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ON RADIATION OF RELATIVISTIC POSITRONS AT CHANNELING

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A b s t r a c t

There have been calculated the spectrum and the intensity of radiation of a high energy positron at planar channeling in crystal with the use of realistic potential.

In recent years the radiation at channeling has been discussed in a series of papers. However, a simplified shape of the potential (usually oscillator or square well potential) has been used and the authors most often confined themselves to the simple estimates of radiation characteristics. Comparison of our results to those obtained earlier is given at the end of this paper. At present, the experimental study of the effect is extensively carried out and a quantitative comparison of the theory with the experiment has become urgent.

Classical description of positron motion in a channel is valid when the phase volume of transverse motion considerably exceeds $\hbar$. Actually, it is the case if $\sqrt{\gamma} \gg 1$ ($\gamma = \frac{\varepsilon}{mc^2}$, ε is the energy of the positron, m is its mass). Let the condition $\gamma U_0 \ll mc^2$ (U_0 is the potential depth) be fulfilled, which is true when $\gamma \ll 10^4$. In this case, for the particles incoming into the crystal at an angle ϑ (the angle between a particle momentum and the planes which build the channel) such that $\vartheta \lesssim \vartheta_{max}$ ($\vartheta_{max} = \sqrt{\frac{2U_0}{\varepsilon}}$), the transverse momentum $p_\perp = \frac{\varepsilon \vartheta}{c} \lesssim \sqrt{\frac{2\varepsilon U_0}{c^2}} \ll mc$. Hence, radiation is dipole. The estimation of the radiated photon energy yields

$$\hbar\omega_c \sim 2\gamma^2 \hbar\omega_0 \sim \varepsilon\, 2\pi \frac{\lambda_c}{d} \sqrt{\frac{2U_0}{mc^2}} \qquad (1)$$

where ω_0 is the characteristic oscillation frequency of the positron in the channel, d is the distance between the planes which build the channel, $\lambda_c = \frac{\hbar}{mc}$. In the region under consideration $\frac{\hbar\omega_c}{\varepsilon} \ll 1$, therefore, the radiation recoil can be neglected and the classical electrodynamics may be used. The dipole radiation under quasiperiodic motion (periodic in the system moving with the particle's average velocity) involving its spectral and polarization characteristics has been studied in detail in the authors paper /1/. The results of the paper /1/ can be used in the problem under study. But it should bear in mind that all expressions depend now on the angle ϑ and the income point x_0. This is due to dependence on the quantity $\varepsilon_\perp = V(x_0) + \varepsilon_0, \varepsilon_0 = \frac{\varepsilon\vartheta^2}{2}$ which at a given potential $V(x)$ determines the characteristics of motion. Since the value of x_0 cannot be, in principle, fixed, it is necessary to carry out the averaging over trajectories with different x_0. Furthermore, if the angular width of the positron beam $\Delta\vartheta \gtrsim \vartheta_{max}$, the averaging over the angle ϑ should be performed as well.

The motion of particles in the channel is determined by an interplane potential. Determination of this potential is a quite complicated problem (see, e.g., /2/). Here the potential of an isolated atom is taken from the Thomas-Fermi model with exponential drop at the distances larger than $a_0 = \frac{\hbar^2}{me^2}$. Then, the potential of the plane consisting of such atoms has been approximated by the exponent. In some sense, this procedure is analogous to that used for obtaining the Molière potential. The latter well describes multiple scattering and bremsstrahlung in solids. Finally, taking into account the potential of two adjacent planes, we have:

$$V(x) = V_0\left[ch\left(\frac{x}{a_s}\right) - 1\right], \quad -\frac{d}{2} \le x \le \frac{d}{2} \qquad (2)$$

For example, for the plane (110) in Si $V_0 \approx 5.9$ eV, $a_s \approx 0.38$ Å. In this case, $U_0 = V_0(ch\delta - 1) \approx 30$ eV, $\delta = d/2a_s \approx 2.5$. All the numerical results are given below for this value of δ.

The positrons for which $\varepsilon_\perp < U_0$ move in the channel (the index (c)), whereas the positrons for which $\varepsilon_\perp > U_0$ move above the barrier in a periodical field formed by a sequence of potentials (2) (correspondingly, the index (nc)).

The motion of a positron in the potential (2) and its radiation are described by elliptic integrals and functions. The period of oscillations is

$$T_c = T_0\sqrt{1-\kappa^2}\,K(\kappa) \qquad (3)$$

where $T_0 = \frac{4a_s}{c}\sqrt{\frac{\varepsilon}{V_0}}$, $\kappa^2 = \left(1 + \frac{2V_0}{\varepsilon_\perp}\right)^{-1}$, $K(\kappa)$ is the complete elliptic integral of the first kind. For Si, when $\varepsilon_\perp$ is varied within $0 \le \varepsilon_\perp \le U_0$, the period lies within the limits $1.115 \le \frac{T}{T_0} \le \frac{\pi}{2}$.

If the motion is determined, the radiation intensity is straightforwardly calculated. At a fixed $\varepsilon_\perp$ we have

$$I_c(\varepsilon_\perp) = I_p\frac{1}{9(1-\kappa^2)}\left[\frac{1+\kappa^2}{1-\kappa^2}\frac{E(\kappa)}{K(\kappa)} - 1\right] \qquad (4)$$

where $I_p = \frac{8e^2V_0^2\gamma^2}{a_s^2m^2c^3}$, $E(\kappa)$ is the complete elliptic integral of the second kind. Figure 1 presents the plot of intensities averaged over x_0: $\langle I_c\rangle(1)$, $\langle I_{nc}\rangle(2)$, $\langle I\rangle = \langle I_c\rangle + \langle I_{nc}\rangle(3)$, as a function of ε_0/U_0 $\left(\varepsilon_0 = \frac{\varepsilon\vartheta^2}{2}\right)$. Note, that intensity I_{nc} is also expressed through elliptic integrals. As seen, although with increase of ε_0, the number of particles $(\varepsilon_0 + V(x_0) < U_0)$ trapping into the channel is decreasing, $\langle I_c\rangle$ increases, that means that the particles with $\varepsilon_0 \sim U_0$ mainly radiate. If the positrons are distributed uniformly over the angle ϑ in the interval $|\vartheta| \le \vartheta_{max}$, the intensity averaged over ϑ is $\langle I_c\rangle = 0.32\, I_p$, $\langle I\rangle = 0.46\, I_p$. It is interesting to compare this value with energy losses at the bremsstahlung, we get $\frac{\langle I_c\rangle}{\langle I_{br}\rangle} \sim 10^{-4}\gamma$.

At a definite $\varepsilon_\perp$ the radiation spectrum is determined by the Fourier component of velocity (see /1/):

$$\int v_c e^{i\tilde{\omega} t} dt = 4\pi i a_s \, sh^{-1}\left[\frac{\pi}{2}(2\ell-1)\frac{K(\sqrt{1-\kappa^2})}{K(\kappa)}\right] \quad (5)$$

where it is taken into account that $\tilde{\omega} T_c = 2\pi(2\ell-1)$. In Fig.2 it is presented the spectral intensity distribution averaged over x_o, as a function of $\xi = \frac{\omega}{\omega_c}$ $(\omega_c = \frac{4\pi\gamma^2}{T_0})$ for the fixed income angle ($\frac{\varepsilon_o}{u_o}$ = 0.2 (1), $\frac{\varepsilon_o}{u_o}$ = 0.9 (2)), each of curves being normalized over 1. The total contribution is given of the particles into the cannel and those moving above the barrier. For the particles in the channel the dominating contribution is given by the first (ℓ = 1) harmonics (the total contribution of all the rest harmonics is less than 1%). The fracture at ξ = 1/1, 115$\simeq$0.9 corresponds to the boundary frequency for this harmonic. The peaks in Fig.2 are at the points where $\xi = \frac{T_o}{T_c}\Big|_{\varepsilon_\perp = \varepsilon_o}$, i.e. they are uniquely determined by the income angle. At $\xi > 0.9$ the spectrum is mainly determined by the particles moving above the barrier, a wide maximum at $\xi \simeq 1.8$ corresponds to their first harmonics. In Fig.3 it is given the spectral distribution of the total intensity, which is averaged over x_o and ϑ, the dotted line denotes the contribution of the particles moving above the barrier, the curve is normalized over 1. All the plots depend on δ, with increase of δ the spread of periods in the channel increases and the contribution of highest harmonics grows. Since the ratio $\frac{d\langle I\rangle/d\omega}{d\langle I_{b2}\rangle/d\omega}\Big|_{\omega\sim\omega_c}$ essentially exceeds 1 already at an energy of 1 GeV (the maximum in the spectrum of Fig.3 lies then at $\hbar\omega$ = 3.5 MeV), so just this characteristic is most convenient for experimental separation of the effect.

The radiation of positrons in the channel has been discussed in various papers (see Refs.[3-5]and the cited references). In /3/ the oscillator potential is used wherein the specific features of the spectrum calculated above disappear. In /3/ the averaging over x_o was not performed and the particles moving above the barrier were not been taken into account. In /4/ the square well potential was used where the radiation spectrum differs, even qualitatively, from that obtained in the present paper and only simple estimates were made. In /5/ adequate formulation of the problem is given, however, the authors write only formal (evident enough) relations, while the explicit results have been obtained only for intensities at ε_o = 0. Moreover, actually it was assumed that $\delta \gg 1$, but according to our estimates, in the most cases $\delta \sim 2$.

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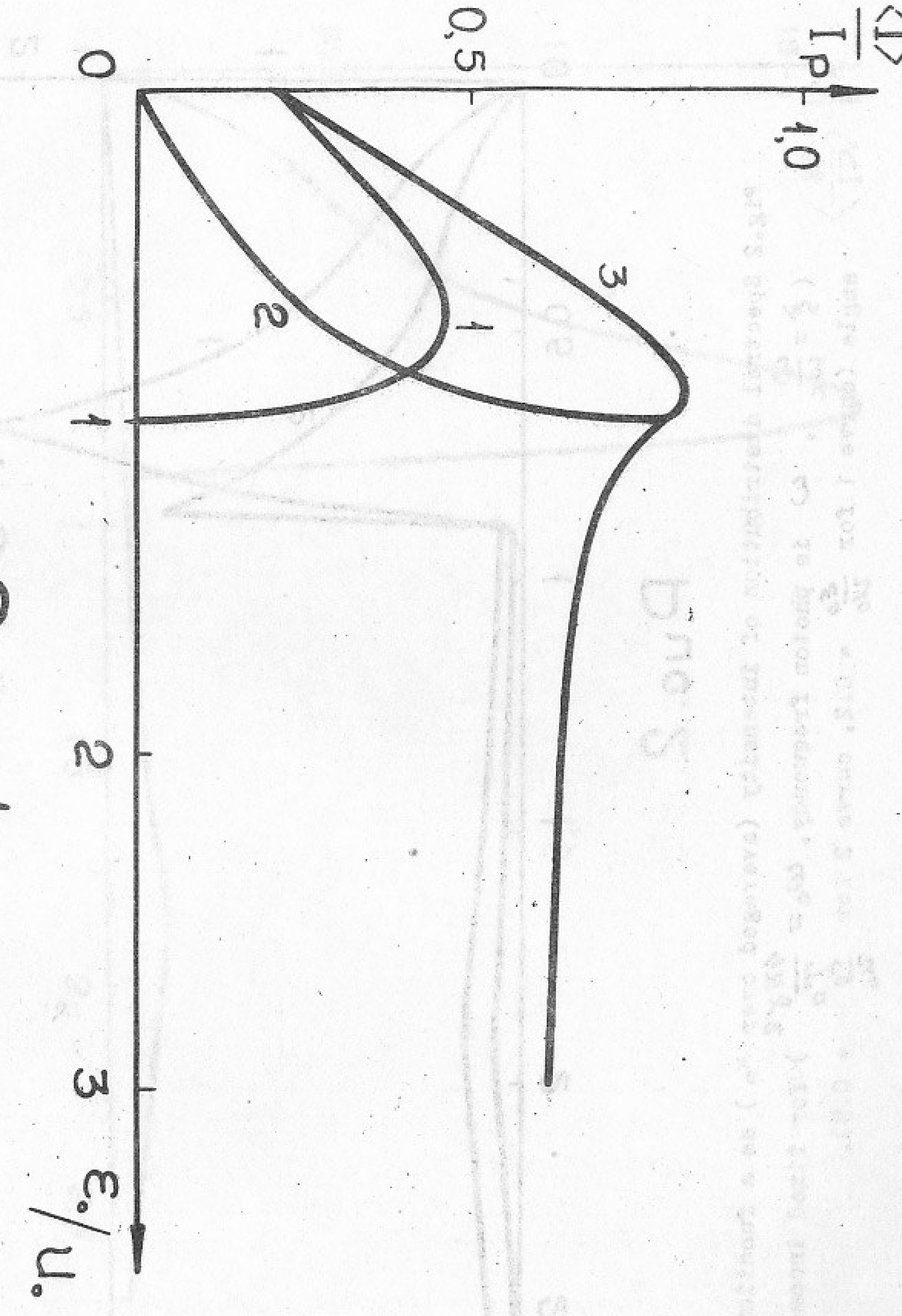

Рис. 1

Fig.1 Intensity of radiation (averaged over x_0) as a function of ε_0/U_0 $(\varepsilon_0 = \frac{\varepsilon\theta^2}{2})$ curve 1 is $\langle I_c \rangle$, curve 2 is $\langle I_{nc} \rangle$, curve 3 is $\langle I \rangle = \langle I_c \rangle + \langle I_{nc} \rangle$

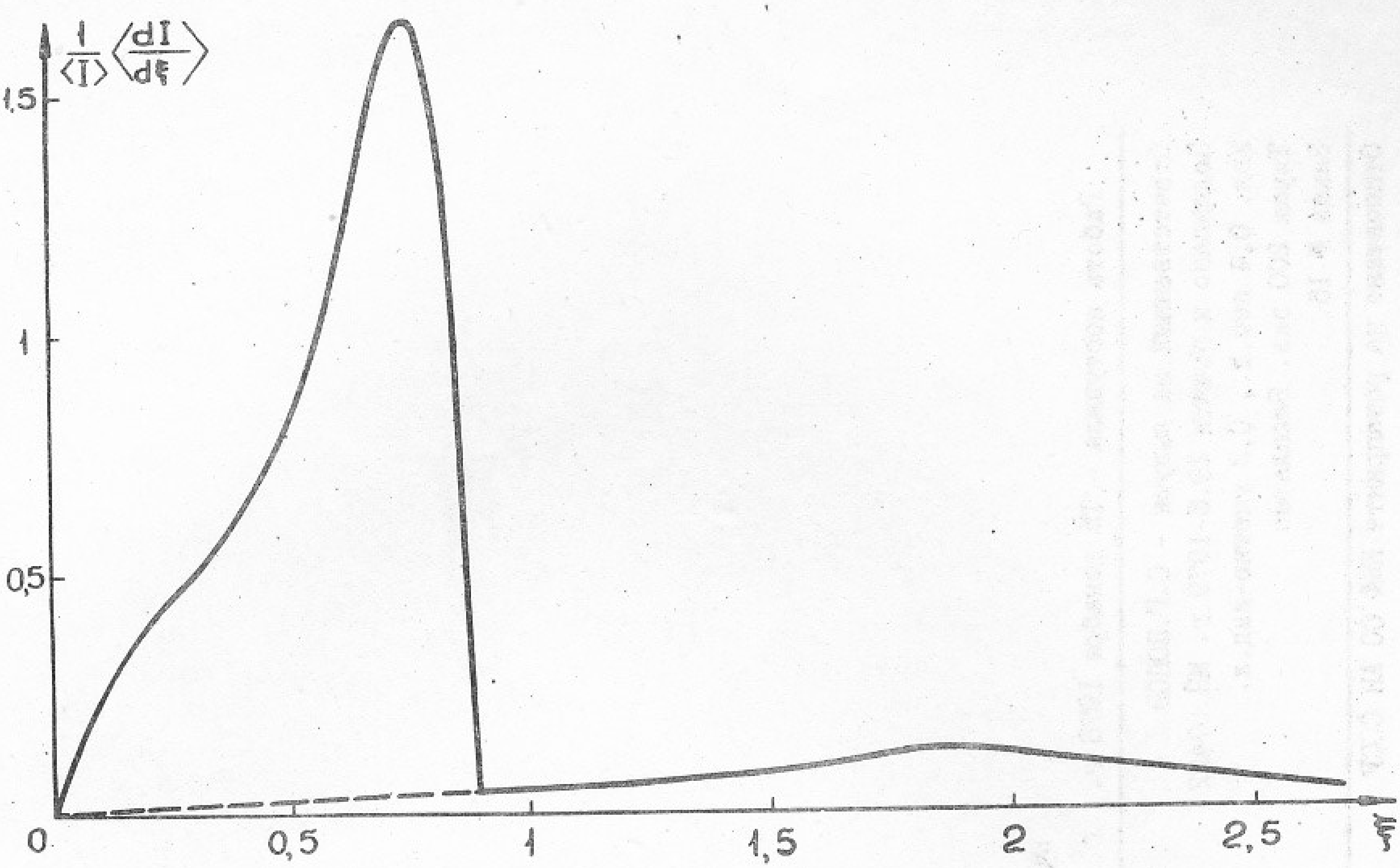

Рис. 3

Fig.3 Spectral distribution of intensity averaged over x_0 and income angle ϑ.

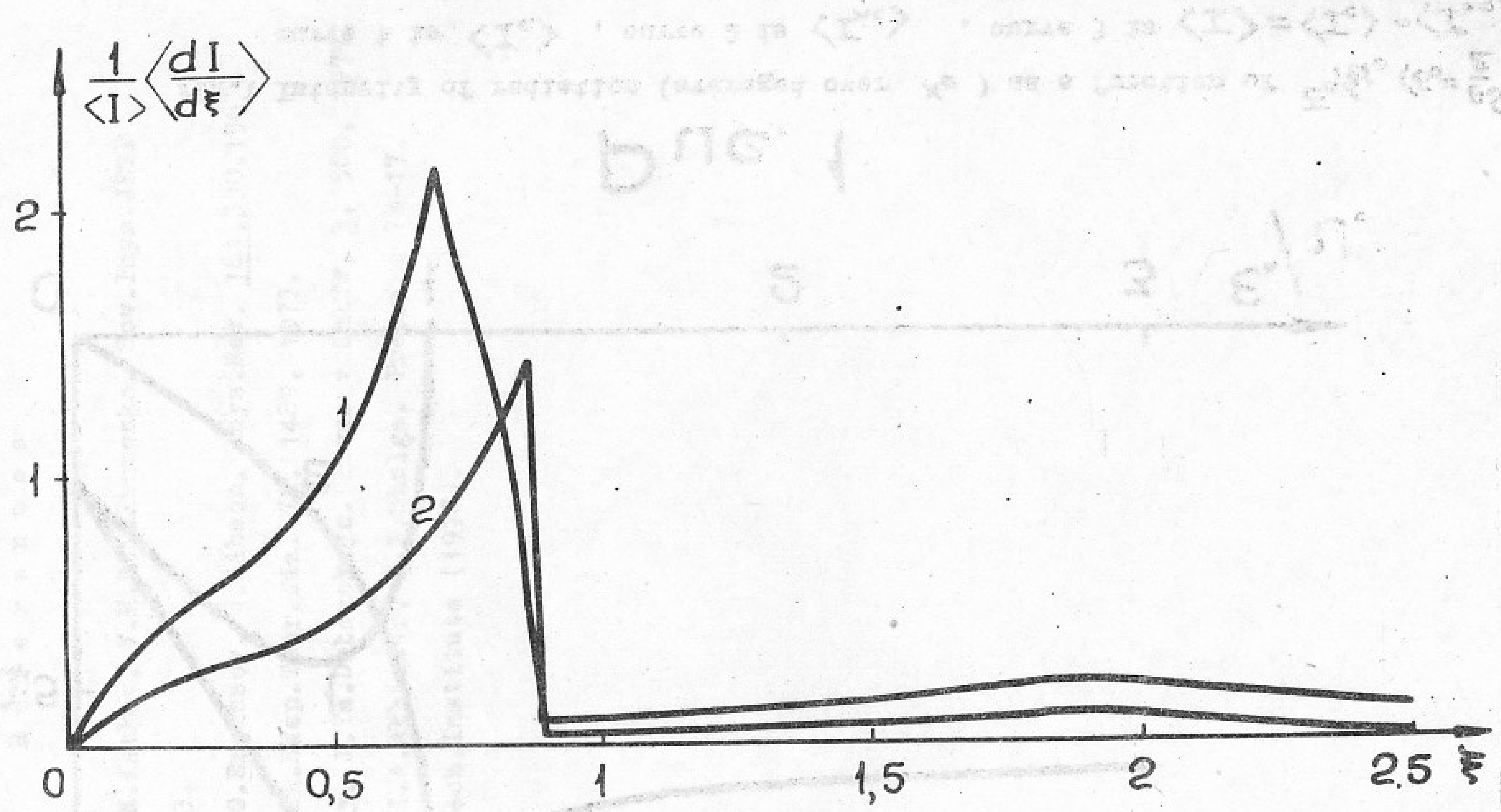

Рис. 2

Fig.2 Spectral distribution of intensity (averaged over x_0) as a function of ξ ($\xi = \frac{\omega}{\omega_c}$, ω is photon frequency, $\omega_c = \frac{4\pi\gamma^2}{T_0}$) for fixed income angle (curve 1 for $\frac{\varepsilon_0}{u_0}$ = 0.2, curve 2 for $\frac{\varepsilon_0}{u_0}$ = 0.9).

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