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PARITY NONCONSERVATION IN HEAVY ATOMS AND WEAK MAGNETISM IN NEUTRAL CURRENT

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abstract

The contribution of the terms $\sim m_p^{-1}$ in the weak eN interaction to the effects of parity nonconservation in heavy atoms dependent on the nucleus spin is calculated. In the Weinberg-Salam model it is comparable with the contribution to these effects from the interaction of electron vector current with nucleon axial current.

Parity nonconserving weak interaction of electrons with nucleus due to neutral currents was discovered in the optical experiment at the Institute of Nuclear Physics in Novosibirsk/1,2/. Then this result was confirmed in fact in the experiment of quite another type carried out at the two miles long linear accelerator in Stanford, in the reaction of deep inelastic scattering of longitudinally polarized electrons on deuterons and protons/3/. High experimental accuracy attained now in Novosibirsk/4/, makes sufficiently real further investigation of the weak neutral currents structure in optical experiments with atoms/5/ and molecules/6/. I mean the parity nonconserving effects dependent on the nuclear spin which up to now have not been observed. These effects arise due to the interaction of the neutral vector current (VC) of electrons with the axial current (AC) of protons and neutrons; in heavy atoms they are roughly speaking Z times smaller than the effects independent of the nuclear spin.

Here I would like to turn attention to the fact that in the Weinberg-Salam (WS) model a comparable contribution to parity nonconserving effects in heavy atoms dependent on the nucleus spin is given by the usually neglected terms in electron-nucleon interaction inversely proportional to nucleon mass m_p. In particular, the contribution is important of the terms in nucleon VC that could be called neutral weak magnetism.

Parity violating, but invariant under time reversal, interaction of electron with nucleon contains four independent spinor structures/7/ and can be written as follows:

$$H=\frac{G}{\sqrt{2}}\Big\{-\bar{u}_e\gamma_\mu\gamma_5 u_e\Big[\varkappa_1\bar{u}_N\gamma_\mu u_N+\frac{\varkappa_3}{2m_p}i\partial_\nu(\bar{u}_N\sigma_{\mu\nu}u_N)\Big]+ \tag{1}$$
$$+\Big[\varkappa_2\bar{u}_e\gamma_\mu u_e+\frac{\varkappa_4}{2m}i\partial_\nu(\bar{u}_e\sigma_{\mu\nu}u_e)\Big]\bar{u}_N\gamma_\mu\gamma_5 u_N\Big\}.$$

Usual arguments in favour of neglect of the terms with derivatives in (1) are as follows. It is natural to expect that the "anomalous weak moment" of electron $\varkappa_4$ arises like anomalous magnetic moment due to radiative corrections only and is therefore extremely small, $\sim\alpha/2\pi$. The term with $\varkappa_3$ can be omitted since it contains large mass in denominator. In such an approximation the only nonvanishing matrix element of the weak interaction of electron with nucleus, leading to the mixing of atomic levels of opposite parity, is equal to/8,9,5/

$$\langle s|H|p_{1/2}\rangle = i\,\frac{Gm^2\alpha^2Z^2R}{\sqrt{2}\,\pi(\nu_s\nu_p)^{3/2}}\,\frac{m\alpha^2}{2}\Big\{\left[Z\varkappa_{1p}+(A-Z)\varkappa_{1n}\right]+$$
$$+\frac{2\gamma+1}{3}\,2\underline{j}\cdot\Big\langle\varkappa_{2p}\sum\underline{\sigma}_p+\varkappa_{2n}\sum\underline{\sigma}_n\Big\rangle+ \qquad (2)$$
$$+(1-\gamma)\,2\underline{j}\cdot\Big\langle\varkappa_{2p}\sum\left(\underline{n}_p(\underline{n}_p\underline{\sigma}_p)-\tfrac{1}{3}\underline{\sigma}_p\right)+\varkappa_{2n}\sum\left(\underline{n}_n(\underline{\sigma}_n\underline{n}_n)-\tfrac{1}{3}\underline{\sigma}_n\right)\Big\rangle\Big\}.$$

Here m is electron mass, $\gamma=\sqrt{1-Z^2\alpha^2}$, $R=\dfrac{4\left(\frac{2Zr_o}{a}\right)^{2\gamma-2}}{\Gamma^2(2\gamma+1)}$, a is Bohr radius, $r_o = 1.2\cdot10^{-13}A^{1/3}$ is nuclear radius, A is atomic number, ν_s and ν_p are effective principal quantum numbers of s and $p_{1/2}$ states; the brackets $\langle\ldots\rangle$ denote the averaging of the spin operators of protons $\underline{\sigma}_p$ and $\underline{\sigma}_n$, and of the structures dependent on the orts of the radius-vectors of these particles $\underline{n}_p$, $\underline{n}_n$, over the state of the nucleus with given angular momentum i. Both vectors with which electron angular momentum $\underline{j}$ is contracted, are directed evidently along $\underline{i}$.

Due to the pairing of nucleons with opposite projections of angular momentum in a nucleus, these vectors do not increase with the growth of Z, in distinction from the term $Z\varkappa_{1p}+(A-Z)\varkappa_{1n}$, so that in heavy atoms the second and third terms in the matrix element (2) are much smaller than the first one. However, due to their dependence on the nucleus spin $\underline{i}$ they lead to very specific effects of parity nonconservation in atoms/5/ and molecules/6/, their investigation lying on the borderline of present experimental possibilities.

In the WS model at the value of mixing parameter $\sin^2\theta\approx0.23$ that follows from the known experimental data, the constants

$$\varkappa_{2p,n} = \mp 1/2(1-4\sin^2\theta)\cdot1.25 \qquad (3)$$

are small, so that the mentioned effects are additionally suppressed. But on the other hand the same circumstance leads to the relative increase of the contribution of the terms with derivatives from the Hamiltonian (1) which also cause the mixing of the levels with

opposite parity dependent on nucleus spin. Another circumstance, increasing now the absolute magnitude of this contribution, is that due to the presence of the derivative in the interaction the corresponding matrix element contains generally speaking the additional, in comparison with the second and third terms in (2), factor $a/(Zr_o)\gg1$. However, this factor of absolute enhancement is absent for the interaction with the constant $\varkappa_4$, as can be checked easily by means of direct computation of the matrix element resembling that leading to the expression (2).

Moreover, in the WS model $\varkappa_4$ is proportional to $\varkappa_2$, so that even relative enhancement does not arise here. To prove the last assertion consider radiative corrections to the amplitude of weak scattering of electron on nucleon. The structure of interest to us breaks γ_5-invariance for electron and hence formally should be proportional to its mass. This mass can appear in denominator instead of nominator only due to the fact that the integral over intermediate momenta, through which the contribution of Feynman diagram is expressed, is convergent at the momenta $q\sim m$. It can be easily seen that the diagram 1 is the only one with this property. Calculation, the same as that for anomalous magnetic

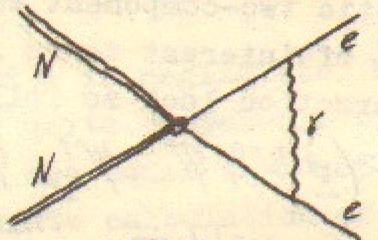

FIGURE 1

moment of electron, gives now

$$\varkappa_4 = \frac{\alpha}{2\pi}\varkappa_2 \qquad (4)$$

And at last, $\varkappa_4$ as well as anomalous magnetic moment of electron depends on the momentum transfer k in such a way that at $k>m$, or at the distances $r<m^{-1}$, it falls off. But since $r_o\ll m^{-1}$, it is clear that the matrix element of this part of weak interaction will be cut off not at the nuclear radius r_o, but at the electron

Compton wave-length m^{-1}. In other words, it is felt here that the magnetic size of electron is much larger than the nuclear radius.*) Taking into account of this fact is equivalent roughly speaking to the following substitution in the matrix element:

$$\left(\frac{2Zr_0}{a}\right)^{2\gamma-2} \longrightarrow \left(\frac{2Z}{ma}\right)^{2\gamma-2} = (2Z\alpha)^{2\gamma-1} \sim 1 \ll \left(\frac{2Zr_0}{a}\right)^{2\gamma-2}$$

All this, put together, allows one to neglect the term with $\varkappa_4$ in the interaction (1).

Pass now to the contribution due to the terms with derivatives in the nucleon current. This term proportional to $\varkappa_3$ it is natural to call neutral weak magnetism by analogy with the corresponding term in charged weak current. Together with it one should retain of course other terms $\sim m_p^{-1}$ in the interaction (1) as well. Their source is space part of the usual vector current of nucleons that can be written in the neglect of the terms of higher order in m_p^{-1} as

$$\bar{u}_N \underline{\gamma} u_N \simeq \frac{1}{2m_p}\left[i\left(\underline{\nabla}\psi^{+}\cdot\psi - \psi^{+}\underline{\nabla}\psi\right) + \underline{\nabla}\times\left(\psi^{+}\underline{\sigma}_N\psi\right)\right]$$

where ψ is nonrelativistic two-component wave function of nucleon. Finally, the terms $\sim m_p^{-1}$ of interest to us in the P-odd Hamiltonian of electron-nucleon interaction look as follows:

$$H_1 = -\frac{G}{\sqrt{2}}\frac{1}{2m_p}\left\{\varkappa_1 i\left(\underline{\nabla}\psi^{+}\cdot\psi - \psi^{+}\underline{\nabla}\psi\right) + (\varkappa_1+\varkappa_3)\underline{\nabla}\times\left(\psi^{+}\underline{\sigma}_N\psi\right)\right\}\underline{\Sigma}, \tag{5}$$

$$\underline{\Sigma} = -\gamma_0\underline{\gamma}\gamma_5 = \begin{pmatrix}\underline{\sigma} & 0\\ 0 & \underline{\sigma}\end{pmatrix}.$$

One more term $\sim m_p^{-1}$ in the interaction (1) is

$$\frac{G}{\sqrt{2}}\varkappa_2\bar{u}_e\gamma_0 u_e\bar{u}_N\gamma_0\gamma_5 u_N \approx -\frac{G}{\sqrt{2}}\frac{\varkappa_2}{2m_p}u_e^{+}u_e\, i\left[\underline{\nabla}\psi^{+}\underline{\sigma}_N\psi - \psi^{+}\underline{\sigma}_N\underline{\nabla}\psi\right] \tag{6}$$

*) Note that the size of nucleon at which other terms in the interaction (1) are cut off is about $A^{1/3}$ times smaller than the radius of nucleus r_0. It justifies taking into account the finite size of nucleus and neglect of the nucleons' size when calculating the matrix elements of weak interaction.

Elementary calculation shows however that at $u_2=u_1$ the expression $u_2\gamma_0\gamma_5u_1$ turns to zero. And without any calculations it is clear that there is no T-even pseudoscalar density in nucleus through which one could express this Hermitian quantity. Therefore, the interaction (6) does not contribute to the mixing of atomic levels of opposite parity.

Pass now to the consideration of the mixing of levels due to the interaction (5). Let us start with a simple estimate. The contribution of the Hamiltonian (5) to the effect differs from the second and third terms in the matrix element (2) first of all by the factor m/m_p. Then, due to the derivative in the interaction (5), the ratio of the characteristic atomic length at small distances, the Bohr radius of K-shell, to the radius of nucleus $a/(Zr_0)$ should arise. And finally, one should take into account that in the nonrelativistic approximation nuclear radius does not enter at all, so that the effect should be proportional to the relativistic parameter at small distances $Z^2\alpha^2$. Taken together, it gives

$$\frac{m}{m_p}\frac{a}{Zr_0}Z^2\alpha^2 = \frac{Z\alpha}{m_p r_0} = 1.3\cdot 10^{-3}\frac{Z}{A^{1/3}} \tag{7}$$

In light atoms this quantity is negligible, but in heavy ones it reaches $(1\div 2)\cdot 10^{-2}$, value quite comparable with the constants $\varkappa_{2p,n}$ which at $\sin^2\theta=0.23$ are equal to ∓ 0.05.

Therefore, more accurate calculation of the effect discussed seems to be justified. It does not cause any difficulties at the atomic level. Relativistic wave functions of electron with principal quantum number n, total angular momentum j, orbital momentum l and the projection of the total angular momentum m can be written as /10/

$$u_{njlm} = \begin{pmatrix} f_{njl}(r)\,\Omega_{jlm} \\ -ig_{njl}(r)\left(\underline{\sigma}\cdot\frac{\underline{r}}{r}\right)\Omega_{jlm}\end{pmatrix} \tag{8}$$

where Ω_{jlm} are spherical functions with spin. Since, unlike the expression (2), matrix element of the Hamiltonian (5) is sensitive to the value of r_0, one should know here radial wave functions f and g inside the nucleus. If one takes, in agreement with experimental data, that the nuclear charge is uniformly distributed inside the sphere of the radius r_0, then the radial functions of the states s and $p_{1/2}$ we are interested in are well approximated at $r \leqslant r_0$ by the following expressions obtained by the authors of the ref./9/:

$$
\begin{aligned}
f_s &= A_s\left[1-\frac{3}{8}Z^2\alpha^2x^2\left(1-\frac{4}{15}x^2\right)\right],\\
g_s &= -A_s\frac{Z\alpha}{2}x\left[1-\frac{1}{5}x^2-\frac{9}{40}Z^2\alpha^2x^2\left(1-\frac{3}{7}x^2+\frac{4}{81}x^4\right)\right],\\
f_p &= -\frac{A_p}{A_s}g_s,\qquad g_p=\frac{A_p}{A_s}f_s,\qquad x=r/r_0,\\
A_s &= \frac{2}{r_0}\frac{2\left(\frac{2Zr_0}{a}\right)^{\gamma-1}}{\Gamma(2\gamma+1)}\left(\frac{Z}{a^3\nu_s^3}\right)^{1/2}\left(1-\frac{1}{40}Z^2\alpha^2\right),\\
A_p &= \frac{Z\alpha}{r_0}\frac{2\left(\frac{2Zr_0}{a}\right)^{\gamma-1}}{\Gamma(2\gamma+1)}\left(\frac{Z}{a^3\nu_p^3}\right)^{1/2}\left(1+\frac{9}{40}Z^2\alpha^2\right).
\end{aligned}
\tag{9}
$$

As to the radial distribution in the nucleus of the nucleons contributing to the effect, we shall assume that their density is distributed in the same way as the charge density. This assumption is supported by the fact that the mean square radius of magnetic moment distribution in all the nuclei where it is known experimentally (H_1^3, He_2^3, Sc_{21}^{45}, V_{23}^{51}, Co_{27}^{59}), coincides practically with the corresponding charge radius/11-13/.

Found in this approximation matrix element of the Hamiltonian (5) over the wave functions (8),(9) is equal to

$$
\begin{aligned}
\langle s|H_1|p_{1/2}\rangle &= -i\frac{Gm^2\alpha^2Z^2R}{\sqrt{2}\pi(\nu_s\nu_p)^{3/2}}\frac{m\alpha^2}{2}\frac{Z\alpha}{m_pr_0}2j\cdot\\
&\Big\{\frac{22}{25}\left(1-0.09Z^2\alpha^2\right)\left\langle \varkappa_{1p}\sum \underline{l}_p+\varkappa_{1n}\sum \underline{l}_n\right\rangle+\\
&+\frac{4}{5}\left(1-0.26Z^2\alpha^2\right)\left\langle(\varkappa_{1p}+\varkappa_{3p})\sum\underline{\sigma}_p+(\varkappa_{1n}+\varkappa_{3n})\sum\underline{\sigma}_n\right\rangle+\\
&+\frac{3}{25}\left(1+1.58Z^2\alpha^2\right)\Big\langle(\varkappa_{1p}+\varkappa_{3p})\sum\left(\underline{n}_p(\underline{n}_p\underline{\sigma}_p)-\frac{1}{3}\underline{\sigma}_p\right)+\\
&\qquad+(\varkappa_{1n}+\varkappa_{3n})\sum\left(\underline{n}_n(\underline{n}_n\underline{\sigma}_n)-\frac{1}{3}\underline{\sigma}_n\right)\Big\rangle\Big\}
\end{aligned}
\tag{10}
$$

Here $l_{p,n}$ are the operators of the orbital angular momentum of proton and neutron. And although approximate nature of the calculations does not allow to insist on high accuracy of the numerical factors in the formula (10), one can be sure that the factors at the first and second brackets $\langle ... \rangle$ are indeed close to unity, and at the last one is much smaller than unity.

It should be noted that the interaction (5), vector in electronic variables, causes mixing not only of the states s and $p_{1/2}$, but s with $p_{3/2}$ as well. However, since at small distances the wave function of $p_{3/2}$ electron decreases more rapidly than that of $p_{1/2}$, in the second case mixing matrix element is roughly speaking $a/(Zr_0)$ times smaller than in the first one. Therefore, the mixing of the levels s and $p_{3/2}$ can be neglected.

The values of the constants $\varkappa_{1p,n}$ in the WS model are well-known (see, e.g.,/7/)

$$
\varkappa_{1p}=1/2(1-4\sin^2\theta),\quad \varkappa_{1n}=-1/2. \tag{11}
$$

Using the values of the anomalous magnetic moments $\mu_{p,n}$ of proton and neutron, one can easily obtain by means of isotopic invariance considerations the constants $\varkappa_{3p,n}$ of interest to us:

$$
\begin{aligned}
\varkappa_{3p}&=1/2(1-2\sin^2\theta)\mu_1-\sin^2\theta\mu_0=1.85-3.59\sin^2\theta,\\
\varkappa_{3n}&=-1/2(1-2\sin^2\theta)\mu_1-\sin^2\theta\mu_0=-1.85+3.83\sin^2\theta.
\end{aligned}
\tag{12}
$$

Here $\mu_1=\mu_p-\mu_n=3.71$, $\mu_0=\mu_p+\mu_n=-0.12$ are isovector and isoscalar parts of the anomalous magnetic moment of nucleon.

Compare now the magnitude of the matrix element (10) with the term in (2) dependent on the spin of the nucleons $\underline{\sigma}_{p,n}$, i.e. on the angular momentum of the nucleus $\underline{i}$. Taking into account the numerical values (3), (11), (12) of the constants $\varkappa_{1,2,3}$, we found that at $\sin^2\theta=0.23$ the contribution of the interaction proportional to m_p^{-1} discussed here to the parity nonconserving effects dependent on the nuclear spin is

in cesium	(Z=55;	A=133;	i=7/2)	~15%
in thallium	(Z=81;	A=203,205;	i=1/2)	~30%
in lead	(Z=82;	A=207;	i=1/2)	~5%
in bismuth	(Z=83;	A=209;	i=9/2)	~20%

These numbers are obtained using shell model of nucleus, and therefore their error can be rather large. It should be noted that due to the small value of the constant $\varkappa_{1p}$ the discussed contribution for cesium, thallium and bismuth is due mainly just to the weak magnetism of the proton neutral current.

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