63

ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
СО АН СССР

A.R.Zhitnitsky

ON POSSIBLE SUPPRESSION OF AXION-HADRON INTERACTIONS

ПРЕПРИНТ ИЯФ 79-81

Новосибирск

ON POSSIBLE SUPPRESSION OF AXION-HADRON INTERACTIONS

A.R.Zhitnitsky

Institute of Nuclear Physics
630090, Novosibirsk 90, USSR

A b s t r a c t

A possible mechanism of the strong suppression of the axion-fermion interaction is considered. Two model realizing this mechanism are described in detail.

I. It is well known that pseudoparticle solutions in quantum chromodynamics leads to an additional term in the Lagrangian of the interaction $\Delta\mathcal{L}_{int}=\Theta\frac{g^2}{32\pi^2}F_{\mu\nu}\tilde{F}_{\mu\nu}$ which violates the P- and CP-invariance /1/. The solution was found in the paper /2/. As it has been shown a global symmetry remedies this situation and conserves the invariance. However, in this case, a light pseudoscalar particle, the axion, must exist /3,4/. The axion coupling to quark in the model/3,4/ is of order $\sim G_F^{1/2}m_q$ and the mass is about 100 keV.

Many authors have discussed possible methods for producing and detecting axions /3-5/. However, the existing experimental evidence indicate that the axion possessing the above properties does not exist /6/. In accordance with this, the problem arises to explain the "negative" results of the experiments. Not focusing on theoretical alternatives, which does not require the existence of the axion at all let us consider a possible suppression mechanism of the axion-quark coupling.

We will show that the axion-quark interaction in the Weinberg-Salam model can be weakened, by extending the Higgs sector of a theory, or by extending a gauge group $SU(2)_L\times U(1)$ Furthermore, the suppression mechanism of question can be included into a wide enough class of models not involving additional Higgs fields and also with no extension of the gauge group if this group is larger in comparison with the minimal required one $SU(2)_L\times U(1)$ (for example, $SU(2)_L\times SU(2)_R\times U(1)$ or $SU(2)_L\times U(1)_L\times U(1)_R$).

The plan of the present paper is as follows. Section 2 discusses the Weinberg-Salam model wherein suppression the axion-quark interaction occurs due to extension of the Higgs sector. Section 3 considers a model based on the gauge group $SU(2)_L\times SU(2)_R\times U(1)$. It is shown that the axion-quark coupling is order $G^{1/2}m_q\frac{V_L}{U_R}$, where V_L, U_R - are the modules of vacuum expectation of the left-handed and right-handed Higgs doublets, respectively. Note that $V_L \ll U_R$. Meanwhile, all of the remaining interactions associated with the left charged currents coincides with the standard model and neutral currents differ only by the terms of order $(\frac{V_2}{U_R})^2$. Appendix discusses the constraints imposed on the Higgs potential.

2. As known /3,4/, to realize a global $U(1)_{RQ}$ symmetry in the Weinberg-Salam model, at least two Higgs doublets φ_1, φ_2. For my purposes, I has to introduce an additional scalar complex field χ. This field is the singlet under the gauge transformations of the $SU(2)_L \times U(1)$ group, but it is transformed nontrivially under $U(1)_{RQ}$. So, let us demand invariance of the Lagrangian under the following transformations $U(1)_{RQ}$:

$$\varphi_1' = e^{i\alpha}\varphi_1 \quad , \quad \varphi_2' = e^{i\beta}\varphi_2 \quad ; \quad \chi' = e^{i(\alpha-\beta)}\chi$$

The quark fields under the transformation $U(1)_{RQ}$ vary in the same way as in the standard model involving the axion /3,4/.

First of all, let us focus on the Higgs sector of a theory. The most general form of the potential energy satisfying both the $SU(2)_L \times U(1) \times U(1)_{RQ}$ symmetry and renormalizability can be written as follows:

$$V(\varphi_1 \varphi_2 \chi) = \mu_1 \varphi_1^+\varphi_1 + \mu_2 \varphi_2^+\varphi_2 + \mu_3 \chi^+\chi + \lambda_1(\varphi_1^+\varphi_1)^2 + \\ + \lambda_2(\varphi_2^+\varphi_2)^2 + \lambda_3(\chi^+\chi)^2 + \Delta_1(\varphi_1^+\varphi_1)(\chi^+\chi) + \Delta_2(\varphi_2^+\varphi_2)(\chi^+\chi) + \\ + \alpha(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) + \beta(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1) + \gamma(\varphi_1^+\varphi_2)\chi + \gamma^*(\varphi_2^+\varphi_1)\chi^+ \qquad (1)$$

In this expression all the parameters, except γ, are real due to hermiticity of the Lagrangian. However, γ can be made a real parameter, redefining any of the fields

$$\varphi_1 \; ; \; \varphi_2 \; ; \; \chi$$

Indeed, as $\gamma = |\gamma| e^{i\vartheta}$, the redistribution $\varphi_1 = e^{i\vartheta}\varphi_1'$ leads to disappearance of ϑ. In what follows the dash at φ_1 is omitted and keep in mind that $\gamma > 0$. I will written γ as instead of $|\gamma|$.

Without loss of generality the vacuum expectation of the fields may be written as follows:

$$\langle \varphi_1 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ V_1 e^{i\theta} \end{pmatrix} \quad \langle \varphi_2 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \sigma \\ V_2 \end{pmatrix} \quad \langle \chi \rangle = u e^{-i\tau} \qquad (2)$$

Here $V_1 > 0$, $V_2 > 0$, $\sigma > 0$, $u > 0$. This results from the gauge invariance of $V(\varphi_1\varphi_2\chi)$. Actually, the $SU(2)_L$ rotation parameters $\omega^\pm$ in the rotation matrix $U^\pm = e^{i\tau^\pm\omega^\pm}$ may always be chosen so that the upper component of the field $\langle\varphi_1\rangle$ becomes zero. Moreover, one can eliminate independent phases of both the upper and lower components in $\langle\varphi_2\rangle$ by means of transformations $U^{0\pm3} = e^{i(1\pm\tau_3)\omega^{0\pm3}}$, In this case, the gauge rotations $U^{0\pm3}$ lead simply to redefining θ in $\langle\varphi_1\rangle$ with no variation in the structure of $\langle\varphi_2\rangle$.

Suppose that the potential V parameters are such that V_1, V_2, u are not equal to zero.

Then, as shown in Appendix, at $\beta V_1 V_2 < 2\gamma u$ the spontaneous charge violation does not occur, i.e. $\sigma = 0$. A minimum for V is fulfilled by a choice of θ and τ such that $\theta + \tau = \pi$. As one should expect, minimization of the potential V does not determine θ and τ independent, what is associated with the additional $U(1)_{RQ}$ symmetry available. Nevertheless, the minimum of the effective potential /2/ with taking into account the instantons will, as usual, define the phases θ and τ separately and the corresponding chiral rotation results in cancelling the CP-nonvariant term in the Lagrangian.

Proceed now to the coupling of the axion to fermions. It is known /7/ that after diagonalization of quark massive terms, the Yukawa interaction between Higgs fields and quarks is written as follows

$$L_Y = \frac{\sqrt{2}\, m_d}{V_1}\bar{L}\varphi_1 d_R + \frac{\sqrt{2}\, m_u}{V_2^*}\bar{L}\tilde{\varphi}_2 u_R + H.c. \qquad (3)$$

Here $L = (u, d)_L$, $\tilde{\varphi}_2 = i\tau_2\varphi_2^*$, V_1, V_2 - are the vacuum values of the corresponding fields, taking account of these phases. In order to choose the axion-quark coupling from (3) one needs to find linear combination of the neutral components of the fields φ_1, φ_2, χ of definite masses. For this purpose, represent the scalar fields after chiral transformations in the following form:

$$\varphi_1 = \frac{1}{\sqrt{2}} e^{i\frac{\vec{\tau}\vec{\xi}_1}{|V_1|}}\begin{pmatrix} 0 \\ V_1\left(1 + \frac{h_1}{|V_1|}\right) \end{pmatrix} \quad \varphi_2 = \frac{1}{\sqrt{2}} e^{i\frac{\vec{\tau}\vec{\xi}_2}{|V_2|}}\begin{pmatrix} 0 \\ V_2\left(1 + \frac{h_2}{|V_2|}\right) \end{pmatrix} \qquad (4)$$
$$\chi = u\left(1 + \frac{\sigma}{|u|} e^{i\frac{\omega}{|u|}}\right)$$

(4)

Here $\vec{\xi}_1, \vec{\xi}_2, \zeta_1, \zeta_2, \omega, \sigma$ - are the Hermitian fields, $\vec{\tau}$ - are the usual Pauli matrices.

Using the gauge transformation with the rotation matrix $U = e^{-i\vec{\tau}\left(\frac{\vec{\xi}_1|v_1| + \vec{\xi}_2|v_2|}{|v_1|^2+|v_2|^2}\right)}$, one can pass to the unitary gauge. The transformed field $\varphi'_{1,2}$ are written as follows:

$$\varphi_1' = U\varphi_1 = \frac{1}{\sqrt{2}} e^{i\vec{\tau}\vec{\rho}\frac{\Delta}{|v_1|^2}} \begin{pmatrix} 0 \\ v_1\left(1+\frac{\zeta_1}{|v_1|}\right) \end{pmatrix}, \quad \varphi_2' = U\varphi_2 = \frac{1}{\sqrt{2}} e^{-i\vec{\tau}\vec{\rho}\frac{\Delta}{|v_2|^2}} \begin{pmatrix} 0 \\ v_2\left(1+\frac{\zeta_2}{|v_2|}\right) \end{pmatrix} \quad (5)$$

Here $\vec{\rho} = \frac{\vec{\xi}_1|v_2| - \vec{\xi}_2|v_1|}{\sqrt{|v_1|^2+|v_2|^2}}$, $\Delta = \frac{|v_1||v_2|}{\sqrt{|v_1|^2+|v_2|^2}}$. It is ready to see that in the case of the fields $\varphi'_{1,2}$, non-diagonal terms the $\langle\varphi'_{1,2}\rangle \vec{W}_\mu \vec{\tau}\partial_\mu \varphi'_{1,2}$ - type in the Lagrangian are cancelled and, hence, the description the fields in the form (5) are actually expressed in terms of the unitary gauge. The fields ρ_0 and ω each taken separately, do not become massive because of the additional $U(1)_{PQ}$ - symmetry; only the combination $H^0 = \rho^0\cos\delta + \omega\sin\delta$ acquires a mass and the combination $a^0 = \omega\cos\delta - \rho^0\sin\delta$, which is orthogonal to H^0, remains massless in a tree approximation and is the axion. Indeed, the part of interest of the potential $\tilde{V}$ which could give masses to the $\vec{\rho}$ - and ω - fields is written as follows:

$$\tilde{V} = \beta(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1) + \gamma(\varphi_1^+\varphi_2\chi + \varphi_2^+\varphi_1\chi^+) \quad (6)$$

Substituying (5) into (6) and remaining only the $\vec{\rho}$ -, ω- - quadratic terms, we obtain to the following expression for $\tilde{V}$:

$$\tilde{V} = \beta\frac{v_1^2v_2^2}{4} - \gamma v_1v_2u + \rho^+\rho^-\left[\frac{\gamma u}{v_1v_2} - \frac{1}{2}\beta\right](v_1^2+v_2^2) +$$

$$+ \frac{1}{2}\gamma\frac{v_1^2u^2+v_1^2v_2^2+v_2^2u^2}{v_1v_2u}(\rho_0\cos\delta + \omega\sin\delta)^2, \quad \sin\delta = \frac{v_1v_2}{\sqrt{v_1^2v_2^2+v_1^2u^2+v_2^2u^2}} \quad (7)$$

Here and below by v_1, v_2, u the modules of corresponding quantities are meant.

As $2\gamma u > \beta v_1v_2$, the charged fields $\rho^\pm$ become massive, the neutral combination $H^0 = \rho^0\cos\delta + \omega\sin\delta$ get a mass $m^2_{H_0} = \gamma\frac{v_1^2u^2+v_1^2v_2^2+v_2^2u^2}{v_1v_2u}$ whereas the axion remains massless. Express ρ^0 and ω via physical fields H^0, a^0 :

$$\rho^0 = H^0\cos\delta - a^0\sin\delta, \quad \omega = a^0\cos\delta + H^0\sin\delta \quad (8)$$

It is seen from (8) that, if a spontaneous break-down of symmetry takes place so that $u \gg v_1, v_2$, the axion-quark interaction will be proportional to $\sin\delta \sim \frac{v_{1,2}}{u} \ll 1$. This is just the result we are striving for.

Let us express this fact more clearly. To this end, let us substitute eq. (8) into eq. (5) and then into (3). Interaction of quarks with the axion is written as follows:

$$L_{int} = 2^{1/4}G_F^{1/2}a^0\sin\delta\left[m_u\frac{v_1}{v_2}\bar{u}i\gamma_5u + m_d\frac{v_2}{v_1}\bar{d}i\gamma_5d\right] \quad (9)$$

As compared to the standard model /3,4/, we have two additional massive neutral particles σ and H^0 and also the factor $\sin\delta$ in expression (9).

Turn our attention to the following circumstance: the fact that the axion interacts weakly both with the upper and lower components of the quark doublet is a direct consequence of the invariance of a field χ under the gauge transformations of $SU(2)_L \times U(1)$. Just this circumstance results in the absence of the contribution $\langle\chi\rangle^2$ to the W - bosons masses and consequently to G_F. Thus, the only requirement to the additional scalar field is that the quantity $\langle\chi\rangle^2$ should be larger in comparison with $\langle\varphi_{1,2}\rangle^2$, that is $\langle\chi\rangle^2 G_F \gg 1$.

If an additional scalar field be introduced which is transformed non-trivially under $SU(2)_L \times U(1)$ (a doublet, for example), of course, an additional parameter in a theory will appear. But weakening of the coupling of the axion to the lower component of the quark doublet thanks to this parameter, would necessarily lead to amplification of the interaction with the upper component of the double (as in the standard model involving the axion /3,4/).

The above mechanism suppression of the quark-axion interaction seems to be somewhat artificial since massive neutral fields H^0 and σ have no any additional leading except suppression. More attractive are the models with a larger group of symmetry (for example, $SU(2)_L \times SU(2)_R \times U(1)$) which already include the Higgs fields with the properties required, namely: $\langle\varphi_{1,2}\rangle_R^2 \ll \langle\varphi_{1,2}\rangle_L^2$. In this case, a natural suppression of the right currents, in comparison with the left ones results in that the axion interacts more weakly with the left sectors than usually in the ratio $\frac{\langle\varphi_{1,2}\rangle_L^2}{\langle\varphi_{1,2}\rangle_R^2} \ll 1$.

For these models there is no necessity in additional Higgs fields. Let us now discuss in detail one of such possibilities.

3. A model is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$. The Higgs sector contains four fields: $\varphi_{1L}, \varphi_{2L} = (1/2, 0, 1)$ $\varphi_{1R}, \varphi_{2R} = (0, 1/2, 1)$. By the notation in the brackets is meant the following: (T_L, T_R, Y) , T_L, T_R - is the multiplicity of a field under the group $SU(2)_L$, $SU(2)_R$ respectively, and Y is the hypercharge of this field.

The left sector's quarks u, d are transformed under a gauge group as follows:

$$\binom{u}{d}_L = (1/2,\ 0,\ 1/3), \quad u_R = (0\ 0\ 4/3), \quad d_R = (0, 0, -2/3).$$

The transformation of the right sector's quarks X, Z is analogous:

$$\binom{X}{Z}_R = (0\ 1/2,\ 1/3), \quad X_L = (0\ 0\ 4/3), \quad Z_L (0\ 0\ -2/3)$$

That is, the model is constructed in such a way that the left-hand part coincides with the standard model and the right-hand part is constructed in the same way as the left one. Let us now impose an additional $U(1)_{PQ}$- symmetry on the Lagrangian at which the scalar fields are transformed as follows:

$$\begin{cases} \varphi_{1L}' = e^{i\alpha}\varphi_{1L} \\ \varphi_{2L}' = e^{i\beta}\varphi_{2L} \end{cases} \qquad \begin{cases} \varphi_{1R}' = e^{-i\alpha}\varphi_{1R} \\ \varphi_{2R}' = e^{-i\beta}\varphi_{2R} \end{cases} \tag{10}$$

The quarks are transformed under $U(1)_{PQ}$ - rotations so that Yukawa interaction remains invariant under transformations (10).

Let us consider, first of all, the model's Higgs sector. The most general form of the potential corresponding to renormalizability and $SU(2)_L \times SU(2)_R \times U(1) \times U(1)_{PQ}$ symmetry is the following:

$$\begin{aligned} &V(\varphi_{1L}, \varphi_{2L}, \varphi_{1R}, \varphi_{2R}) = V^L + V^R + V^{LR} \\ &V^L = \mu_{1L}\varphi_{1L}^+\varphi_{1L} + \mu_{2L}\varphi_{2L}^+\varphi_{2L} + \lambda_{1L}(\varphi_{1L}^+\varphi_{1L})^2 + \lambda_{2L}(\varphi_{2L}^+\varphi_{2L})^2 + \\ &+ \alpha_L(\varphi_{1L}^+\varphi_{1L})(\varphi_{2L}^+\varphi_{2L}) + \beta_L(\varphi_{1L}^+\varphi_{2L})(\varphi_{2L}^+\varphi_{1L}) \\ &V^{LR} = \delta_{11}(\varphi_{1L}^+\varphi_{1L})(\varphi_{1R}^+\varphi_{1R}) + \delta_{22}(\varphi_{2L}^+\varphi_{2L})(\varphi_{2R}^+\varphi_{2R}) + \delta_{12}(\varphi_{1L}^+\varphi_{1L})(\varphi_{2R}^+\varphi_{2R}) + \\ &+ \delta_{21}(\varphi_{2L}^+\varphi_{2L})(\varphi_{1R}^+\varphi_{1R}) + \gamma(\varphi_{1L}^+\varphi_{2L})(\varphi_{1R}^+\varphi_{2R}) + \gamma(\varphi_{2L}^+\varphi_{1L})(\varphi_{2R}^+\varphi_{1R}) \end{aligned} \tag{11}$$

V^R has the same form as V^L with the corresponding variation $L \to R$. Just as in eq. (1), all the parameters are real and, without loss of generality, we shall assume that $\gamma > 0$.

As it is shown in Appendix, the condition for the absence of the spontaneous charge violation may be written as follows:

$$-\beta_L v_1 v_2 + \gamma u_1 u_2 > 0$$
$$-\beta_R u_1 u_2 + \gamma v_1 v_2 > 0$$

where $v_{1,2}$, $u_{1,2}$ are the absolute values of the vacuum *expectation* of left and right fields, respectively. These requirements are fulfilled automatically if $\beta_L < 0$, $\beta_R < 0$.

Then, if one acts in a similar manner, it is ready to derive the following expression for the Lagrangian of the axion-quark interaction:

$$\begin{aligned} L_{int} = {}& 2^{1/4} G_F^{1/2} a^0 \sin\tilde\delta \left[m_u \frac{v_1}{v_2} \bar u i\gamma_5 u + m_d \frac{v_2}{v_1} \bar d i\gamma_5 d + \ldots \right] + \\ & - 2^{1/4} G_R^{1/2} a^0 \cos\tilde\delta \left[m_X \frac{u_1}{u_2} \bar X i\gamma_5 X + m_Z \frac{u_2}{u_1} \bar Z i\gamma_5 Z + \ldots \right] \end{aligned} \tag{12}$$

where $G_F = \frac{1}{\sqrt{2}}\frac{1}{(v_1^2+v_2^2)}$ $G_R = \frac{1}{\sqrt{2}}\frac{1}{(u_1^2+u_2^2)}$ $tg\hat{\delta} = \frac{v_1 v_2 \sqrt{u_1^2+u_2^2}}{u_1 u_2 \sqrt{v_1^2+v_2^2}}$

Supposing that $u_1 \sim u_2 \gg v_1 \sim v_2$, one obtains that $\sin\hat{\delta} \ll 1$ and the interaction of the axion with left quarks is strongly suppressed.

Let us now show that the Lagrangian of fermion-heavy $W^\pm, Z$ - boson interaction coincides with the standard model to the terms $(\frac{v_{1,2}}{u_{1,2}})^2 \ll 1$. It is clear that the expressions for charged current entirely coincides with the standard model because no mixing of $W_L^\pm$ with $W_R^\pm$ occurs. Meanwhile, neutral components of vector fields are mixed and get the following masses:

$$m_A^2 = 0 \quad m_{Z_1}^2 = \tfrac{1}{4}(v_1^2+v_2^2)(1+\sin^2\theta)g^2$$
$$m_{Z_2}^2 = \tfrac{1}{4}(u_1^2+u_2^2)(g^2+g'^2) \gg m_{Z_1}^2 \tag{13}$$

where $g_L = g_R = g$ is the constant associated with $SU(2)_L$, $SU(2)_R$, and $\frac{1}{2}g'$ is the constant associated with the group $U(1)$; the angle θ is introduced, as usual, by the relation: $tg\theta = g'/g$ With the above notation the quark-Z_1-boson coupling $L_{\bar{q}qZ_1}$ takes the form:

$$L_{\bar{q}qZ_1} = \frac{g}{2}\frac{Z_{1\mu}}{\sqrt{1+\sin^2\theta}}\Big\{\bar{u}\gamma_\mu u(-\tfrac{1}{2}+\tfrac{5}{6}\sin^2\theta) - \tfrac{1}{2}\bar{u}\gamma_\mu\gamma_5 u(1+\sin^2\theta) + \bar{d}\gamma_\mu d(\tfrac{1}{2}-\tfrac{1}{6}\sin^2\theta) + \tfrac{1}{2}\bar{d}\gamma_\mu\gamma_5 d(1+\sin^2\theta)\Big\} \tag{14}$$

It is easy to see from eqs. (13) and (14) that the replacement $\sin^2\theta \to tg^2\theta_W$, where θ_W -is the standard value of Weinberg angle, leads in the standard model's Lagrangian up to the terms $(\frac{v_{1,2}}{u_{1,2}})^2 \ll 1$. A similar statement is of course, valid for leptons too.

In conclusion, note that from the experiments on a search for the axion it follows the restriction on a suppression parameter $\delta \sim \frac{v}{u} \sim \frac{v_L}{u_R} \lesssim 10^{-2}$. In this case, the mass of the axion $m_a \sim 100\,\delta$ keV.

The author is indebted to A.I.Vainshtein and I.B.Khriplovich for usefull discussions and remarks.

Appendix

In this appendix we discuss some conditions which should be imposed on c - numerical potentials $V(\Phi_1,\Phi_2,\chi)$ and $V(\Phi_{1L},\Phi_{2L},\Phi_{1R},\Phi_{2R})$ the latter determining by expressions (1) and (11), respectively.

Let us begin with the Higgs potential determined by eq. (1). As it is previously explained, the vacuum expectation fields Φ_1, Φ_2, χ can be always written in the form:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1 e^{i\theta}\end{pmatrix} \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\sigma\\ v_2\end{pmatrix}, \quad \langle\chi\rangle = u e^{-i\tau} \tag{15}$$

where $v_1 > 0$, $v_2 > 0$, $\sigma > 0$, $u > 0$. Substituting eq. (15) into eq. (1), one gets the following expression for potential energy:

$$V(v_1,v_2,\sigma,u,\theta,\tau) = \tfrac{1}{2}\mu_1 v_1^2 + \tfrac{1}{2}\mu_2(v_2^2+\sigma^2) + \mu_3 u^2 + \tfrac{1}{4}\lambda_1 v_1^4 + \tfrac{1}{4}\lambda_2(v_2^2+\sigma^2)^2 + \lambda_3 u^4 + \tfrac{1}{4}\alpha v_1^2(v_2^2+\sigma^2) + \tfrac{1}{4}\beta v_1^2 v_2^2 + \gamma v_1 v_2 u\cos(\theta+\tau) + \tfrac{1}{2}\Delta_1 v_1^2 u^2 + \tfrac{1}{2}\Delta_2 u^2(v_2^2+\sigma^2) \tag{16}$$

as $\gamma > 0$ the minimum of energy is satisfied by the condition $\theta + \tau = \pi$. That's why the following holds:

$$\frac{\partial V}{\partial \sigma^2} = \frac{\partial V}{\partial v_2^2} + \frac{1}{2}\frac{v_1}{v_2}(\gamma u - \tfrac{1}{2}\beta v_1 v_2) \tag{17}$$

As $\frac{\partial V}{\partial v_2^2}$ in the minimum point of the requirement $\gamma u - \frac{1}{2}\beta v_1 v_2 > 0$ results in the absence of spontaneous charge violation, i.e. $\sigma = 0$.

In order to set a lower limit for the potential energy it is sufficient to require the fulfilment of the following conditions:

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 > 0$$
$$\begin{cases}\lambda_1\lambda_2 > (\alpha+\beta)^2\\ \lambda_1\lambda_3 > \Delta_1^2\\ \lambda_2\lambda_3 > \Delta_2^2\end{cases} \tag{18}$$

Proceed now to the potential which is determined by expression (11). Without loss of generality the vacuum expectation of fields can be written as follows:

$$\langle \varphi_{1L} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 e^{i\theta_L} \end{pmatrix} \qquad \langle \varphi_{2L} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 \sin\varphi \\ v_2 \cos\varphi \end{pmatrix}$$

$$\langle \varphi_{1R} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ u_1 e^{i\theta_R} \end{pmatrix} \qquad \langle \varphi_{2R} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u_2 \sin\psi \\ u_2 \cos\psi \end{pmatrix} \tag{19}$$

Here $v_1 > 0,\ v_2 > 0,\ u_1 > 0,\ u_2 > 0 \qquad 0 \le \varphi \le \frac{\pi}{2}, \quad 0 \le \psi \le \frac{\pi}{2}$

Substituting (19) into (11), one gets the following expression for V

$$V(\langle \varphi_{1,2} \rangle_L, \langle \varphi_{1,2} \rangle_R) = \frac{1}{2}\mu_{1L} v_1^2 + \frac{1}{2}\mu_{2L} v_2^2 + \frac{1}{4}\lambda_{1L} v_1^4 + \frac{1}{4}\lambda_{2L} v_2^4 + \tag{20}$$
$$+ \frac{\alpha_L}{4} v_1^2 v_2^2 + \frac{1}{4}\beta_L v_1^2 v_2^2 \cos^2\varphi + (L \to R,\ v_{1,2} \to u_{1,2},\ \varphi \to \psi) +$$
$$+ \frac{1}{4}\delta_1 v_1^2 u_1^2 + \frac{1}{4}\delta_2 v_2^2 u_2^2 + \frac{1}{4}\delta_{12} v_1^2 u_2^2 + \frac{1}{4}\delta_{21} v_2^2 u_1^2 + \frac{1}{2}\gamma \cos(\theta_L + \theta_R) v_1 v_2 u_1 u_2 \cos\varphi \cos\psi$$

The condition under which the spontaneous charge violation does not occur, i.e. the minimum V corresponds to $\varphi = 0,\ \psi = 0$, can be written in the form:

$$-\beta_L v_1 v_2 + \gamma u_1 u_2 > 0. \quad ; -\beta_R u_1 u_2 + \gamma v_1 v_2 > 0 \tag{21}$$

Inequalities (21) are fulfilled automatically for $\beta_L < 0, \beta_R < 0$

In order to find the low limits for the potential, let us use of the inequality: $-\gamma v_1 v_2 u_1 u_2 \ge -\frac{1}{2}(v_1^2 v_2^2 + u_1^2 u_2^2)\gamma$.

Then, one rearranges the terms of expression (20) in such a way that the combinations of the fields of the type $\frac{1}{12}\lambda_{1L} v_1^4 + \frac{1}{12}\lambda_{2R} u_2^4 + \frac{1}{4}\delta_{12} v_1^2 u_2^2$ be included. Then the condition of a lower limit for potential energy V can be written in the form:

$$\lambda_{1L} > 0,\ \lambda_{1R} > 0,\ \lambda_{2L} > 0,\ \lambda_{2R} > 0$$

$$\begin{cases} \lambda_{1L}\lambda_{2L} > 9/4(\alpha_L + \beta_L - \gamma)^2 \\ \lambda_{1L}\lambda_{1R} > 9/4\ \delta_1^2 \\ \lambda_{1L}\lambda_{2R} > 9/4\ \delta_{12}^2 \end{cases} \qquad \begin{cases} \lambda_{1R}\lambda_{2R} > 9/4(\alpha_R + \beta_R - \gamma)^2 \\ \lambda_{2L}\lambda_{2R} > 9/4\ \delta_2^2 \\ \lambda_{2L}\lambda_{1R} > 9/4\ \delta_{21}^2 \end{cases} \tag{22}$$

References

/1/ A.A.Belavin et al. Phys. Lett. 59B, 85, 1975.

/2/ R.O.Peccei, H.R.Quinn. Phys. Rev. Lett. 38, 1440, 1977; Phys. Rev. D16, 1791, 1977.

/3/ S.Weinberg, Phys. Rev. Lett. 40, 223, 1978.

/4/ F.Wilczek, Phys. Rev. Lett. 40, 279, 1978.

/5/ T.Goldman, C.H.Hoffman, Phys. Rev. Lett. 40, 220, 1978
W.A.Bardeen, S.H.Tye, Phys. Lett. 74B, 229, 1978
A.R.Zhitnitsky, Yu.I.Skovpen', Preprint INP 78-73, Novosibirsk.

/6/ T.W.Donnelly et al., Phys. Rev. D18, 1607, 1978.
I.Ellis, M.K.Gaillard, Phys. Lett. 74B, 374, 1978.
I.Kandaswamy et al. Phys. Lett. 74B, 377, 1978.
P.Alibram et al. Phys. Lett. 74B, 137, 1978.
T.Hanse et al. Phys. Lett. 74B, 139, 1978.

/7/ S.Weinberg, Phys. Rev. Lett., 37, 657, 1976.

Работа поступила - 24 апреля 1979 г.

Ответственный за выпуск - С.Г.ПОПОВ
Подписано к печати 14.9-1979г. МН 16506
Усл. 0,8 печ.л., 0,7 учетно-изд.л.
Тираж 150 экз. Бесплатно
Заказ № 81.

Отпечатано на ротапринте ИЯФ СО АН СССР