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In the present note the dependence of an anomalous magnetic moment of the electron moving in the magnetic field H has been found upon the field intensity. It is shown that at $H \geq H_0$ ($H_0 = m^2/|e| = (m^2c^3/|e|\hbar) = 4.41 \cdot 10^{13}$ Oe) the value of the anomalous magnetic moment is essentially different from the Schwinger value $\alpha/2\pi$.

To determine the anomalous magnetic moment of the electron[1] it is necessary to know the spin-dependent part of a mass operator. The mass operator in the magnetic field was found recently in the authors' paper[2] (see also [3] where another classification of states was used). Let's represent Eq.(4.18) Ref.[2] in the form of

$$\langle M_{RH}^{(1/2)} \rangle = \langle M_{RH}^{(1/2)} \rangle_0 + \langle M_{RH}^{(1/2)} \rangle_\zeta \qquad (1)$$

where $\langle M_{RH}^{(1/2)} \rangle_\zeta$ is the part of the mass operator depending on spin variable ζ ($\zeta = \pm 1$). For the electron which is at the n-th level[4] in the magnetic field H the value of $Re\langle M_{RH}^{(1/2)} \rangle_\zeta$ is connected with the anomalous magnetic moment resulting from its interaction with the radiation field (compare[5] p.184 for quasiclassical approximation)

$$Re\langle M_{RH}^{(1/2)} \rangle_\zeta = -\zeta \frac{eH}{2m} \sqrt{1 + 2n\frac{H}{H_0}} \cdot \frac{\alpha}{2\pi} \mu_n(\lambda) \qquad (2)$$

here

$$\mu_n(\lambda) = 4\lambda \int_0^\infty \frac{dx}{x} \int_0^1 du\cdot u(1-u)\,\frac{1}{\Delta}\cdot\left(\sin x - \frac{1-\cos x}{x}\right)\sin\left[(\lambda+n)ux - 2na(x)\right] \qquad (3)$$

where

$$\lambda = \frac{H_0}{2H}, \quad a(x) = \operatorname{arctg}\left[\frac{1-\cos x}{\sin x + x\frac{1-u}{u}}\right],$$

$$\Delta = 2(1-\cos x)u^2/x^2 + 2u(1-u)\frac{\sin x}{x} + (1-u)^2 \qquad (4)$$

The quantity $\frac{\alpha}{2\pi}\mu_n(\lambda)$ is the anomalous magnetic moment of the electron in the field in α-order of the perturbation theory. Its behaviour is determined by the properties of the function $\mu_n(\lambda)$.

In the weak field $H \ll H_0$ ($\lambda \gg 1$) and at large quantum numbers n the main contribution into integral is given by $x \ll 1$, so that one can expand the integrand in powers of x. This yields[6]

$$\mu_n(\lambda) \to \mu(\chi) = \frac{2}{\chi}\int_0^\infty dz \int_0^\infty \frac{dv\, v}{(1+v)^3}\sin\left[\frac{v}{\chi}\left(z + \frac{z^3}{3}\right)\right] \qquad (5)$$

where $\chi^2 = 2n\left(\frac{H}{H_0}\right)^3$, the substitutions are made $x = \frac{z}{\lambda\chi(1-u)}$, $v = \frac{u}{1-u}$.

The expression $\frac{\alpha}{2\pi}\mu(\chi)$ coincides with the anomalous magnetic moment of the electron in quasiclassical approximation (see Ref.[5] p.184 and also [7],[8]), its asymptotic expansions have the form:

$$\mu(\chi) = 1 - 12\chi^2\left[\ln\frac{1}{\chi} + C + \frac{\ln 3}{2} - \frac{37}{12}\right] + \ldots, \quad \chi \ll 1$$

$$\mu(\chi) = \frac{2\pi}{9\sqrt{3}}\Gamma\left(\tfrac{1}{3}\right)(3\chi)^{-2/3}\left[1 + 6\frac{\Gamma(2/3)}{\Gamma(1/3)}(3\chi)^{-2/3} + \ldots\right], \quad \chi \gg 1 \qquad (6)$$

$\mu(\chi)$ decreases monotonically with increase of χ.

The behaviour of $\mu_n(\lambda)$ in essentially quantum case (strong fields, lower levels) is of sufficient interest. At asymptotic values of λ one succeeds in finding explicit representation of $\mu_n(\lambda)$. For $H \ll H_0$ ($\lambda \gg 1$) the main contribution into integral (3) is given by region $ux \ll 1$. An explicit form of the

expansion (following from the spin-dependent part of the mass operator calculated first in this region in Ref.[9] see also[3]) is[10]

$$\mu_n(\lambda) = 1 - \frac{1}{\lambda^2}\left(\frac{7}{3}\ln\lambda - \frac{16}{5}\ln 2 + \frac{83}{180}\right) + \ldots \qquad (7)$$

For strong field $H \gg H_0$ ($\lambda \ll 1$) the main contribution into integral (3) is given by region $x \gg 1$, $x(1-u) \sim 1$. Then[11]

$$\mu_1(\lambda) = -2\lambda\left(\ln\frac{1}{\lambda} - 1.0\right); \quad \mu_n(\lambda) = -\frac{2\lambda}{n}\ln\frac{1}{\lambda} \qquad (8)$$

In the intermediate region the function $\mu_n(\lambda)$ may be found only by numerical integration. Calculation of the integral in the form (3) appears to be very cumbersome, since the integrand contains oscillating functions. Accordingly it is desirable to transform the integral. E.g. it is convenient to rewrite $\mu_1(\lambda)$ in the form:

$$\mu_1(\lambda) = 4\lambda\,\mathrm{Im}\left[\int_0^\infty dx\,(1 - \cos x - x\sin x)\int_0^1 \frac{du\cdot u(1-u)e^{-i(\lambda+1)ux}}{[x(1-u) - iu(1-e^{-ix})]^2}\right] \qquad (9)$$

which, after some transformations, allows one to rotate the integration path in the complex plane x (note, that the denominator has no zeroes in the 4-th quarter). Finally, one obtaines the representation of $\mu_1(\lambda)$ containing the exponential functions of real arguments only, that essentially simplifies a numerical calculation. Its result is shown in Fig. It is seen that at $H \sim H_0$ $\mu_{1,2}(\lambda)$ decrease sharply passing through zero, reach their minimum value ($\mu_{1\,min} \approx -0.17$ at $\lambda \approx 0.1$ that is at $H = 5H_0$) and then tend to zero being negative according to asymptotic expansion. One can easily assure that at $H \ll H_0$ the main contribution into the integral (3) gives the region $k^2 \sim m^2$ which determines the contribution to the anomalous magnetic moment in the absence of the field that explains the expantion (7).

Beginning from $H\sim H_0$ the main contribution gives the region $k^2\sim eH$ that leads to the effective integral cutoff since the cyclotron radius $m/|e|H$ becomes smaller than Compton wavelength $1/m$,then $\mu_n(\lambda)$ is decreasing. In quasiclassical limit (where electron energy $\varepsilon \gg m$) the contribution into anomalous magnetic moment gives only diagram with an electronic intermediate state (in terms of the noncovariant perturbation theory), while the contribution of the diagram cotaining electron-positron pair is small ($\sim m^2/\varepsilon^2 \backsim 1/n$) and according to Eq.(6) $\mu(\lambda) \to 0$ being positive. In the situation considered the contribution of the diagram containing electron-positron pair is significant and this is a possible reason for the appearence of negative values of $\mu_n(\lambda)$ at $H \gtrsim H_0$. With further increase of H the value of $|\mu_n(\lambda)|$ decreases because of contribution region fall off. Note, that at $\lambda > 10$ and at $\lambda < 10^{-2}$ the function $\mu_n(\lambda)$ is described satisfactorily by expansions (7,8).

From the results above it follows that the anomalous magnetic moment of the electron at the n-th level ($n \geqslant 1$) in the intense magnetic field ($H \gg H_0$) becomes negative and $\mu_n(\lambda) \backsim -\frac{H_0}{H}\ln\frac{H}{H_0}$,so that its contribution to the energy of interaction with the field is positive and increases logarithmically with the increase of the field. As it has already been pointed out decomposition (1) has no sense for ground state (formal extrapolation of expression (3) at n=0 leads to contradictions), thus it is necessary to consider the whole mass operator. Its asymptotie expansion at $H \gg H_0$ [13] gives $\langle M_{RH}^{(\psi_2)}\rangle(n=0) = \frac{\alpha}{4\pi} m\left(\ln\frac{1}{\lambda} - C - \frac{3}{2}\right)^2 > 0$,so that correction to the ground state energy is positive. Let's compare these results with those of paper [14] in which the motion of the electron with fixed anomalous magnetic moment in the intense magnetic field has been

considered. The anomalous magnetic moment contribution to the energy is negative and at $H > \frac{4\pi}{\alpha}H_0$ it exceeds mass m, so that the energy gap between the ground state and vacuum disappears (this might appear to be important for astrophysical applications). From above it follows that this conclusion [14] is based on erroneous assumptions that in electron ground state it is possible to use concept of anomalous magnetic moment and that the latter does not depend on the field intensity. In fact the mentioned energy gap extends. In the paper [15] at the same assumptions a modified effective Lagrangian was calculated. Such formulation of the problem is wrong since anomalous magnetic moment is only one of the radiative corrections which besides sufficiently changes in the strong fields.

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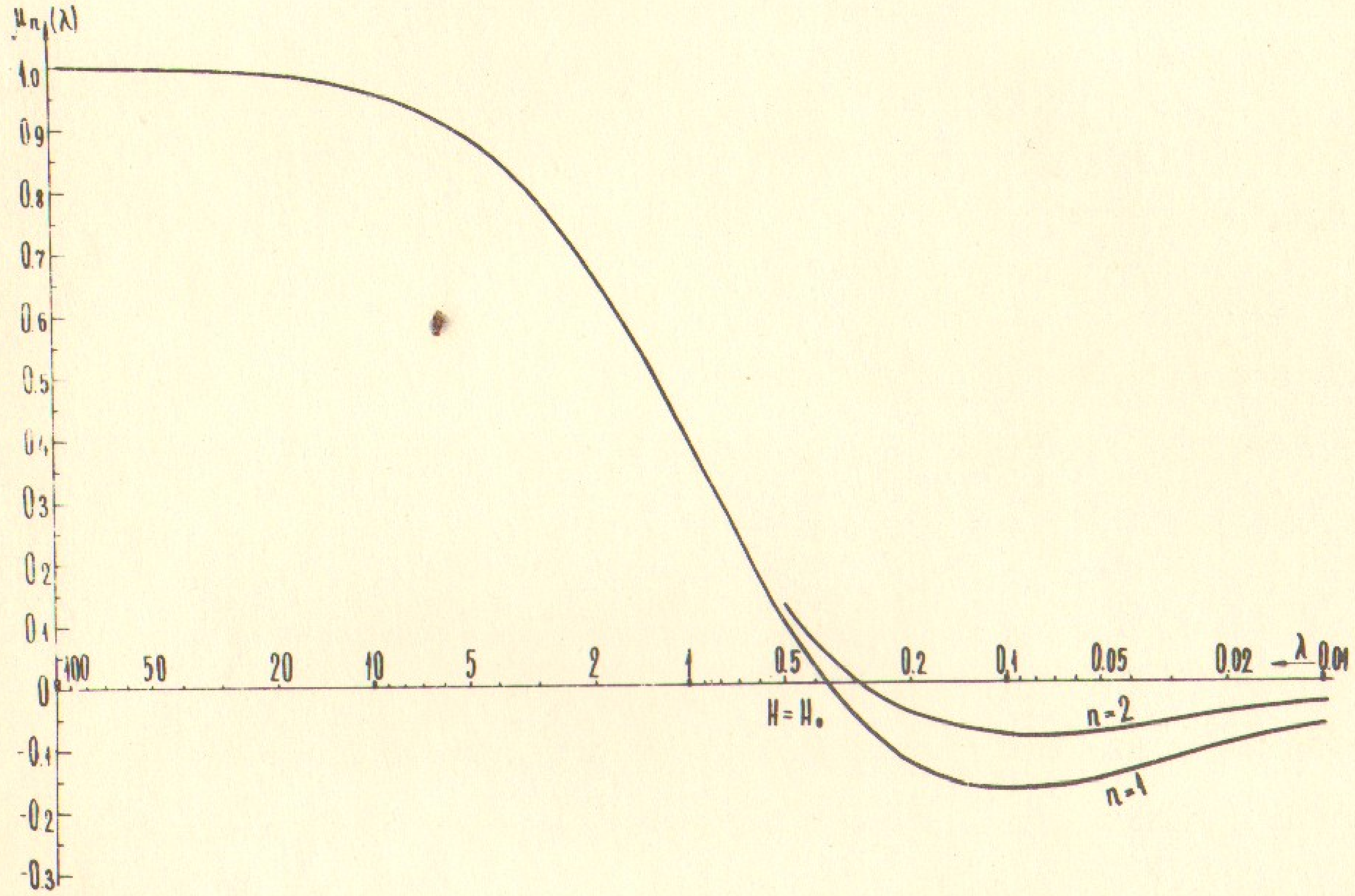

Figure.Plot of μ_1,μ_2 as a function of $\lambda = \frac{H_0}{2H}$.

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