_\mu^2 \ll \varepsilon^2 \ll M_Z^2$ we consider the Z_0 - exchange contribution to cross section. The resulting expressions (see sections 2-5) for cross sections have the form

$$\frac{d\sigma}{dO} = \frac{d\sigma_0}{dO}\left(1 + \Delta_W + \Delta_{QED}\right) \qquad (1.1)$$

where $d\sigma_0/dO$ - is the cross-section in Born approximation, Δ_W, Δ_{QED} the corrections from the Z_0 exchange and virtual and soft in c.m. system photons with energy ω, $\omega < \Delta\varepsilon \ll \varepsilon$ emission. Each of quantities $d\sigma_0/dO$, $\Delta_W\, d\sigma_0/dO$ and $\Delta_{QED}\, d\sigma_0/dO$ is the sum of two terms, one of which corresponds to unpolarized case and second takes into account polarization effects and is proportional to quantity

$$a = |\vec{\zeta}_1||\vec{\zeta}_2| \cos(\varphi_0 + 2\varphi) \qquad (1.2)$$

where $\vec{\zeta}_{1,2}$ are the polarization vectors of initial $e^{\pm} e^{-}$, vectors with momenta p_1, p_2 :

$$P_1=(\varepsilon,0,0,P)\ ,\ s_1=(0,\vec{s}_1,0)\ ,\ q_1=(\varepsilon,\vec{q}_\perp,q_u)$$
$$P_2=(\varepsilon,0,0,-P)\ ,\ s_2=(0,\vec{s}_2,0)\ ,\ q_2=(\varepsilon,-\vec{q}_\perp,-q_u) \qquad (1.3)$$

$q_{1,2}$ - the 4 - momenta of final particles, $\varphi_0=\widehat{\vec{s}_1\vec{s}_2}$, $\varphi=\widehat{\vec{s}_1\vec{q}_\perp}$ - the azimuthal angles between the polarization vectors $\vec{s}_1,\vec{s}_2$ and polarization vector $\vec{s}_1$ and the transverse to beam axes component of one of final particles.

The quantities Δ_w are proportional to the weak interactions parameter

$$y=\frac{G\varepsilon^2}{\pi\alpha\sqrt{2}}$$

The results of our calculations of Δ_w for range of energies

$$m_\mu^2 \ll \varepsilon^2 \ll M_Z^2$$

are in agreement with computations of Bundy et al. [2,4], as well as an E.R. limit for Δ_{QED} for process $e^+e^-\to\mu^+\mu^-$ [1]. As to Δ_{QED} for $e^-e^-\to e^-e^-$ our result contradicts to that obtained by Gastmans [4], whose result contained a term of kind $\ln^2(2E/m_e)\cos(2\varphi)$, inadmissible (they lead to $d\sigma/do<0$).

In section 6 we give the differential cross sections of processes $e^\pm e^-\to e^\pm e^-\gamma$, $e^+e^-\to\mu^+\mu^-\gamma$, $\gamma\gamma\gamma$ in extreme relativistic (ER) limit. Corresponding expressions have comparatively simple form, moreover the terms which describes the polarization are proportional to one of three quantities

$$\alpha_i=|\vec{s}_1||\vec{s}_2|\cos(\varphi_0+2\varphi_i)\ ,\ \varphi_i=\widehat{\vec{s}_1,\vec{q}_{i\perp}},\ i=1,2,3, \qquad (1.4)$$

where q_i - 4 - momentum of one of final particles.

In the case when photon in final state in processes $e^\pm e^-\to e^\pm e^-$, $e^+e^-\to\mu^+\mu^-\gamma$ is emitted not close to any charged particles ($\theta_i \gg m/\varepsilon$), these cross-sections have the form of product of two multipliers, one of which, W, is universal and coincides with that obtained by Berends, Gastmans and W_u for unpolarized case [3].

The polar angle, θ, dependence of Δ_W, Δ_{QED} for

$2\varepsilon = 30$ GeV, $\frac{\Delta\varepsilon}{\varepsilon} = 0.3, 0.1$ is presented in Fig. 1.

The expression for virtual-photon exchange correction to $e^+e^-\to\mu^+\mu^-$, valid for $\beta=\sqrt{1-(m_\mu^2/\varepsilon^2)}\sim 1$ and some algebraic identities which may be used in section 6 are given in Appendices I, II.

2. The matrix element of process

$$e^+(p_+)+e^-(p_-)\to\mu^+(q_+)+\mu^-(q_-)$$

in Born approximation has the form

$$M=\frac{e^2}{s}\bar{V}_{p_+}\gamma_\mu U_{p_-}\cdot\bar{U}_{q_-}\gamma_\mu V_{q_+}-\frac{G}{\sqrt{2}}\bar{V}_{p_+}\gamma_\nu(g_v^e+g_a^e\gamma_5)U_{p_-}\cdot\bar{U}_{q_-}\gamma_\nu(g_v^\mu+g_a^\mu\gamma_5)V_{q_+}, \qquad (2.1)$$

where $e^2=4\pi\alpha$, $s=(p_++p_-)^2=4\varepsilon^2$; the second term in r.h. 2.1 corresponds to the Z_0 - exchange, and the G, g_v, g_a quantities in accordance with standard model have the form (θ_W - the Weinberg's angle $\sin^2\theta_w=0.23$)

$$\frac{G}{2\sqrt{2}}=\frac{\pi\alpha}{M_Z^2\sin^2(2\theta_w)}\ ,\ g_v^e=g_v^\mu=4\sin^2\theta_w-1\ ,\ g_a^e=g_a^\mu=-1. \qquad (2.2)$$

Using the density matrix of electron

$$\sum_\lambda u_p^\lambda\bar{u}_p^\lambda=\frac{1}{2}(\hat{p}-m)(1-\gamma_5\hat{s})$$

one can obtain from (2.1) in E.R. limit:

$$\frac{d\sigma^{e^+e^-\to\mu^+\mu^-}}{dO}=\frac{d\sigma_0}{dO}(1+\Delta_W)\ ,\ \frac{d\sigma_0}{do}=\frac{\alpha^3}{2s}A\ ,$$
$$\Delta_W=-y[g_v^2+g_a^2(1-2x)A^{-1}]\ ,\ A=x^2+(1-x)^2+2x(1-x)\alpha \qquad (2.3)$$

where $x=\sin^2\theta/2$, $\theta=\widehat{\vec{p}_-,\vec{q}_-}$ and $y=G\varepsilon^2/\pi\alpha\sqrt{2}$.

The Q E D corrections due to emission of virtual and soft ($\omega<\Delta\varepsilon\ll\varepsilon$) real in c.m. system photons in E R limit have the form

$$\Delta_\mu^{QED} = \frac{4\alpha}{\pi}\Big\{\frac{1}{2}(\rho_e+\rho_\mu-2)\Big(\ln\frac{\Delta\varepsilon}{\varepsilon}+\frac{13}{12}\Big)+\frac{\pi^2}{6}-\frac{17}{36}+\ln\frac{x}{1-x}\ln\frac{\Delta\varepsilon}{\varepsilon}+$$
$$+\frac{1}{4}\ln\frac{x}{1-x}\ln x(1-x)-\frac{1}{2}\int_x^{1-x}\frac{dx}{x}\ln(1-x)\Big\}+\frac{\alpha}{\pi}A^{-1}\Big\{(1-x)\ln x-x\ln(1-x)-\frac{1}{2}(1-2x)\big(\ln^2 x+$$
$$+\ln^2(1-x)\big)+\frac{1}{2}\alpha\Big[(1-2x)\Big(\frac{x}{1-x}\ln^2 x+\frac{1-x}{x}\ln^2(1-x)\Big)+2(1-x)\ln(1-x)-2x\ln x\Big]\Big\}, \qquad (2.4)$$

where $\rho_e=\ln(4\varepsilon^2/m_e^2)$, $\rho_\mu=\ln(4\varepsilon^2/m_\mu^2)$ and the other quantities are defined in 1.2 and 2.3.

The expression 2.4 is in agreement with that obtained by Khriplovich (see [1]). In appendix I we give the expression for Q E D cross-section in α^3 order of perturbation theory (p.t.), which is valid when $m_\mu/\varepsilon \sim 1$, and the only terms of order $(m_e/m_\mu)^2, (m_e/\varepsilon)^2$ are neglected.

3. Combining the known expressions for integrals on virtual 4 - momenta with the new expressions for traces one may obtain the cross-section of $e^+e^- \to e^+e^-$ in form (1.1) where

$$\frac{d\sigma_0}{dO}=\frac{\alpha^2}{3x^2}B, \qquad B=(1-x+x^2)^2-x^2(1-x)^2\alpha, \qquad (3.1)$$

$$\Delta_W=\frac{1}{2}\gamma x(1-x)B^{-1}\Big[g_v^2\big(3(1-x+x^2)+x(1-3x)\alpha\big)+g_a^2\big(1-5x+x^2-x(1+x)\alpha\big)\Big]$$

$$\Delta_{QED}=\frac{4\alpha}{\pi}\Big[\Big(\rho_e+\ln\frac{x}{1-x}-1\Big)\ln\frac{\Delta\varepsilon}{\varepsilon}-\frac{23}{18}+\frac{11}{12}\rho_e-\frac{1}{2}\int_x^{1-x}\frac{dx}{x}\ln(1-x)\Big]+\frac{\alpha}{\pi}B^{-1}\big[f_1(x)+\alpha f_2(x)\big],$$

where

$$f_1(x)=\frac{\pi^2}{12}(-4+8x+3x^2-10x^3+8x^4)+\frac{1}{6}(22-30x+33x^2-11x^3)\ln x-\frac{1}{2}x(1+x^2)\ln(1-x)+$$
$$+\frac{1}{4}x(3-x-3x^2+4x^3)\ln^2 x+\frac{1}{4}(-4+10x-14x^2+10x^3-4x^4)\ln^2(1-x)+\frac{1}{2}(4-8x+7x^2-2x^3)\ln x\ln(1-x),$$

$$f_2(x)=-4x(1-x)\Big[\frac{\pi^2}{6}\frac{x(1-4x)}{4}+\frac{x(2-3x+4x^2)}{16(1-x)}\ln^2 x+\frac{1}{16}(-1+3x-6x^2+4x^3)\frac{\ln^2(1-x)}{x}+$$
$$+\frac{1}{4}x\ln x\ln(1-x)+\frac{11}{24}x\ln x-\frac{1}{8}(1-x)\ln(1-x)\Big].$$

4. For the process of electron-electron scattering the cross-section has the form (1.1) where

$$\frac{d\sigma_0}{dO}=\frac{\alpha^2}{3x^2(1-x)^2}B, \qquad B=(1-x+x^2)^2-x^2(1-x)^2\alpha,$$

$$\Delta_W=\frac{1}{2}\gamma x(1-x)B^{-1}\Big\{g_v^2\big[3(1-x+x^2)-x(1-x)\alpha\big]+g_a^2\big[1+3x(1-x)+x(1-x)\alpha\big]\Big\},$$

$$\Delta_{QED}=\frac{4\alpha}{\pi}\Big\{-\big(1-\rho_e-\ln x(1-x)\big)\ln\frac{\Delta\varepsilon}{\varepsilon}+\frac{11}{12}\rho_e-\frac{23}{18}\Big\}+\frac{\alpha}{\pi}B^{-1}\big\{\varphi_1(x)+\alpha\varphi_2(x)\big\},$$

$$\varphi_1(x)=-\frac{3}{8}x(1-x)\pi^2+\frac{1}{6}x(11-3x^2+14x^3)\ln(1-x)+\frac{1}{4}x(-3+8x-4x^2+3x^3)\ln^2 x$$
$$+\frac{1}{4}\big(-8+19x(1-x)-8x^2(1-x)^2\big)\ln x\ln(1-x)+\{x\to(1-x)\},$$

$$\varphi_2(x)=-\frac{1}{4}x(1-x)\Big[x(6-5x)\ln^2 x+\frac{2}{3}x(11-14x)\ln x-8x(1-x)\ln x\ln(1-x)\Big]+$$
$$+\{x\to 1-x\}.$$

5. Consider now the two-quantum annihilation of e^+e^- pair. In the lowest order of p.t. the deviations from Q E D due neutral current don't exist. The result of calculations has the form

$$\frac{d\sigma}{dO_1}=\frac{d\sigma_0}{dO_1}(1+\Delta_{QED})$$

where

$$\frac{d\sigma_0}{dO_1} = \frac{\alpha^2}{2s\chi(1-\chi)}\cdot A \;, \quad A = (1-\chi)^2+\chi^2+2\chi(1-\chi)a \;,$$

$$\Delta_{QED} = \frac{2\alpha}{\pi}\left[(\rho_e-1)\ln\frac{\Delta\varepsilon}{\varepsilon} + \frac{\pi^2}{6} + \frac{3}{4}(\rho_e-1)\right] + \frac{\alpha}{2\pi}A^{-1}\Big\{\chi(2+\chi)\ln(1-\chi) +$$
$$+ (1+\chi^2)\ln^2\chi + a\chi(1-\chi)\left[\frac{2}{\chi}\ln(1-\chi) + \frac{1+\chi^2}{\chi^2}\ln^2(1-\chi)\right] + (\chi\to 1-\chi)\Big\}$$

The polar angle θ dependence of Δ_W, Δ_{QED} calculated with the aid of formulae from sections 2-5 are drawn in Fig. 1 where we take ε = 15 GeV, $\Delta\varepsilon/\varepsilon$ = 0.3. At these conditions the Δ_W quantity reaches the order of $\Delta_{QED} \sim$ 10%. Such considerable contribution of weak currents permits to distinguish it reliably in experiment organization when the corrections to Born cross-section of Q E D nature are suppressed. We believe that organization of experiment of this kind will be one when the energy of additional quanta may be arbitrary (roughly speaking it leads to vanishing of terms proportional to $\ln\frac{\Delta\varepsilon}{\varepsilon}$ in Δ_{QED}).

6. The differential cross sections of processes $e^\pm e^- \to e^\pm e^-\gamma,\ e^+e^-\to\mu^+\mu^-\gamma,\ \gamma\gamma\gamma$ are known for a long time [1]. As a rule corresponding expressions are very cumbersome even in ER limit. Recently, however the apt expressions were found out by Berends, Gastmans and Wu [3] who had considered the unpolarized case. Here we present the cross sections of this processes in E R limit for the case of transversely polarized initial $e^\pm e^-$ beams. This cross-sections may be written in the form

$$d\sigma_a = \frac{\alpha r_0^2}{2\pi}\left(\frac{m_e}{\varepsilon}\right)^2(\varepsilon^2\Gamma_a)\,d\Phi$$

where r_0 = 2.8·10^{-13}cm is the classical electron radius, ε - beam energy (in s.c.m.),

$$d\Phi = \frac{d^3k\,d^3q_1\,d^3q_2}{4\pi\,\varepsilon^2\,\omega\,\varepsilon_1\varepsilon_2}\,\delta^{(4)}(p_1+p_2-q_1-q_2-k)$$

is the phase volume element of final state, Γ_a is proportional to the summed over spin states of final particles square of module of matrix element

$$\Gamma_a = \frac{1}{4}\sum|M_a|^2$$

The quantity Γ_γ for process $e^+(p_+)+e^-(p_-)\to\gamma(q_1)+\gamma(q_2)+\gamma(q_3)$ has the form:

$$\Gamma_\gamma = s\left[\chi_3^2+\chi_3'^2-(3\chi_1\chi_1'+3\chi_2\chi_2'-\chi_3\chi_3')a_3\right](\chi_1\chi_2\chi_1'\chi_2')^{-1} -$$
$$-2m_e^2\left[\frac{\chi_1^2+\chi_2^2}{\chi_3'^2\chi_1\chi_2} + \frac{\chi_1'^2+\chi_2'^2}{\chi_3^2\chi_1'\chi_2'}\right] + (3\to1,2\to3,1\to2)+(3\to2,2\to1,1\to3), \tag{6.1}$$

where the quantities a_i are defined in 1.4, $\chi_i = p_-q_i$, $\chi_i' = p_+q_i$, $s = 4\varepsilon^2$. For the case of process $e^+(p_+)+e^-(p_-)\to\mu^+(q_+)+\mu^-(q_-)+\gamma(k)$ one has

$$\Gamma_\mu = \frac{1}{4}W\left[t^2+t_1^2+u^2+u_1^2+2t_1ua_+ + 2tu_1a_-\right](\chi_+\chi_-\chi_+'\chi_-'ss_1)^{-1} -$$
$$-\frac{m_\mu^2}{2s^2}\left[\frac{t_1^2+u^2+2t_1ua_+}{\chi_-'^2} + \frac{t^2+u_1^2+2tu_1a_-}{\chi_+'^2}\right] - \frac{m_e^2}{2s_1^2}\left[\frac{t^2+u^2}{\chi_+^2} + \right. \tag{6.2}$$
$$\left. + \frac{t_1^2+u_1^2}{\chi_-^2}\right] - \frac{1}{2ss_1}\left[tu_1a_- + t_1ua_+\right]\left(\frac{m_e^2}{\chi_+^2}+\frac{m_e^2}{\chi_-^2}\right),$$

$$W = \frac{1}{4}\left[u(st+s_1t_1)+u_1(s_1t+st_1)+2ss_1(t+t_1)+2tt_1(s+s_1)\right],$$

and the invariants are defined as

$$s=(p_++p_-)^2,\ t=(p_--q_-)^2,\ u=(p_--q_+)^2,\ \chi_\pm = kp_\pm,$$
$$s_1=(q_++q_-)^2,\ t_1=(p_+-q_+)^2,\ u_1=(p_+-q_-)^2,\ \chi_\pm' = kq_\pm. \tag{6.3}$$

The quantity W enters in the $\Gamma_{e^+e^-}$ for process $e^+(p_+)+e^-(p_-)\to e_+(p_+')+e_-(p_-')+\gamma(k)$

$$\Gamma_{e^+e^-} = \frac{1}{4}W\left[ss_1(s^2+s_1^2)+tt_1(t^2+t_1^2)+uu_1(u^2+u_1^2)+\chi_+\chi_-(3t_1ua_+ + 3tu_1a_- -\right.$$
$$\left. -4\chi_+\chi_-a_\gamma) - \frac{1}{2}t_1^2u^2a_+ - \frac{1}{2}t^2u_1^2a_-\right](\chi_+\chi_-\chi_+'\chi_-'tt_1ss_1)^{-1} - \tag{6.4}$$
$$-\frac{m_e^2}{\chi_+'^2}\left[\left(\frac{s}{t}+\frac{t}{s}+1\right)^2-\frac{u_1^2}{s^2}a_-\right] - \frac{m_e^2}{\chi_-'^2}\left[\left(\frac{s}{t_1}+\frac{t_1}{s}+1\right)^2-\frac{u^2}{s^2}a_+\right] -$$

$$- \frac{m_e^2}{\varkappa_+^2}\left[\left(\frac{s_1}{t}+\frac{t}{s_1}+1\right)^2-\frac{u}{2ss_1t}\left(t_1u\,a_+ + tu_1a_-\right)\right]-$$
$$- \frac{m_e^2}{\varkappa_-^2}\left[\left(\frac{s_1}{t_1}+\frac{t_1}{s_1}+1\right)^2-\frac{u_1}{2ss_1t_1}\left(t_1u\,a_+ + tu_1a_-\right)\right],$$

where $a_\gamma = |\vec{s}_+||\vec{s}_-|\cos(\varphi_0+2\varphi_\gamma)$, $\varphi_\gamma = \widehat{\vec{s}_-\vec{k}_\perp}$ and the remaining quantities are defined in (6.3) where we must to exchange $q_- \to p_-'$, $q_+ \to p_+'$.

Lastly the quantity $\Gamma_{e^-e^-}$ for the process $e_-(p_1)+e_-(p_2)\to e_-(p_1')+e_-(p_2')+\gamma(k)$ has the form

$$\Gamma_{e^-e^-} = \frac{1}{4}W'\Big[ss_1(s^2+s_1^2)+tt_1(t^2+t_1^2)+uu_1(u^2+u_1^2)+\varkappa_1\varkappa_2(3tu_1a_1+3t_1ua_2-4\varkappa_1\varkappa_2a_\gamma)$$
$$-\frac{1}{2}t^2u_1^2a_1-\frac{1}{2}t_1^2u^2a_2\Big](\varkappa_1\varkappa_2\varkappa_1'\varkappa_2'tt_1uu_1)^{-1}-\frac{m_e^2}{\varkappa_1'^2}\left[\left(\frac{t}{u_1}+\frac{u_1}{t}+1\right)^2-a_2\right]- \qquad (6.5)$$
$$-\frac{m_e^2}{\varkappa_2'^2}\left[\left(\frac{t_1}{u}+\frac{u}{t_1}+1\right)^2-a_1\right]-\frac{m_e^2}{\varkappa_2^2}\left[\left(\frac{t}{u}+\frac{u}{t}+1\right)^2-\frac{s_1}{2stu}(ut_1a_2+u_1ta_1)\right]-$$
$$-\frac{m_e^2}{\varkappa_1^2}\left[\left(\frac{t_1}{u_1}+\frac{u_1}{t_1}+1\right)^2-\frac{s_1}{2st_1u_1}(ut_1a_2+u_1ta_1)\right],$$

where

$$W' = \frac{1}{4}\left[s(tu+t_1u_1)+s_1(t_1u+u_1t)+2uu_1(t+t_1)+2tt_1(u+u_1)\right],$$
$$s=(p_1+p_2)^2,\quad t=(p_1-p_1')^2,\quad u=(p_1-p_2')^2,\quad \varkappa_{1,2}=kp_{1,2}$$
$$s_1=(p_1'+p_2')^2,\quad t_1=(p_2-p_2')^2,\quad u_1=(p_2-p_1')^2,\quad \varkappa_{1,2}'=kp_{1,2}'$$

and the quantity $a_{1,2}$ and a_γ were defined earlier in 1.4 and 6.4.

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Appendix I

Here we give the expression for cross-section of $e^+(p_+) + e^-(p_-) \to \mu^+(q_+) + \mu^-(q_-)$ up to α^3 order of p.t. for transversally polarized e^+e^- valid in the intermediate relativistic region $m_e \ll \varepsilon \gtrsim m_\mu \equiv m$ (When inferring it only terms of order $(m_e/\varepsilon)^2$, $(m_e/m)^2$ were omitted). This expression is the sum of charge-odd and charge-even parts (odd and even under the interchange $\theta \to \pi - \theta$, $\theta = \widehat{\vec{p}_-, \vec{q}_-}$).

The charge odd part is connected with the interference of amplitudes in first M_0 and second B_1 Born approximations:

$$\frac{d\sigma^{odd}}{dO_\mu} = \frac{\alpha^3\beta}{2\pi s^2}\left(R(s,t) - R(s,u)\right), \quad R(s,t) = \frac{s}{64\pi\alpha^3}\sum_{\ell} \mathrm{Re}\left(\frac{M_0}{i}\right)^{*}(-iB_1) \qquad (1.1)$$

where β is the velocity of muon (in s.c.m) $\beta = (1 - m^2/\varepsilon^2)^{1/2}$,

$t = (p_- - q_-)^2 = m^2 - 2\varepsilon^2(1 - \beta z)$, $u = (p_- - q_+)^2 = m^2 - 2\varepsilon^2(1 + \beta z)$, $z = \cos\theta$

θ - scattering angle,

$$R(s,t) = 16 J(\tau - \Delta^2)[\overset{+\tau^2}{\Delta^4} - m^2\Delta^2 + a\mathcal{D}] + 4F(\tau - \Delta^2)[2\tau - m^2 + a\mathcal{D}^{-1}(2\tau(\tau - \Delta^2)^2 -$$
$$- m^2(\tau - \Delta^2)(\tau + 3\Delta^2) + \Delta^2 m^4] + 4F_\Delta[2\Delta^4 - 2\Delta^2\tau + 2\tau^2 - m^2\Delta^2 + a\mathcal{D}^{-1}(\Delta^4 m^4 -$$
$$- m^2(\tau - \Delta^2)(\Delta^4 + 3\Delta^2\tau) + 2(\tau - \Delta^2)^2(\Delta^4 + \Delta^2\tau + \tau^2))] + 4F_Q[(\tau - \Delta^2)(2\tau - \Delta^2) -$$
$$- \tau m^2 + a\mathcal{D}^{-1}((\tau - \Delta^2)^2(\Delta^4 + \Delta^2\tau + 2\tau^2) - m^2(\tau - \Delta^2)(\tau^2 + 3\tau\Delta^2 + \Delta^4) +$$
$$+ \tau\Delta^2 m^4)] + 4G_Q[\Delta^4 + \Delta^2\tau + \Delta^2 m^2 - a\Delta^2(\tau - \Delta^2 - m^2)] + 4H_Q[\tau^2 -$$
$$- \Delta^4 - 2m^2\Delta^2 - a(\Delta^2 - \tau)^2],$$

where $a = |\vec{s}_+||\vec{s}_-|\cos(\varphi_0 + 2\varphi)$,

$\Delta^2 = -\frac{1}{4}s$, $s = (p_+ + p_-)^2 = 4\varepsilon^2$, $Q^2 = m^2 + \Delta^2$, $\tau = \frac{1}{4}(u - t)$, $\mathcal{D} = \Delta^2 Q^2 - \tau^2$,

$$(\tau - \Delta^2) J = -\frac{1}{s}\ln\frac{s}{\lambda^2}\ln\frac{m^2 - t}{m m_e}, \quad (\tau - \Delta^2) F = -\frac{1}{2}\ln\frac{s}{m^2}\ln\frac{m^2 - t}{m m_e} +$$
$$+ \frac{1}{4}\ln^2\left(\frac{m^2 - t}{m^2}\right) - \frac{1}{8}\ln^2(m^2/m_e^2) + \frac{1}{2}\int_0^{-t/m^2 - t} dz \ln(1 - z)/z,$$

$$F_\Delta = \frac{1}{s}\left(\frac{\pi^2}{6} + \frac{1}{2}\ln^2(s/m_e^2)\right), \quad H_Q = \frac{1}{t}\ln\left(\frac{m^2 - t}{m^2}\right),$$

$$F_Q = \frac{1}{s\beta}\left(\frac{1}{2}\ln^2(s/m^2) + \pi^2/6 + \ln^2\left(\frac{1+\beta}{2}\right) + 2F\left(\frac{1-\beta}{2}\right)\right),$$

$$G_Q = \frac{1}{s\beta^2}\left(-2\ln\frac{s}{m^2} + sF_Q\right).$$

The charge - even part has the form:

$$\frac{d\sigma_{even}}{dO} = \frac{\alpha^2\beta}{4s}\left(2 - \beta^2 + \beta^2 z^2 + \beta^2(1 - z^2)a\right)\left[1 + 2F_1 + 2\widetilde{F}_1\right] + \frac{\alpha^2\beta}{s}\widetilde{F}_2, \qquad (1.2)$$

where

$$F_1 = \frac{\alpha}{\pi}\left\{\left(1 - \ln\frac{s}{m_e^2}\right)\left(\ln\frac{m_e}{\lambda} - 1\right) + \frac{\pi^2}{3} - \frac{1}{4}\ln^2\left(\frac{s}{m_e^2}\right) - \frac{1}{4}\ln\left(\frac{s}{m_e^2}\right)\right\},$$

$$\widetilde{F}_1 = \frac{\alpha}{\pi}\left\{\left(1 + \frac{1+\beta^2}{2\beta}b\right)\left(\ln\frac{m}{\lambda} - 1\right) + \frac{1+\beta^2}{2\beta}\left[\frac{1}{3}\pi^2 - \frac{1}{4}\ln^2 b + \ln b\ln(1-b) - \int_0^b \frac{dt}{t}\ln(1-t)\right] + \frac{\ln b}{4\beta}\right\},$$

$$\widetilde{F}_2 = \frac{\alpha}{\pi}\,\frac{1-\beta^2}{4\beta}\ln b, \quad b = \frac{1-\beta}{1+\beta}.$$

The E R limit of I.I, I.2 with the E R limit expression for soft photon emission (see [5]) leads to 2.1.

Appendix II

Here we give some algebraic identities which are useful for calculation of hard photon cross-sections.

With the definition of invariants (6.4) one can obtain in E R limit:

$$s + s_1 + t + t_1 + u + u_1 = 0,$$

$$2\chi_- = s + t + u\,, \quad 2\chi'_- = s + t_1 + u\,,$$

$$2\chi_+ = s + t_1 + u_1\,, \quad 2\chi'_+ = s + t + u_1\,,$$

$$s_1\chi_+\chi_- + s\chi'_+\chi'_- + u_1(\chi_+\chi'_- + \chi_-\chi'_+) - t(\chi_+\chi'_+ + \chi_-\chi'_-) + 2(\chi_- - \chi_+)\chi'_+\chi'_- =$$

$$= s_1\chi_+\chi_- + s\chi'_+\chi'_- + u(\chi_+\chi'_- + \chi_-\chi'_+) - t_1(\chi_+\chi'_+ + \chi_-\chi'_-) + 2(\chi_+ - \chi_-)\chi'_+\chi'_- =$$

$$= \frac{1}{2}\left[\chi_- s_1 t_1 + \chi_+ s_1 t - \chi'_- s t_1 - \chi'_+ s t\right] =$$

$$= \frac{1}{4}\left[u(st + s_1 t_1) + u_1(s t_1 + s_1 t) + 2 s s_1 (t + t_1) + 2 t t_1 (s + s_1)\right] \equiv W\,,$$

$$s_1 t_1 \chi_- \left[t\chi'_- + u_1(\chi_+ - \chi'_+) - 2\chi'_+\chi'_-\right] + s_1 t \chi_+\chi_- (t - 2\chi'_+) - s t_1 \chi'_- (s_1\chi'_+ +$$

$$+ u_1\chi_+) + s t \chi'_+ \chi_+ (t + 2\chi'_-) = (s s_1 - u u_1 - t u_1) W\,.$$

The dependence of magnitudes Δ_W , Δ_{QED} (in %) from the scattering angle θ for 2ε = 30 GeV, $\Delta\varepsilon/\varepsilon$ = 0.3. The signs $\pm$; 0 corresponds to the values of polarization parameter α = $\pm$ 0.85; 0. (Δ_{QED}-solid curve, Δ_W-dashed curve)

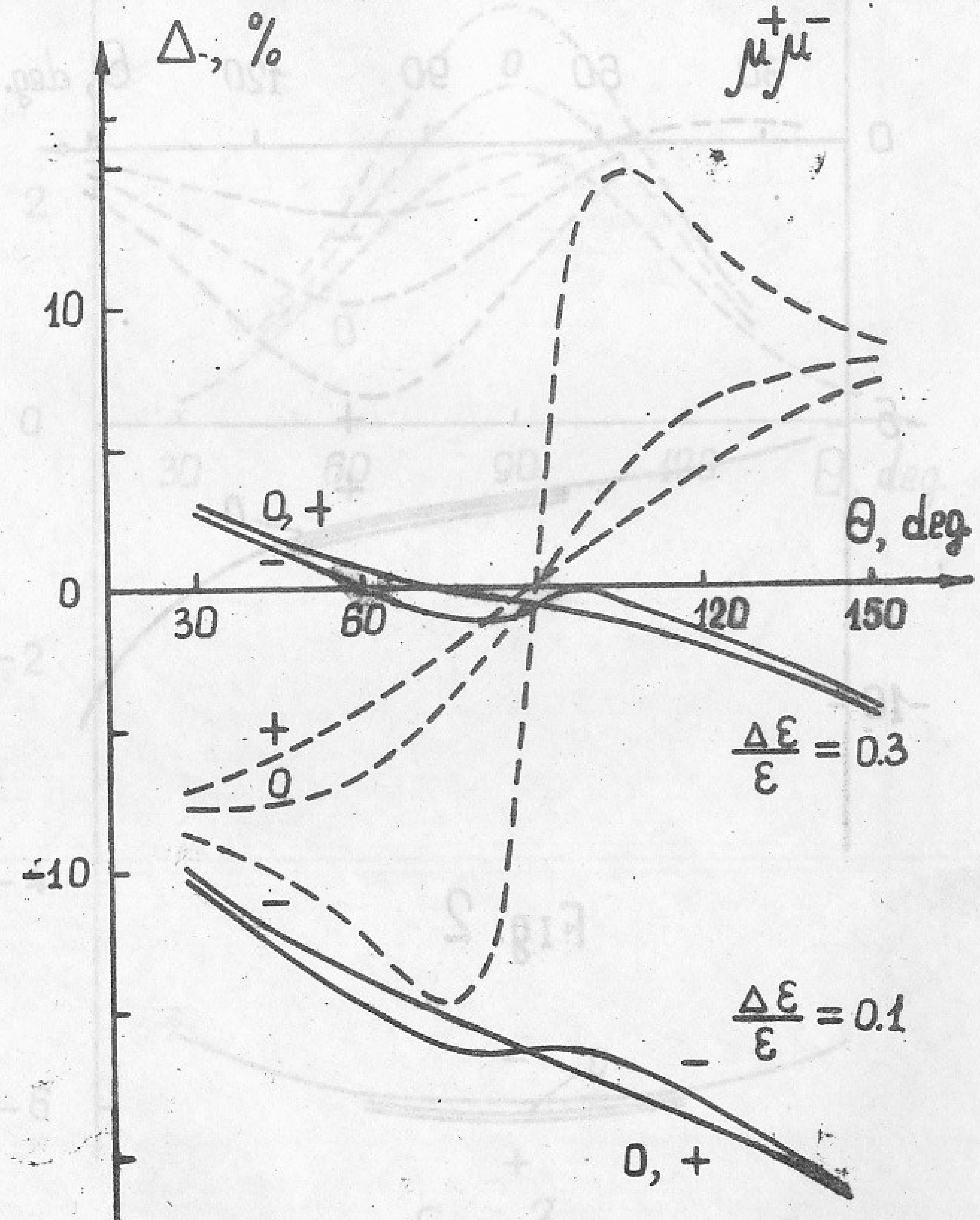

Fig. 1

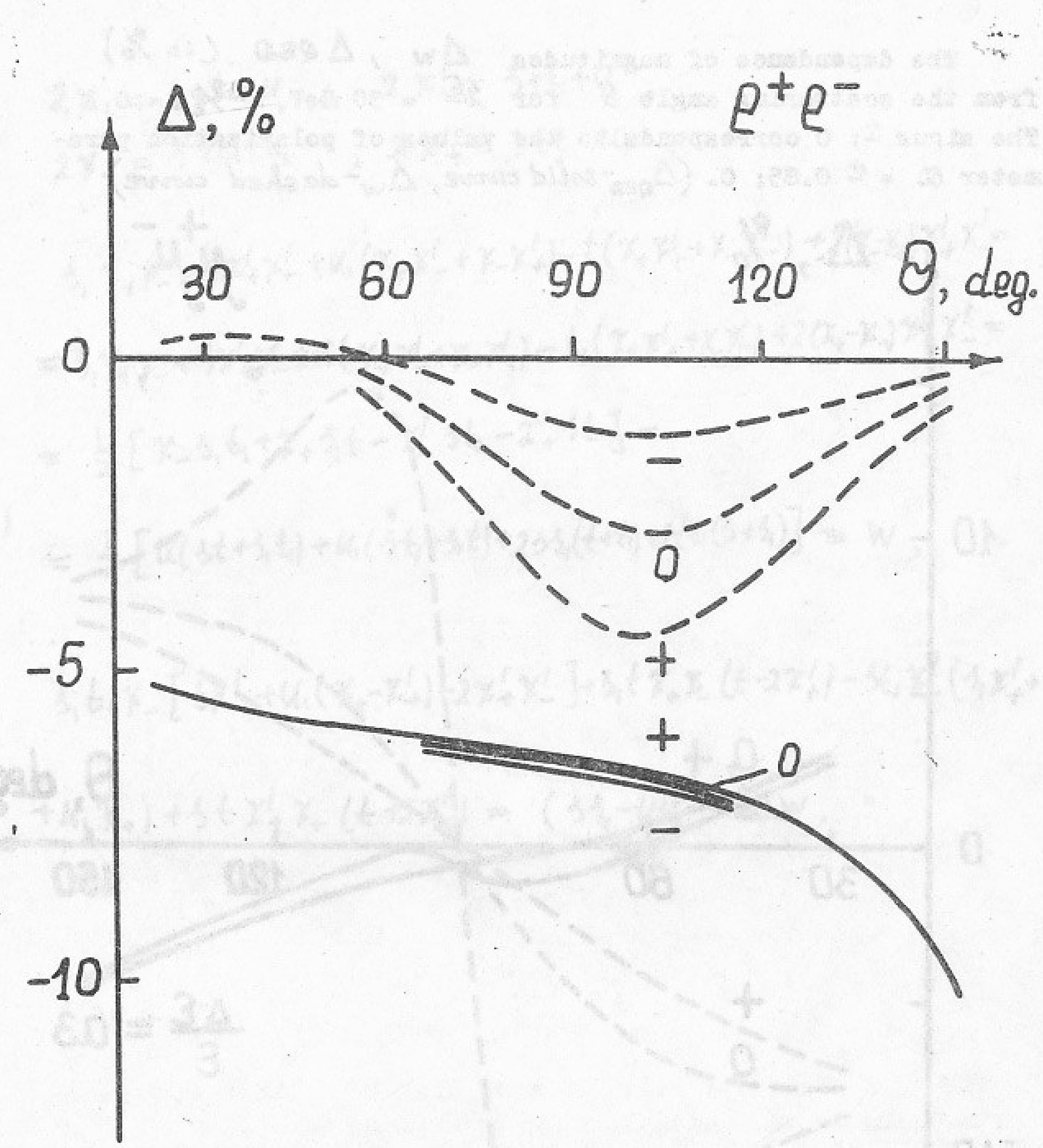

Fig. 2

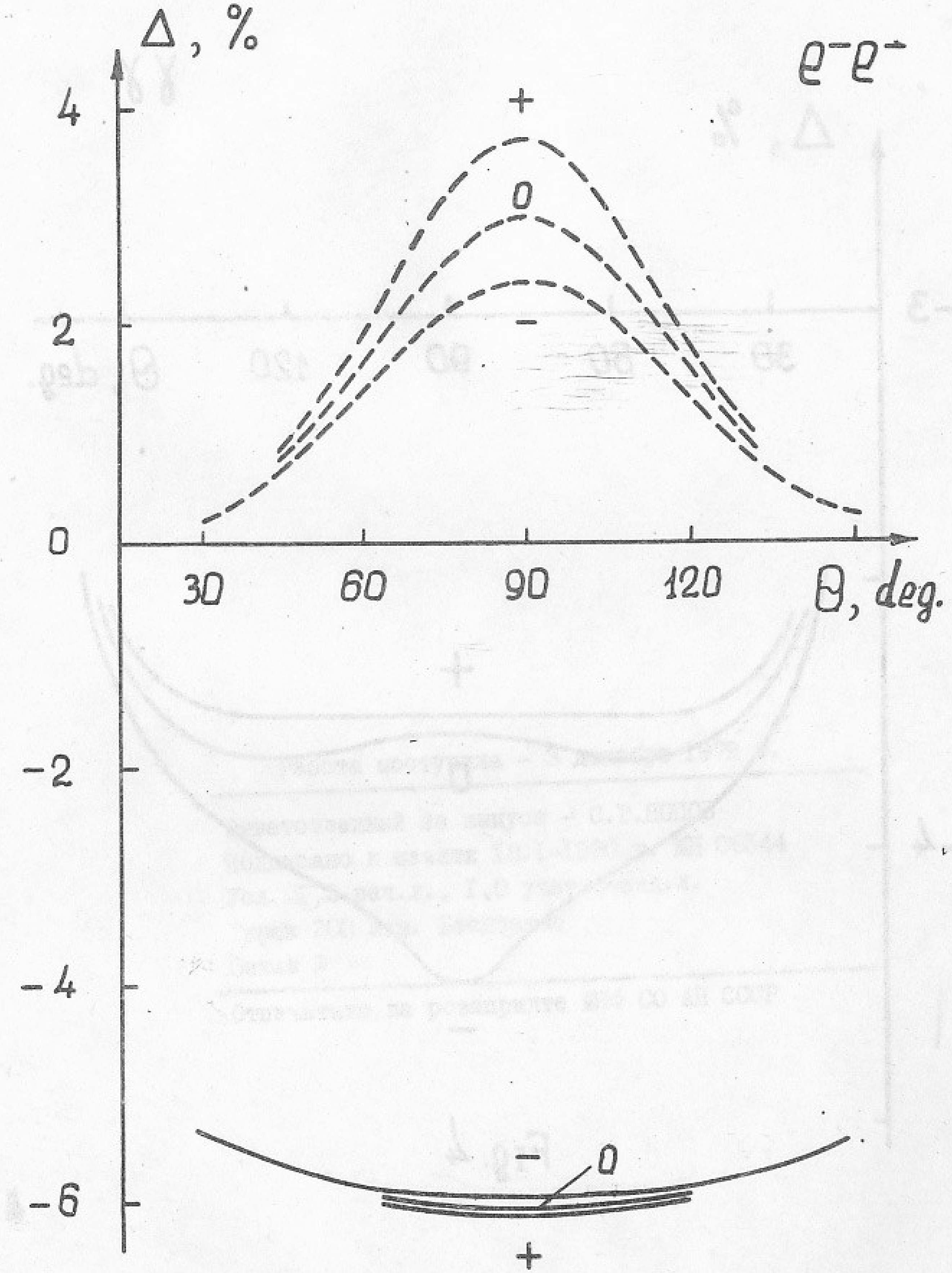

Fig. 3

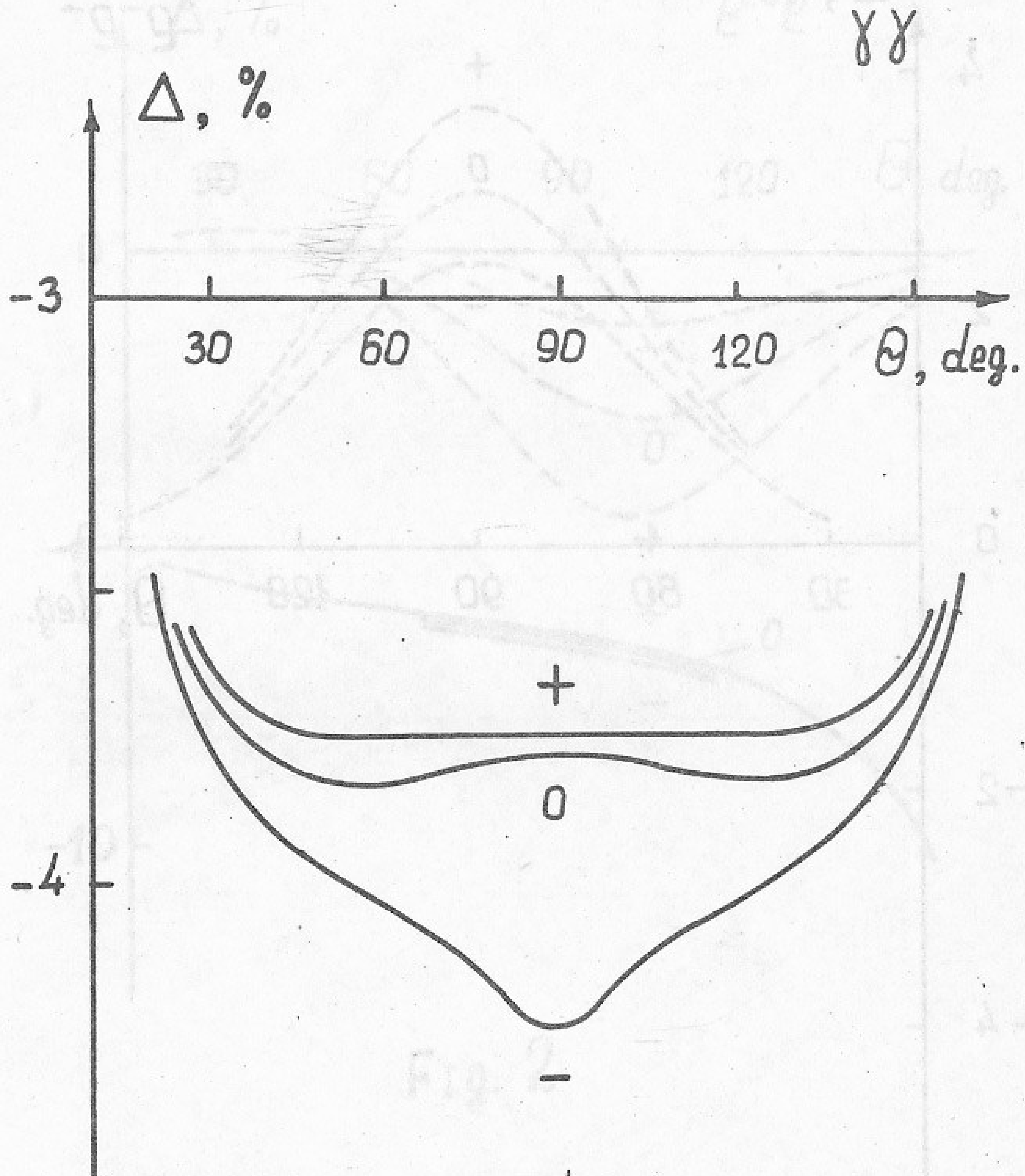

Fig. 4

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