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High Energy Multiple Production of Hadrons and Landau Hydrodynamical Theory.

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Abstract.

In the present paper the following topics are discussed: 1) The inclusive and exclusive particle spectra; 2) The account of the interaction in the equation of state; 3) The role of the leading particles; 4) The multiplicity distribution; 5) Collisions with nuclei; 6) e^+e^- annihilation into hadrons. In all these cases the predictions of the hydrodynamical model are in good agreement with data.

The multiple production of hadrons in high energy collisions have been very intensely studied during the last several years. A lot of efforts has been made in order to reveal the most significant features of this complicated phenomenon and to constract some phenomenological models fitting the whole lot of data available. Although some questions still remain , the general picture of the hadronic production in various reactions and rather wide energy range seems to be now clarified due to this work. But the theoretical understanding of the phenomenon is not so advanced, it is still incomplete and controversial. So there is great need for theories which can provide explanations of the main features of the process from some general viewpoint.

The aim of this paper is to show, that the theory created by L.D.Landau /1/ more than twenty years ago, turns out to be in nice agreement with the wide range of recent observations on the production processes. This theory is a consistent development of the approach, proposed by W.Heisenberg /2/, E.Fermi /3/, I.Ya. Pomeranchuck /4/. The basic assumption is that strong interaction at small distances is strong enough to mix the system up to thermodynamical equilibrium. Note, that this assumption differs drastically from the ideas of the multiperipheral and parton models, which assume that the interaction at small distances asymptotically vanishes. Nevertheless the resulting predictions of such different approaches turn out to be rather close in the available energy region. So this principal dilemma remains unsolved in the studies of the hadronic collisions. The deep inelastic processes like $e^+e^- \to$ hadrons seem to be more promising.

The hydrodynamical theory has attracted considerable attention of the cosmic ray physicists in late fifties, but the interest to this approach has decreased much in the next decade mainly due to the intensive studies at that time of the two-body reactions and Regge phenomenology, as well as the peripheral and multiperipheral models development. But when the regular studies of the multiple production at accelerators began, especially the inclusive measurements, this theory again has appeared at the stage. In the papers /5-7/ it is shown that the average multiplicity and inclusive spectrum for hadronic collisions agrees with the hydrodynamical model quite well. The theory has been applied to the description of the process $e^+e^- \to$ hadrons /8,9/, the results are in very good agreement with preliminary SPEAR data /10/.

In the present paper we are going to discuss these applications of the theory, paying more attention to important physical effects like the corrections in the equation of state due to interaction, leading particle effects etc., than it was done previously. In this paper we deal with a lot of questions, so their presentation is rather brief. We do not discuss much the details of calculations, paying more attention to physical reasoning. The results are presented mainly as simple approximate formulae, expressing the dependence on the variables in question but not claiming for high accuracy. Such presentation seems to be adequate to the present status of the theory, in which many questions like viscosity, nonequilibrium and quantum corrections are open and much more work is needed in order to make its predictions really quantitative.

A lot of results of the hydrodynamical theory, such as particle composition, transverse momentum distribution etc., are discussed elsewhere /11,12/. The reason for this separation is that these predictions do not depend on the hydrodynamical stage and are completely determined by the final stage interaction (or in more modern terminology- by the short range effects). Thus their discussion can be made in more general context. In particular, they can be easily combined with multiperipheral and parton models.

After the very short presentation of the principles of the hydrodynamical model (chapter 2) we discuss the inclusive and exclusive spectra of secondaries (chapter 3).The various channels are considered as a thermodynamical fluctuations(as in /12/ where low energy reactions have been considered).

The equation of state of the hadronic plasma is discussed in chapter4. The corrections to interaction are obtained with the help of the Beth-Uhlenbeck method /13/. We discuss also the applicability limits of this method and make some critical discussion of the statistical bootstrap model /14/.

The chapter 5 is devoted to the discussion of the leading particle effects and their role in producing large fluctuations in the system. The more detailed consideration of this point is made in the chapter 6 for the multiplicity distribution.

The inclusive rapidity distribution is analyzed in chapter 7. The discussion of the data is made first, then they are compared with the theoretical predictions.

The last two chapters deal with hadron-nucleus collisions and $e^+e^- \rightarrow$ hadrons. The intense theoretical and experimental studies of these processes have begun quite recently. But even the first observations are of great interest. We are going to show, that they agree with the behaviour expected in the framework of the discussed theory.

2. The main ideas of the hydrodynamical theory.

The main assumption of this theory is the Fermi hypothesis that the system, created in the collision, reaches equilibrium by the time at which particle pass each other in CM reference frame. So the initial form is the thin disk with the width E_{CM}^{-1} * and radius m_π^{-1}. This estimate is used as an initial condition in the theory. It is importent to note, that the final results depend on it rather weakly (logarithmically) and it can to some extend justify its crudness.

It is not clear now whether this strong assumption is true or not** and what kind of the dynamics can lead to such a result. It must be the field theory with very strong interaction at small distances, may be nonrenormalizable. It must be opposite to more popular now possibility - the asymptotical freedom, leading to another picture - the parton model. The observed trend of the data on $e^+e^- \to$ hadrons seems to be more favourable to the first possibility.

The next after the described above initial stage comes the hydrodynamical expansion, which is described by the relativistic hydrodynamical equations:

$$\frac{\partial T^{ik}}{\partial x^k} = 0 \qquad T^{ik} = (\varepsilon + p) u^i u^k - p g^{ik} \tag{1}$$

where u^i is the 4-velocity of the liquid, ε and p are energy density and pressure. They are connected by the equation of state to which we return later. Note, that since the initial distribution is a thin disk, the expansion is mainly longitudinal.

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*We use units $\hbar = c = 1$

**There are some indirect indications. For example, the long range asimuthal correlations /16/ can be ascribed to the production of some intermediate system. Some phenomena at large $p_\perp$ can be explained as nonequilibrium effects with temperature higher then final /11,12/.

If the hadronic plasma does not decay into separate particles, then the system will expand untill the pressure vanishes. But the hydrodynamics (and, by the way, thermodynamics) have the following applicability condition /1/: the mean free path ℓ must be much smaller than the minimum system dimension L_{min}. As soon as these quantities become comparable, the system decays into physical particles.* The stage of the process when $\ell \sim L_{min}$ we call the final stage, all quantities at this stage we mark by the index "f". Simple but crude estimate provides their order of magnitude: $L_{min} \sim 1/m_\pi$; $\ell \sim 1/n\sigma$ $\sigma \sim m_\pi^{-2}$ so the final density n_f is $\sim m_\pi^3$, also $T_f \sim m_\pi$. These results themselves lead to importent predictions, see /11-13/.

The first solution of the hydrodynamical equations (1) have been given by Landau /1/ for the equation of state $C^2 \equiv \frac{dp}{d\varepsilon} = \frac{1}{3}$ (the physical meaning of C is the velocity of the sound). Then I.M.Khalatnikov /17/ has found the exact solution of the onedimensional problem. G.A.Milekhin has studied the arbitrary C^2 case /18/ and have solved numerically the problem for $C^2 = \frac{1}{3}$ in full threedimensional case /19/, in which little deviations from /1/ have been found. The approximate threedimensional solution for any C^2 has been found in /5/. Let us warn the reader, that these solutions are made in the asymptotics $\ln E_{CM} \gg 1$, which is not well satisfied at available energies. So the good agreement with data in /5-7/ is to some extent occasional and is due to cancellation of the nonasymptotic effects. We shall discuss this point below in more details.

The equations (1) lead to entropy conservation and the average number of secondaries can be estimated directly from the initial

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* The data on two body correlations indicate, that the system first decays into several smaller drops (clusters) and only then into secondaries. Something like surface tension is necessary to add in order to explain this effect. We do not discuss it in the present paper.

condition /1/. For arbitrary c^2 the result is /18/:

$$\langle N\rangle \propto E_{CM}^{\frac{1-c^2}{1+c^2}} \tag{2}$$

where E_{CM} is the total energy in CM frame.

3. The secondary particles spectra.

Suppose one has solved the equations (1) and has obtained the dependence of the energy density $\mathcal{E}(\vec{x},t)$ and 4-velocity $U_\mu(\vec{x},t)$ on coordinates and time. The final stage condition $\mathcal{E}(x,t)=\mathcal{E}_f$ determines some 4-surface σ. On this surface in the rest frame of volume elements moving with velocity $U_\mu(x,t)$ the particle distribution is the equilibrium one:

$$E'\frac{dN}{d^3p'}=\int\frac{g\,dv'}{(2\pi)^3}E'\left[\exp\left(\frac{E'}{T_f}\right)\pm 1\right]^{-1}\equiv\int gE'\frac{dv'}{(2\pi)^3}f(E'/T_f) \tag{3}$$

Here g is the statistical weight of the given particle, E and p are its energy and momentum, the primed quantities are taken in the accompanying frame. Since the left part of (3) is invariant, the right one can also be rewritten in the invariant form /20/:

$$E\frac{dN}{d^3p}=\frac{g}{(2\pi)^3}\int f(E'/T_f)\,p^\mu d\sigma_\mu \tag{4}$$

Here $d\sigma_\mu$ is the surface element, σ has been determined above.

The expressions used in /5,7/, correspond to approximation $p^\mu\approx mu^\mu$. If one parametrizes the surface σ by the rapidity y' of the hydrodynamical motion, then its normalized distribution $\varphi(y')$ is:

$$\varphi(y')=\frac{1}{V_{eff}}\,U^\mu\frac{d\sigma_\mu}{dy'}\ ;\quad V_{eff}=\int U^\mu d\sigma_\mu \tag{5}$$

We call V_{eff} the effective volume. Since the thermodynamical fluctuations do not depend on the motion of the volume elements as a whole, our system is equivalent in some respect to plasma at rest in the box with volume V_{eff} at temperature T_f. This idea is explored in /11,12/.

In the discussed approximation the particle spectrum is:

$$E\frac{dN}{d^3p} = \frac{g}{(2\pi)^3}\int f(E'/T_f)\,E'\,\varphi(y')\,dy' \qquad (6)$$

Both (4) and (6) lead to the same particle number

$$N_{tot} = \int E\frac{dN}{d^3p}\,\frac{d^3p}{E} = n_f V_{eff} \qquad (7)$$

but the total energy, given by (6), disagrees /20/ with the exact value $E_{tot} = \int T^{0\mu} d\sigma_\mu$. The difference is caused by the pressure term, so the accuracy of this approximation can be astimated as $p_f/\varepsilon_f = C_f^2$ at final stage which, according to calculations in chapter 4, is of the order of 10-15%.

The studies of the hydrodynamical problem have shown, that in the pionisation region the collective rapidity distribution (5) is approximately of gaussian shape:

$$\varphi(y') = \frac{1}{\sqrt{2\pi L_{hydr}}}\exp\left(-\frac{y'^2}{2L_{hydr}}\right) \qquad (8)$$

According to the numerical solution /19/ for $C^2 = 1/3$

$$L_{hydr} = 0.56\,\ln\frac{s}{4m^2} + 1.6 \qquad (9)$$

The leading term for any C^2 according to /5/ is:

$$L_{hydr} = \frac{4}{3}\frac{C^2}{1-C^4}\ln\frac{s}{4m^2} \qquad (10)$$

Note, that if one uses the formulae (2), (10) for the check of the sum rule $E_{tot} = \langle N\rangle\langle E\rangle$ he finds, thet the power of the energy in the r.h.s. is not unity, but a little less. This is because the transverse hydrodynamical motion must be taken into account also, which leads to small increase of the average $p_\perp$, like $s^{1/14}$ according to /19/. This sume rule is not a good tool for the calculation of this power, as it is done in /21/, for it is found as a small difference of two large numbers and the precision is poor. Nevertheless, the estimate in /21/ agrees resonably with that of author, based on the solution /5/. But the statement in /21/ that the theory contradicts to data is completely wrong and is due just to misunderstanding of the variable on the plot, E_{CM} instead of E_{LAB}. In fact, the agreement with data on $\langle p_\perp \rangle$ dependence on energy is quite well (see e.g. /24/) even for this delicate effect. We shell neglect the $\langle p_\perp \rangle$ growth in the discussions to follow.

The formulae (8-10) are valid in the asymptotics $\ln \frac{s}{4m^2} \gg 1$ * and in the rapidity range $y \lesssim L_{hydr}$. For larger rapidity values the decrease of $\varphi(y')$ is more strong due to further terms in the exponent like y^4/L^3_{hydr} with negative coeffisient. The distribution of the type $\varphi(y') \propto \exp(\sqrt{L^2 - y'^2})$, present already in /1/, is better approximation and it also gives better fit to ISR data. But we still use (8) first because the difference at present energy is mainly in the fragmentation region, were the nonhydrodynamical effects may also be of importence, and second, because it is easier to deal with gaussian. In particular, the gaussian has the useful property to conserve its shape in various integrations with some modification of the width only. For example, the approximate account of the thermal motion can be made if one performs the integration in (7) (or in (4)) by the saddle point method. Then one obtains the result for the width L:

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* This is due to the ultrarelativistic approximation for the hydrodynamical motion $|U_\mu| \gg 1$ used in the solution. E.L.Feinberg /23, 24/ has repeatedly asserted that the hydrodynamics can not be applied if this is not so, that is below the ISR energies. We do not understend this reasoning and believe that hydrodynamics has the same applicability condition as thermodynamics, and thus it can be used irrespective whether the collective motion is much larger or comparable with the thermal one, although no accurate calculations of the low energy case has been made untill now.

$$L \approx L_{hydr} + \frac{T_f}{M_\perp} \; ; \; M_\perp^2 = p_\perp^2 + m^2 \qquad (11)$$

The thermal motion is more essential for pions than for heavy particles. The data really shows such "Doppler broadening" of the pion spectrum compared to that of kaons and antinucleons in agreement with (11). This fact shows that the very idea of the two component of the motion, the collective and thermal ones, proved correct. The same conclusion has been obtained in the fit in the so called thermodynamical model of Hagedorn and Ranft /22/, in which the collective motion is treated phenomenologically, while in the discussed theory it is calculated from the hydrodynamical equations.

Now we proceed to a brief discussion of the exclusive spectra. Since we have assumed the thermal equilibrium, it is natural to treat the various reaction channels, which are the deviations from the average system behaviour, with the help of the standard theory of the thermodynamical fluctuations. Such an approach has been successfully developed in /13/ for the low energy case, when the hydrodynamical motion can be neglected and Pomeranchuck statistical model can be used. One can easily generalize this approach with the following rather natural assumption: the fluctuations average each other during the hydrodynamical expansion. It means that the solution of the equations $\mathcal{E}(x,t)$ and $U_\mu(x,t)$ remains unaffected and only fluctuations at final stage must be considered. It is necessary to find out what physical condition characterizes these fluctuations. It is the same condition of the final stage which we have discussed in the chapter 2 $l \sim L_{min}$ which leads to $n(x,t) = n_f$. Thus, the fluctuations take place at fixed density $n_f \sim m_\pi^3$.

Suppose we deal with some reaction channel "a" with definite set of particles of the kind "i" N_i^a. The state of such system can be described by the set of chemical potentials μ_i^a and temperature T_a. The quantities like the energy density are given by the usual formulae of statistical mechanics as a function of μ_i^a and T_a. The final stage surface σ_a is determined by the condition $\mathcal{E}(x,t) = \mathcal{E}_a$ as before, but the value of this constant is not connected with the condition $n(x,t) = n_f$ (as in the inclusive case) so directly. First it is necessary to fix the μ_i^a and T_a value from the normalization condition as a function of $\mathcal{E}_a$, and only then the value of $\mathcal{E}_a$ can be found from the condition $n = n_f$. The normalization conditions we speak about are:

$$N_i^a = \int f((E' - \mu_i^a)/T_a)\, p^\mu d\sigma_\mu^a \frac{g_i d^3p}{E(2\pi)^3} \; ; \; E_{tot} = \int f((E' - \mu_i^a)/T_a)\, p^\mu d\sigma_\mu^a \frac{g_i d^3p}{(2\pi)^3} \qquad (12)$$

What can be obtained after this procedure, which seems to be rather cumbersome ? First one finds the exclusive spectrum of particles in this channel:

$$E\frac{dN_i^a}{d^3p} = \frac{g_i}{(2\pi)^3} \int f((E' - \mu_i^a)/T_a)\, p^\mu d\sigma_\mu^a \qquad (13)$$

Even more important is the fact, that the probability of this channel production is given by the Einstein formula for the fluctuation probability $\frac{\sigma_a}{\sigma_{tot}} \propto \exp\left[\mathcal{S}(\mu_i^a, T_a, V_a)\right]$, where $\mathcal{S}(\mu, T, V)$ is the entropy, given by the usual expression for Bose gas.

The described theory has very large predictive power, but up to now it was used only in some particular examples and its general connection with data are not clear. In the case of high energy hadronic collisions additional complications appear due to the leading particle effects, which we discuss below. The case of low energy $p\bar{p}$ annihilation is discussed in /13/, as well as the correlation between the charged multiplicity and average transverse momentum and the average number of π_0. The calculation of the N_{K^-} and $N_{\bar{p}}$ in /11,12/ also is based on this approach, as well as the discussion of the multiplicity distribution in the chapter 6. In all these cases this theory gives results in agreement with data, but much more wide and accurate test of it is still needed.

The question can be posed, whether such a thermodynamical approach can be applied in real cases in which the particle number is not large. This point is discussed in /13/ with the conclusion: the accuracy is agreeable even if the total particle number is only $4 \div 5$. It is not so surprising since all the thermodynamical formulae are close relative to the Sterling formula known for its high accuracy even in nonasymptotical region.

4. The equation of state.

In the Landau original work /1/ the equation of state $p = \frac{1}{3}\varepsilon$ was used, which corresponds to the ideal ultrarelativistic gas. But the collision energy now available at accelerators corresponds to initial temperatures $T \lesssim 1$ Gev. In this temperature range the hadronic plasma is neither ideal nor ultrarelativistic gas. In this chapter we discuss the possibilities to calculate the corrections in the equation of state due to interaction with our present knowledge of the strong interaction dynamics. This chapter continues the discussion of this point in /5/.

In1956 Belenky and Landau /24/ have proposed to use the Beth - Uhlenbeck method /14/ to this aim. This method in some approximation reduces the problem of interacting particles to the simple case of ideal gas of the mixture of stable particles with unstable ones - the resonances.

Before we proceed to calculations based on this idea, let us discuss first one principal question arising in this way. Suppose a volume element of matter is taken and the number of state is calculated in the usual thermodynamical way, like the nuclear state density in the Fermi gas model. The model used contains some number of particles and resonances. But the states of the volume element also manifest themselves as resonances in scattering on this element. Is it necessary to take them into account also?

Hagedorn /15/ gave the positive answer to this question and proposed the so called statistical bootstrap condition: the density of resonances and the whole system coincides. This assumption leads to exponential growth of the resonance spectrum and the divergency of the statistical sum at temperatures higher than some limiting value.

We can not agree with such a solution of the problem. The interaction must be renormalized in the media, and the "compound" resonances must disappear, if they are "less dense" than the media in average, just like molecules in condensed matter. As Ya.B. Zel'dovitch has noted /25/, the very similar situation is known in the plasma physics: the hydrogen-like ions have an infinite number of states which causes the divergency of the statistical sum. But most of these states are fictitious and do not exist in plasma.

Another point of the critisizm of this model is connected with limited range of the applicability of the Beth-Uhlenbeck method. We remind to the reader, that this method gives only the second virial coefficient, so the result is valid only if the virial expansion parameter- the interaction range r_{int} divided on the particle separation $n^{-1/3}$ - is small. In the hadronic plasma $r_{int} \sim m_g^{-1}$ (not m_π^{-1} since pions and kaons are pseudoscalars and can not exchange pions), and $n^{-1/3} \sim T^{-1}$. In the calculations below we courageously use this method till this parameter approaches unity, that is the nonideal gas becomes more like liquid, but at arbitrary large density (as in bootstrap model) it is wrong. Although we must note, that in all low energy applications of this model the results are practically indistinguishable from those of Pomeranchuck model /25,26/. Thus for practical applications the simplicity of the Pomeranchuck model is even more important than the principal questions discussed above.

In the calculations presented below ~~we~~ have assumed that the spectrum of the resonances in the matter does not grow too strongly with their mass, so the $exp(-\frac{m}{T})$ makes the cutoff and only the low mass region is important. Note that in the bootstrap model this exponent is cancelled by the exponent in the resonance spectrum and all masses contribute to the statistical sum. This makes the system behaviour vary unstable like that in phase transition phenomena, which seems to be rather strange.

Our next assumption is that up to temperature $T \leq m_\rho$ the known resonances do not disappear in the matter. This assumption may be partly violated, but fortunately such integral property as C^2 in which we are mainly interested, does not depend much on the particular set of the resonances used.

We have calculated the C^2 defined as $C^2 \equiv \frac{dP}{d\varepsilon}$ where

$$P = \sum_i \int \frac{g_i d^3p}{(2\pi)^3} \frac{p^2}{3E} \left[exp\left(\frac{E}{T}\right) \pm 1\right]^{-1} ; \quad \varepsilon = \sum_i \int \frac{g_i d^3p}{(2\pi)^3} E \left[exp\left(\frac{E}{T}\right) \pm 1\right]^{-1} \qquad (14)$$

The sum is taken over particle kinds, $g_i = (2S_i+1)(2I_i+1)$ where S_i and I_i are spin and isospin. The results are plotted in Fig.1 as C^2 versus T. Three variants of the calculation are plotted: 1) The account of pions only, this is shown for the comparison; 2) 16 first mesons with mass less than 1.7 Gev, are taken into account; 3) 31 resonances in the same mass range. The results do not depend on the resonances used, but the results differ from estimates in /5/where resonances were changed by some smooth function and the value $C^2 = 0,14$ were found.

We conclude with the comment that at temperatures higher than m_ρ the Beth-Uhlenbeck method is not reliable and the calculation of the equations of state in this region, which is of interest for superhigh energy collision and cosmology, hardly can be made without better understanding of the strong interaction dynamics.

5. The leading particle effects.

The existence of the secondaries which have spectra quite different from those of other particles, and quantum numbers close to those of the primary particles shows that the mixing at the initial stage is not complete. Thus such particles, called the leading ones, must be excluded from the comparison of the statistical theories with data. Our idea to describe by such theories the nonleading particles only is similar to the nuclear reaction theory, in which also the secondaries are divided into direct reaction products and those of the statistical compound nuclei decay.

The model we deal with takes into account only the kinematical effect of the leading particles. Any prediction of the theory $F(M^2,\dots)$ (M is the invariant mass of nonleading particles) is averaged with the leading particle distribution:

$$\tilde{F}(s,\dots) = \int d^3p_1\, d^3p_2\, g(p_1 p_2 s)\, F(M^2,\dots) \qquad (15)$$

where $g(p_1, p_2, s)$ is the probability for two leading particles, the forward and backward in CM, to have momenta p_1 and p_2 . Integrating over $p_\perp$ we rewrite (15) as $\left(x = \frac{2p_{\parallel}}{\sqrt{s}}\right)$

$$\tilde{F}(s,\dots) = \int dx_1\, dx_2\, g(x_1, x_2, s)\, F(M^2,\dots) \qquad (16)$$

At high enough energy the mass M^2 is approximately

$$M^2 \approx s(1-x_1)(1-x_2) \qquad (17)$$

The calculations presented below have been made for pp collisions. The function $g(x_1 x_2 s)$ in (16) is taken as $g_1(x_1 s)\, g_2(x_2, s)$ The smallness of the leading particles correlations outside the diffraction region is seen, for example, in data in /27/ of the two arm detector. The kinematical correlations in the diffractive region are accounted by some production threshold in M . The function $g(x, s)$ is taken from the experimental proton spectrum with the substraction of that for antiprotons, in order to exclude the $p\bar{p}$ pair production. Of course, the leading particle can be the excited state of the proton, as can be seen from the energetic neutrons presence and the excess of π^+ over π^-. But the data do not allow yet the quantitative analysis of this point, so we assume that $g(x,s)$ is the same as for protons.

The most essential qualitative effect of the leading particles is the large nonstatistical fluctuations in the system which they produce. In the statistical system the relative fluctuation (the ratio of the dispersion to the average value) of all quantities must vanish as the system becomes larger, something like $N_{tot}^{-1/2}$. But the broad distribution of the invariant mass M (17) makes it always to be of the order of unity. The more clearly it is seen in the multiplicity distribution, which we discuss in the next chapter. This effect is also expressed in the large variety in the individual events, which have caused doubts of experimentalists about the applicability of any statistical approach. Our aim is to show that there is large subsystem which can be described in a statistical way in agreement with data.

6. The multiplicity distribution.

In this chapter we show that the account of the leading particles in kinematical way, described above, together with statistical treatment of other particles leads to results in very nice agreement with data /29/.

Assume that in the statistical subsystem the independent charged pion pairs are produced. This ass umption is a very crude approximation to the results of the fluctuation theory described in chapter 3 , but the corrections are not significant since the fluctuations caused by the leading particles are much larger than those in the statistical system. We also neglect the contribution of kaons. The n charged pair production probability W_n we write as

$$W_n = \frac{\bar{n}^{2n}}{(n!)^2} \frac{1}{I_0(2\bar{n})} \tag{18}$$

Here I_0 is a Bessel function which makes the normalization, $\bar{n}$ is a parameter, proportional to the effective volume of the statistical system (5), at large $\bar{n}$ it is the average particle number directly. Its dependence on the total mass M we parametrize as

$$\bar{n}(M) = A\cdot(M^2)^B \tag{19}$$

The averaging over the leading particles proceeds as in (15), so the probability of the n_{ch} charged particle creation is $\widetilde{W}_{n_{ch}}$

$$\widetilde{W}_{2n+2} = \int dx_1\, dx_2\, g(x_1 s)\, g(x_2 s) \frac{\bar{n}^{2n}}{(n!)^2} \left[I_0(2\bar{n})\right]^{-1} \tag{20}$$

The numerical integration gives the results* putted in table 1 in the form of the moments of the multiplicity distribution C_q

$$C_q = \frac{\langle n_{ch}^q \rangle}{\langle n_{ch} \rangle^q} \tag{21}$$

The value of these moments depends on the paremeter B (19), the better agreement is for $B \approx 0.33$, which corresponds to numbers in the table. The agreement with data is very nice, it takes place up to the highest moment which has been computed. The average multiplicity is compared with data at Fig. 2.

Note that in the case of the exact scaling in the proton spect-

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* These calculations have been also published separately in /28/.

rum $g(x,s) = g(x)$ one can easily find from (18-20) that at high energies $\langle n_{ch}^{q} \rangle \propto s^{B \cdot q}$ and C_q are energy independent constants. Such result corresponds to the so called KNO scaling /30/. But the deviations from scaling in proton spectrum change this result somehow, in particular the average charged multiplicity grows with energy more slowly than s^{B}, as is shown at Fig.2. Since the proton spectrum has two peaks, at $x \approx 0,4$ and $x = 1$, its qualitative presentation as two delta functions can be made, which corresponds to the two component model /31/. It is interesting to note, that since our approach gives the correct integrated multiplicity moments, connected directly with the integrated correlations, it must describe partly the two body correlations, although the short range effects need, of course, additional considerations.

If one considers formula (19) not just as a parametrization, but a hydrodynamical model prediction for V_{eff}, then one can connect the parameter B with c^2 according to (2):

$$c^2 = \frac{1-2B}{1+2B} \tag{22}$$

The value for B found in the fit to multiplicity moments C_q gives $c^2 \approx 0,20$ in nice agreement with the results of chapter 4.

7. The inclusive rapidity distribution.

We begin this chapter with the analysis of the data available. We are not going to discuss some topics well discussed previously. The transverse momentum distribution both in small and large $p_\perp$ region is discussed in /12,13/. The dependence of the rapidity distribution on $p_\perp$ is well described by the simple threshold cutoff in (6) at y'_{max} determined as $ch\, y'_{max} = \frac{\sqrt{s}}{2m_\perp}$ /7/. So we concentrate our attention on the global properties of the inclusive rapidity distribution.

The data shows wide evidence for gaussian form of $\frac{dN}{dy}$ for any type of the secondaries. There are deviations in some ISR data, plotted at Fig.3. Unfortunately no good measurements of slow particles have been made. The data of variouse groups, working at small and large angles do not join smoothly. The more accurate data are still needed.

In order to see how the inclusive spectrum behave with energy we plot at Fig.4 the parameter of the gaussian fit

$$\frac{dN}{dy} = \frac{\langle N \rangle}{\sqrt{2\pi L}} \exp\left(-\frac{y^2}{2L}\right) \qquad (23)$$

versus energy for $\pi^-, K^-, \bar{p}$. Most of these data are integrated over $p_\perp$, but if such data are absent we took those with the smallest $p_\perp$ given which at high energy do not deviate much from the integrated distribution. The figure clearly shows two energy regions. Below Serpukhov energies L depends weakly on the energy and is well described by the thermal motion only for any kind of the secondaries. So in this energy region the simple statistical model (with the account of the leading particles) can be used.

With the further increase in energy L increases due to collective (hydrodynamical) motion. As we have noted above, the direct comparison of the data with (9,10) is not possible for the asimptotics $\ln\left(\frac{S}{4m^2}\right) \gg 1$ is not yet reached. One must take into account not only terms of the order $O(\ln\frac{S}{4m^2})$, but also O(1). There are several effects leading to such terms: the thermal motion, leading particles, also the hydrodynamical equations must be solved to this accuracy. None of this was done in /6/, and only thermal motion is taken into account in /5,7/, so the conclusions of these papers are not well grounded. In the present paper we choose an indirect way, based on the formula for the average multiplicity (I8,I9):

$$\langle N \rangle = \frac{\sum n W_n}{\sum W_n} = \frac{I_1(2\bar{n})}{I_0(2\bar{n})} \,, \quad \bar{n} \propto M^{\frac{1-c^2}{1+c^2}} \tag{24}$$

If we <u>assume</u> the gaussian shape of the inclusive rapidity distribution, then its width L can be found from the total energy sum rule:

$$M = \langle N \rangle m \cdot \exp\left(\frac{L}{2}\right) \tag{25}$$

The reason for this assumption is that it is valid both in high and low energy limits. With this simple assumption we do the integration over leading particle distribution. Note, that the maximum of the gaussian is at rapidity $y_1 \approx \frac{1}{2} \ln \frac{1-x_1}{1-x_2}$, corresponding to the rest frame of all nonleading particles. The final expression used is:

$$E\frac{dN}{d^3p} = \int dx_1 dx_2\, g(x_1,s)\, g(x_2,s)\, \frac{\exp\left[-\frac{(y-y_1)^2}{2L}\right]}{(2\cdot L)^{1/2}} \langle N(M) \rangle \tag{26}$$

where $\langle N \rangle, L, M$ depend on x_1, x_2 according to (I7,I9, 24,25). The curves are compared with data at Fig.3,4 for $c^2=0.2$.

Now we pass to the discission of the heavy particle production Their absolute yield is discussed in /II,I2/, so here we discuss only the shape of the inclusive spectra. Their computation can be done with the use of our general fluctuation approach to exclusive channels, discussed in the chapter 3, but such computation is complicated and is not yet made. So we make here only some remarks. The consequence of hydrodynamical approach is that the rapidity distribution of the collective motion is <u>the same</u> for heavy particles and pions in the <u>subset</u> of events in wich this particles are produced. As far as the whole system becomes large, it is not much affected by the fact of $K\bar{K}$ or $p\bar{p}$ production, so at high energy the collective rapidity distribution is <u>the same</u> for all particles inclusivly. The total width L differs only due to different thermal motion (different "Doppler broadening") , so (11):

$$L_{K,\bar{p}} \approx L_{\pi} - T\left(\frac{1}{\langle m_{\perp}^{\pi}\rangle} - \frac{1}{\langle m_{\perp}^{K,\bar{p}}\rangle}\right) \tag{27}$$

As can be seen from Fig.4 , this is indeed the case for K^- at any energy, but $L_{\bar{p}}$ is smaller than given by(27). The difference decreases rapidly in the ISR energy range, a nd will probably disappear at higher energy.

<u>8. The collisions of hadrons with nuclei.</u>

In the works /I,I8,I9/ rather detailed theory of this phenomenon has been developed. The essential part of this theory is the consideration of the shock waves on the primary stage of the process, at which the nonequilibrium and quantum effects are essential and the accurate calculations are hardly possible. It is more resonable now to give simple estimates, valid at least qualitatively.

We use the idea presented by K.Gottfried /33b/ that the tube in the nucleus, hitted by the incoming hadron, behave as a single hadron, That means that it is mixed with initial hadron into common statistical system. With this assumption all results can easily be obtained from known ones for hadron-hadron collisions just by the substitution instead of S the hadron -tube invariant $S' \propto S \cdot A^{1/3}$ where A is the atomic weight. The average multiplicity is then:

$$R_A \equiv \frac{\langle N \rangle_{hA}}{\langle N \rangle_{hh}} \propto A^{\frac{1}{6}\frac{1-C^2}{1+C^2}} \tag{28}$$

The data /32/ give for this power the value 0.131 ± 0.005, which is in reasonable agreement with $C^2 \approx 0.2$ according to (28). The shape of inclusive distribution is predicted to be also gaussian, with maximum in the hadron-tube rest frame, moved from hp CM frame to $\Delta y_A \approx \frac{1}{6} \ln A$. The rapidity distribution is compared with data at Fig.5.

9. The e^+e^- annihilation into hadrons.

The reaction $e^+e^- \to$ hadrons is very important since it gives most directly the properties of thé strong interactions at small distances. The application of the hydrodynamical model to this process have been made first in /8,5a/.

The initial conditions in this process are quite different from those in hadronic collisions. In the one-photon approximation, dominant in annihilation, the e^+ and e^- annihilate in the point. This causes the crucial difference in the geometry of the process, the hydrodynamical explosion is radial in this case rather than the quasi onedimensional in hadronic collisions. Due to this the system cools much more quickly and the hydrodynamical effects are less prominent and become essential at higher energies.

Assume that the dissipative phenomena mix the system in some initial volume V_c. Unfortunately we can not estimate it in such a simple way as in the case of hadronic collisions. We remain it to be a free parameter. Many observable quantities depend on it, so it can be obtained from fit to data. For example, the estimation of the average multiplicity can be made with the help of entropy conservation, the result /5a/ is:

$$\langle N \rangle \propto E_{tot}^{\frac{1}{1+C^2}} \, V_0^{\frac{C^2}{1+C^2}} \tag{29}$$

Here E_{tot} is the total CM energy. The average ~~energy of the~~ energy of secondaries is $\langle E\rangle \propto (E_{tot}/V_0)^{\frac{c^2}{1+c^2}}$. At Fig.6 this is compared with the preliminary SPEAR data /10/. The increase with energy is clearly seen. The thermodynamical model /35/ predicts the constant value of this quantity. More generally, the statistical treatment connect the hadron momenta in e^+e^- annihilation with transverse momentum in the hadronic collisions, while in our approach it is more like longitudinal one, although the hydrodynamical effects are here less prominent due to different geometry.

The derivation of the shape of the spectrum of secondaries needs the solution of the hydrodynamical equations, which are in this case much more complicated. The asymptotic estimates have been made analitically with the assumption that the system is close to scaling regime $v \propto z/t$. This estimates can be found for any c^2 /5a/. The SPEAR data recently have caused some interest to the hydrodynamical approach and the numerical studies of the problem have been made /9/. It has been shawn that this approximation is valid for low c^2 but for $c^2 = \frac{1}{3}$ it is violated, at least for enrgy range studied. The origin of this is not yet clear and more studies are needed. There are also some other questions open, like the sensivity of the result to the particular form of the initial condition, the possible role of viscosity etc.

That is why we present here the semi qualitative dissucussion of the hydrodynamical theory prediction of the secondary particle spectrum. In the low energy region (E_{tot} is several Gev.) the rapidity of the thermal motion is larger than that of collective one. In this region the invariant cross section is approximately exponential with the slope decreasing slowly according to (29) as energy increases. Note that it is not the temperature growth which one can check by the study of the heavy particle production as in

/12,13/. At higher energies the hydrodynamical motion becomes more essential. It is interesting, that the collective rapidity distribution has peak at nonzero rapidity /8,9/. If one neglect its width, which according to /9/ is small, the very simple approximate formula for the spectrum can be obtained, Let y_0 be the position of the peak of $\frac{dN}{dy_{coll}}$, then the spectrum is boosted thermal distribution:

$$E\frac{dN}{d^3p} \propto E' exp(-\frac{E'}{T}), \quad E' = E\,ch y_0 - p\,sh y_0 \qquad (30)$$

The normalization constant and y_0 can be obtained from the normalization to particle number and total energy. The corresponding curves are compared with spectra at SPEAR /34/ at Fig.7. The agreement is even more than reasonable since the accuracy of both the formula (30) and the data is not high.

10. Summary.

1. The hydrodynamical theory is based on the assumption that the strong interactions quickly mix the system up to equilibrium. It remains for future studies to see whether this is the case. But the derivation of the conclusions from this assumption is made consistently and with the help of methods well developed and approved in other problems of physics. The possibility to use these familiar methods and intuition supplies this theory with large heuristic value.

2. The use of the Beth-Uhlenbeck method makes it possible to estimate the equation of state of the hadronic plasma. The result is $c^2 \equiv \frac{dp}{d\varepsilon} \approx 0.2$. This method is not reliable at temperatures higher than m_ρ and densities higher than m_ρ^3.

3. We propose to use this theory to all particles but leading ones. The main effect of the leading particles is the large fluctuations which reveal themselves both in average quantities, like the multiplicity distribution, and in individual events.

4. The phenomenological account of the leading particles and the statistical treatment of all others leads to very good description of the multiplicity distribution, ten moments of the distribution agrees with data. The parameter of the fit is in good agreement with $C^2 = 0,2$.

5. The inclusive rapidity distribution is approximately gaussian. The dependence of its width on total energy is studied and the conclusion is, that the theory predictions are in agreement with data for various kinds of secondaries.

6. Simple estimates on the hadron- nuclei collisions are presented in the assumption that the "nuclear tube", hitted by incoming hadron, behaves as a single hadron. The main results are: $\langle N_{h,A} \rangle \propto A^{\frac{1}{6}\frac{1-C^2}{1+C^2}}$. the value $C^2 = 0,2$ again agrees with data since this power is 0.13 /32/. The rapidity distribution is gaussian, its width and position dependence on atomic number are in correspondence with data /33/.

7. The theory predicts isotropic distribution of the secondaries in e^+e^- hadrons. The average particle energy increases as a small power of the total energy. The exponential shape of the invariant cross section is predicted for low total energy, and the dip at zero momentum at higher ones. Simple approximate formula is presented and compared with preliminary SPEAR data /34/. The reasonable agreement is found.

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Table I.

q	S=96 Gev^2 theory	S=96 Gev^2 exp.		S=I32 Gev^2 theory	S=I32 Gev^2 exp.	
2	I.255	I.236 ±	0.0I4	I.260	I.24I5 ±	0.0084
3	I.852	I.784	0.054	I.868	I.806	0.030
4	3.08	2.89	0.0I5	3.III	2.959	0.084
5	5.59	5.I6	0.04I	5.66	5.32	0.22
6	I0.9	9.9	I.I	II.07	I0.32	0.57
7	22.7	20.5	2.9	23.0	2I.3	I.5
8	49.4	44.6	7.9	50.I	46.2	4.0
9	II3	I02	22	II4	I05	II
I0	266	24I	6I	27I	245	30
	S=I94 Gev^2			S=388 Gev^2		
2	I.264	I.249 ±	0.0I4	I.268	I.258 ±	0.0I9
3	I.874	I.828	0.052	I.878	I.856	0.065
4	3.II	3.0I	0.I5	3.I0	3.08	0.I8
5	5.6I	5.43	0.42	5.53	5.60	0.46
6	I0.8	I0.6	I.I	I0.5	II.0	I.2
7	22.I	2I.8	3.I	2I.0	22.8	3.I
8	47.0	47.3	8.4	43.9	50.I	8.3
9	I03	I07	23	95.4	II5	22

Table I. The multiplicity moments C_q (2I) at variouse energies compared to calculations described in the text.

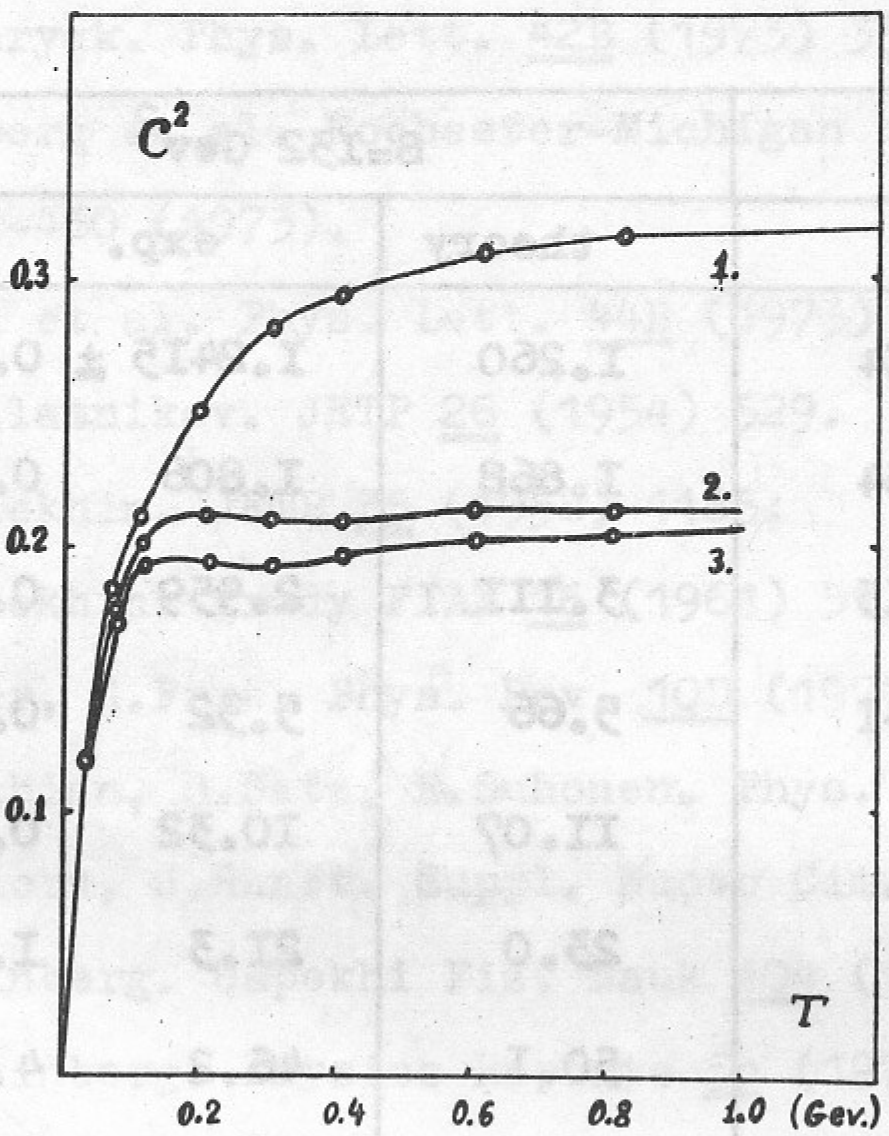
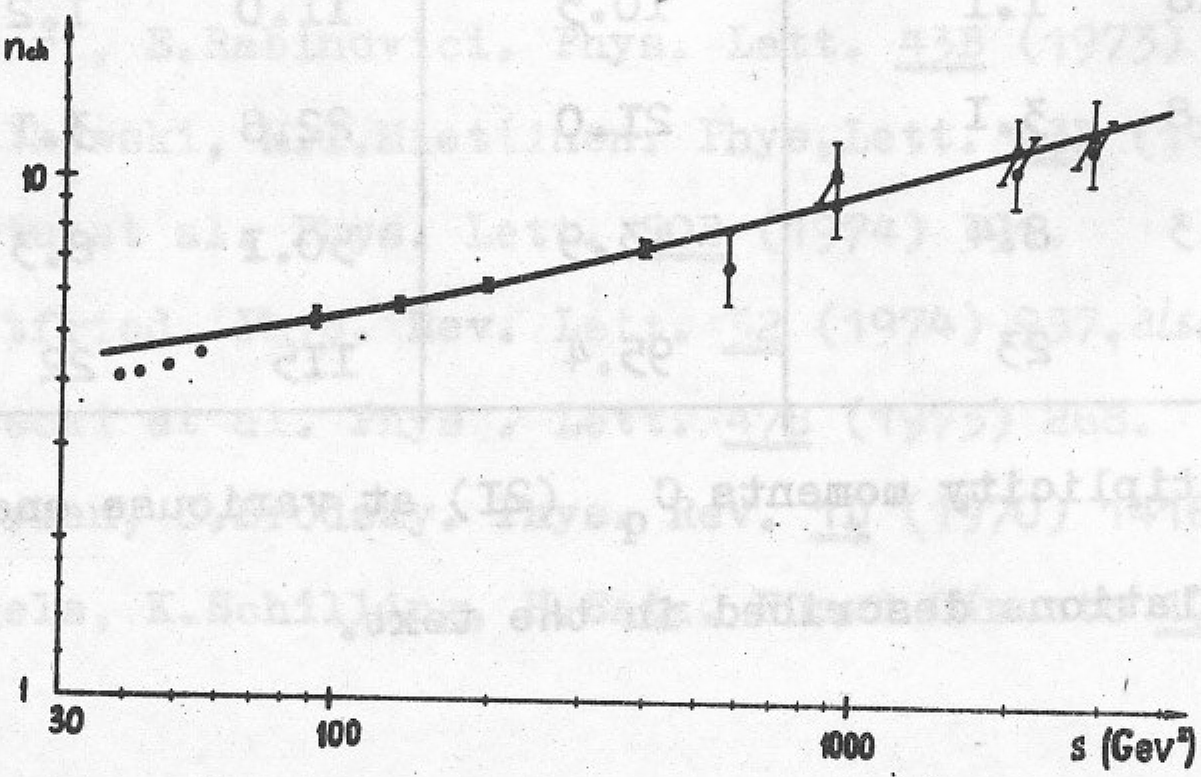


Fig.I. The squared sound velocity C^2 versus temperature according to calculations described in the text. The curves correspond to the account of the I)pions only, this one is shown for the comparison; 2) I6 meson resonances with $M \leq I.7$ Gev; 3) all 3I known resonances in the same mass range.

Fig.2. The average charged multiplicity versus energy, the curves correspond to calculations described in the text.



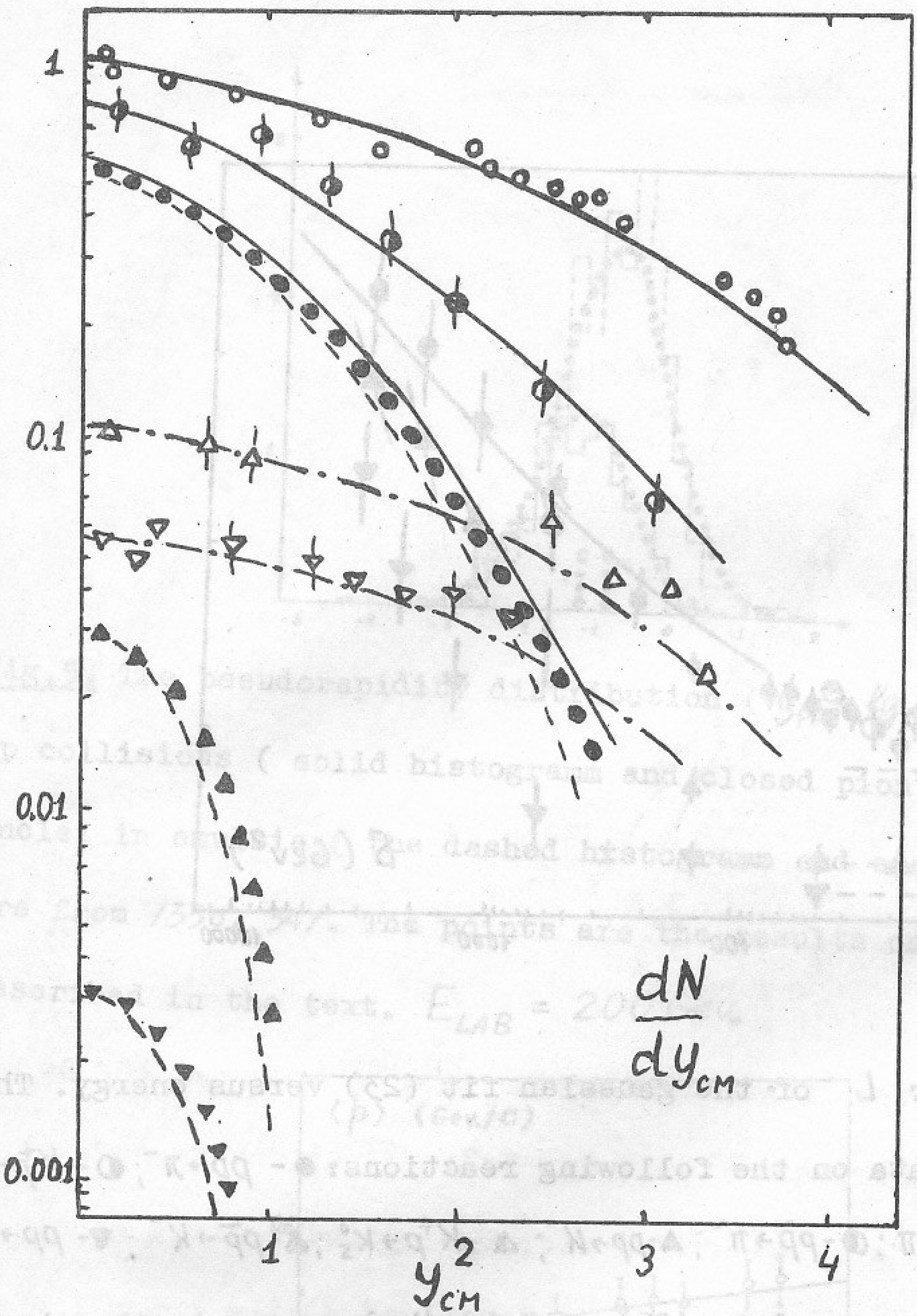


Fig.3. The inclusive rapidity distribution of π^- (o, ◐, ●); K^- (△, ▲); $\bar{p}$ (▽, ▼) at energies s=2800 Gev^2 (o, △, ▽) ; 388 Gev^2 (◐); 46.8 Gev^2 (●, ▼, ▲) (See e.g. /29/). At s=2800 data points are in fact at $p_\perp$=0.4Gev/c normalized to total multiplicity. We made so because the integrated over $p_\perp$ data are absent, but the difference in the rapidity dependence is not large. The solid curve correspond to (26) , the point-dashed to (26,27) and the dashed ones to the thermal distribution with $T = m_\pi$. The dashed and point-dashed curves are arbitrary normalized.

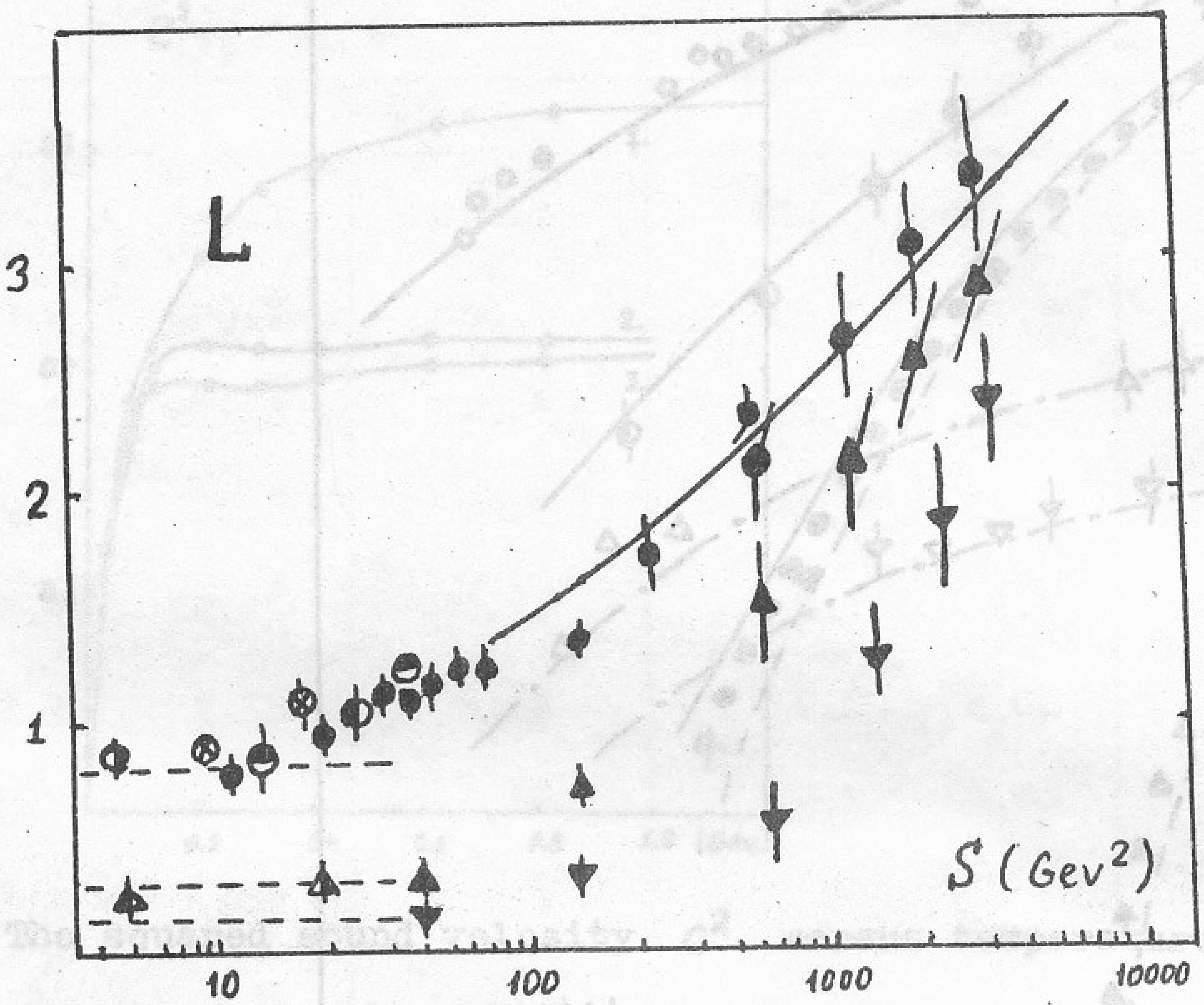


Fig4. The parameter L of the gaussian fit (23) versus energy. The points represent data on the following reactions: ● - $pp\to\pi^-$; ◐ - $K^+p\to\pi^-$; ⊖ - $\pi^+p\to\pi^-$; ⊗ - $\gamma p\to\pi^-$; ◐ - $p\bar{p}\to\pi^-$; ▲ - $pp\to K^-$; △ - $K^+p\to K^0_S$; ▲ - $p\bar{p}\to K^-$; ▼ - $pp\to\bar{p}$.

The solid curve correspond to (26) , the dashed ones - to the thermal distribution with $T=m_\pi$ for $\pi, K, \bar{p}$.

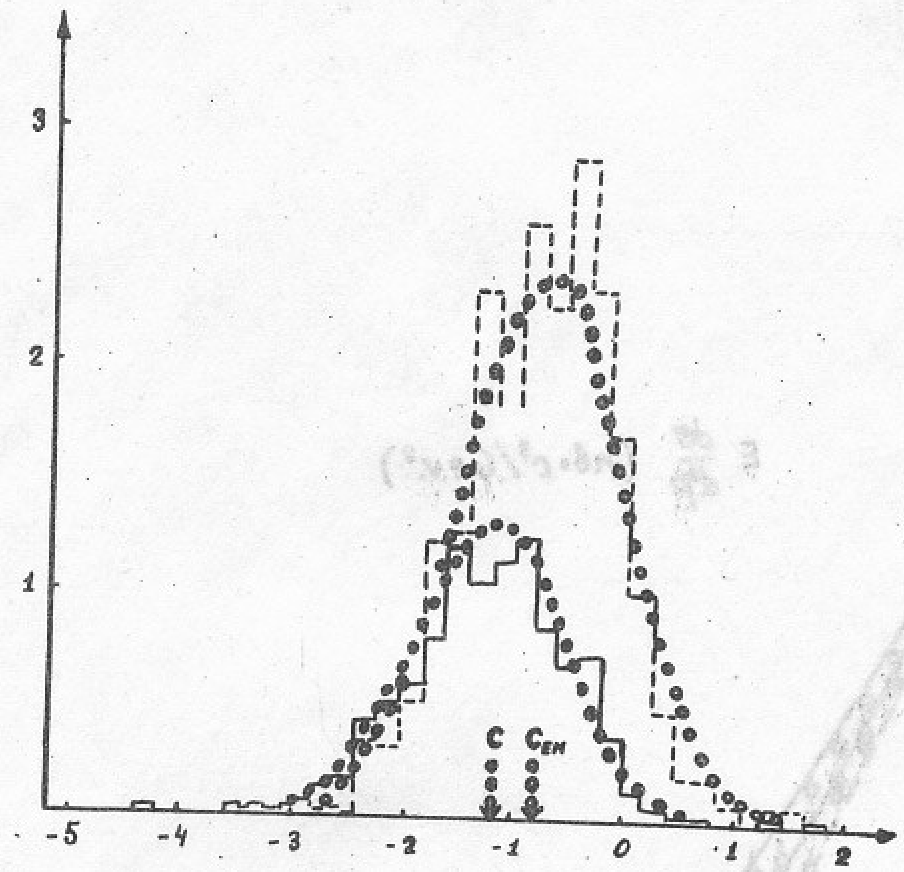


Fig.5. The pseudorapidity distribution ($y_{ps} = lg\, tg\, \theta$) for pp collisions (solid histogramm and closed pionts) and those with nuclei in emulsia (the dashed histogramm and open points). The data are from /33b, 34/. The points are the results of the calculations described in the text. $E_{LAB} = 200$ Gev.

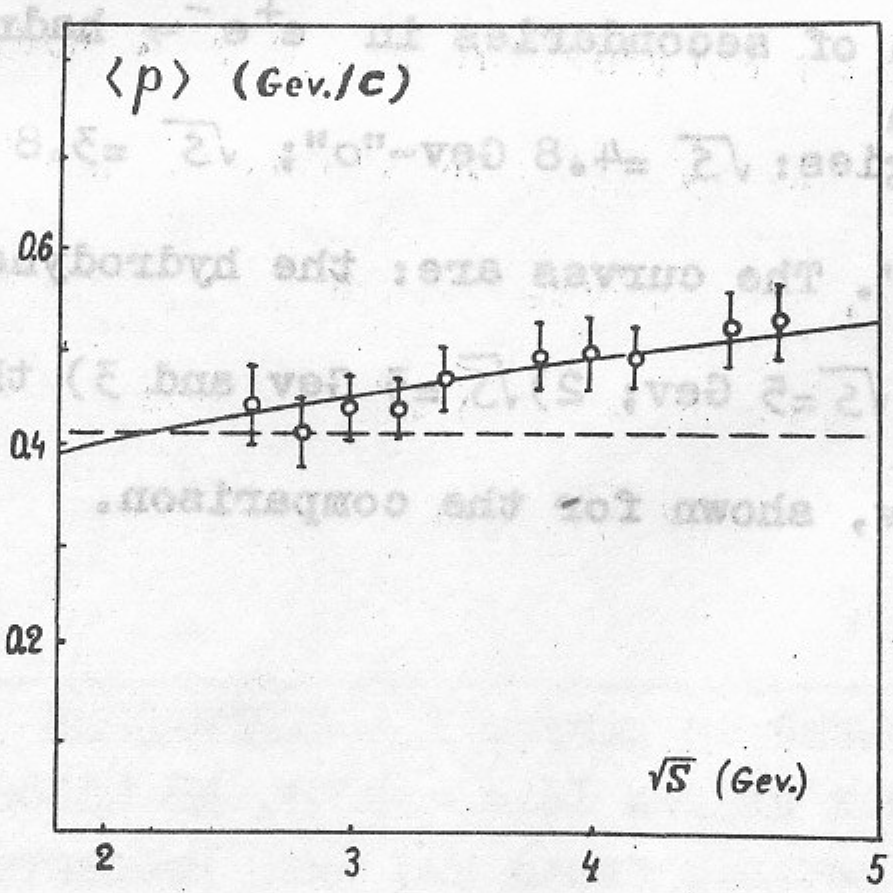


Fig. 6. The average momentum of secondaries in e^+e^- annihilation into hadrons versus the total energy $\sqrt{S}$. The data are from SPEAR /IO/, the solid line is the fit $\langle p \rangle \sim S^{1/8}$; the dashed one corresponds to the thermal distribution with $T = m_\pi$.

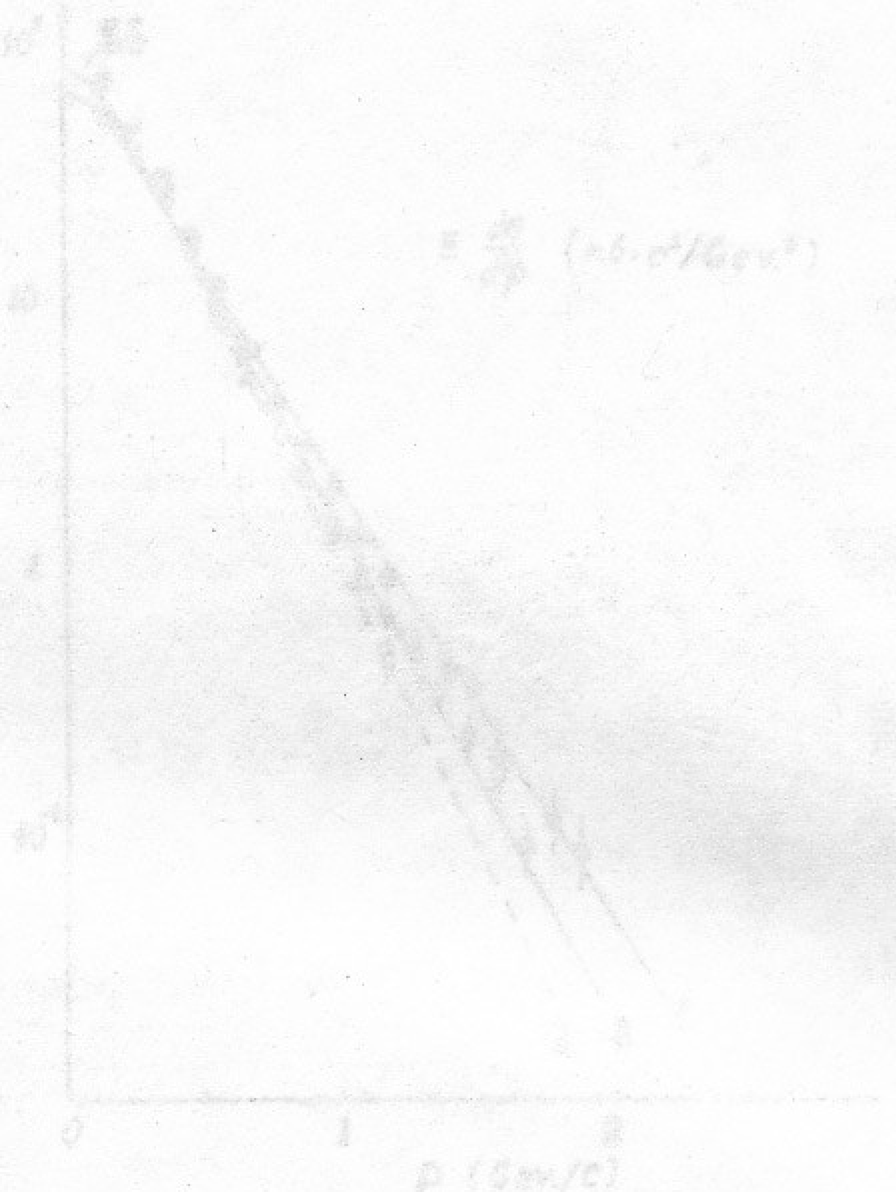


Fig. 7. The spectra of secondaries in $e^+e^- \to$ hadrons /10/ at the following energies: $\sqrt{s}$ =4.8 Gev–"o"; $\sqrt{s}$ =3.8 Gev–"x"; $\sqrt{s}$ =3.0 Gev –"Δ". The curves are: the hydrodynamical model predictions for 1) $\sqrt{s}$ =5 Gev; 2) $\sqrt{s}$ =3 Gev and 3) the thermal spectrum with T=160 Mev, shown for the comparison.

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