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# ON THE ACTION OF A FREE ELECTRON LASER AT HIGH GAIN

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A b s t r a c t

The system of equations is derived and solved which describes the action of a free electron laser in case of arbitrary gain G and small spatial charge when the plasma effects can be neglected. Spectral curves are obtained and analysed. Their evolution is studied when G is changed. It is shown that the growth of phase oscillations leads to the limitation of output signal power. A value of the output signal is obtained. The limitations imposed on the parameters of electrons participating in the generation are analysed.

An interest to the problem of generation of coherent radiation with the use of relativistic electron beams traveling through a periodic external field, the so-called free electron lasers (FEL) is intensified in last years. This is caused by a series of unique properties of the systems of this kind, namely: continuous tunability in a broad range of frequencies, prospective to operate in a region of short wavelengths (up to ultraviolet part of spectrum) and also a possibility of high power operation. Presently, considerable experimental data are available. The results both of the amplification and generation of radiation in the infrared part of the spectrum are obtained in Stanford University /1,2/, in the submillimeter region there are the results of NRL (Washington /3/ and École Polytechnique /4/.

Theoretical analysis of the action of FEL within the range of small gain G (that corresponds to the experimental conditions /1,2/) was made in the paper by the authors /5/ (see also /6,7/). In that paper the set of equations describing the action of FEL at $G \ll 1$ was derived and solved. In addition, the spectral characteristics in the region of low and strong signals were studied. And finally, it was shown that there exists a mechanism of saturation caused by the growth of phase oscillations resulting in stabilization of the output signal level. Of large interest is the analysis of the case of a large gain $G \gtrsim 1$ as well. The present paper is devoted just to this question.

Let an electron beam move through the helicoidal magnetic lattice of period $\lambda_0$ and magnetic field strength $H_0$. The vector-potential of the lattice is taken in the form:

$$\mathcal{A}_0^1 = a_0 \cos \frac{\nu_0 z}{c}, \quad \mathcal{A}_0^2 = a_0 \sin \frac{\nu_0 z}{c} \tag{1}$$

where $\nu_0 = \frac{2\pi c}{\lambda_0}$. Let us consider the interaction of the beam with the circularly polarized electromagnetic wave propagating along the lattice axis in the direction of a beam motion,* the vector-potential of which may be represented as follows

$$A^1 = a(t)\cos\left[\nu\left(t-\frac{z}{c}\right)+\chi(t)\right],\quad A^2 = a(t)\sin\left[\nu\left(t-\frac{z}{c}\right)+\chi(t)\right] \qquad (2)$$

where $\nu$ is the wave frequency, $a(t)$, $\chi(t)$ are the slowly varying amplitude and phase of the wave. In the region $G \ll 1$ one can neglect the distinction between the phase velocity of a wave and the velocity of light and the phase increment $\chi(t)$ connected with this. At $G \gtrsim 1$ the mentioned phase increment becomes significant.

The transverse part of the generalized momentum $\vec{P}_\perp(P^1, P^2)$ is conserved in the superposition of the fields (1) and (2). Due to this, one can put $\vec{P}_\perp = 0$. Then, expressing the kinetic momentum $\vec{p}_\perp$ (and the current) through the vector-potential, substituting this current into the Maxwell equation for the transverse part of the vector-potential and carrying out an appropriate averaging over phase, we get the system

$$\Omega' = \frac{\Omega_0 \omega_p^2}{2\nu_0}\overline{\sin(\phi+\chi)}$$

$$\Omega\chi' = \frac{\Omega_0 \omega_p^2}{2\nu_0}\overline{\cos(\phi+\chi)} + \frac{\omega_p^2\Omega}{2\nu} \qquad (3a)$$

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* The effect under study occurs for any polarization of a lattice, the wave polarization must correspond to lattice polarization.



where $\phi = \nu\left(t-\frac{z}{c}\right) - \frac{\nu_0}{c}z + \phi_0$; $\Omega = \frac{eH}{mc}$, $H = \frac{\nu a}{c}$ is the wave field; $\Omega_0 = \frac{eH_0}{mc}$, $H_0 = \frac{\nu_0 a_0}{c}$; $\omega_p^2 = \frac{4\pi e^2 n}{m}$; $n$ is the electron beam density. Differentiation in (3a) is carried out on a proper time $d\tau = dt/\gamma$ ($\gamma = \varepsilon/mc^2$, $\varepsilon$ is the electron energy), the line denotes the initial-phase averaging. The term $\frac{\omega_p^2\Omega}{2\nu}$ in the second equation of the system (3a) takes into account the constant part of the dielectric constant of the electron beam. To derive a closed set of equations, an equation for $\phi$ is required too. For this purpose, the transformation of equations of electron motion may be used (see /5/):

$$\phi'' = 2\Omega\Omega_0 \sin(\phi+\chi) \qquad (3b)$$

The above system (3) is full. It describes the amplification and generation in the region both of the low and strong signals.

In order to obtain universal characteristics of the evolution of the electromagnetic wave under study, the transition to new dimensionless variables is convenient:

$$s = \Omega_0 \tau a^{1/3}, \quad y = \frac{\Omega}{\Omega_0} a^{-2/3} \qquad (4)$$

where $a = \omega_p^2/2\nu_0\Omega_0$. The system (3) takes the following form in these variables:

$$\frac{dy}{ds} = \overline{\sin(\phi+\chi)}$$

$$y\frac{d\chi}{ds} = \overline{\cos(\phi+\chi)} + y\delta \qquad (5)$$

$$\frac{d^2\phi}{ds^2} = 2y\sin(\phi+\chi)$$

where $\delta = \frac{\nu_0}{\nu} a^{2/3}$.

The region $G \ll 1$ corresponds to $\delta \ll 1$. In this case the phase increment $\chi$ may be neglected and we have from (5) a system of equations just as that in the paper /5/.

Description of the operation regime at which $y \ll 1$ (linear regime) can be made in an analytical form. Let $\frac{d\phi_0}{ds} = \varkappa$,

$$\varkappa = \gamma \frac{\nu_0}{\Omega_0 a^{1/3}} \left[ \frac{\nu}{2\nu_0 \gamma^2} \left( 1 + \frac{\Omega_0^2}{v_0^{e}} \right) - 1 \right] \tag{6}$$

We put

$$\phi = \phi_0 + \varkappa s + \phi_L \tag{7}$$

At $y \ll 1$ the value $\phi_L \ll 1$. From the last equation of the system (5) we have

$$\phi_L = 2 \int_0^s y(s')(s-s') \sin[\phi_0 + \varkappa s' + \chi(s')] ds'$$

After substituting this expression into (7) and then eq.(7) into the first two equations of (5), and averaging over $\phi_0$, we get

$$\frac{dy}{ds} = \int_0^s y(s')(s-s') \sin[\varkappa(s'-s) + \chi(s') - \chi(s)] ds'$$
$$y\frac{d\chi}{ds} = -\int_0^s y(s')(s-s') \sin[\varkappa(s'-s) + \chi(s') - \chi(s)] ds' + y\delta \tag{8}$$

We shall find a solution of the system (8) in the form $Z = y e^{i(\chi + \varkappa s)}$. For the function $Z$ the following equation is derived from (8):

$$\frac{dZ}{ds} - iZ(\varkappa + \delta) + i \int_0^s (s-s') Z(s') ds' = 0 \tag{9}$$



It should bear in mind that the process is here considered in a single-particle approximation, when the electrons interact with each other through electromagnetic wave and the electrostatic interaction between the electrons in a bunch is not taken into account. Since $G \gg 1$ is attained at high density of electrons $n$, the type of operation determined by plasma effects can be also realized (three-wave instability; see, e.g., /3/). The approximation accepted is true if a wavelength of Langmuir oscillations is long as compared to a typical length of the growth of an electromagnetic wave (signal) $\ell_g$ ( $\ell_g$ is estimated below, formula (20)), or

$$\frac{\omega_p \ell_g}{c \gamma^{3/2}} \ll 1 \tag{10}$$

Under this condition the allowance for a parameter $\delta$ leads to insignificant shift of the resonance curve. In what follows it will be omitted. Subsequent differentiation of eq.(9) yields

$$\dddot{Z} - i\varkappa \ddot{Z} + iZ = 0 \tag{11}$$

In this case the initial conditions are: $Z(0) = y(0)$, $\dot{Z}(0) = i\varkappa y(0)$, $\ddot{Z}(0) = -\varkappa^2 y(0)$. We put $Z = Z(0) e^{-i\lambda s}$ and obtain the characteristic equation

$$\lambda^3 + \varkappa \lambda^2 + 1 = 0 \tag{12}$$

similar to the known equation for the lamp of traveling wave.*

Given the initial conditions, the solution is

$$Z = y(0) \left( B_1 e^{-i\lambda_1 s} + B_2 e^{-i\lambda_2 s} + B_3 e^{-i\lambda_3 s} \right) \tag{13}$$

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* The same equation (in connection with the stimulated scattering of waves from the relativistic beams) was discussed in /7/.

Here $\lambda_1$, $\lambda_2$, $\lambda_3$ are the roots of the characteristic equation,

$$B_1 = \frac{\varkappa^2 + \varkappa(\lambda_2+\lambda_3) + \lambda_2\lambda_3}{\lambda_1^2 - \lambda_1(\lambda_2+\lambda_3) + \lambda_2\lambda_3}$$

$B_2$ and $B_3$ are obtained from $B_1$ by cyclic permutation of indices. Signal amplification on a "length" $s$ is characterized by $\left|\frac{z}{y(0)}\right|^2 = 1+G$ the explicit representation for which follows from (13). It has a simple analytical form when $\varkappa \ll 1$ and $\varkappa \gg 1$ :

$$\left|\frac{z}{y(0)}\right|^2 = \begin{cases} f + \varkappa f_1 & |\varkappa| \ll 1 \\ 1 + f_2 & |\varkappa| \gg 1 \end{cases} \tag{14}$$

where

$$f = \frac{1}{9}\left[1 + 4ch^2\left(\frac{\sqrt{3}}{2}s\right) + 4ch\left(\frac{\sqrt{3}}{2}s\right)\cos\frac{3}{2}s\right],$$

$$f_1 = \frac{4}{27}\left[-ch(\sqrt{3}s) + ch\left(\frac{\sqrt{3}}{2}s\right)\cos\frac{3}{2}s + \sqrt{3}\,sh\left(\frac{\sqrt{3}}{2}s\right)\sin\frac{3}{2}s\right] \tag{15}$$

$$f_2 = \frac{4}{\varkappa^3}\left[\frac{1}{4}sh^2\left(\frac{s}{\sqrt{\varkappa}}\right) + \frac{\varkappa^{3/2}}{2}sh\left(\frac{s}{\sqrt{\varkappa}}\right)\sin\varkappa s + ch\left(\frac{s}{\sqrt{\varkappa}}\right)\cos\varkappa s - 1\right]$$

Note, that at $s \ll 1$ $f = 1 + \frac{s^6}{40}$, $f_1 = \frac{s^4}{6}$, and at $s \gg 1$ $f = \frac{1}{9}e^{\sqrt{3}s}$, $f_2 = -\frac{2}{27}e^{\sqrt{3}s}$.

In case when $s \ll 1$ ( $G \ll 1$ ) it is rather easy to get a general expression for $G$ (see /5/):

$$\left|\frac{z}{y(0)}\right|^2 = 1 + G = 1 + 2s^3 R\left(-\frac{\varkappa s}{2}\right), \quad R(x) = -\frac{1}{4}\frac{d}{dx}\left(\frac{\sin^2 x}{x^2}\right) \tag{16}$$



Note, that under experimental conditions /2/ $s = 0.48$. In case when $s \gg 1$ , G can take very large values:

$$1 + G = \left|\frac{z}{y(0)}\right|^2 = \frac{1}{9}e^{\sqrt{3}s}$$

Linear regime takes place until phase increment $\phi_L$ becomes of order of unity. In case of a single passage (superradiant oscillator) for the output signal $y \sim 1$ , independently upon $y(0)$ . Taking into account (4), we have the estimate for maximal strength of the output signal field:

$$H = H_0\, a^{2/3} \tag{17}$$

Due to building up of phase oscillations, the growth of wave amplitude is limited. Using (17), we can calculate the efficiency $\eta = \Delta\gamma/\gamma$ :

$$\eta \simeq \frac{a^{1/3}}{2}\frac{\Omega_0}{\nu_0\gamma} \tag{18}$$

Given fixed length $s$ , the spectral distribution of emitted radiation is determined by a dependence of $\left|z/y(0)\right|$ on $\varkappa$ (see (6)). In linear regime this dependence follows from (13) and at $\varkappa \ll 1$ and $\varkappa \gg 1$ we have derived above the explicit expressions (14) and (15). In a general case the system (5) can be only solved numerically. The result of our calculation for $y(0) = 0.01$ is shown in Fig.1, where the evolution of spectral curves upon variation of $s$ ( $s$ = 5;6;7,5). Note, that in the region $s \ll 1$ studied in /5/, the maximum of spectral curve lies at $\varkappa < 0$ ( $\frac{|\varkappa| s}{2} \sim 1$ ) and at $\varkappa = 0$ the gain turns to zero (cf. the above asymptotics of functions $f$ and $f_1$ (15) at $s \ll 1$ ). As seen from Fig.1 (and also from formulas (14) and (15)) the spectral curve is considerably deformed at $s \gtrsim 1$ , its maximum lying, as before, at $\varkappa < 0$ but the antisymmetric curve becomes nearly symmetric one and the point $\varkappa$ = 0 is not far from the maximum. A width of the resonant curve is determined by the condition $|\varkappa| \sim 1$, or

$$\frac{\Delta\nu}{\nu} \simeq \frac{2a^{1/3}\Omega_0}{\gamma\nu_0} \sim \eta \tag{19}$$

The dependence of the output signal (at $y(0)$ = 0.01) upon the "length" $S$ at fixed values $\varkappa$ = 0 and $\varkappa$ = -1 (near the maximum of the spectral curve) is given in Fig.2. It is seen that at $S > 1$ the output signal grows rapidly (exponentially) up to the values $y \sim 1$ and then there grow phase oscillations causing the oscillation of a signal. As $\left|\frac{Z}{y(0)}\right| = \frac{1}{3}e^{\frac{\sqrt{3}}{2}S}$, the characteristic growth length $S = 2/\sqrt{3}$, or $\ell_g$ (see (14))

$$\ell_g = \frac{2}{\sqrt{3}}\frac{c\gamma}{\Omega_0 a^{1/3}} \tag{20}$$

and the length on which $y \sim 1$ is attained is $L = \ell_g \ln\left(\frac{3}{y(0)}\right)$.
The curve 3 in Fig.1 gives a shape of the signal at $S$ = 7.5 when $\varkappa$ = 0, for this value of $S$ the gain is maximum. Note, that the values $S$ = 5 and $S$ = 6 lie within the region of linear regime (see Figs. 1 and 2, and eqs. (13)-(15)) and at $S$ = 7.5 the regime is essentially non-linear.

If the length of a magnetic lattice $L_e \sim L$, the maximal value of the signal is already attained upon a single passage, as this has above been discussed. If $L_e \ll L$, it is fairly reasonable to use multiple amplification of the signal, for instance, using the quasi-optic type cavity (just as in /2/).

For illustration let us consider the following example. Let the current $J = 10^4$ A with the cross-section $S = 1$ cm$^2$ and energy $\gamma$ = 20 pass through the magnetic lattice with $H_0$ = 



= 2.4 kGauss and $\lambda_0$ = 3.2 cm. In this case the gorwth length (20) $\ell_g = 15$ cm and the criterion (10) is fulfilled. Let an incident wave appear, due to bremsstrahlung when passing the initial part of the lattice. Then the length on which the output signal attains its maximum $L \simeq$ 170 cm, radiation power $W = 2\cdot10^9$ W and the efficiency $\eta \simeq$ 0.02 (see (18)), in accordance with (19), the line width $\Delta\nu/\nu$ is of the same order.

In this study we consider the fixed energy ( $\gamma$ ) and the angle of inlet of the electrons into the lattice ( $\vartheta$ ). By using formula (6) and Fig.1, for the permissible spreads of energy and angle, we have

$$\frac{\delta\gamma}{\gamma} \lesssim \frac{c}{\nu_0\ell_g} \sim \eta, \quad \delta\vartheta \lesssim \frac{c}{\Omega_0\gamma\ell_g}\left(1+\frac{\Omega_0^2}{\nu_0^2}\right) \tag{21}$$

Electromagnetic wave generation involves only those electrons which satisfy the above conditions.

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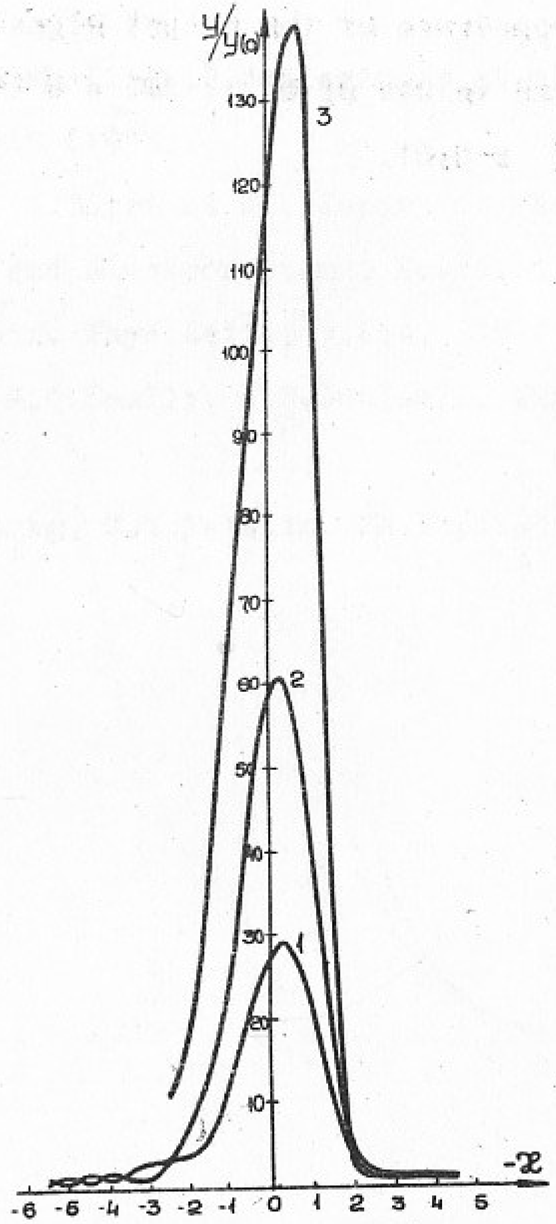
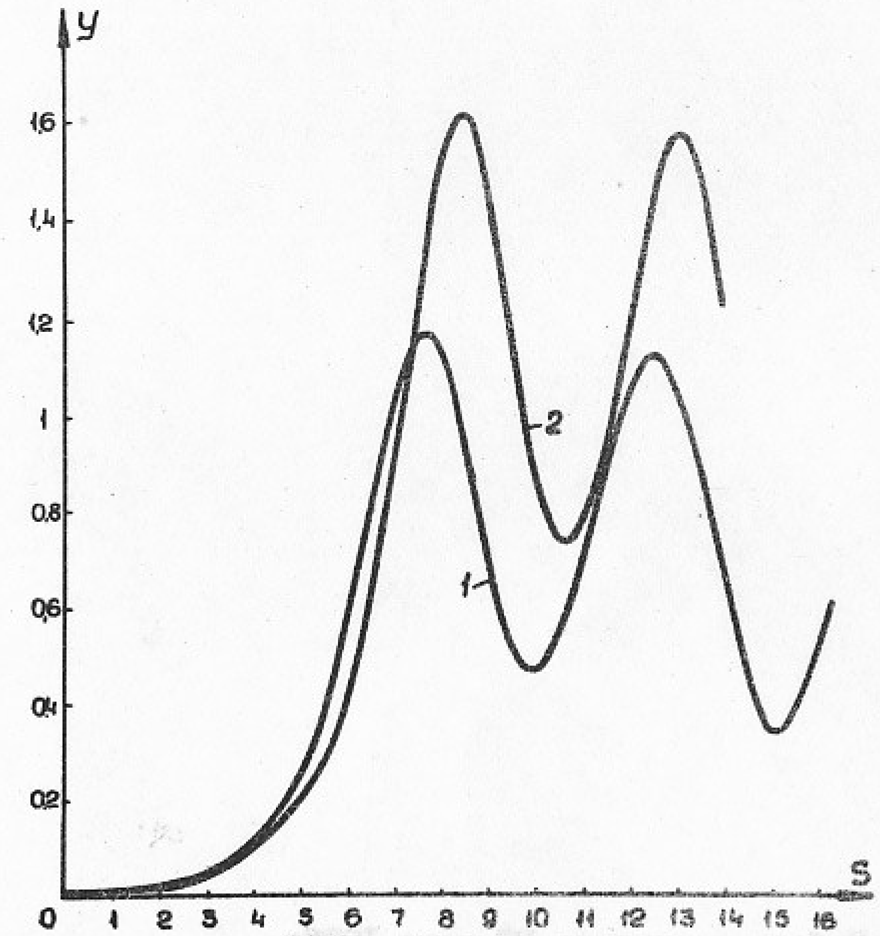
FIGURE CAPTIONS

Fig.1 The dependence of the output signal on $\mathscr{X}$ at different $S$ : $S = 5$ (1), $S = 6$ (2), $S = 7.5$ (3), $y(0) = 0.01$.

Fig.2 The dependence of the output signal on a length $S$ at fixed values of $\mathscr{X}$ : $\mathscr{X} = 0$ (1), $\mathscr{X} = -1$ (2), $y(0) = 0.01$.


y/y(0)
130
120
110
100
90
80
70
60
50
40
30
20
10
3
2
1
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5
Fig.1


y
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
2
1
S
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Fig.2

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