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A b s t r a c t

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On Symmetry of the Homogeneous Schwinger-Dyson Equations

B.G.Konopelchenko

It is well known that quantum field theory admit a formulation in terms of Green's functions. A renormalized set of the equations for Green's functions - Schwinger-Dyson equations [1-4] possess, in general, the same symmetry as the Lagrangian of the theory. But the homogeneous Schwinger-Dyson equations which arise after elimination of bare vertex (i.e. when the field's renormalization constants are equal to zero) possess higher symmetry. As it was shown in [5,6] the homogeneous equations are conformal invariant and therefore permit the solutions with symmetry higher then Poincare, viz. conformal invariant solutions. Homogeneous Schwinger-Dyson equations are conformal invariant also for arbitrary interactions [7].

At the present paper we investigate a symmetry of the renormalized equations for Green's functions. We will show that the homogeneous Schwimger-Dyson equations for arbitrary interactions are invariant under arbitrary Lie groups. Therefore, these equations permit the solutions with arbitrary symmetry. Hence, any solution of the homogeneous Schwinger--Dyson equations is continuously degenerate.

Let us consider for definiteness a theory of the one interacting scalar field. Schwinger–Dyson equations contain as full Green's functions so and amputated Green's functions. We will now obtain the equations for these functions which follows from the demand of the symmetry.

Let the Green's functions are invariant under Lie group G with generators F_i $(i=1,\dots,N)$ which satisfy commutation relations

$$[F_i, F_k] = C^j_{ik} F_j \,, \quad (i,k,j=1,\dots,N) \quad (1)$$

where C^j_{ik} are the structure constants of the group G.

The scalar field $\varphi(x)$ has as known the following transformation law in linear realization of symmetry

$$\mathcal{U}(c)\,\varphi(x)\,\mathcal{U}^{-1}(c) = \varphi'(x) = S(c,x)\,\varphi(x'),$$

where C_i are the parameters of the group G, $\mathcal{U}(c)$ – operator of the unitary representation of G, $S(c,x)$ – some function. For infinitesimal transformations we have

$$\delta\varphi(x) = \varphi'(x) - \varphi(x) = iC_i[F_i, \varphi(x)] = -iC_i\mathcal{D}_i(x)\varphi(x) \,. \quad (2)$$

As it follows from (1) the differential operators $\mathcal{D}_i(x)$ satisfy the commutation relations ($|0\rangle$ denote a vacuum):

$$[\mathcal{D}_i(x), \mathcal{D}_k(x)]\,\varphi(x)|0\rangle = C^j_{ik}\mathcal{D}_j(x)\varphi(x)|0\rangle \,. \quad (3)$$

The general form of the operators $\mathcal{D}_i(x)$ for field's space–time symmetries is

$$\mathcal{D}_i(x) = f_{i(\mu)}(x)\frac{\partial}{\partial x^\mu} + \Phi_i(x) \quad (\mu=0,1,2,3) \quad (4)$$

Operators $f_{i(\mu)}(x)\frac{\partial}{\partial x^\mu}$ are the generators of the group G in space–time realization and functions $\Phi_i(x)$ are determined from $S(c,x)$.

By virtue of (2) invariance conditions ($\delta G=0$) for Green's functions $G(x_1,\dots,x_n) = \langle 0|T(\varphi(x_1)\dots\varphi(x_n))|0\rangle$ may be represented in the usual form

$$\left(\mathcal{D}_i(x_1) + \dots + \mathcal{D}_i(x_n)\right)G(x_1,\dots,x_n) = 0 \quad (i=1,\dots,N) \quad (5)$$

The amputated Green's functions are determined by the relation

$$G(x_1,\dots,x_n) = \int dy_1\dots dy_n\, G(x_1,y_1)\dots G(x_n,y_n)\,\Gamma(y_1,\dots,y_n) \quad (6)$$

Let us introduce the adjoint field $\tilde{\varphi}(x)$ so that

$$\tilde{\varphi}(x) = \int dy\,\Gamma(x,y)\varphi(y) \quad , \quad \varphi(x) = \int dy\, G(x,y)\tilde{\varphi}(y) \quad (7)$$

Amputated Green's functions are the vacuum expectation values of these fields

$$\Gamma(x_1,\dots,x_n)=\langle 0|T(\tilde{\varphi}(x_1)\dots\tilde{\varphi}(x_n))|0\rangle \qquad (8)$$

and

$$\langle 0|T(\varphi(x)\tilde{\varphi}(y))|0\rangle=\delta(x-y).$$

We now obtain the equations for $\tilde{\varphi}(x)$ and $\Gamma(x_1,\dots,x_n)$ analogous to (2) and (5). Taking into account (4),(5) and integrating by part, we obtain

$$\delta\varphi(x)=\int dy\, G(x,y)\,\delta\tilde{\varphi}(y)=$$

$$=-ic_i\int dy\,\mathcal{D}_i(x)G(x,y)\tilde{\varphi}(y)=-ic_i\int dy\,G(x,y)\tilde{\mathcal{D}}_i(y)\tilde{\varphi}(y) \qquad (9)$$

where

$$\tilde{\mathcal{D}}_i(y)=f_{i(\mu)}(y)\frac{\partial}{\partial y^\mu}+\frac{\partial f_{i(\mu)}(y)}{\partial y^\mu}-\Phi_i(y). \qquad (10)$$

Thus

$$\delta\tilde{\varphi}(x)=-ic_i\tilde{\mathcal{D}}_i(x)\tilde{\varphi}(x),\quad [F_i,\tilde{\varphi}(x)]=-\tilde{\mathcal{D}}_i(x)\tilde{\varphi}(x). \qquad (11)$$

Using the equations for functions $f_{i(\mu)}(x)$ and $\Phi_i(x)$ which follow from (3), it isn't difficult to show that the operators $\tilde{\mathcal{D}}_i(x)$ satisfy the relations

$$[\tilde{\mathcal{D}}_i(x),\tilde{\mathcal{D}}_k(x)]\tilde{\varphi}(x)|0\rangle=C^j_{ik}\tilde{\mathcal{D}}_j(x)\tilde{\varphi}(x)|0\rangle.$$

Using the equations for functions $f_{i(\mu)}(x)$ and $\Phi_i(x)$ which follow from (3), it isn't difficult to show that the operators $\tilde{\mathcal{D}}_i(x)$ satisfy the relations

$$[\tilde{\mathcal{D}}_i(x),\tilde{\mathcal{D}}_k(x)]\tilde{\varphi}(x)|0\rangle=C^j_{ik}\tilde{\mathcal{D}}_j(x)\tilde{\varphi}(x)|0\rangle.$$

Then, the Casimir operators of the group G don't change by the substitution $\mathcal{D}_i(x)\to\tilde{\mathcal{D}}_i(x)$. Therefore, fields $\varphi(x)$ and $\tilde{\varphi}(x)$ are transformed by the equivalent representations of the group G*. From (7) and (8) follows that transformation $\varphi(x)\to\tilde{\varphi}(x)$ is analogous to rising of index in the tensor analysis. The fields $\varphi(x)$ and $\tilde{\varphi}(x)$ may be considered as co- and contravariant components of the same field. The propagator $G(x,y)$ is analogous to metric tensor.

From (11) it follows the equations for amputated Green's functions

$$\left(\tilde{\mathcal{D}}_i(x_1)+\dots+\tilde{\mathcal{D}}_i(x_n)\right)\Gamma(x_1,\dots,x_n)=0 \qquad (12)$$

$$(i=1,\dots,N)$$

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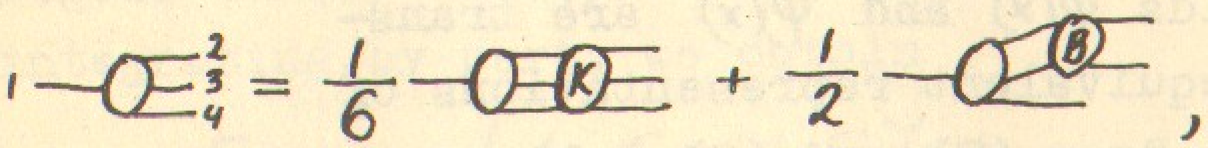
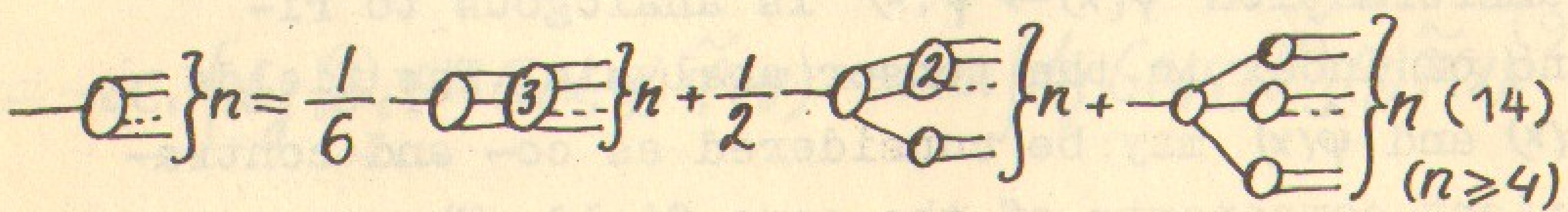
*For Poincare group $\tilde{\mathcal{D}}_i(x)=\mathcal{D}_i(x)$ and, hence. fields $\varphi(x)$ and $\tilde{\varphi}(x)$ are transformed by the identical representations. For conformal group $\tilde{\mathcal{D}}_{i\,d}=$ $=\mathcal{D}_{i\,4-d}(x)$, i.e. the field $\tilde{\varphi}(x)$ has the scale dimension $\tilde{d}=4-d$ [7].

m - particle irreducible amputated Green's functions also satisfy the equation (12).

The generalization of the equations (4),(5),(10) and (12) for arbitrary field (with many components) is straightforward and obvious.

Let us now consider the homogeneous Schwinger-Dyson equations. We consider for definiteness the φ^4 theory. These equations have the form [3,4] (notations follows to [4]):


(13)


(14)

In the coordinate space (13) have the form

$$G(x_1,x_2,x_3,x_4) = \frac{1}{6}\int dy_1\, dy_2\, dz_1\, dz_2\, dt_1\, dt_2\, G(x_1,t_1,z_1,y_1)\times$$
$$\times\,\Gamma(y_1,y_2)\,\Gamma(z_1,z_2)\,\Gamma(t_1,t_2)\,G_K(t_2,z_2,y_2,x_2,x_3,x_4) + \qquad (15)$$
$$+\frac{1}{2}\int dy_1\, dy_2\, dz_1\, dz_2\, G(x_1,y_1,z_1,x_4)\,\Gamma(y_1,y_2)\,\Gamma(z_1,z_2)\,G_B(z_2,y_2,x_2,x_3).$$

Let functions $G(x_1,\ldots,x_n)$ and $\Gamma(x,y)$ are invariant under arbitrary group G. Using (5) and (12), it isn't difficult to show, integrating by part that

$$\left(\mathcal{D}_i(x_1)+\mathcal{D}_i(x_2)+\mathcal{D}_i(x_3)+\mathcal{D}_i(x_4)\right)G(x_1,x_2,x_3,x_4) = \qquad (16)$$
$$=\left(\mathcal{D}_i(x_1)+\mathcal{D}_i(x_2)+\mathcal{D}_i(x_3)+\mathcal{D}_i(x_4)\right)\left(\text{r.h.s. of}\,(15)\right)=0.$$

From (16) it follows that r.h.s. of the equation (15) have the same transformation properties under group G as the l.h.s. of (15). Therefore, the equation (15) is invariant under the group G. The invariance of the equations (14) under the group G is proved by the same manner. We emphasize that group G is arbitrary Lie group (it may be a group of space-time transformations or a group of the internal symmetries such as $SU(n)$ or their combinations).

Thus, the homogeneous Schwinger-Dyson equations are invariant under arbitrary Lie group*. This result is easy generalized to the case of the arbitrary interactions (at least polynomial type, such as φ^n, $\bar\psi\psi\varphi$, $\bar\psi\psi\bar\psi\psi$ and so on).

Since just the same homogeneous Schwinger-Dyson equations (for example, equations (13), (14) possess the invariance under different groups, these equations permit solutions with different (in general, arbitrary) symmetries.

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*It also follows from the invariance of the generating functional under arbitrary Lie group.

Hence it follows that any solution of the homogeneous Schwinger-Dyson equations is continuously degenerate. Indeed, a solution which is not invariant under a group is degenerate by the usual reason: acting on this solution by some group of symmetry of the homogeneous equations we obtain a continuous set of the solutions. Let us now consider the solution of these equations which is invariant under some group G_1. Among all groups of the symmetries of the homogeneous equations always exist the group G_2 (really, infinite number of groups) which isn't a subgroup of the group G_1, so that the considered solution is not invariant under the group G_2. Acting on this solution by the transformations of the group G_2 we obtain also the solutions of the homogeneous equations. As a result the continuous degeneracy of the solution appears.

In particular from the continuous degeneracy it follows that the homogeneous Schwinger-Dyson equations have an infinite set of the conformal-invariant solutions, i.e. the system of equations for scale dimensions and coupling constant [6, 8, 9] is degenerate. This result follows from the previous reasoning if G_1 is the conformal group and G_2 is the group which can change the scale dimension.

At last, so long as the continuous degeneracy of the translational invariant solutions is equivalent to spontaneous break down of the symmetry [10] any translational invariant solution of the homogeneous Schwinger-Dyson equations has to contain Goldstone particles.

In conclusion we note the following. Just as from the conformal invariance follows the form of the two- and three-point Green's functions one may consider the group which give the possibility to determine the four-point function. Then, for the theory φ^4 (for equations (13) and (14)) it is possible to formulate the program analogous to the bootstrap program for three-linear interactions [6,8]. The equations (13-14) may be rewrite in the terms of the spectral functions just as it has done in [9].

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