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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
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A b s t r a c t

To obtain high luminosity in colliding linear electron-positron beams the very dense singles bunches of particles and antiparticles must be collided. In addition, to provide the needed monochromaticity a bunch length must be about 1 cm. The possibility is studied to create the bunches of $\sim 10^{12}$ particles with a radial phase volume of $\sim 10^{-6}$ cm.rad in a 1 GeV storage ring for their injection into the accelerating structure.

The forces acting on the particles of a single bunch in the accelerating structure are calculated. It is shown that by selecting the longitudinal distribution of a bunch it is possible to extract a significant fraction of the energy stored in the accelerating structure at a comparatively high monochromaticity of the accelerated beam.

The transverse forces appearing upon deflection of the bunch from the centre of the system lead to instability of a bunch motion. A method is found to suppress this instability.

An analysis is made of the requirements for the placement precision of the accelerating structure and focusing lenses, which are necessary for obtaining the needed phase volume of the beam at its outlet.

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To obtain high luminosity ($L \sim 10^{32}$ per shot) an electron or positron bunch of $N \sim 10^{12}$ particles with a longitudinal dimension of $\ell \sim 1$ cm and a phase volume of $\mathcal{E} \sim 10^{-7}$ cm must be injected into the VLEPP accelerating structure.

In principle, a particle beam with these parameters may be produced in the storage ring by using radiation damping. It is known that the phase volume determined by the quantum fluctuations of synchrotron radiation strongly depends on the frequency of radial betatron oscillations ν. In a smooth approximation:

$$\mathcal{E} \simeq \lambda\!\!\!^{-} \frac{\bar{R}}{R} \frac{\gamma^2}{\nu^3} , \qquad (1)$$

where $\lambda\!\!\!^{-}$ - is the Compton wavelength of the electron, γ - the relativistic factor, $\bar{R}/R$ - the ratio of the average radius of the storage ring to the "magnetic" one. The required value of $\mathcal{E}$ is readily attained if, for example, $\gamma = 10^3$, $\nu = 10$, $\bar{R}/R = 2.6$.

However at very high beam density the effect of multiple particle scattering of a bunch (the Touschek effect) must be also taken into account /1/.

If the constribution of scattering is decisive,

$$\mathcal{E} \approx \left(\frac{3\sqrt{\pi}}{2} \frac{N r_e}{\gamma^6 \ell} \frac{R}{\bar{R}} \right)^{2/5} \cdot \frac{\bar{R}}{\nu} \left(\ln 137^{3/4} \left(\frac{r_e}{\bar{R}}\right)^{1/4} \left(\frac{\bar{R}}{R}\right)^{3/4} \frac{\gamma^{5/2}}{\nu^2} \right)^{4/5} , \qquad (2)$$

where r_e – is the classical radius of the electron and the coupling factor of the vertical and radial oscillations is taken to be equal to $1/2\pi$. Since the dependence of $\mathcal{E}$ on ν in the case (2) is much weaker compared to the case (1), the value of ν found from the equality condition (1) and (2) will be optimal:

$$\nu_{\text{опт}} \approx \left(\frac{2\cdot 137^{5/2}\, r_e^{3/2}\, \ell}{3\sqrt{\pi}\, N \bar{R}^{5/2}}\right)^{1/5} \left(\frac{\bar{R}}{R}\right)^{3/10} \gamma^{11/5} \left(\ln 137^{3/4} \left(\frac{r_e}{\bar{R}}\right)^{1/4} \left(\frac{\bar{R}}{R}\right)^{3/4} \frac{\gamma^{5/2}}{\nu^2}\right)^{-2/5}. \quad (3)$$

Assuming the validity of formula (2) we obtain for luminosity per shot:

$$L = \frac{K N^{8/5} \nu \cdot \gamma_i^{9/5} \gamma_f}{\bar{R}\ell} \left(\frac{1}{6\sqrt{2\pi}} \frac{\ell}{r_e} \frac{\bar{R}}{R}\right)^{2/5} \left(\ln 137^{3/4} \left(\frac{r_e}{\bar{R}}\right)^{1/4} \left(\frac{\bar{R}}{R}\right)^{3/4} \frac{\gamma^{5/2}}{\nu^2}\right)^{-4/5}, \quad (4)$$

where $K = 10$ is the longitudinal compression factor of the bunch prior to the extraction and the indices i and f denotes the values of γ -factor at the beginning and the end of acceleration.

Estimation for $\gamma_i = 4\cdot 10^3$, $\gamma_f = 2\cdot 10^5$, $\bar{R} = 2\cdot 10^3$ cm, $\bar{R}/R = 4$, $N = 10^{12}$, $\ell = 10$ cm, $\nu = 20$ ($\nu_{\text{опт}} = 16$) yields: $\mathcal{E} = 3\cdot 10^{-7}$ cm, $L = 0.84\cdot 10^{32}$ cm^{-2}. The beam dynamics in the accelerating structure are determined by the radiation field. Indeed, the total field in the structure can be represented as the sum of radiation fields proportional to the charge, and the accelerating field:

$$E = q E_{\text{изл.}} + E_{\text{ускор.}}$$

The energy extracted is

$$W = q \bar{E}_{\text{ускор}} - q^2 \bar{E}_{\text{изл.}},$$

where q – is the charge of the bunch, $\bar{E}_{\text{изл.}}$ – the average field amplitude radiated by a unit charge, and $\bar{E}_{\text{ускор.}}$ – the average accelerating field amplitude. Thus, to accelerate the largest possible charge in a given accelerating field, the radiated energy must be minimal, and since this energy is proportional to the number of diaphragms per unit length, a π -structure is optimal. The time pattern of the lines of force of the electric field of a charge traveling past a diaphragm, which was found by integration of Maxwell equations by means of the difference method according to Lax's scheme /2/, is shown in Fig.1.

Analysis of the fields for an extracted bunch shows that by selecting the longitudinal distribution of charge and the phase of inlet into the cavity it is possible to provide high energy monochromaticity. Thus, for a structure in the 5-cm range with an effective accelerating field amplitude of 1 MeV/cm and a charge of 10^{12} electrons distributed according to the law

$$q = q_0 \sin^2 \pi z/L$$

where $L = 1$ cm and the effective charge size is $2\sigma = 0.36$ cm, the relative energy spread is 1% for 90% of the particles. Here the extraction of stored energy is 18%, and the total amplitude is reduced by 17%.

A graph of the energy distribution along the bunch against the sinusoidal background of the accelerating field and the charge distribution along the bunch (the dot-dash line) are shown in Fig. 2a. It should be noted that for a charge equivalent to 4 x 10^{12} electrons distribututde, as in the previous case, with parameters $L = 2$ cm, the energy extraction is 45% at a 4% non-monochromaticity for 90% of the particles.

When the bunch is deflected from the axis of the structure nonsymmetric fields whose reverse reaction leads to the appearance of transverse forces are radiated. For small deflections we may confine ourselves to fields with a single variation in azimuth, and the field amplitude is proportional to the charge deflection.

The transverse force, averaged over the period of the structure, may be represented as the effective gradient

$$\bar{F} = \bar{G} \cdot \delta,$$

$$\bar{G}(z_0) = \frac{1}{a} \int_{-a/2}^{a/2} \left(E_z(0,z,t) - H_\varphi(0,z,t) \right) dz,$$

$$ct = z - z_0$$

where δ – is the charge deflection, a – the period of the structure, and z_0 – the coordinate in the bunch.

A graph of the gradient upon deflection of a bunch as a whole and the charge distribution are shown in Fig. 2b. A calculation showed that the magnitude of the gradient is proportional, to good accuracy, to the value of the relativistic dipole field at the edge of the aperture in the diaphragm, i.e., it is inversely proportional to the square of the radius of the aperture, and as in the case of longitudinal forces is proportional to the number of diaphragms per unit length of the structure. To reduce the value of the gradient a π-structure with large apertures in the diaphragm therefore must be used.

A characteristic feature of the action of transverse forces is the fact that the field radiated by some section of a bunch acts only on the following part, i.e., the magnitude of the

force acting on the "tail" is determined by the deflection of the "head". This interaction leads to a rapid increase in the deflection of the "tail" when there is a slight initial deflection of the "head", i.e., an instability is observed analogous to that of the sequence of bunches in the linear accelerators /3/.

Focusing with quadrupole lenses does not suppress the instability. In this case the "head" of a bunch acts on the "tail" with the frequency of free oscillations in the focusing field, the frequency being determined by the lens gradient and particle energy. This leads to a resonance buildup of the oscillations of the "tail". To suppress the resonance instability a spread over the frequencies of the oscillations must be introduced. This can be done by introducing a linear particle energy distribution over the length of the bunch. By starting from general concepts of the character of the instability it is possible to obtain the following estimate for the value of the needed spread $\Delta\lambda/\lambda$:

$$\frac{\Delta\lambda}{\lambda} \gtrsim \frac{eG}{2E} \left(\frac{\lambda}{2\pi} \right)^2,$$

where λ – is the wavelength of the oscillations, E – the particle energy.

A numerical calculation was made of the motion of particles of a bunch in the structure with allowance for transverse forces. The calculation confirmed the qualitative arguments. Shown in Fig.3 is a graph of the relative phase volume of a bunch at the exit from a 1000-m accelerator section as a function of the energy spread. As can be seen from the graph, the excited phase volume depends on the sign of the spread. It turns out that to suppress the instability, or more accurately, to prevent the action of transverse forces from resulting in an increase in the initial phase volume, an initial spread of $\pm 10\%$ must be introduced, and as acceleration proceeds the spread may be smoothy decreased to $\pm 3\%$ as the energy changes from 2 to 100 GeV, since the effect of transverse forces decreases with increasing energy. For the same reason the spread can be reduced in the final section to the minimum value with an insignificant increase in phase volume. For our calculation the magnetic structure with the increasing wavelength $\lambda \sim E^{0.35}$ was taken and the energy spread decreased with the energy $\sim E^{0.3}$, according to the above formula.

It should be noted that introducing a linear energy distribution along a bunch is not inconsistent with the requirement of high monochromaticity, since by having a uniform distribution it is possible to obtain the required linear distribution by shifting the phase of entry into the cavity.

An analysis was made of the influence of the imprecision of lens placement on the excited phase volume. With allowance for adiabatic damping in the structure with increasing distance ($\ell \sim \sqrt{E}$) between lenses, the phase volume is

$$\mathcal{E} = 8/\pi \left(\frac{\Delta X}{\ell_o} \right)^2 L_S \frac{\gamma_o}{\gamma_K},$$

where L_S - is the accelerator length, ℓ_o - the initial distance between lenses, and γ - the relativistic factor. For the parameters $L_S = 10^5$ cm, $\ell_o = 10^2$ cm, and $\gamma_K / \gamma_o = 50$, to obtain a phase volume $\mathcal{E} \sim 5 \times 10^{-9}$ cm·rad the tolerance for lens placement is $\Delta X \sim 1$ micron.

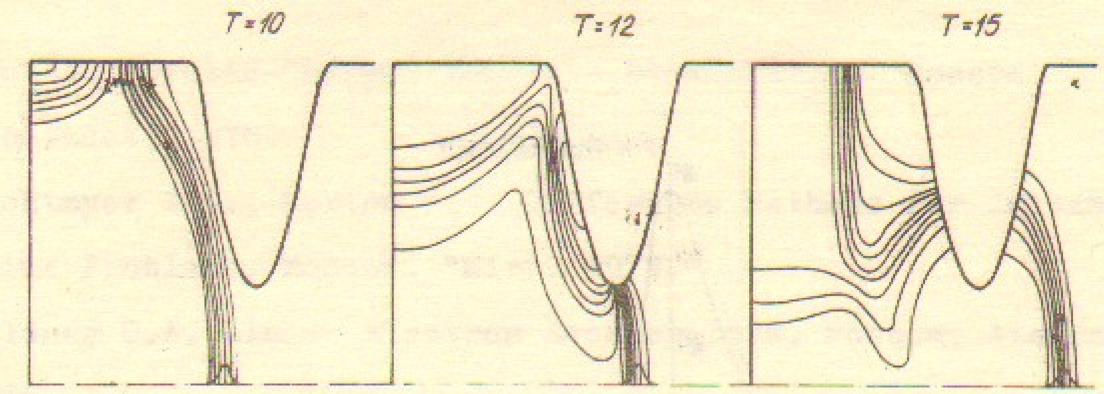

Fig.1 The time pattern of the lines of force of the electric field of a charge traveling past a diaphragm.

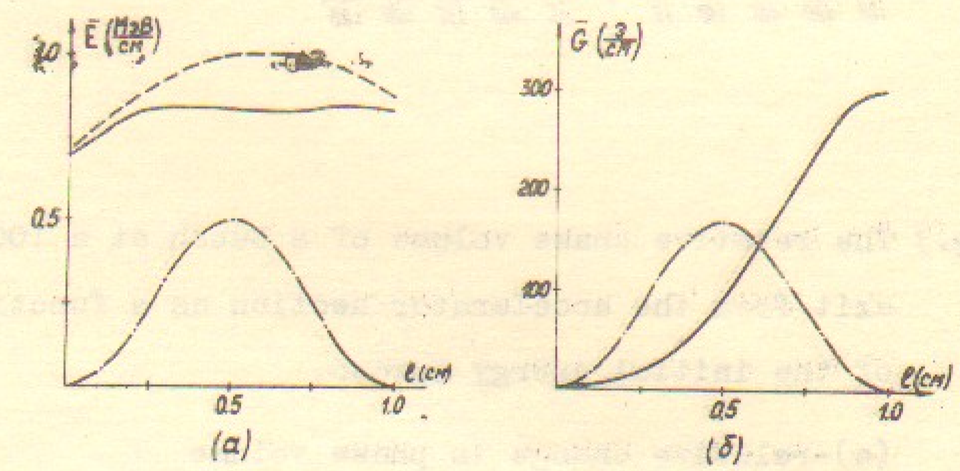

Fig.2 (a) The energy distribution of particles along the bunch and the sinusoid of the accelerating field (the dot line);

(b) The distribution of the gradient of transverse force. The charge distribution along the bunch (the dot-dash line).

1 - E (MeV/cm)

2 - G (Oe/cm^2)

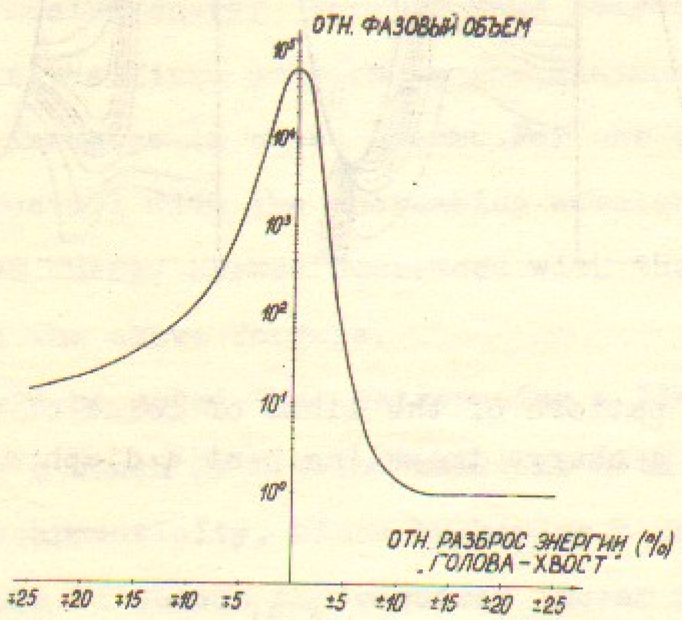

Fig.3 The relative phase volume of a bunch at a 100-GeV exit from the accelerator section as a function of the initial energy spread.

(a)-relative change in phase volume

(b) - "head-tail" relative energy spread, %

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