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THE POLYNOMIAL SPECTRAL PROBLEM OF ARBITRARY ORDER: A GENERAL FORM OF THE INTEGRABLE EQUATIONS AND BACKLUND-TRANSFORMATIONS

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THE POLYNOMIAL SPECTRAL PROBLEM OF ARBITRARY ORDER: A GENERAL FORM OF THE INTEGRABLE EQUATIONS AND BACKLUND-TRANSFORMATIONS

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A b s t r a c t

The general form of the partial differential equations, integrable by the general polynomial with respect to spectral parameter, linear matrix spectral problem of the order N, and the corresponding Backlund-transformations are described.

I. INTRODUCTION

The inverse spectral transform (IST) method allows a comprehensive study of a great number of various partial differential equations (see, e.g. [1-3]). The general scheme of this method was discussed in Refs. [4] and [5].

All the differential equations to which the IST method is applicable are united in the classes of equations integrable by the same linear spectral problem. A simple and convenient description of the class of equations which are integrable with the help of the linear (in spectral parameter) spectral problem of the second order was presented in Ref. [6]. This class of equations is characterized by the (n-1)th arbitrary functions (n is the number of independent variables) and by certain integro-differential operator [6,7]. The analogous results was obtained for the class of equations which are associated with matrix stationary Schrodinger equation [8], the general linear spectral problem of arbitrary order [9-11] and also with the second-order linear problem quadratic in spectral parameter [12]. Within the framework of this approach the wide classes of Backlund-transformations (BT) which are playing a significant role in the study of nonlinear differential equations are also found [7,8,11].

In the present paper we construct Backlund-transformations and describe the general form of differential equations integrable by the general polynomial spectral problem of the order N

$$\frac{\partial \psi}{\partial x} = \tilde{P}\psi = \Big(i\lambda^m A + \sum_{\alpha=1}^{m} \lambda^{m-\alpha} P^{\alpha}_{(x,\dots)} \Big)\psi, \quad (1.1)$$

where λ is the spectral parameter, A is the constant diagonal matrix N x N ($A_{ik} = a_i \delta_{ik};\ i,k = 1,\dots, N$), the "potentials" $P^{\alpha}_{(x,\dots)}$ are the matrices N x N with zero diagonal elements ($P^{\alpha}_{ii} = 0,\ i=1,\dots, N;\ \alpha=1,\dots,m$), N and m are any numbers and $\alpha = 1,\dots, m$. We will clarify some functional relations between "potentials" P^{α} and consider the group properties of BT as well.

II. A general form of the integrable equations and Backlund-transformations.

The system of the linear differential equations (1.1) gives a mapping $\tilde{P} \to \psi$. Let us consider arbitrary transformations $\tilde{P} \to \tilde{P}'$, $\psi \to \psi'$ conserving this mapping (i.e. $\lambda \to \lambda' = \lambda$). There is no difficulty to be convinced that (at $N = 2$, see Ref. [13])

$$\psi' - \psi K = -\psi \int_x^{\infty} dy\, \psi^{-1} (\tilde{P}' - \tilde{P}) \psi', \tag{2.1}$$

where the constant matrix K is determined by the asymptotic properties of the matrix-solutions ψ.

We shall suppose that all $P^{\alpha} \to 0$ at $|x| \to \infty$. Then $\psi_{|x| \to \infty} \to \exp i\lambda^m A x = E$. Let us introduce, following to [14], fundamental matrices-solutions F^+ and F^- with asymptotics $F^+_{x \to \infty} \to E$, $F^-_{x \to -\infty} \to E$ and the transition matrix S;

$F^+ = F^- S$.

Putting $\psi = F^+$ and proceeding to the limit $x \to -\infty$ we get

$$S' - S = -S \int_{-\infty}^{+\infty} dx\, F^{+-1} (\tilde{P}' - \tilde{P}) F^{+\prime}, \tag{2.2}$$

The formula (2.2) connecting the variation of the "potentials" P^{α} with variation of the transition matrix S is a basis for what a further discussion.

Let us suppose that the transition matrix is transformed under $\tilde{P} \to \tilde{P}'$, $\psi \to \psi'$ as follows

$$(S^{-1} S')_{in} = -\frac{1}{i} (S^{-1} B S')_{in} \qquad i, n = 1, \ldots, N;\ i \neq n, \tag{2.3}$$

where B is some diagonal matrix independent of the variable x.

From comparison (2.2) and (2.3) we have

$$\int_{-\infty}^{+\infty} dx \left\{ F^{+-1} (\tilde{P}' - \tilde{P}) F^{+\prime} \right\}_{in} = \frac{1}{i} (S^{-1} B S')_{in}. \quad (i \neq n) \tag{2.4}$$

Taking into account the relation

$$\{S^{-1} B S'\}_{in} = -\int_{-\infty}^{+\infty} dx \frac{\partial}{\partial x} \{F^{+-1} B F^{+\prime}\}_{in} = \int_{-\infty}^{+\infty} dx \{F^{+-1} (\tilde{P} B - B \tilde{P}') F^{+\prime}\}_{in}$$

we find that

$$\int_{-\infty}^{+\infty} dx \left\{ F^{+-1} \left(\tilde{P}' - \tilde{P} + i (\tilde{P} B - B \tilde{P}') \right) F^{+\prime} \right\}_{in} = 0 \qquad (i \neq n). \tag{2.5}$$

Rewriting formula (2.5) in the components and designating $\varphi^{(in)}_{k\ell} = (F^{+-1})_{ik} (F^{+\prime})_{\ell n}$ we obtain $\left(B_{ik}(\lambda) = B_i(\lambda) \delta_{ik} \right)$

$$\int_{-\infty}^{+\infty} dx \sum_{k,\ell}{}' \sum_{\alpha=1}^{m} \lambda^{m-\alpha} \left\{ P'^{\alpha}_{k\ell} - P^{\alpha}_{k\ell} + i \left(B_\ell(\lambda) P^{\alpha}_{k\ell} - B_k(\lambda) P'^{\alpha}_{k\ell} \right) \right\} \varphi^{(in)}_{k\ell} = 0 \qquad (i \neq n),\ k, \ell = 1, \ldots, N \tag{2.6}$$

Formula (2.6) contain the product $f(\lambda) \varphi^{(in)}_{k\ell}(x, \lambda)$ which is given locally in each point λ of the bundle (1.1). The spectral problem (1.1) give the possibility to transform this local (in λ) product into the global which is determined on the whole bundle.

Let us pointed out, that the sum over k, ℓ in the formula (2.6) does not contain the contribution from the diagonal quantities $\varphi^{(in)}_{kk}$ $(k = 1, \ldots, N)$. The equations for quantities $\varphi^{(in)}_{k\ell}$ are easily obtained from (1.1) and have the form

$$\frac{\partial \varphi^{(in)}_{k\ell}}{\partial x} = i \lambda^m (a_\ell - a_k) \varphi^{(in)}_{k\ell} + \sum_{\alpha=1}^{m} \lambda^{m-\alpha} \Big\{ \sum_{p \neq k} P'^{\alpha}_{\ell p} \varphi^{(in)}_{kp} - \sum_{q \neq \ell} P^{\alpha}_{qk} \varphi^{(in)}_{q\ell} + P'^{\alpha}_{\ell k} \varphi^{(in)}_{kk} - P^{\alpha}_{\ell k} \varphi^{(in)}_{\ell\ell} \Big\}, \quad (i \neq n) \tag{2.7}$$

It follows from formula (2.7) that $\frac{\partial \varphi^{(in)}_{kk}}{\partial x}$ is proportional only to nondiagonal quantities $\varphi^{(in)}_{k\ell}$. Integrating the equations for $\varphi^{(in)}_{kk}$ and taking into account the asymptotic properties of F^+ we obtain

$$\varphi^{(in)}_{kk}(x) = \delta_{ik} \delta_{kn} - \int_x^{\infty} dy \sum_{\beta=1}^{m} \lambda^{m-\beta} \Big\{ \sum_{p \neq k} P'^{\beta}_{kp}(y) \varphi^{(in)}_{kp}(y) - \sum_{q \neq k} P^{\beta}_{qk} \varphi^{(in)}_{qk} \Big\}. \tag{2.8}$$

Substitution of the expression (2.8) into the formula (2.7) give

$$\frac{\partial \varphi^{(ln)}_{ke}}{\partial x} = i\lambda^m (a_e - a_k)\varphi^{(ln)}_{ke} + \sum_{\alpha=1}^{m} \lambda^{m-\alpha}\Big\{\sum_{p\neq k} P'^{\alpha}_{ep}\varphi^{(ln)}_{kp} - \sum_{q\neq e} P^{\alpha}_{qk}\varphi^{(ln)}_{qe}\Big\}$$

$$-\sum_{\alpha=1}^{m}\lambda^{m-\alpha}\sum_{\beta=1}^{m}\lambda^{m-\beta}\Big\{P'^{\alpha}_{ek}(x)\int_x^{\infty} dy\Big[\sum_{p\neq k} P'^{\beta}_{kp}(y)\varphi^{(ln)}_{kp}(y) - \sum_{q\neq k} P^{\beta}_{qk}(y)\varphi^{(ln)}_{qk}(y)\Big] \qquad (2.9)$$

$$- P^{\alpha}_{ek}(x)\int_x^{\infty} dy\Big[\sum_{p\neq e} P'^{\beta}_{ep}(y)\varphi^{(ln)}_{ep}(y) - \sum_{q\neq e} P^{\beta}_{qe}(y)\varphi^{(ln)}_{qe}(y)\Big]\Big\} \quad \begin{matrix}(l\neq n)\\(k\neq e)\end{matrix}$$

Let us emphasize that the equations (2.9) contain only nondiagonal quantities $\varphi^{(ln)}_{ke}$ ($l\neq n,\ k\neq e$) . It is convenient to renumate P^{α}_{ke} and $\varphi^{(ln)}_{ke}$: $P^{\alpha}_{ke}\to P^{\alpha}_{A}$, $\varphi^{(ln)}_{ke}\to\varphi^{(ln)}_{A}$, $A=1,\dots, N^2-N$. Then the equation (2.9) can be rewritten in the compact form

$$\mathcal{F}(\lambda)\,\varphi^{(ln)} = \Big(\sum_{\gamma=0}^{2m-2}\lambda^{2m-2-\gamma}\,Q_\gamma\Big)\varphi^{(ln)} = 0, \qquad (2.10)\quad (l\neq n)$$

where Q_γ are the matrices with operator elements. The explicit form of the quantities Q_γ in the terms P^{α} and P'^{α} can be easily obtained from the equations (2.9) and (2.10).

In virtue of the generalyzed Bezout theorem (see e.g. [15]) the equation (2.10) can be reprezented as follow

$$Q(\lambda)\prod_{g=1}^{M}\big(\lambda-\Lambda_{(g)}\big)\varphi^{(ln)} = 0 \qquad (2.11)$$

where operators $\Lambda_{(g)}$ are the "roots" of the operator polynomial $\mathcal{F}(\lambda)$: $\mathcal{F}(\Lambda_{(g)})=0$, $g=1,\dots, M$. In the case $m = 1$ we have $(\lambda-\Lambda)\,\varphi=0$ [9,11] .

The equivalency of the right and left division [15] leads to the commutativity of the factors $\lambda-\Lambda_{(g)}$ in the product $\prod_g(\lambda-\Lambda_{(g)})$. In virtue of this property the solutions of the equation (2.11) can be represented in the form (assuming the nondegeneracy of operator $Q(\lambda)$).

$$\varphi^{(ln)} = \sum_{g=1}^{M}\varphi^{(ln)}_{(g)},$$

where

$$(\lambda-\Lambda_{(g)})\,\varphi^{(ln)}_{(g)} = 0 \qquad g=1,\dots, M.$$

As a result the equality (2.6) can be rewritten in the following form

$$\int_{-\infty}^{+\infty} dx\sum_{g=1}^{M}\sum_{\alpha=1}^{m}\sum_{q,p,s,t}\Big\{P'^{\alpha}_{qp} - P^{\alpha}_{qp} + i\sum_{k,e}\Big[P^{\alpha}_{ke}\big[B_e(\Lambda_{(g)})\big]_{keqp} - \qquad (2.12)$$

$$- P'^{\alpha}_{ke}\big[B_k(\Lambda_{(g)})\big]_{keqp}\Big]\Big\}\Lambda^{m-\alpha}_{(g)qpst}\,\varphi^{(ln)}_{(g)st} = 0\ , \quad (l\neq n)$$

The expression in the round bracket in the (2.12) does not depend on λ i.e. it is given globally on the whole bundle (1.1).

At last, integration by parts and changing of the order of integration in (2.12), i.e. the transition from operators $\Lambda_{(g)}$ to adjoint operators $\Lambda^{+}_{(g)}$ give

$$\int_{-\infty}^{+\infty} dx\sum_{g=1}^{M}\sum_{\alpha=1}^{m}\sum_{q,p,s,t}\varphi^{(ln)}_{(g)st}(x)\cdot\Lambda^{+\,m-\alpha}_{(g)stqp}\Big\{P'^{\alpha}_{qp} - P^{\alpha}_{qp} + \qquad (2.13)$$

$$+ i\sum_{k,e}\Big[\big[B_e(\Lambda^{+}_{(g)})\big]_{qpke}P^{\alpha}_{ke} - \big[B_k(\Lambda^{+}_{(g)})\big]_{qpke}P'^{\alpha}_{ke}\Big]\Big\} = 0 \quad (l\neq n)$$

Thus, the transformations $P^{\alpha}\to P'^{\alpha}$ conserving (1.1) have the form

$$\sum_{\alpha=1}^{M}\sum_{q,p}\Lambda^{+\,m-\alpha}_{(g)stqp}\Big\{P'^{\alpha}_{qp} - P^{\alpha}_{qp} + i\sum_{k,e}\Big[\big[B_e(\Lambda^{+}_{(g)})\big]_{qpke}P^{\alpha}_{ke} - \qquad (2.14)$$

$$- \big[B_k(\Lambda^{+}_{(g)})\big]_{qpke}P'^{\alpha}_{ke}\Big]\Big\} = 0 \qquad (s\neq t).$$

Let us pointed out that in the general case operators $\Lambda^{+}_{(g)}$ are functions of all P^{α} and P'^{α} $(\alpha=1,\dots, m)$. The arbitrary functions B_e can also depend on any number of additional parameters (t , $\vec{y}$).

The differential equations integrable by the linear problem (1.1) are obtained from (2.14) if one considers the transformations of P^{α} , which are generated by the infinitesimal displacements alone t and $\vec{y}$:

$$\Delta P^{\alpha} = \frac{\partial P^{\alpha}}{\partial t} + \vec{H}(\lambda,t,\vec{y})\,\frac{\partial P^{\alpha}}{\partial \vec{y}}$$

where $\vec{H}(\lambda, t, \vec{y})$ are arbitrary functions.

The general form of the integrable equations is the following

$$\sum_{\alpha=1}^{m}\sum_{q,p} L^{+\,m-\alpha}_{(\varrho)stqp}\left\{\frac{\partial P^{\alpha}_{qp}(x,t,\vec{y})}{\partial t}+\sum_{k,e}\left\{\left[\vec{H}(L^{+}_{(\varrho)},t,\vec{y})\right]_{qpke}\frac{\partial P^{\alpha}_{ke}}{\partial \vec{y}}\right.\right. \tag{2.15}$$
$$\left.\left.+i\left[Y_e(L^{+}_{(\varrho)},t,\vec{y})-Y_k(L^{+}_{(\varrho)},t,\vec{y})\right]_{qpke}P^{\alpha}_{ke}\right\}\right\}=0,$$

where $L^{+}_{(\varrho)}=\Lambda^{+}_{(\varrho)}\,(P'^{\alpha}=P^{\alpha})$; $\alpha=1,\ldots,m$; $\varrho=1,\ldots,M$, $k,e,q,p,s,t=1,\ldots,N$; $s\neq t$ and $Y_e(\lambda,t,\vec{y})$ and $\vec{H}(\lambda,t,\vec{y})$ are arbitrary functions.

Thus, the polynomial bundle (1.1) permits the integration of the M classes of differential equations*. Each class of the equations are characterized by the integro-differential operator $L^{+}_{(\varrho)}$ and by $N+n-3$ arbitrary functions $Y_k(\lambda,t,\vec{y})-Y_e(\lambda,t,\vec{y})$ $(k,e=1,\ldots,N)$, $\vec{H}(\lambda,t,\vec{y})$.

For the explicit description of the integrable equations it is necessary to know the explicit form of the operators $L^{+}_{(\varrho)}$. The explicit form of the operators $L^{+}_{(\varrho)}$ was found in the following cases: 1) m = 1, N = 2 [6], N is arbitrary [9,11]; 2) m = 2, N = 2 [12] - in this case $L_{(1)}=-L_{(2)}$ and operator $L^{2}_{(1)}$ plays a role of operator L.

It is possible to show, analogously to Ref. [9], that the equations (2.15) are the Hamiltonian equations, and as in the case N = 2 [16,17] the infinite number of symplectic structures are connected with these equations.

III. Transformations properties of the transition matrix and functional relations.

Transformations $P^{\alpha}\to P'^{\alpha}$ of the form (2.14) induce $S\to S'$. Rewriting in virtue to (2.2) the transformations

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* If some of operators $L^{+}_{(\varrho)}$ coincide each with other, then the number of classes, of course, decreases.

(2.2) in the components and taking into account (2.3) we obtain

$$S'_{in}-S_{in}=-S_{in}I_n-\frac{1}{i}\sum_{k\neq n}S_{ik}(S^{-1}BS')_{kn}, \tag{3.11}$$

where
$$I_n=\int_{-\infty}^{+\infty}dx\sum_{\alpha=1}^{m}\lambda^{m-\alpha}(P'^{\alpha}-P^{\alpha})_{ke}\,\varphi^{(nn)}_{ke}.$$

It isn't difficult to verify (using (2.8)) that
$$\mathcal{F}(\lambda)\,\varphi^{(nn)}_{ke}=i\sum_{\beta=1}^{m}\lambda^{m-\beta}\left(P'^{\beta}_{ek}\delta_{nk}-P^{\beta}_{ek}\delta_{ne}\right).$$

As a result we have
$$I_n=i\sum_{k,e,q,p}\sum_{\alpha,\beta=1}^{m}\int_{-\infty}^{+\infty}dx\,\lambda^{2m-\alpha-\beta}(P'^{\alpha}-P^{\alpha})_{ke}\cdot \tag{3.2}$$
$$\cdot\left(\left[Q(\lambda)\prod_{\varrho=1}^{M}(\lambda-\Lambda_{(\varrho)})\right]^{-1}\right)_{keqp}\left(P'^{\beta}_{pq}\delta_{nq}-P^{\beta}_{pq}\delta_{np}\right).$$

Thus, the changing of the transition matrix S under the transformations (2.14) is determined by the relation

$$\sum_{e}\left\{\delta_{ie}+\frac{1}{i}\sum_{k\neq n}S_{ik}(S^{-1}B)_{ke}\right\}S'_{en}=(1-I_n)S_{in} \tag{3.3}$$

where I_n is given by the formula (3.2).

Let us pointed out that the transformation properties (3.3) of the elements of the transition matrix are the same for all equations of different classes (with any $\Lambda_{(\varrho)}$).

The fairly complicated transformation law (3.3) can be rewritten in a compact form if one introduce a diagonal matrix $\mathcal{D}$, for which

$$\mathcal{D}_{nn}=(S^{-1}BS')_{nn}-iI_n.$$

Then

$$S\to S'=(1-iB)^{-1}S(1-i\mathcal{D}). \tag{3.4}$$

If the condition

$$(S^{-1}BS')_{nn}-iI_n=B_{nn} \tag{3.5}$$

is fulfilled, then

$$S' = (1 - i B)^{-1} S (1 - i B).$$

The evolution of the transition matrix S is the transformation of the type (3.5) and in the infinitesimal form is determined by the equation

$$\Delta S = \frac{1}{i} [S, Y]. \qquad (3.6)$$

For finite transformations (in the case of two independent variables X, t) we have

$$S(\lambda, t') = \exp\Big(i \int_t^{t'} ds\, Y(\lambda, s)\Big) S(\lambda, t) \exp\Big(-i \int_t^{t'} ds\, Y(\lambda, s)\Big). \qquad (3.7)$$

In particular, the diagonal elements of matrix S are independent of time: $S_{ii}(\lambda, t) = S_{ii}(\lambda, 0)$ $(i = 1, \ldots, N)$. Therefore, quantities $S_{ii}(\lambda)$ are the integrals of motion. Using the standart procedure of the expanding of $\ln S_{ii}(\lambda)$ in the series on λ^{-1} one can obtain the infinite set of the explicit (in the term of $P^{\alpha}(x, t)$) integrals of motion.

The transformations of the type (3.5) does not change the diagonal elements of the transition matrix (and, therefore, the Hamiltonian) too. So, they are the symmetry transformations for the equations (2.15). It is obvious from formula (3.5) that these transformations form the infinite group. It is possible to show that the existence of the infinite sets of the integral of motions for the equations (2.15) is connected with the invariance of these equations under the infinite group of transformations of the type (3.5).

Transformations $P^{\alpha} \to P'^{\alpha}$ which does not satisfy the condition (3.5) change, in general case, the diagonal element of S. One can divide these transformations following to Ref. [7] into two types. The first type of transformations is the transformations (2.14) with matrix B independent on t and $\vec{y}$. It isn't difficult to show using (3.3) that these transformations does not change the form of the dependence of $S(\lambda, t)$ on time. Therefore, they transform the solutions of so-

me equations of the type (2.15) into the solutions of the same equations; i.e. they are usual (auto) BT. If B depends on t and (or) $\vec{y}$, then the transformations (2.14) are generalized BT: they transform one to another the solutions of different equations of the type (2.15) (with different functions Y_e and $\vec{H}$).

BT (2.14) which change the number of zeros of the diagonal elements S_{ii} of the transition matrix are solitons BT. They add one or several solitons to the initial solution. Transformations (2.14) which does not change the number of zeros of $S_{ii}(\lambda)$ are continual (nonsoliton) BT.

In the case of two independent variables (X, t) one can transform any solution of the equation (2.15) into any other solution by the transformation of the type (2.14). In particular, the transformation for which

$$B = \frac{1}{i}\Big(1 - \exp\Big\{-i \int_t^{t'} ds\, Y(\lambda, s)\Big\}\Big)$$

displace in time (as it follows from (3.7)) the transition matrix: $S(\lambda, t) \to S(\lambda, t')$. The inverse mapping $P^{\alpha}(x, t) \to S(\lambda, t)$ induce corresponding transformation $P^{\alpha}(x, t) \to P'^{\alpha}(x, t) = P^{\alpha}(x, t')$. This transformation have the form (in case $N = 2$ see Ref. [7])

$$\sum_{\alpha=1}^{m} \sum_{q,p,k,e} \Lambda^{+\,m-\alpha}_{(\beta) stqp} \Big\{ \Big[\exp\Big(-i \int_t^{t'} ds\, Y_e(\Lambda^{+}_{(\beta)}, s)\Big)\Big]_{qpke} P^{\alpha}_{ke}(x, t) - \Big[\exp\Big(-i \int_t^{t'} ds\, Y_k(\Lambda^{+}_{(\beta)}, s)\Big)\Big]_{qpke} P^{\alpha}_{ke}(x, t') \Big\} = 0 \qquad (3.8)$$

where in the operator $\Lambda^{+}_{(\beta)}$ one must put $P'^{\alpha}(x, t) = P^{\alpha}(x, t')$. If one consider displacement $P^{\alpha}(x, t) \to P^{\alpha}(x, t + \varepsilon)$ where $\varepsilon \to 0$, then it is easy to show that the transformation (3.8) reduce to the equation (2.15).

Thus, the differential equations of the type (2.15) are non other than the infinitesimal form of the transformations (3.8).

IV. Conclusion

In the conclusion some remarks.

1. BT (2.14) are determined by the matrix $B(\lambda, t, \vec{y})$. Diagonal matrices B form the infinite abelian group. Therefore, BT for the equations (2.15) form abelian infinite group too. This group (analogously to the case $N = 2$ [18]) is the tensor product of $B_c \otimes B_d$ of the infinitedimensional continuous group B_c of continual BT and infinite discrete group B_d of solitons BT. Follows to [18] one can consider the transformation properties of spectral parameter λ, quantities $\psi(x,t,\lambda)$ and BT under the symmetry group of the integrable equations.

2. The results obtained in the present paper can be transfered to the case of nondiagonal matrix A. In this case transformations (2.14) and equations (2.15) are determined by the group of matrices B, commuiting with matrix A.

In any cases transformations (2.14) is group of transformations conserving the mapping $\tilde{P}_1 \to \psi$ (1.1) and asymptotic E of spectral problem (1.1) ($B^{-1} E B = E$).

3. Many aspects of the theory of the integrable equations of the type (2.15) have manifestly group character. Indeed, BT (2.14) and, in particular, the evolution of the "potentials" $P^{\alpha}(x,t)$ in time (3.8) is the transformations under which the spectral problem (1.1) is invariant. Transformations $\psi(x,t,\lambda) \to g(x,t)\,\psi(x,t,\lambda)$ are the gauge transformations connected the different equations integrable by the problems of the type (1.1) [5,19,20]. Local in λ transformations $\psi(x,t,\lambda) \to \psi'(x,t,\lambda) = G(x,t,\lambda)\,\psi(x,t,\lambda)$ conserving the form of the dependence of $\tilde{P}$ on λ, underlie the method of construction of the explicit (soliton) solutions [3,5].

Thus it seems that the theory of nonlinear partial differential equations of the type (2.15) is a constituent of the general theory of the group of the nonlinear transformations connected with the spectral problem of the type (1.1)

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