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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
СО АН СССР

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ПРЕПРИНТ И Я Ф 78-79

Новосибирск

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1. Introduction

The inverse scattering transform method enabled one to analyse in detail a wide class of nonlinear equations from various domains of physics (see reviews /1-3/). Particularly, the relativistic-invariant equations being of interest in the field theory have been recently studied. Among them are the two-dimensional space-time models, namely: the sine-Gordon equation /4-6/, the massive Thirring model /7/, the theory of chiral fields /8/ and the others. The inverse scattering transform method is also applicable to the relativistic-invariant equations in the four-dimensional space-time /9/.

An initial point of all these papers is: an equation possesses the Lorentz invariance; the solutions of this equations are covariant under the Lorentz group too. However, all intermediate steps and formulae possess no Lorentz invariancy, or their Lorentz-invariancy is far from to be clear. There is no doubt that such a version of the solution method at which the Lorentz-invariancy is conserved at each step (and moreover, this invariancy is manifest) is urgent.

In this paper it is proposed a relativistic-invariant formulation of the inverse scattering transform (IST) method. As examples, the sine-Gordon model and the massive Thirring model in the two-dimensional space-time are considered. The basic steps and the formulae of the IST method - the linear problem, the direct and inverse problems, the triangular representation, the Gelfand-Levitan-Marchenko formulae, etc. are written in the Manifestly Lorentzinvariant form, that provides the Lorentz-invariancy of the results obtained. Particularly, all integrals of motion can be obtained as tensors of different range.

The possibility to formulae the IST method in a relativistic-invariant fashion is due to the following. First, the initial nonlinear relativistic-invariant equation can be related to the relativistic-invariant linear problem:

$$T_\mu \Phi = 0 \qquad (1.1)$$

where the differential (matrix) operator T_μ $(\mu=0,1)$ is a vector under the Lorentz group in the two-dimensional space-time. The relativistic-invariant condition of commutativity

$$[T_\mu, T_\nu] = 0 \qquad (1.2)$$

is equivalent to the initial equation. Second, the spectral parameter in the linear problem (1.1) possesses the definite transformation properties under the Lorentz group. For the sine-Gordon equation the spectral parameter is a vector, and for the massive Thirring model - a two-component spinor under the Lorentz group[1]. Third, the surface $X_0 = t = const$ may be replaced by an arbitrary space-like one thorouhout.

In the next sections we shall use the notation and the conditions adopted in Refs. /5-7/. For this reason, all the intermediate formulae and constructions are omitted.

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1) The fact that the spectral parameter and operators T_0, T_1 in the massive Thirring model is transformed under the Lorentz transformations was pointed out in Ref. /7/. For the general case see /10/.

For sine-gordon equations linear problem in the form $T\Phi = 0$, $X\Phi = 0$ is also used in review paper of L.D.Faddeev and V.E.Korepin "Quantum theory of solitons in Phys. Reps, 42C 1, (1978)

II. The Sine-Gordon Model

For the sine-Gordon equation

$$\Box \varphi(x) + m^2 \sin \varphi(x) = 0 \qquad (2.1)$$

the operator T_μ has the form[2]:

$$T_\mu = i\frac{\partial}{\partial x^\mu} + \frac{1}{4}\epsilon_{\mu\nu}\frac{\partial\varphi}{\partial x^\nu}\sigma_1 + \frac{1}{2}\cos\varphi/2\, a_\mu \sigma_2 + \frac{1}{2}\sin\varphi/2\, \epsilon_{\mu\nu} a_\nu \sigma_3, \qquad (2.2)$$

where $\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices, $\epsilon_{\mu\nu}$ is the anti-symmetric tensor ($\epsilon_{01} = 1$). Here and below the indexes $\mu, \nu, \rho, \ldots$, are Lorentz ones ($\mu, \nu, \rho, \ldots = 0,1$) and the summation is performed over the repeated indexed. The vector a_μ being independent of co-ordinates satisfies the condition

$$a_\mu a_\mu = m^2 \qquad (2.3)$$

The vector a_μ plays a role of spectral parameter. By Eq. (2.3) the latter may be represented as follows:

$$a_0 = \frac{m}{2}\left(\frac{1}{\lambda} + \lambda\right), \quad a_1 = \frac{m}{2}\left(\frac{1}{\lambda} - \lambda\right), \qquad (2.4)$$

where λ is an arbitrary number. The number λ is a spectral parameter in the noninvariant formulations of the IST method for Eq. (2.1) /5,6/.

Let us consider the direct problem for Eqs. (1.1). The involution relation follows from the form of the operator T_μ and is the following:

$$\bar{\Phi}(x,a) = \sigma_2 \overset{*}{\Phi}(x,a). \qquad (2.5)$$

The asymptote of Φ is determined by the behaviour of T_μ at $x_\mu x_\mu \equiv x^2 \to -\infty$. One assumes that $\varphi(x) \to 0$ at $x^2 \to -\infty$. As a result, we have the equation for asymptote e

$$\left(i\frac{\partial}{\partial x^\mu} + \frac{1}{2} a_\mu \sigma_2\right) e = 0 \qquad (2.6)$$

This equation has two linearly-independent solutions:

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2) Note that Eqs. (1.1), (1.2) are invariant under the SL(2) group of transformations $\varphi \to \varphi' = S\varphi$, $T_\mu \to T_\mu' = S T_\mu S^{-1}$. Thus, the whole class of operators T_μ corresponds to Eq.(2.1).

$$e=\begin{pmatrix}\exp(-\frac{i}{2}a_\mu x_\mu)\\ i\exp(-\frac{i}{2}a_\mu x_\mu)\end{pmatrix},\quad e'=\begin{pmatrix}\exp\frac{i}{2}a_\mu x_\mu\\ -i\exp\frac{i}{2}a_\mu x_\mu\end{pmatrix}=-\bar{e} \tag{2.7}$$

Let us form the matrix

$$E(x,a)=(e,-\bar{e}). \tag{2.8}$$

Next, one defines the matrix solutions F and G of Eq.(1.1), by their asymptotics on space-like infinities (at real a_μ)

$$\begin{aligned} F(x,a)&\to E(x,a) \quad \text{at } x^2\to-\infty,\ x_1>0 \\ G(x,a)&\to E(x,a) \quad \text{at } x^2\to-\infty,\ x_1<0 \end{aligned} \tag{2.9}$$

At a real a_μ the matrices-solutions F and G are fundamental and, therefore, there exists such a matrix T (the transition matrix) that

$$F(x,a)=G(x,a)\,T.$$

There is no difficulty to see that $\det F=\det G=\det E=-2i$

Whence

$$\det T=1.$$

Then, using a number of relations for F, G, E, T, it is possible to show that the transition matrix is representable in the form:

$$T=\begin{pmatrix}A(x,a), & -B^*(x,a)\\ B(x,a), & A^*(x,a)\end{pmatrix} \tag{2.10}$$

The following relations take place as well:

$$A^*(x,-a)=A(x,a),\quad B^*(x,-a)=-B(x,a) \tag{2.11}$$

and

$$A=\frac{1}{2i}\det\begin{pmatrix}F_{11}, & G_{12}\\ F_{21}, & G_{22}\end{pmatrix}. \tag{2.12}$$

The dependence of A and B on co-ordinates can be also found from the form of T_μ :

$$A(x,a)=A(0,a),\quad B(x,a)=\exp(-\tfrac{i}{2}a_\mu x_\mu)B(0,a). \tag{2.13}$$

Thus, A depends only on the spectral parameter and, as known, just this determines the existence of an infinite set of the integrals of motion in the problem.

Using the relations (2.9), (2.11), and (2.12), it is easy to identify the analytical properties of F, G and $A(a_\mu)$, as well as the zero positions of A ($A(a^i_\mu)=0;\ i=1,\dots,N$). Just as in Refs. /6,7/, the scattering data are introduced, i.e. the set:

$$S=\{R(x,a),\ a^i_\mu,\ m^i_\mu(x);\ i=1,\dots,N\}$$

where

$$R(x,a)=\frac{B(x,a)}{A(a)},\quad m^i_\mu(x)=\frac{1}{i}\,b_i\Big/\frac{\partial A(a^i)}{\partial a^\mu}.$$

The quantities $A(a)$ and $B(x,a)$ are restored completely from scattering data /5,6/.

For solutions F and G one can write the following integral representations ('triangular' ones):

$$\begin{aligned} F(x,a)&=E(x,a)+\int d\sigma_\mu\,\theta(y_1-x_1)\,\hat{K}_\mu(x,y)\,E(y,a)\\ &\quad+(a_\mu-\epsilon_{\mu\nu}a_\nu)\int d\sigma_\mu\,\theta(y_1-x_1)\,\hat{K}(x,y)\,E(y,a), \end{aligned} \tag{2.14}$$

$$\begin{aligned} G(x,a)&=E(x,a)+\int d\sigma_\mu\,\theta(x_1-y_1)\,\hat{L}_\mu(x,y)\,E(y,a)\\ &\quad+(a_\mu-\epsilon_{\mu\nu}a_\nu)\int d\sigma_\mu\,\theta(x_1-y_1)\,\hat{L}(x,y)\,E(y,a), \end{aligned} \tag{2.15}$$

where integration is carried out over the space-like hypersurface σ : $(x_0-y_0)^2-(x_1-y_1)^2<0$. The relativistic invariance of the representations (2.14) and (2.15) is guaranted by the fact that the kernels $\hat{K}_\mu$, $\hat{L}_\mu$ are the vectors, and the kernels $\hat{K}$ and $\hat{L}$ are the scalars under the Lorentz group, and also by the Lorentz invariance of a function $\theta(z_1)$ ($\theta(z_1)$ = 1 at $z_1>0$, $\theta(z_1)=0$ at $z_1<0$) for space-like intervals $z_0^2-z_1^2<0$. Choosing the hypersurface $x_0-y_0=t=const$ as σ , one arrives at triangular representations of Refs. /5,6/.

Matrix kernels $\hat{K}_\mu$ and $\hat{K}$ may be represented through scalar kernels as follows:

$$\hat{K}_\mu = \begin{pmatrix} K_\mu & 0 \\ 0 & K_\mu^* \end{pmatrix}, \quad \hat{K} = \begin{pmatrix} 0 & K \\ K^* & 0 \end{pmatrix}.$$

Among the consistency conditions of equations (1.2) with representations (2.14), (2.15) there is a relation expressing $\varphi(x)$ through the kernel K explicitly:

$$\varphi(x) = -i \ln \frac{1+2i\,K^*(x,x)}{1-2i\,K(x,x)}.$$

The equations of the inverse scattering problem enable one to restore the kernels K_μ and K, and also the solution $\varphi(x)$ of Eq.(2.1) from scattering data, using linear integral equations. In the relativistic-invariant form these equations (Gelfand-Levitan-Marchenko ones) look as follows:

$$K_\mu(x,y) + P_\mu(x+y) + \int d\sigma_\varsigma\, \theta(z_1-x_1) K_\varsigma(x,z) P_\mu(z+y) + \int d\sigma_\mu\, \theta(z_1-x_1) K(x,z) P(z+y) = 0, \tag{2.16}$$

$$K(x,y) - P(x+y) - \int d\sigma_\mu\, \theta(z_1-x_1) K_\mu(x,z) P(z+y) + \int d\sigma_\mu\, \theta(z_1-x_1) K(x,z) P_\mu(z+y) = 0, \tag{2.17}$$

where integration is performed over a space-like hypersurface $(z_0-x_0)^2-(z_1-x_1)^2<0$, and the scalar kernel $P(z)$ and the vector kernel $P_\mu(z)$ are constructed through scattering data by the formulae:

$$P(z) = \int d^2a\, \delta(a_\mu a_\mu - m^2) R(z,a) + \sum_{i=1}^{N} m_\mu^i(z)(g_{\mu\nu}-\epsilon_{\mu\nu}) a_\nu^i,$$

$$P_\mu(z) = \frac{1}{i}(g_{\mu\nu}-\epsilon_{\mu\nu})\frac{\partial}{\partial z^\nu} P(z) = \int d^2a\, \delta(a_\tau a_\tau - m^2) \times \tag{2.18}$$

$$\times (g_{\mu\nu}-\epsilon_{\mu\nu}) a_\nu R(z,a) + \sum_{i=1}^{N} m_\varsigma^i(z)(g_{\mu\lambda}-\epsilon_{\mu\lambda})(g_{\varsigma\nu}-\epsilon_{\varsigma\nu}) a_\nu^i a_\lambda^i.$$

Analogous expressions may be also written for the kernels L_μ and L.

Using the factorization of the kernels K_μ and K

$$K_\mu(x,y) = \sum_{j=1}^{N} \tilde{K}_\mu(x) \exp \tfrac{i}{2} a_\nu^i y_\nu; \quad K(x,y) = \sum_{j=1}^{N} \tilde{K}(x) \exp \tfrac{i}{2} a_\nu^i y_\nu$$

N - soliton solutions may be derived in a standard fashion. The single-soliton solution has the form:

$$\varphi(x) = 4\, \mathrm{arctg} \{ \exp(-\varepsilon(p_0)\, \epsilon_{\mu\nu} p_\mu x_\nu + const) \}$$

where $\varepsilon(p_0) = \frac{p_0}{|p_0|}$, p_μ is the momentum of soliton with a mass M $(p_\mu p_\mu = M^2)$.

The angle-action-type variables and the traces identities leading to integrals of motion /5,6/ can be also written in the relativistic-invariant form. In the relativistic-invariant approach the recurrent formulae for conserving quantities make it possible to derive them as tensors of different ranges directly. Both this question and a number of other questions dealing with the relativistic-invariant formulation of the IST method will be studied elsewhere. Here we give only the expressions of integrals of motion $I_{\mu_1 \cdots \mu_n}$ via the action variable $\varsigma(a)$:

$$I_{\mu_1\cdots\mu_n} = \int d^2a\, \delta(a_\nu a_\nu - m^2) a_{\mu_1} \cdots a_{\mu_n} \varsigma(a) + \sum_{i=1}^{N_1} p_{\mu_1}^i \cdots p_{\mu_n}^i + \sum_{i=1}^{N_2} \pi_{\mu_1}^i \cdots \pi_{\mu_n}^i \qquad (n=1,3,5,\ldots) \tag{2.19}$$

The integral in expression (2.19) corresponds to the contribution from the continuous spectrum (of the particle with a mass m); the first sum - to the contribution of solitons (p_μ^i is the momentum of soliton), the second sum - to the double solitons (π_μ^i is the momentum of double soliton). The expression (2.19) makes the particle spectrum interpretation proposed in Ref. /6/ more obvious.

And finally, let us present the invariant form of Backlund transformation $\varphi \to \varphi'$, which plays, as known, an important role in the theory of eq. (2.1):

$$\frac{\partial \varphi'}{\partial x^\mu} + \epsilon_{\mu\nu} \frac{\partial \varphi}{\partial x^\nu} = 2 \frac{m}{M} p_\mu \cos \varphi'/2 \cdot \sin \varphi/2 - 2 \frac{m}{M} \epsilon_{\mu\nu} p_\nu \sin \varphi'/2 \cdot \cos \varphi/2$$

It is seen from the above writing that the parameter involved in Backlund transformation is transformed as a vector under the Lorentz transformations.

III. The Massive Thirring Model

For the classical massive Thirring model described by the equation

$$i\gamma_\mu \frac{\partial \psi}{\partial x^\mu} - m\psi - g\gamma_\mu \psi \cdot \bar{\psi}\gamma_\mu \psi = 0$$

the operator T_μ has the form[3]:

$$T_\mu = i\frac{\partial}{\partial x^\mu} + \frac{\sqrt{g}}{4}(\bar{\psi}\gamma_\mu \xi + \bar{\xi}\gamma_\mu \psi)\sigma_1 + \frac{i\sqrt{g}}{4}(\bar{\psi}\gamma_\mu \xi - \bar{\xi}\gamma_\mu \psi)\sigma_2 \qquad (3.1)$$

$$+ \frac{1}{4}(2\bar{\xi}\gamma_\mu \xi - g\bar{\psi}\gamma_\mu \psi)\sigma_3 ,$$

where γ_μ are the two-dimensional Dirac matrices ($\gamma_0 = \sigma_1$, $\gamma_1 = i\sigma_2$, $\gamma_5 = \gamma_0\gamma_1$; $\bar{\psi} = \psi^+\gamma^0$). The spectral parameter ξ is a two-component spinor under the Lorentz group and satisfies the relation

$$\bar{\xi}\gamma_\mu \xi \cdot \bar{\xi}\gamma_\mu \xi = m^2 \qquad \text{or} \qquad \bar{\xi}\xi = m . \qquad (3.2)$$

In virtue of (3.2) the spinor ξ may be presented in the form

$$\xi = \sqrt{\frac{m}{2}}\begin{pmatrix} \lambda^{-1} \\ \lambda \end{pmatrix}$$

In Refs. /7/ the parameter λ is used as a spectral parameter.

Let us consider the direct and inverse problems for Eqs. (1.1) with operator (3.1). $\psi(x)$ is assumed to go to zero quite fastly at $x^2 \to -\infty$.

As in the foregoing section, one defines the fundamental solutions F and G with asymptotics:

$$F(x,\xi) \to E^+(x,\xi) = \begin{pmatrix} 0 \\ \exp(-\frac{i}{2}\bar{\xi}\gamma_\mu \xi \cdot x_\mu) \end{pmatrix} \quad \text{at } x^2 \to -\infty ,\ x_1 > 0$$

$$G(x,\xi) \to E^-(x,\xi) = \begin{pmatrix} \exp \frac{i}{2}\bar{\xi}\gamma_\mu \xi \cdot x_\mu \\ 0 \end{pmatrix} \quad \text{at } x^2 \to -\infty \ x_1 < 0 . \qquad (3.3)$$

[3] Similarly to the sine-Gordon equation, the operator T_μ is not single with an accuracy to transformations $T_\mu \to T_\mu' = S T_\mu S^{-1}$ where $S \in SL(2)$.

Following Ref. /7/ and taking into account the considerations involved in relativization, it is possible to introduce the transition matrix in a Lorentz-invariant fashion and to show that its diagonal elements are independent of the coordinates x_μ. One can also identify the analytical properties of solutions F and G, and the transition matrix, introduce the scattering data and find their dependence on x_μ. These questions will be studied in another paper in more detail. Here we give the basic equations only.

The triangular representation for solution F has the following form in the relativistic-invariant approach:

$$F(x,\xi) = \exp(-\tfrac{i}{2}Q(\sigma))E^+(x,\xi) + \int d\sigma_\mu\, \theta(y_1 - x_1) \times$$

$$\times\left(\bar{\xi}\gamma_\mu \hat{K}(x,y) + \hat{M}_\mu(x,y) + (\bar{\xi}\gamma_\mu \xi - \bar{\xi}\gamma_\mu\gamma_5 \xi)\hat{M}(x,y)\right)E^+(y,\xi) , \qquad (3.4)$$

where $Q(\sigma) = \int d\sigma_\mu\, \theta(y_1 - x_1)\bar{\psi}(y)\gamma_\mu \psi(y)$, σ is the space-like hypersurface, the matrix kernels $\hat{K}, \hat{M}_\mu, \hat{M}$ are the two-component spinor, the vector, and the scalar under the Lorentz group, respectively.

The matrix kernels $\hat{K}, \hat{M}_\mu, \hat{M}$ can be expressed through scalar kernels K, M_μ, and M :

$$\hat{K} = \begin{pmatrix} 0 & K^* \\ K & 0 \end{pmatrix}, \quad \hat{M}_\mu = \begin{pmatrix} M_\mu & 0 \\ 0 & M_\mu^* \end{pmatrix}, \quad \hat{M} = \begin{pmatrix} M & 0 \\ 0 & M^* \end{pmatrix} \qquad (3.5)$$

Particularly, substituting (3.4) and (3.5) into eq. (1.1), one gets:

$$\psi(x) = 2\exp\left\{2i\gamma_5 \int d\sigma_\mu\, \theta(y_1 - x_1)\bar{K}(y,y)\gamma_\mu K(y,y)\right\} \cdot \gamma_5 K(x,x) .$$

The Gelfand-Levitan-Marchenko equations have the following form:

$$M(x,y) = \int d\sigma_\mu\, \theta(z_1 - x_1)\bar{K}(x,z)\gamma_\mu \cdot P(z+y),$$

$$M_\mu(x,y) = \int d\sigma_\nu\, \theta(z_1 - x_1)\left\{\bar{K}(x,z)\gamma_\nu(1+\gamma_5)P_\mu^-(z+y) + g_{\mu\nu}\bar{K}(x,z)(1-\gamma_5) \cdot P(z+y)\right\}, \qquad (3.6)$$

$$K(x,y) + \exp(-\tfrac{i}{2}Q(\sigma))P(x+y) + \int d\sigma_\mu\, \theta(z_1 - x_1)\left\{M_\mu^*(x,z)P(z+y) + \right.$$

$$\left. + \tfrac{1}{2}M^*(x,z)(1+\gamma_5)P_\mu^+(z+y) + \tfrac{1}{2}M^*(x,z)(1-\gamma_5)\gamma_\mu P(z+y)\right\} = 0$$

where the spinor kernel $P(z)$ and the spin-vector kernels $P^{\pm}_{\mu}(z)$ are expressed via scattering data ($R(z,\xi)=\frac{B(z,\xi)}{A(\xi)}$, ξ_i, m_i) as follows ($d\xi \equiv d\xi_1 d\xi_2$):

$$P(z)=\frac{1}{8\pi i}\int d\xi\, \delta(\bar{\xi}\xi-m)\, \xi\, R(z,\xi)-\frac{1}{2}\sum_{i=1}^{N} m_i(z)\Big/\frac{\partial A(\xi_i)}{\partial \xi},$$

$$P^{\pm}_{\mu}(z)=\frac{1}{i}\left(g_{\mu\nu}\pm\epsilon_{\mu\nu}\right)\frac{\partial}{\partial z^{\nu}}P(z).$$

Equations (3.6) offer, as known, the possibility to find N-soliton solutions of eq. (3.1). One can also find an infinite set of conserving tensor quantities, to introduce the angle action variables in an invariant fashion. The conserving tensors expressed through the action-type variables $\rho(\xi)$, P^{i}_{μ} have the form:

$$I_{\mu_1\dots\mu_n}=\int d\xi\,\delta(\bar{\xi}\gamma_{\mu}\xi\cdot\bar{\xi}\gamma_{\mu}\xi-m^2)\,\bar{\xi}\gamma_{\mu_1}\xi\cdots\bar{\xi}\gamma_{\mu_n}\xi\cdot\rho(\xi)+\sum_{i=1}^{N}P^{i}_{\mu_1}\cdots P^{i}_{\mu_n} \qquad (n=1,2,3,\dots) \tag{3.7}$$

IV. Conclusion

The manifestly relativistic-invariant formulation of the IST method can be given also for other relativistic-invariant equations in the two-dimensional space-time. As seen from formulae of the foregoing sections, the specificity of two-dimensionality is essentially used.

However, the IST method may be formulated for the relativistic-invariant equations in the four-dimensional space-time (for example, for the equations of self-duality /9/) in a relativistic invariant fashion as well. In the relativistic-invariant formulation of the IST method the spectral parameter, as seen from four-dimensional analogues of formulae (2.19) and (3.7), is transformed on the basis of a non-scalar representation of the Lorentz group.

In conclusion note that in the general case of an equation invariant under some group G, the IST method may be formulated in the form manifestly invariant under the group G, the spectral parameter possessing definite transformation properties under the group G /10/.

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Работа поступила - 2I сентября I978 г.

Ответственный за выпуск - С.Г.ПОПОВ
Подписано к печати 20.X-I978 г. МН 07739
Усл. I,0 печ.л., 0,8 учетно-изд.л.
Тираж 200 экз. Бесплатно
Заказ № 79.

Отпечатано на ротапринте ИЯФ СО АН СССР